Mechanics of Sheet Metal Forming

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Week- 03
Lecture- 06
Sheet Deformation in Plane Stress

Okay, so let us continue our discussion from the previous module. So today, now we are going to start module 3. Okay, so module 1 and 2 are completed. So we are extending our discussion from module 3, which I have named as deformation of sheet in plane stress, deformation of sheet in plane stress. So in this is actually a small module. So in this we are going to discuss about some theoretical and some practical aspects, okay and some calculations, you know like that.

Okay, so to start with in the previous chapter, we were discussing about we can evaluate strains, principal strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$, which is going to depend on the initial and original dimension of a particular element. We worked out some problem, 2 problems also in the previous chapter, right. So now then further strains if you calculate, you can calculate effective strain, then effective stress by knowing some flow stress model, from effective stress by knowing a yield function, you can get σ_1 and by knowing β, α , you can get a σ_2. Okay, so from there onwards, you can get a hydrostatic stress and principal stress, your deviatoric stresses.

So all this can be calculated along with the work done. That is the way we started, right. So now the first important thing is to evaluate the, you know, strains at different locations of a sheet. So how do we do it practically? So, so we start with that discussion and then we go ahead with some important aspects in this particular chapter. So now, so here I have given 3 figures A, B and you have C.

Okay, so you can imagine that A is basically an undeformed sheet. Okay, this is basically an undeformed sheet and this is a deformed sheet. Okay, and this is a deformed sheet. And this is a deformed sheet. Okay, so now the question is, how do you calculate your strains? So if an undeformed sheet of thickness t_0, let us say initial thickness t_0 is marked with grid of circles of diameter d_0 .

Okay, so or a square mesh or a square grid of, you know, dimension, let us say d_0. Okay, then during uniform deformation, the circles will become ellipse. Okay, with major and minor diameters as d_1 and d_2. Correct. So what does that mean? That means, suppose you have a sheet, you know, through which I mean using that you are going to do deep drawing or stretching or any sheet forming operations.

First of all, you have to put lot of circles on the sheet surface. Okay, you have to itch or you have to print several circles of known dimensions. Say for example, this d_0. Okay, so I have just given you some example 4 circles, 1, 2, 3, 4 of diameter d_0. d_0 could be maybe about, let us say, 20 mm, let us say 2 centimeter, you can say 20 mm.

Okay, so let us say 2 centimeter, you can say. Okay, so, you know, small, you know, circles, you can, you can put, okay, or maybe like, you know, 10 mm, you know, circles, you know, like which is 1 centimeter dimension, like that you can put, okay, small dimensions. Okay, and smaller the diameter of the, you know, these grids, you can calculate strains at a very localized region. Okay, that is a, you know, thing. Okay, so I just given you a reference, okay, like this.

So, you can look into, you know, different standards how to do it. So, anyway, so now given a sheet, you know, you are going to put lot of circles on the sheet surface. Okay, and what will happen to the circles when you deform it, it is going to become an ellipse like this depending on whatever β or whatever α you are going to deform it. Okay, so, d_0 is going to become d_1 and d_2 and d_2, we are going to call it as your minor axis, which is nothing but minor diameter and d_1 as a major diameter. Okay, so, sometimes what we do is instead of circles, we can put square also, square grids are also possible, okay, which is going to become d_1 and d_2 like a rectangle or any other distorted rectangle you can have, but it is always better to put the grids of circles, grids of circles.

Okay, so, now what will happen? Okay, so, given, you know, deformation or α, β this ellipse is going to have some dimension, okay, you will get d_1 and d_2 and from d_0, which is original dimension, you can get $\varepsilon_1, \varepsilon_2$. So, $\varepsilon_1 = ln\frac{d_1}{d_0}$ because the major, no. So, let us say you have major and minor axis. So, $\varepsilon_1 = ln\frac{d_1}{d_0}$ and $\varepsilon_2 = ln\frac{d_2}{d_0}$. Okay, so, you have to be a little bit careful in which orientation you are going to pick up these two strains, but we have seen some examples accordingly you can take it.

Okay, so, now the new thickness t and the deformation stresses are σ_1, σ_2, okay, which are also referred here. So, this has become t, small t and you have σ_1 and σ_2, these are two principal stresses acting on the sheet, which is responsible for deformation, okay and this σ_1, σ_2 will give rise to T_1 and let us say T_2, which are actually called tension or traction, tension or traction. Okay, so, that T can be evaluated from σt. Okay, if you want T_1, it will be $\sigma_1 t$, if you want T_2, it will be $\sigma_2 t$, which is what I have given here. Okay, t is known as tension or traction, which means it is always pulled in one direction.

