Mechanics of Sheet Metal Forming

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Week- 05
Lecture- 09
Load Instability and Tearing

So, let us continue our discussion with respect to the this particular course and in the previous module ok, one last problem we solved in that you have to note down a small change here. So, this particular problem Q2 we solved in the previous one ok, previous module you will see that we calculated ε_1 and then we calculate a blank holding force B is not it. So, this B is actually in kilo Newton we missed it, it was kept as Newton. So, you have to be little bit careful in units this is to be in kilo Newton. So, that change you basically note down ok. So, so now we are going to start discussing about the next two module in mechanics of sheet metal forming ok, that is basically your load instability and tearing ok.

So, load instability and tearing. So, this particular situation load instability and tearing comes ok. So, when you deform a sheet beyond uniform plastic deformation. So, we start with 0 deformation and then we cross elastic portion we enter into plastic deformation ok.

So, and so wherever the transit happens you are going to have the significance or use of yield locus or yield surface to know the onset of plasticity. Then we have very nice uniform plastic deformation the strain hardening region described by any power law we have used ok. And then we are going to reach a deformation stage where instability is going to start ok. So, when we say instability and tearing in this in this particular you know course it means that the material is actually going to fail, the material is actually going to physically fail ok. So, beyond yielding beyond plastic deformation we crossed the entire uniform plastic deformation and then we are going to reach let us say for example closer to UTS and that is where your instability is going to start and after that if you deform it will convert into full fracture. So, that is a situation. So, we are going to call this as a load instability and tearing. So, instability can be a variety of types. So, I have just noted here few of them. So, one is when the load reaches a maximum the deformation will concentrate on a particular region and diffused neck will be generated ok.

The uniform deformation is limited by this load that is P_{max} after which deformation is going to be non uniform ok. So, we are going to reach a maximum load that is nothing but your P_{max} ok in the load displacement graph you can imagine ok. We are going to reach P_{max} ok and you will see that once you reach that particular stage we are going to have diffused necking, diffuse necking means the neck is generated over a larger gauge length, larger span ok in the gauge region that is called diffused neck ok. And the moment this happens you will have your uniform deformation is going to cease ok and after which what you are going to have is a non uniform deformation that we already discussed in the first section. Now, this is a diffused necking you can also have localized necking ok.

So, localized necking means what localized necking could be next stage of diffuse necking ok in which there is a neck that is basically constructed in a small region ok, very small gauge region you will see this neck and this could suddenly happen ok and this can quickly cause tearing and full fracture ok. This is a form of local instability, local instability means in a outside this region you are going to have you may not be able to find any such information, but then locally it will create some sort of instability in a small section of gauge length or material ok. And if you want to analyze more on this aspect you can concentrate only on that location and not necessarily the other locations in the material ok. For example, tensile test and after this localized necking you will see that the material is going to fracture fully ok. So, since the material that we are going to discuss or of ductile nature.

So, you will have a fracture after sufficient amount of uniform deformation and brittle fracture type of you know material we are not going to consider at all in this course because you are not going to make any useful component shear component from that ok. So, that is what I have written here. So, how do you quantify fracture ok, how do you quantify necking. So, all these things will have some theoretical basis that is what we are going to so going to see. Other than this you are going to have something called as wrinkling ok. So, wrinkling you know that it is going to suppose like for example in a deep drawn product in a deep drawn cup. So, you will see that wrinkling is going to happen on the flange region of the cup and that could be because of compressive nature of one of the principle stresses ok. It is like buckling ok, it is like buckling of a column ok. Similarly you are going to have wrinkling of the sheet ok away from the its one plane ok. So, that waviness ok it could be because of this.

So, instability can be of variety of types depending on what metal forming problems you have with respect to sheet metal forming some of these are described in this particular slide ok. So, now we will discuss you know one after another how we are going to quantify instability and when we speak about load instability what do you mean by load instability means instability in terms of load requirement. We are going to put some condition for load requirement ok, when instability is going to happen ok, load is going to satisfy this particular condition and then we keep on deriving some equation finally, lead to a very useful equation through which you can predict you know appearance of neck during deformation. So, first section that we are going to pick up is basically uniaxial tension of a perfect strip ok, uniaxial tension of a perfect strip. So, perfect strip means sheet you can say per strip means sheet ok.