Okay, suppose in principle direction 1, you have $T_1 = \sigma_1 t$, assuming t to be a constant value, new thickness but constant. Okay, σ_1, $T_1 = \sigma_1 t$, which is nothing but major tension, generally it is positive in nature. Okay, and you have $T_2 = \sigma_2 t$, which is generally called as minor tension, it is positive in stretching, but opposite negative when you go for compression type of deformation. Okay, so, T can be found out as σt. Okay, so, now, you know how to calculate $\varepsilon_1, \varepsilon_2$ and from there you can calculate ε_3 or you can directly calculate $\frac{t}{t_0}$.

Okay, with respect to this figure, you can get $\varepsilon_3 = ln\frac{t}{t_0}$. We have seen some examples before in the previous chapter. Okay, so, now, such evaluation of strains will give you something called as a strain distribution. Okay, in the entire deformed sheet, which will give you some important information during the deformation process. Okay, so, what do we do? Say for example, you take a deep drawing, I just given you as example, cup deep drawing, cylindrical cup deep drawing.

Okay, so, you have a blank and you have punch, you have die and there is a blank holder. So, you know what is the meaning of blank holder? Blank holder is going to hold the sheet between the die and the blank holder and you are going to give some force let us say, you give a force of BHF, let us say a blank holding force is given here. Okay, and you know that there is no restriction for the moment of sheet in the radial direction, it moves actually. Okay, but only restriction is through friction. Okay, there is otherwise there is no constraint.

So, finally, you are going to get a cylindrical cup like this, you have seen this example before. Okay, so, now, I have a partially drawn cup. Okay, similar diagram we have seen before I have partially drawn cup. Okay, I have just shown you one sector of that. So, this you know is a flange region is a cup wall region and this is a cup bottom region.

Okay, that is known to you. So, now, we want to evaluate strain. Now, we want to we are going to pick up this deep drawing as a process, we are going to evaluate strain at six different locations 1, 2, 3, 4, 5, 6. Okay, so, we are starting from cup center to the know your this, your punch corner, then in the cup wall, then in the die corner and somewhere in the flange and at the edge, six different locations we are picking and you imagine that this entire sheet on the sheet surface from top view if you see, okay on the sheet surface is going to be a sheet is going to be a circular one and there are a lot of circles, you know printed on it and you are doing deep drawing and you are going to pick up this six locations to get strain distribution. Okay, and that is what they simply referred as we evaluate strain in all these locations and by measuring the grids.

Okay, and I can plot a graph between . Okay, $\varepsilon_1, \varepsilon_2$ and you should note down here that ε_2 can have negative and positive value and ε_1 we are keeping always as a positive value. Okay, so, ε_2 is in X axis and ε_1 is in Y axis and you are going to calculate all the strains and you are going to plot here I have shown two different stages. Okay, so, this is your stage number 1 and this distribution is for stage number 2 at six different locations. So, it starts with the edge, this edge is here.

Okay, it starts with the edge and it moves to a center. So, this is your center of the cup. Okay, so, you can see that you know maybe in the stage 1. Okay, let us pick up stage 1. Okay, which has got the lower strains as compared to stage 2 naturally.

Okay, and stage 1 is center location you have strain here and then here and then the third location is somewhere on the Y axis. Okay, and the fourth location is here, fifth location is here and the edge it is somewhere in the negative you know ε_2. Okay, probably β is going to be minus in there. So, now if you further deform it and the same location same grids I am going to monitor the new dimensions, new diameters and I am going to get $\varepsilon_1, \varepsilon_2$ and I am going to plot here in the six different locations and you will see that this is a second stage. So, what do you get from this? You can get the present status of deformation in the six different salient locations, six different salient locations and six different locations will have six different β and α .

You can see this dotted line tells let us say this is one β, this is another β, this is 1, 4, 5, 6, 6 different β s you can imagine. Okay, this will give you the entire feel for how strain is getting distributed in this. And when you compare first stage and second stage, the regions are not equally deformed. The regions are not equally deformed, you see that. So, here the strains are closer and when you move towards the edge it is deviating a lot, it is deviating a lot.

So, it will also tell you which location is going to deform significantly as compared to the other locations. Okay, so such a strain distribution can be evaluated for any sheet deformation or even for tube deformation process also to get an idea where you are going to have, what type of strain distribution, where you are going to have strain localization which is going to be responsible for let us say instability or you know fracture which you are going to see later on or necking like that. Okay. So, I just summarized you know whatever I have discussed with you here. Okay, the strains are located at locations mentioned in figure b, the strains are plotted on these locations.

Okay, on these locations, okay in strain space $\varepsilon_1, \varepsilon_2$ graph and strain locus can be obtained in a particular stage. So, this is called as one strain locus, this is called as another strain locus, same location, two different stages. You can also have third stage which could be something like this. So, fourth stage, something like that until you have a full cup form.

Okay. The strain locus may expand uniformly till a stage and later some points may stop straining. Okay, then a process limit is reached. That is what I gave an example. Probably this region will stop straining if you further deform it because the strain gradients are very low here but here you will see that there is significant deformation going on in these locations.

Okay. So this typical strain pattern is called as a strain signature. Okay, the strain signature can be obtained for different processes at different stages and there are certain advantages of that. Okay. So, just to summarize how do you calculate the principal strains, you have ε_1 which is nothing but $\frac{d_1}{d_0}$, $\varepsilon_1 = \frac{d_1}{d_0}$, what did I write? Yeah, so $\varepsilon_1 = ln\left(\frac{d_1}{d_0}\right)$, $\varepsilon_2 = ln\left(\frac{d_2}{d_0}\right)$ and $\varepsilon_3 = ln\left(\frac{t}{t_0}\right)$ or you can get it from $\varepsilon_1, \varepsilon_2$ at a particular location like this you are going to get for all the grids. Okay, so now if you know $\varepsilon_1, \varepsilon_2$, okay, so then you can get β.

That is why we have 6 different locations, 6 different you know β, 6 different strain ratios or strain paths, $\beta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\ln\left(\frac{d_2}{d_0}\right)}{\ln\left(\frac{d_1}{d_0}\right)}$. Okay. And if β is a constant we call that as a linear strain path. What do you mean by strain path? Strain path is nothing but the mode of deformation.

Okay, nothing but the mode of deformation. So, strain path means this is one strain path. Okay, let us say we are going to pick up from 0, okay, this particular limit is following this particular strain path like this. Okay, essentially β. Okay, that is considered as linear. It is not going to change, it is not going to become non-linear.

If it becomes non-linear which can happen then you have to be little bit careful. There is something happening when there is a change from you know one slope to another slope. You have to be careful. Okay, so how do you get ε_3? We are actually summarizing what is required for such strain signature. So, $\varepsilon_3 = \ln\left(\frac{t}{t_0}\right) = -(1+\beta)\varepsilon_1 = -(1+\beta)\ln\left(\frac{d_1}{d_0}\right)$. Okay, so you do not need to measure this. Rather you can get d_0 which is known to you. Okay, let us say 10 mm and let us say you have d_1 you can measure it and get it. So, β is fixed. Okay, so you can substitute it and you will get ε_3. And from ε_3 you can get the current thickness which is nothing but $t = t_0\exp(\varepsilon_3) = t_0\exp[-(1+\beta)\varepsilon_1]$.

So, ε_3 you can substitute here. Okay, alternatively you can get t as you know volume remains constant okay in a particular grid. Okay, you can get this equation and from there onwards you can get t. $t = \frac{t_0 {d_0}^2}{d_0 d_1}$, where d_0 *is* nothing but diameter of the initial circle grid and t_0 is at the thickness original thickness at that location and d_1 and d_2 are the new dimensions of that ellipse. That will give you t. So, either you can get t from this or you can get t from this.

So, the entire summary of this is you can get just by putting circle grid so you can get $\varepsilon_1, \varepsilon_2, \varepsilon_3$. Okay, and you can plot it at any number of locations which will give you a strain locus or strain distribution in the entire section and that will give you some idea of what type of deformation you are going to have and whether it is equally deforming at different stages or it is going to be different type of deformation, different locations that can be obtained. Okay, this I think I told you. Okay, the whole summary is this assuming β to be constant.