So, when we say sheet it could be of any dimension, but then when we speak about strip we are speaking about like for example, a tensile strip ok and we are going to pick up the gauge region which is typically rectangular shape is not it. So, you have a tensile strip ok with the gauge region like this right. So, you have a tensile strip like this ok and this is your shoulder region ok which has got a particular thickness. So, sheet will have particular thickness right, sheet will have particular thickness t_0 let us say ok. So, we are going to pick up a gauge region ok for example, like this let us say l_0 and this is we are going to call it as strip ok.

So, you can also call it as sheet, but you have to be little bit careful with respect to how you imagine the dimension ok. So, what is perfect strip here we will see that what is perfect, what is the meaning of perfect we will see in due course and how this perfect strip looks like is given in this particular figure. You will see that this is a nothing but a gauge region with length l and width w which has got a thickness t ok and this will have a uniform cross section area let us say A or A_0 ok. It will have uniform cross section area let us say A or A_0 ok and you are going to displace the material and there will be a corresponding load required with respect to displacement ok. So, now what we are saying is when this strip is stretched in tension ok, the volume remains constant we know due to plastic deformation and the following equations apply.

Cross sectional area is given by $A = wt$ ok, ok, where w is a width of the strip and t is the strip thickness and volume remains same ok. At any instantaneous deformation stage you will have Al which is equal to $A_0\ l_0$ ok. So, these two will remain same and from this you can get the relationship between you know the new area of cross section A and the original you know volume where l is the final length and l_0 is the initial length you can say instantaneous length also ok. So, now what I am going to do I am going to differentiate this equation to get this particular equation $\frac{dA}{A} + \frac{dl}{l} = 0$ or I can also say that $\frac{dl}{l} = d\varepsilon_1 = -\frac{dA}{A}$ right. So, as per original definition of strain increment is nothing but $\frac{dl}{l}$ ok. This we discussed in the probably in the second module and you will see that $\frac{dl}{l} = d\varepsilon_1 = -\frac{dA}{A}$ ok. So, now from this you can get $\varepsilon_1 = ln\frac{l}{l_0}$ this is the original definition of your the principle strain $\varepsilon_1 = ln\frac{l}{l_0}$. So, $ln\frac{l}{l_0}$ new length divided by the original gauge length and since it is a tension test ok the state of stress is given here. So, you are going to have only σ_1, σ_2, σ_3 are not there until instability starts. So, $\sigma_1 = \frac{P}{A}$ and $A = \frac{A_0\ l_0}{l}$ so $A_0\ l_0$ will be the denominator so it will be $\frac{PL}{A_0\ l_0}$ ok.

So, with this basic description what we are going to do is we are going to put a condition that when the load reaches maximum ok. For example you draw a graph between load versus displacement it reaches a maximum load ok. Let us say it is a P_{max} when load reaches maximum P_{max} we are going to say that $dP = 0$ this is the condition for instability ok. This is the condition for instability we can say so at maximum load $dP = 0$ this can also be written as $\frac{d\sigma_1}{\sigma_1} + \frac{dA}{A}$ why because P is a function of σ_1 and A. So, you can say $\frac{d\sigma_1}{\sigma_1} + \frac{dA}{A} = 0$ ok and you can also rewrite this as $\frac{d\sigma_1}{\sigma_1} - d\varepsilon_1 = 0$ ok.

So, $dP = 0$ or $\frac{d\sigma_1}{\sigma_1} - d\varepsilon_1 = 0$. So, this can also be written as $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1$ right $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1$ ok. This is an important you know condition for us to give some idea about when instability is going to start or when maximum load is reached. So, when maximum load is reached ok when you reach P as P_{max} then this fellow will be satisfied $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1$ ok. So, this equation on the left hand side it is actually called as a non dimensional strain hardening parameter ok, non dimensional strain hardening parameter and you can get this values from a true stress strain data right. So, load stress displacement graph will give you true stress strain data right. So, typical curve like this ok and what you need to get is actually $\frac{d\sigma_1}{d\varepsilon_1}$. So, at every point in the deforming zone you have to get this particular you know ratio and divided by the σ_1 ok and you can plot that with respect to ε_1 ok. You can plot that with respect to ε_1. So, you will get this particular data ok.