Okay, β can change. Okay, β can change. Okay. So, now let us come to modes of deformation. We will give some, you know take more ideas into these modes of deformation or modes of deformation is nothing but β or α. Okay, so what all the information that we are going to get from this. Each point in the strain diagram indicates a magnitude of final major and minor strain and the assumed linear strain path to reach this particular point. Right, suppose you pick up this particular point let us say, okay this is again a plot between $\varepsilon_1, \varepsilon_2$.

Correct, let us pick up this particular point let us say A. Okay, it has got a magnitude of $\varepsilon_1, \varepsilon_2$ assuming this linear strain path to reach that particular point. Same way for OB, OC, OD and OE. Right. So, now this ellipse which I have shown here, this ellipse which I have shown here, okay is nothing but a contour of equal effective strain $\bar{\varepsilon}$ just to give some idea.

It is a contour of equal effective strain. Okay, it is a contour of equal effective strain. That means you deform the sheet along OA and you stop here, OB you stop here, OC you stop here, OD you stop here and OE you stop here and they have to be stopped at almost same effective strain. Okay, and then you join it to create a contour. Okay, so and from the work hardening hypothesis they will all have same σ_f at that particular location. Okay, so the ellipse shown in the contour is a contour of equal effective strain $\bar{\varepsilon}$, each point on the ellipse will represent strain in the material limit that from work hardening hypothesis has the same flow stress σ_f.

Okay, anyway so that is just one small information but what is very important for us is what is this signify? OA, OB, OC, OD and OE. That is what we are going to discuss now in the next this particular sections. So, before discussing just get a you know some idea from this diagram. So, this anyway is a plot between $\varepsilon_1, \varepsilon_2$ and you will see that I am representing OA path which is denoted by $\beta = 1$. Okay, that means $\frac{\varepsilon_2}{\varepsilon_1} = 1, \varepsilon_2 = \varepsilon_1$.

Okay, and here OB when you go along this path you are going to have $\beta = 0$. Okay, which is nothing but $\varepsilon_2 = 0$ which could be called as a plane strain process. Okay, because as I already told you when you have OA this particular point. Okay, there is one ε_2 and ε_1 associated with this point A and there is ε_3 inside that. Okay, so ε_3 has to be calculated from this location ε_2 and ε_1.

So, ε_3 exist inside. Okay, so which means that you can call this as $\varepsilon_2 = 0$. So, that is why you have only ε_1 but ε_3 exist. So, it is you can call this as a plane strain deformation. Okay, similarly OC you can say $\beta = -1/2$, OD you can say $\beta = -1$ and OE which is also a critical strain path $\beta = -2$ we get and you can also see how this ellipses are going to how the circles are going to get deformed.

So, here the circle will become a larger circle. Okay, so that if you get $\beta = 1$. Right, so this dimension d_0 is known to you. Okay, and the new dimension let us say this d_1, d_2 both are same, d_2 both are same. So, you can calculate your ε_1, ε_2 from the previous formula. Okay, and you will get the strains to be same that is why $\beta = 1$.

If you come to this particular plane strain you will see that it is going to become ellipse but the minor axis will be same as that of your circle that is why you have strain to be 0 in that direction but otherwise you have ε_1 and ε_3 and here onwards ellipse is going to be inside the circle. Okay, so you can see ellipse has gone inside further inside and further inside ellipse is actually getting compressed. Okay, in the width direction. What is this thin and thickens we will come to this after this particular discussion. Okay, so now there is a limit for this β, there is a limit for this β, β generally changes from 1 to -2.

Okay, that is what I am saying $\beta = 1$ is one limit, $\beta = -2$ is another limit. Okay, and in between you can have any number of β depending on what deformation you are going to do but we are just shown here 5 different β. Okay, 5 different β, 1, 0, $-1/2$, -1 and -2. Okay, so now let us go into some important details about this each strain path OA, OB, OC, OD and OE and where do you see this type of situations that also we can have to some extent we can have some idea. Okay, let us go for point A, point A is along that means you are picking up OA path.

Okay, you are picking up OA path, this path is actually called as balanced biaxial stretching or equal biaxial stretching. Okay, we call it as $\beta = 1$. Okay, so what are the salient features in this, this $\beta = 1$ can be seen in this type of situations. Okay, what is the situation? Situation is there is a circle and circle is going to become a bigger circle, so it will become a concentric circle. Okay, so and you will see this type of situation in a circle which is inscribed on the uppermost point when you do this kind of deformation like there is a sheet which is kept in a blank holder.