And we are going to say that this is the condition for maximum load and this is also called as considered condition for maximum load in a tensile strip ok. So, when maximum load is reached this condition is going to be satisfied ok. So, we are not going to stop with this. Now what we are going to do is any further evaluation depends on what kind of material law you choose for the σ_1 right. So, our standard equation power law equation is $\sigma_1 = K{\varepsilon_1}^n$ ok. If we choose that particular strain hardening law ok $\sigma_1 = K{\varepsilon_1}^n$ if we choose that then what will happen is $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = \frac{nK}{\sigma_1}(\varepsilon_1)^{n-1} = \frac{n}{\varepsilon_1}$ ok. So, just before we make a conclusion from this we will get the one more equation which is already introduced to you in the previous modules before we get that this particular situation ok can be described by this particular graph which I was just explaining from this particular one is not it. So, you draw a graph between $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1}$ with respect to ε_1. If you plot that you will see this particular left hand side of that equation will actually decay or decrease in this manner in this manner ok.

So, there will be one particular location where this fellow is going to be equal to 1 and if you draw horizontal line and if you come and meet here in X axis ε_1 will be equal to one particular value and that value can be obtained by further derivation. So, what is that? So $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = \frac{n}{\varepsilon_1}$ and in the previous equation you will see that it is you know $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1$. So, these two equations can be equated and I am going to say that ok $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = \frac{n}{\varepsilon_1} = 1$. So, I can say that $n = \varepsilon_1$ and since I am going to put a condition for maximum load I am going to call this as ε_1^* which is the same ε_1^* which we introduced in the previous module ok. So, there also we got I think n by we had a general equation know I think ${\varepsilon_1}^* = \frac{n}{1+\beta}$ we introduced and then if you choose a pre strain you know in place of this equation if you also include pre strain in this equation then this equation got changed I think n minus you know there is a factor of pre strain into it and when pre strain is not there it becomes n that we discussed in the previous model.

So, here also we are going to introduce a similar equation we derived in this fashion that is $\varepsilon_1{}^* = n$ and here actually you know that star indicates maximum load condition or $\varepsilon_1{}^*$ is nothing but limit strain only when you discuss in terms of tensile instability and this is a condition for diffuse necking in a tensile strip this is a condition for tensile diffuse necking in a tensile strip. So, now when this fellow is equal to 1 you draw horizontal line it will meet X axis and you will see that the $\varepsilon_1 = n$ as per this particular equation ok. So, when you do tensile test get true stress strain data and get this particular you know data and you plot that with respect to ε_1 you will get this particular similar graph like this and whenever this becomes 1 you will see that $\varepsilon_1 = n$ ok. If the instability is going to happen for a condition $dP = 0$ only if that happens then this all equations are valid ok. So, instability condition for tensile strip if you want then you can tell these two conditions $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1$ or if you follow the power hardening law $\sigma_1 = K\varepsilon_1{}^n$ then you can say $\varepsilon_1{}^* = n$ ok or two equations that describe the condition for diffuse neck in a tensile strip ok.

So, now what we are going to do is you are going to introduce the term imperfect or perfect strip we introduced here perfect strip is what you mean by perfect strip perfect strip basically means it is not a real strip it is not a real strip. So, where is the question of real strip what you mean by real strip real strip means it is a strip which you actually see in practice ok like what we do in experiments like a tensile samples with lot of heterogeneity in the material geometric heterogeneity in the material like that. So, that is a real sample ok, but if you really look into this type of you know gauge region which you have picked up and analyzed you will see that the entire strip is actually perfect there is no defect available in this material ok there is no defect it is not defective at all ok. So, it is actually not a real strip it is actually a perfect strip real strips are actually imperfect strips ok either we use real strip or imperfect strip and opposite to that is perfect strip like what we see here like what we have seen here ok. So, that is why I have written unlike in a real strip real strip means like what we witness in actual tensile test experiments because it is a perfect strip because it is a perfect strip we are going to say that necking will not happen at all.

So, diffuse necking cannot occur in all the elements ok in the perfect strip why because it will behave in a similar fashion and uniform deformation will continue to occur right. If it is a perfect strip ok then there is no location which is actually considered weak if that is the case then all the elements in the deforming zone ok for example, you pick up these elements. So, there are there could be a number of elements you can discretize that all the elements in this strip will behave in identical fashion because it is perfect one ok it is not a real one then your diffuse necking will not happen at all and uniform deformation will continue to occur ok. In that case you can describe your load by a simple equation $P = \sigma_1 A = K\varepsilon_1^n A_0 \frac{l_0}{l} = KA_0\varepsilon_1^n \exp(-\varepsilon_1)$. So, what does that mean? That means you can draw a graph between P and ε_1 ok why because A is a material parameter n is also material parameter they are constant values and A_0 is initial area of cross section which is also constant.