Okay, and the punch is you know getting displaced here and your sheet is getting deformed here right, so this is the height of deformation, this is your height of deformation, this much it has deformed and you will see $\beta = 1$situation somewhere in the mid portion, in the uppermost portion that is the pole region. That element is actually if you put a circle on that particular location it is pulled equally in both the directions, so it will be like this. Okay, so $\beta = 1$. Okay, what will happen to thickness strain? Thickness strain can be obtained, so if $\beta = 1$ what is thickness strain? So, thickness strain with β what is the equation we have? We have this equation isn't it. So, thickness strain is equal to $-(1+\beta)\varepsilon_1$correct.

So, $\beta = 1$ isn't it, so 1, $-2\varepsilon_1, \varepsilon_3 = -2\varepsilon_1$. What does it mean? That means so you have a particular strain okay and your thickness is going to decrease rapidly with respect to your strain in principle direction 1. Okay, so it is -2 times of $-$ signifies the thickness strain is negative which means you are going to have thinning and twice, twice means the thickness is going to reduce more rapidly with respect to ε_1. Okay, so ε_1 is you know like in that location ε_1 you can get from d_0 and d_1, d_2 but for a particular ε_1 okay if the element is actually stretching like this then you will have thickness decreases more rapidly as per this particular equation twice, it is going to decrease twice. Okay, and there is one more important feature that we need to understand $\bar{\varepsilon} = 2\varepsilon_1$, $\bar{\varepsilon} = 2\varepsilon_1$.

So, how do you get the moment you go for $\bar{\varepsilon}$ which is nothing but your effective strain by using von Mises equation we already derived this okay, we already derived this equation in the last section and you can put $\beta = 1$ here so this is going to be 1, 1 so 3, 3 will be cancelled it will be yeah so $2\varepsilon_1$. Okay, so $\bar{\varepsilon} = 2\varepsilon_1$ what does that mean? That means the sheet is going to work out and rapidly with respect to ε_1. So, you give strain in principle direction 1 and you calculate it but you will see $\bar{\varepsilon}$ effective strain will be twice than that of that. Okay, so now you can get $\bar{\varepsilon}$ from this and $\bar{\sigma} = K\varepsilon^n$ you can substitute for constant K and n you can get $\bar{\sigma}$. Okay, and you will see that the sheet work hardening is going to be rapid in this particular type of deformation that means you have a sheet that is component that is made okay and at one particular location let us say a set of elements are deforming in equi-biaxial stretching and strain path where $\beta = 1$ then in that location you will see thinning is going to be significant and work hardening is also going to be significant.

Okay, so with respect to what because when we are saying you know thickness strain is going to be we know the thickness is going to decrease you know rapidly and strain hardening is going to be rapid now with respect to what? With respect to ε_1 but then we are also comparing with other strain paths. Let us go for point B this point B okay is like this so wherever you have this OB strain path you will see that if there is a circle, circle will become an ellipse but the change in dimension in this direction okay is going to be negligible so it will remain as a plane strain type of analysis it will become plane strain. So where is OB in this diagram? OB is here you go along Y axis but ε_3 is there inside huh so here $\frac{\varepsilon_2}{\varepsilon_1} = 0$ so $\varepsilon_2 = 0$ so $\varepsilon_1 = -\varepsilon_3$. Okay, so now here I just simply made a single circle expands only in one direction and circle becomes ellipse in which minor axis is unchanged like this and you can see such situation in a channel a side wall you can see okay suppose this is a channel sheet channel that is made okay channel type of deformation okay you will see somewhere in the cup wall region you may see that kind of situation. Let us come to point C path OC, path OC is what? Path OC is this path $\beta = -1/2$ this is known to us $\beta = -1/2$ for that $\alpha = 0$ which means it is a uniaxial type of deformation it is uniaxial tension we have seen that predominantly.

So when $\sigma_2 = 0$ like in the uniaxial tensile test that is the situation you see here okay that means your street stretches in one direction and contracts in the other direction so ellipse started going inside the circle dimension and generally you can see such type of situation exists in the whole expansion test at the edge okay. So one has to really look into it but some idea you will get from this kind of application okay you can see suppose a whole stretched actually pushed in this direction you may see this kind of situations here. Point D this point you already discussed in one of the problem your point D that means you are following path OD and you are reaching this point D okay which is described by $\beta = -1/2$ okay. This OD path is also called as drawing or constant thickness process. Constant thickness process I think a constant thickness process this example we have seen in the second problem of the previous module is not it where we are trying to compare two different you know β right one of that is basically drawing or constant thickness process I also referring to the calculation and say that $\varepsilon_3 = 0$ there which means thickness is not going to change at all and you will see in such cases ellipse is going to further compressed inside okay in the your minor dimension side okay minor dimension it has gone is further compressed and you can see such situation existing in the flange region of the cup that is formed okay.

So here you will see that the principal strains are equal and opposite $\frac{\varepsilon_2}{\varepsilon_1} = -1$ so equal and opposite okay. So observed in flange region of drawing that is what I was telling you and work hardening is gradual work hardening is gradual how will you find out okay you will see that here okay. So before that you can find out $\varepsilon_3 = -(1+\beta)\varepsilon_1$ and β=-1 so $\varepsilon_3 = 0$. What does it mean? That means if you deform a material in this particular strain path you can deform the material path to a particular strain but without much change in the thickness.

So thickness strain is going to be almost 0 okay. So now for this also you can get $\bar{\varepsilon}$ right. So what is $\bar{\varepsilon}$? $\bar{\varepsilon}$ is this equation and your $\beta = -1, \bar{\varepsilon} = \sqrt{\frac{4}{3}[1+\beta+\beta^2]\varepsilon_1}$, $\sqrt{\frac{4}{3}[1-1+1]\varepsilon_1}$, $\sqrt{\frac{4}{3}\varepsilon_1}$. So $\frac{2}{\sqrt{3}}\varepsilon_1$which is nothing but $1.155\varepsilon_1$ and you when you compare this with the previous fellow that is your 0.8 is 2 times of $2\varepsilon_1$ this is what I was telling you. If you compare these two process $\bar{\varepsilon} = 2\varepsilon_1$ and $\bar{\varepsilon} = 1.155\varepsilon_1$ and you can say that here work hardening is actually very gradual in the other case when you go for balanced biaxial stretching or equi biaxial stretching is going to be rapid okay. So one has to be very very careful in this type of deformation which deformation you are going to pick up and there is one more point called as point E okay and if you pick up a point OE that is the least one okay -2 okay here also you can see that $\bar{\varepsilon} = -\varepsilon_2$ and of course you can also get this type of relationship okay if you put a $\bar{\sigma} = \sqrt{(1-\alpha+\alpha^2)\sigma_1}$ is not it is there if you put you can get $\bar{\sigma}$ and you will see that $\bar{\sigma} = -\sigma_2$ and $\bar{\varepsilon}$ equation can also be obtained $\beta = -2$, $\beta = -2$, 1 -2 +4 $-2 = 2$ okay into this 1. So you will get $-\varepsilon_2$, $\bar{\varepsilon} = -\varepsilon_2$. So now there is one more point to this here this point E is generally seen in the edge of the flange okay we call it as uniaxial compression that means the ellipse is going to further compressed in the width direction okay and this is actually a location where sheet is going to thicker okay we have seen one example know in the first problem where sheet thickness is actually a little bit increased okay.

So I think initial thickness was 0.8 and after calculating the new thickness it was 0.8 0.8 or 0.84 like that so which means the sheet thickness has increased that type of situation can come probably at the edge of the you know this flange and you have to be very careful with point OE because that can create wrinkling okay it is pulled in one direction and pushed in this direction know so in plane right. So what will happen now the sheet will try to move up out of plane okay which is what you are going to call it as wrinkling on the flange region which is a defect actually which is a defect that is why you are actually providing sufficient blank holding force to suppress it okay so that will happen here okay. So these are the five important you know modes of deformation okay β or α you can say of course now we are going to see how to get α from β and this all this β is going to have different situations okay from OA being you know equi-biaxial okay then you have OB which is a pain strain then you have uniaxial then you have drawing then you have uniaxial compression okay this five are going to be important and of course you can have any strain paths in between in between also you can have any strain paths to pick up your deformation okay.