So, only variable is ε_1 ok and you will see that this kind of equation theoretical equation will give you load versus strain load versus strain graph in this format in this way ok. So, this equation will give this equation will give load versus strain graph for a perfect strip for a perfect strip ok load to deform a perfect strip obeying our standard power law $K\varepsilon_1^n$ is given by this equation and if you follow this equation you will get a load versus strain graph like this. So, it will start going like this and it will continue to go like this the red color one ok the red color portion the red color curve ok. But if you want to compare this with respect to real strip real strip means like what we do in experiments ok the real strip which are actually imperfect in nature ok then you will see that it will there will be a good difference between the load reduction and you know the displacement at which full fracture is going to happen the strain at which full fracture is going to happen ok the difference is given here you can see that. So, what is the main difference in main difference is in real strip the load actually falls off pretty rapidly ok when compared to the perfect strip why because a load is not going to come down so easily in perfect strip because the material is actually perfect without any defects.

But in real strip there are going to be defects quite naturally ok and hence there will be you know different locations going to undergo different levels of strain there will become there will be one weaker region which is actually going to govern the further instability process and you will have sudden drop in load once your P_{max} is reached which is what is shown here. So, variation in load in real strip and perfect strip is not same in real strip after reaching maximum load the load falls off rapidly while this is not so in the case of perfect strip ok you can you can see there is some small decrease but then it is very gradual ok what does that mean that means this indicates that we need to include an imperfection in the strip and develop theoretical models for load instability or if you want to calculate let us say $\varepsilon_1{}^*$ you can say right. So, if you want to actually calculate for the real strip ok which is going to have imperfection then you have to really put some theoretical imperfection in the strip for further analysis ok so that is why I mentioned very clearly that you are going to have uniform area of cross section A here uniform area of cross section A here ok. So, now this is the situation with respect to your perfect strip ok so now we are going to convert this perfect strip into imperfect strip by including some imperfection in the strip and how are we going to consider it is given in this particular section. So what is given here there is a figure here you can understand this ok so this figure is nothing but again a gauge region ok in a tensile strip you can imagine ok only thing is like we are going to introduce an imperfection here the grey region which introduced here has got an imperfection ok that is why given here as imperfection.

And the outside region the colored one is actually uniform region which is similar to the previous sample which you have seen ok. So, as usual you are going to get some load ok with respect to strain you can plot for this particular sample and from the thickness direction point of view you can see that ok what is the difference between imperfect region uniform region is basically in the imperfect region you have a small change in thickness as compared to your uniform region and that will lead to change in area of cross section here if it is A here it is $A + dA$ if it is σ_1 it is $\sigma_1 + d\sigma_1$ here it is ε_1 it is $\varepsilon_1 + d\varepsilon_1$ ok. So, that is why I have written here it is an imperfect strip equivalent to real test piece in experiments. So, whatever defect you have in the real test piece you are going to have an equivalent imperfection that is called your thickness imperfection like this ok. So, if the thickness here is let us say t ok here is going to be slightly less than this particular t ok actually this dA actually is going to be negative quantity ok.

So, suppose this thickness is let us say 2 mm which we 1.2 mm, 2 mm, 0.8 mm like that ok then you can see that the thickness in this region could be let us say 1.999 mm something like that ok 1.999 mm something like that. So, there is a small imperfection here that is a meaning ok. So, now once we introduce this imperfection now it is equivalent to a real test piece real strip. So, and we are going to now further put some you know discussion on this derivation on this what we are going to do we will see it now ok. So, now for simplicity of analysis we are saying that this imperfect region ok is actually perpendicular to the loading axis actually perpendicular to the loading axis is not it. So, it is easy to understand ok. So, I have written here that the imperfection ok is characterized by slight lesser cross sectional area ok.

The initial area of most of the strip is A_0 or you can say A either way is fine. So, you can take it as A_0 also that means uniform region it is A_0 then imperfection is initially of area A_0 plus dA_0 where dA_0 is a small negative quantity ok. You can call this as A_0 also it is ok it is going to refer original area of cross section nothing else ok. So, if this is A_0 this is going to be in this region it is $A_0 + dA_0$ ok. So, now how do you calculate load for this particular situation ok.

The load is going to act ok perpendicular to this and these two regions ok this uniform region you can combine one as one and this is another one these two regions are connected in this fashion then you can say that load transmitted in these two regions could be equal. So, you can say $P = KA_0\varepsilon_1^n\exp(-\varepsilon_1) = K(A_0 + \mathrm{d}A_0)(\varepsilon_1 + \mathrm{d}\varepsilon_1)^n\exp[-(\varepsilon_1 + \mathrm{d}\varepsilon_1)]$ ok.

This is the equation that we can frame to for load requirement with respect to ε_1. So, now you given K, A_0, n and how much imperfection you have here ok. So, you can get a graph between ε_1 versus P for the entire strip ok. So, now what we are going to do is we are going to discuss a situation how these two regions going to behave ok when you give deformation to it ok. So, what we are going to say is the imperfect region is actually a weaker region correct.

So, this imperfect region is actually weaker region and it is sensible to know that your failure is going to happen in this particular location which we already we can understand that and since it is going to happen so I am going to simply say that $\varepsilon_1^* = n$ or $\varepsilon_1 = n$ for instability develop in allocation right. So, now if this is going to happen in the in the imperfect region I can simply say $\varepsilon_1 + d\varepsilon_1 = n$ which is what I am writing here. The imperfection will reach a maximum load why because it is weaker ok when the strain is

$\varepsilon_1 + d\varepsilon_1 = n$ same equation only we are going to add this part why because imperfection is actually going to have a larger strains here when compared to the uniform region ok. So, now if this is the situation whenever this is going to happen there will be one particular load that will be reached now at this particular load ok what will happen to the uniform region if you see uniform region would have reached some strain let us call it as ε_{1U} that is called as a uniform strain $\varepsilon_1 < n$ ok which is the maximum uniform strain you can have in the entire stream ok which is the maximum you can uniform strain you can have in the entire stream ok which is actually situation is actually drawn in this particular figure. So, this figure is our usual P versus ε_1 ok which is nothing but with respect to your uniform region same graph has got P versus $\varepsilon_1 d\varepsilon_1$ also which is for the imperfect region and there are two curves one is a blue one uniform region other one is a imperfect one red region.

So, that means what we are going to pick up somehow we are going to monitor load in these two locations we are going to monitor load in these two locations separately material is same. So, that is one important thing this your uniform region and imperfection has got same material that is why you are going to have same properties K, n all are going to remain same in both the locations material is same only thing there is a imperfection at the center you can imagine like that. So, we are going to have load versus strain for uniform region and load versus strain for imperfect region and this is the way it looks like and you will see that since imperfection is going to dominate deformation. So, we expect this F which is nothing but maximum load I have drawn here this is the level of maximum load when this F is reached in red color portion you will see that this fellow will be satisfied this equation is satisfied $\varepsilon_1 + d\varepsilon_1 = n$ ok. So, now when F is reached I was telling that when F is reached you will see the situation in the uniform region it will go to G ok it will reach a point G and G is quantified by $\varepsilon_{1U} < n$ this fellow would be less than n and this is the maximum uniform strain you can have in the entire sample in the entire sample ok.

So, now what will happen is this uniform region cannot strain beyond G ok if at all it has to happen it will happen only if you give a larger load in imperfect region more than F more than F ok if you give more than F then only uniform region can strain beyond G ok that is what I have written here. The uniform region cannot strain beyond point G as it require a higher load that can be transmitted by the imperfection which is not going to happen here because we fixed maximum load happens at F when you speak about imperfection ok. Now we have only one imperfect region so now you can imagine that there are several imperfect regions like this ok this is number 1 let us say there are there could be another imperfection 2, 3, 4 like that so out of this 4 imperfect regions ok other than uniform region there will be one imperfect region which is going to be severe in nature ok that is going to be one imperfect region which is going to be the weakest and that is actually going to dominate the entire deformation ok and that is going to dominate the entire deformation and you will see that you are going to have localized deformation in that particular imperfection and that is going to govern the entire load occurment when you do deformation. So, I am going to call that imperfection which is weakest is actually called as I am going to call it as a greatest imperfection becomes a focal point ok reaching maximum load capacity causing concentrated deformation in the neck while the uniform region unloads elastically as the load decreases. Uniform region if you see that the load actually when it is decreasing it is going to elastically deform and this is the entire situation that is going to happen in these locations.