So now there are there is one more point in this diagram I have copied the same figure here ε_1 versus ε_2 in this you will see you can divide this β into two parts okay one a $\beta > -1$ one greater than you know $\beta > -1$ that means a -1 -2 0 and 1 in these strain parts if you see the material will actually thin down okay material actually thin down okay and $\beta = -1$ is a transit okay there is a transit region and that is why you have 0 thickness strain that means it is not going to thin it is not going to be thickening so no thinning and no thickening okay this is a transit region and if you go on this side right hand side okay of this diagram then you will have a thinning in all these strain parts and if you go on this side of β that means your $\beta < -1$ that is between -1 and -2 you will see the sheet becomes thicker you will see sheet becomes thicker. So β with respect to β you can divide the sheet forming deformation into rather two parts one is those deformation process which involves thinning and that will be $\beta > -1$ this side okay and the other part is actually $\beta < -1$ which is going to be sheet getting thicker. Okay so now we will come back to this after this discussion so now this part this small part we are going to discuss about different effective stress strain loss okay so effective stress effective strain loss means how do you relate $\bar{\sigma}$ to $\bar{\varepsilon}$ okay so one or two equations we have already seen okay but we will see that in the effective terms okay. So the first law that we are going to see and we are we already seen this several locations including problems that is nothing but $\bar{\sigma} = \mathrm{K}\,\bar{\varepsilon}^n$ it is called power law it is called power law okay and this equation is predominantly used in all the calculations and we have also used this equation similar equation in our problem also when you are trying to find $\bar{\sigma}$ from $\bar{\varepsilon}$ and you remember $\bar{\sigma}$ can be related to σ_1 and σ_2 by any yield function like von Mises is yield function which you already derived okay. So now you know how to find K and n from this right so standard procedure is there you do uniaxial tensile test get the load displacement graph convert that into true stress strain data right so and for isotropic material we say that effective stress strain curve is going to be coinciding with uniaxial stress strain curve that we already derived okay $\bar{\sigma} = \sigma_1$ and $\bar{\varepsilon} = d\varepsilon_1 = \varepsilon_1$ is we already discussed so if that is the case then you can get K and n from the slope of natural logarithm plot of σ as a natural logarithm plot of ε and that will be n and the intercept will give you K value correct.

So now if you substitute this equation this equation is ready let us $\bar{\sigma} = 200\ \bar{\varepsilon}^{0.25}$means by giving different $\bar{\varepsilon}$ values you can get different $\bar{\sigma}$ values and that can be compared with experimental data like this okay and depending on how you fit you will have agreement between these two curves okay. But there is one problem to this problem is for 0 strain you will have 0 stress for 0 strain you will have let us say 0 stress okay it does not predict the actual yield stress what does it mean that means suppose the material is already undergone some deformation before coming to tensile test that means the material is already hardened to some extent is not it that will not be captured by this fit this equation why because when you put $\bar{\varepsilon} = 0$ you will get the strength as 0. So that means it is going to start from here okay as if like there is no yielding happened okay actually there is a initial yielding that has happened which is going to actually start from here which will not be predicted by this particular power law but still it is predominantly used. Now to capture that the change in strength when you do tensile test you introduce something called as a pre strain okay this is what we used it in the previous problem as probably 0.008 or 0.0008 which a small value we used okay it is generally a small value okay. So now if you put $\bar{\varepsilon} = 0$ you will see that there is a small you know effective stress staying with that material and they may coincide somewhere here okay what is the physical meaning of this this pre strain meaning $\bar{\varepsilon}$ is called as pre strain okay it is going to take care of the materials hardened in the prior process. If the material is hardened or not in the prior process, prior process means let us say for example rolling okay the material is rolled and then you are going to characterize its tensile properties okay or you know several stages of deep drawing let us say there are several different stages of deep drawing that means with respect to second stage first stage is prior form with respect to third stage there are two stages before. So how do you capture the change in strength one way is to model the stress strain behavior using this equation just by putting this constant ε_0 it is going to capture the initial strength of the material and the physical meaning of ε_0 is this difference okay. So you will see that this ε_0 is such a small strain that you are going to give and $\bar{\varepsilon} = 0$ it is going to start with a particular strength which is nothing but the yield strength of a material which actually is equivalent to metal it is already hardened ability before process in the prior process.

So but in this how do you get K and n? K and n can be obtained by fitting it okay. So what you can do is like you can use this equation okay to know K and n that you know already and you can use that K and n here okay and calibrate your you know stress strain data such that your experiment and you know the data from this equation are going to match there will be one particular value of ε_0 from each you know at which you are going to have you know very good agreement between these two that could be the value of ε_0. But how to get K and n? K and n you can get from the the other hardening law $\bar{\sigma} = \mathrm{K}\,\bar{\varepsilon}^n$ okay. These two are other simple equations generally we do not use these days okay. So again you can see the plot actual stress strain behavior is this only the dotted ones are actual stress strain behavior you can say from experiments okay you can say this is from experiments okay this is also from experiment okay.

So now you can have an equation $\bar{\sigma} = Y + P\bar{\varepsilon}$. So when you have strain 0 $\bar{\sigma} = Y$ and that is this point which is nothing but your yield strength okay. It starts from yield strength but it is not going to have you know a power law based strain hardening behavior it is going to vary linearly with the strain and the fitting is going to be varying significantly at different locations you can see the difference. But if you do not have any material constant one can use it. The fourth one is further simple $\bar{\sigma} = Y$ this is also called as rigid perfectly plastic model. Your actual experimental curve data point is this but you are modeling it with the horizontal line which is equal to $\bar{\sigma} = Y$ and this is nothing but your Y only nothing but the yield strength and it is not going to harden at all okay which is called as rigid perfectly plastic model.

So these are four equations one can use but other than that there are several other forms of equation okay there are maybe another four five important forms are there one can look into literature for that we are not going to discuss it here okay. And just to complete this particular discussion okay so here what I have done is for different βs which you have already seen okay 1, 0, $-1/2$, -1, -2 we can calculate α is not it. So that you know α ,α relationship with respect to β you know is not it. So we already derived it using Levi Mises flow rule is not it. So in that way you can calculate different α and you will see that for equi-biaxial stretching this is one and this is also one plane strain this is 0 this is 1/2 uniaxial tension this is $-1/2$ this is 0 for drawing it is -1, -1 uniaxial compression it is -2 and if you convert that into for α it is going to be $-\infty$ okay.

So now this diagram can be converted to this diagram okay this diagram is we already know it is basically nothing but a plot between $\varepsilon_1, \varepsilon_2$. Now I want to convert that into σ_1 versus σ_2 which is our well-known yield locus which is our well-known yield locus okay since we are speaking more of von Mises type then it will be in the form of an ellipse of course $\sigma_3 = 0$ it is plane stress process okay. So and the same paths are noted here OA, OB, OC, OD and OE are noted here OA, OB, OC, OD and OE but you have to be very very careful in comparison okay OA will have almost the same location of what you see here is not it because both are one okay. So now of course you know that this element is actually pulled equally in both the directions so $\alpha = 1$. So now you pick up OB, OB is actually coinciding with Y axis which is a plane strain mode of deformation okay for that if you get α it is not going to coincide with Y axis we have to be very careful it is going to be little on the right hand side of Y axis that is your OB okay where $\alpha = 1/2$ it is also pulled in both the directions but in different proportions.

OC if you pick up it is on the left hand side of Y axis here in the strain diagram but that is going to coincide with Y axis here why because your $\alpha = 0$ which means your σ_2 is not there okay and σ_3 is anyway not there and you are going to have you know along the Y axis and here you will see that it is actually pulled in only one direction okay. So OD here and OD here are you know looks same because it is -1, -1 okay and you will see that the square element is actually pulled in one direction and pushed in other direction or compressed in other direction maintaining $\alpha = -1$ okay. So you have to be a little bit you know very careful in this okay. So I think in when we discuss about yield locus I shown some example of different locations of yield locus with respect to deep drawing and I was referring there in the yield locus we are referring to plane strain right. So you have to be little bit very careful that this plane strain OB is actually here is actually here in your σ_1 σ_2 plot.

This OB is actually plane strain is actually OB here okay. So the last one is actually you know purely compression that is along the X axis okay when you have σ_1 and σ_2 which is going to $\beta = -2$ okay. So in this way you can convert all this βs into αs okay we have done some numerical problems also where once you got β you can get α and these are the 5 prominent αs and βs you know through which you can deform material to any strain. You can have in between also it depends on the material okay and what type of deformation you are giving you can have in between these two you can have between plane strain in the axial in the axial to drawing, drawing to in the axial compression okay. So you can have a variety of values in this okay and this can be separated and this can be separated as I was telling you one is going to be thinning and that is β in this side β this side $\beta > -1$, $\beta < -1$ will be divided into two parts one for thinning one for thickening like what we discussed here this slide so okay. So we will stop here and we will continue our discussion further.