So now what will happen if the test is continued beyond maximum load P_{max} only the imperfection will deform and it will do under falling load and it will do under falling load the load will decrease like this ok. This is what will happen in the entire strip ok. So now you can imagine though we are saying that here it is an imperfection you are going to introduce all those things this region itself can be referred as a necked region you can imagine that this as a necked region ok you can imagine because we know that this is what is going to have instability or neck. So now you can simply compare what is going to happen in the necked region and the uniform region ok. When you are comparing imperfect region and uniform region you can as well compare what is going to happen in the neck region and in the neighboring region ok.

This is what will happen. This is what will happen ok. So now the question is the difference between maximum strain in the uniform region of an imperfect strip and the strain ε_1^n at the maximum in a perfect strip can be found from the by following any material law $\sigma = K\varepsilon^n$ ok. So for that we are going to use our previous equation $P = KA_0\varepsilon_1^n\exp(-\varepsilon_1) = K(A_0 + \mathrm{d}A_0)(\varepsilon_1 + \mathrm{d}\varepsilon_1)^n\exp[-(\varepsilon_1 + \mathrm{d}\varepsilon_1)]$. So now what I am going to do is I am going to replace this part by n and this part by n and this ε_1 I am going to keep it as ε_U as per my this one right $\varepsilon_1 + d\varepsilon_1 = n$ if that is the case the imperfect region F is reached at that time if you see what is going to happen in G or in the uniform region is nothing but $\varepsilon_{1U} < n$. So ε_1 is nothing but ε_{1U} here $\varepsilon_1 + d\varepsilon_1 = n$ ok. So if I replace ε_1 here by ε_U and $\varepsilon_1 + d\varepsilon_1$ by n here ok you can follow some steps and finally I will get a simple equation like this $\left(\frac{\varepsilon_\mathrm{u}}{n}\right)^n \exp(n - \varepsilon_\mathrm{U}) = 1 + \frac{\mathrm{d}A_0}{A_0}$ ok and you will see that I also drawn this particular figure which will be easy for you to understand the same figure which I referred before in perfect strip ok and real strip if you compare these two real strip is going to have sudden fall in load ok rapid decrement in load ok and this is your ε_U and the total strain is actually denoted by ε_t ok this total strain is actually governed by the real strip situation rather than a perfect strip situation that is what is given here and anyway we will come back to this particular equation ok.

In this equation if you see this $(n - \varepsilon_\mathrm{U})$ what is n? n is nothing but $\varepsilon_1 + d\varepsilon_1$ that is the strain that you have in the neck region or imperfect region ok when maximum load is going to happen is going to be reached ok $-\varepsilon_U$ at that particular you know situation what is the principle strain in the uniform region this difference actually ok these are all going to be small quantities of course this depends on your what is the imperfection level ok $\frac{\mathrm{d}A_0}{A_0}$ that is why I am saying this $\frac{\mathrm{d}A_0}{A_0}$ is going to be a small quantity ok you can refer this I have just mentioned here this is 2 mm and t is 1.999 mm so you can get a from this and A from this area of cross section what is it ok A for uniform region you can get a for imperfect region you can get these are all actually if you get $\frac{dA_0}{A_0}$ is going to be a small quantity is going to be a small quantity ok and $-\varepsilon_U$ also is going to be a small quantity but it depends on how severe is your change in area of cross section ok. So, now what you can do is you can do some simple mathematical calculations you can expand this ok this part series expansion you can do and this is available in resources finally I will get into one particular equation ok which is nothing but this part $n - \varepsilon_U$ which is what we want $n - \varepsilon_U$ in this particular graph is this distance $-\varepsilon_U$. So, what is the difference in strain ok between the net region and outside region ok $n - \varepsilon_U$ is given by such a small equation easy equation $\sqrt{-n\frac{dA_0}{A_0}}$ here you will say that the equation tells us that the difference between maximum strain in the uniform region and n ok maximum strain in the uniformly maximum strain in the uniform region is nothing but ε_U, n is nothing but the strain available in the neck region or in the imperfect region ok it depends on actually n value of the material and your change in area of cross section how severe is your imperfection ok. So, this dA_0 or the thickness imperfection you have in this particular figure you can see this thickness imperfection you have in this particular material in this particular figure ok.

So, this can be actually adjusted ok such that you are the entire behavior of material is going to closely resemble that of the practical you know data ok. So, theoretically you can assume let us say dA_0 as one particular value and you can compare ok with respect to the real test piece data ok and you can little bit adjust this you know dA_0 or the difference in thickness ok such that the outputs can match which will also tell you how much of imperfection you can take for one particular material. So, this equation is going to be very very important for us ok and this will give you an idea how a real test piece is going to behave when you do tensile test ok. So, again there are two quantities in this the difference actually depends on one material parameter n ok which is nothing but your strain hardening exponent ok this is one material property and your A_0 or dA_0 is actually a geometrical one that you pick up geometrical one you pick up ok. So, now just to understand the significance of this particular equation ok this problem I have given ok.

So, this problem what is this problem this problem is. So, the length width and thickness of the parallel reduced section of tensile test piece are given as 100 mm, 12.5 mm and 0.8 mm ok. So, you have a test piece and there is a rectangular region let us say gauge length you can say this is 100 mm this is 12.5 mm ok and it is thickness is actually given as 0.8 mm thickness is given as 0.8 mm ok. And the material follow this particular strain hardening law $\sigma_1 = 700\varepsilon_1{}^{0.22}$ and 700 is nothing but K which is in mega Pascal ok.

And what is also given is in a small length the width is 0.05 mm less than the other locations that means, ok. So, what I am going to do is I am going to have let us say this is my initial strip and at one particular location I will see that there is a small change in width actually small location I will see a small change in width small change in width. So, in this particular figure I am referring here this particular figure which I already introduced there is a small change in thickness we said this is t and this fellow is actually less than t right. Instead of that what they are saying is in this problem there is a small change in width situation remains same that is also an imperfection that is also an imperfection and that region where your width is less is going to be a weaker region right.

So, that is the only difference here. So, how much difference the width is 0.05 mm less. So, this will directly give you dA_0. So, now what is the question? Question is estimate the strain in uniform region of test piece as the strip has been tested to fracture. So, when you deform the sheet it will reach fracture failure at that particular stage what is the strain in uniform region that means, you have to get ε_U ok.

So, simple calculations you can do. So, initial area of cross section that is in the uniform region is you can say you know 0.8 into 12.5 ok which is about 10 mm square you can get and dA_0 you can directly from this width is 0.05 mm less ok. So, you can say 0.8 into thickness remains same this thickness is going to remain same only width is reduced in this particular problem.

So, I can say $dA_0 = 0.8 \times 0.05 = 0.04mm^2$. So, $\frac{dA_0}{A_0} = -0.004$ is a negative quantity 0.004. So, I can straight away take this particular equation which we have derived just now $n - \varepsilon_U \approx \sqrt{-n\frac{dA_0}{A_0}}$. So, n is already given here $0.22 - \varepsilon_U \approx \sqrt{-0.22 \times (-0.004)}$. So, if you calculate it is 0.03 and if you get $\varepsilon_U = 0.22 - 0.03 = 0.19$. So, it will become plus. So, $\varepsilon_U = 0.19$. So, now the interpretation is if there is no imperfection if there is no imperfection the material is a perfect the material is a perfect there is no imperfection at all then as per our previous theory your $\varepsilon_1 = n = 0.22$ or you can call this $\varepsilon_1{}^*$ ok. This is nothing but your ε_U only when there is no imperfection right.

Now this 0.22 has become 0.19 because you are putting an imperfection because you are having an imperfection of such a small value 0.05 mm ok. So you can get this difference percentage difference is not it. So, what is the difference in this value you can get ok.

So, you can see that there is about 13.5% decrement in uniform strain ok. When you compare this with this because you are putting in a small imperfection very small you see that 0.05 mm ok. So, width of 12.5 mm in place of that the one small region is there where you have a reduction in width of 0.05 mm accordingly there is a small change in area of cross section dA_0 ok which is like saying similar to what I said before which is like saying ok. Here you have thickness of 2 mm and here you have small thickness change 1.999, 2—1.999 that is such a small change in thickness ok. If you see that such a small change in width you are going to have a 13.5 % decrement in uniform strain due to a small defect of the order of 0.05 mm width change ok. So, you can see that the uniform strain has decreased by about 13.5 % if you put a small imperfection of this order. Now you can imagine that how such a small imperfection is going to play a big role in determining the uniform you know strain or you can say limiting strain or instability stage ok. So, if you do not consider this imperfection then you would end up in actually telling people that your material is going to fail when strain is actually 0.22 which is actually not true it is going to fail at strain of 0.19 ok. So, we will stop here and then we will continue in the next session. Thank you.