**Mechanics of Sheet Metal Forming**
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**Week- 02**
**Lecture- 05**
**Sheet Deformation Processes (contd)**

So, let us continue our discussion in this class. So, just to refresh what we have done in the previous class, we derived a Tresca yield function, Von Mises yield function and then we discussed about normality convexity concepts ok. Then we discussed about why hydrostatic stress does not influence the yielding of metallic materials ok. So, and then we solved one small problem ok. So, now we will continue our discussion in this section. There are couple of small topics that we have to complete for this module ok.

So, now before we start our new you know heading. So, this one thing which you have to briefly discuss that is basically we derived this equation for Von Mises yield function in the previous section ok. So, this we derived in terms of principle stresses that is whenever this equation is satisfied that is $\sqrt{\frac{1}{2}\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\}}=\sigma_f$ uniaxial flow strength ok or uniaxial yield stress. We say that material is going to yield right.

So, this we derived in principle coordinate system and now in general coordinate system for yielding to happen as per Von Mises yield function, this equation has to be satisfied ok. This is just for our information. We have not derived it, but this will be useful for solving problems. $(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma3_3)^2+(\sigma_{33}-\sigma_{11})^2+6\,({\sigma_{12}}^2+{\sigma_{23}}^2+{\sigma_{31}}^2)=6k^2=2{\sigma_f}^2$ ok. This is actual you know equation for Von Mises yield function ok.

When the left hand side of this equation equals to $6k^2$ where $k$ is your you know shear yield strength ok and then yielding will start ok and the same left hand side is equal to $2{\sigma_f}^2$ where $\sigma_f$ is your uniaxial yield strength then yielding will start. So, that is the meaning of this equation and in this if you want to write in principle coordinate system then only the first part will come that is 11 will become 1, 22 becomes 2, 33 becomes 3 and this equation one can derive right. So, this is a general equation which will be useful for solving some problems later on maybe during assignments ok and the $k$ and $\sigma_f$ has got same meaning like what we discussed in Tresca yield function ok and the $6k^2 = 2\sigma_f{}^2$ will yield this particular relationship between $k$ and $\sigma_f$ ok. So, this is nothing but $k = \frac{\sigma_f}{\sqrt{3}}$ ok. So, this relates shear yield strength to uniaxial yield strength and this is from Von Mises yield function.

So, $k = \frac{\sigma_f}{\sqrt{3}}$. In Tresca yield function I think we have evaluated we got $k = \frac{\sigma_f}{2}$ ok. So, depending on the yield function relationship between your shear yield strength and uniaxial yield strength can change ok. So, with this brief we are going to next important section for us that is called work of plastic deformation ok. So, how to get a work done during plastic deformation very briefly we are discussing ok.

Suppose you take a principal element ok of a unit side ok and in this force acting on each face is shown ok for a small displacement ok which is given by a small strain $d\varepsilon_1$ ok. So, it is a initial one this dotted one is basically the new dimension or the new deformed element and all forces are shown here for a unit side ok. So, since this is a principal element we already made a point that is a principal element we can directly write ok $\frac{dW}{vol.} = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3$ ok. And if you want to get work done per unit volume ok the absolute value ok you can integrate it and I have given example for plane stress process. Suppose if it is a plane stress as per our definition ok your $\sigma_3$ will go off ok then you will have $\sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2$ and if you want work done per unit volume you can integrate it between 0 to $\varepsilon_1$ and then 0 to $\varepsilon_2$ which will give you this particular equation and $\sigma_1, \sigma_2$ or one can have an equation for that as a function of $\varepsilon$ ok.

So, now if you want to go for tensile test ok we know then we can say that $\sigma_1 = \sigma_2 = 0$ then only this fellow will exist. So, $\frac{W}{vol.} = \int_0^{\varepsilon_1} \frac{dW}{vol.} = \int_0^{\varepsilon_1} \sigma_1 d\varepsilon_1$ . So, this is for a general principal element the work done for deforming that element and you can minimize the problem to a plane stress problem that that will become this 1 and 2 will exist if it is uniaxial only 1 is going to exist and you can get and we all know that this integral is nothing, but area under the stress strain behavior to stress strain behavior ok. Similarly you can imagine that area under this one also this one together ok then it becomes a work done per unit volume in plane stress process you will see the diagram now. So, this is a basic for work done per unit volume and given a stress strain curve we can get work done I will show you an example when we work out a problem today at the end of this particular module.

And there is something called as work hardening hypothesis which is an extension of this you know how to get a you know work of plastic deformation and of course, I have given some introduction here. So, ok so, but as per work hardening hypothesis ok we can say that it has been found by experiment that flow stress increases in any process according to the amount of plastic work done during this process correct that is known. But the main point is in two different processes ok if the work done in each is the same ok the flow stress end of the process will be same regardless of the stress path this statement is true for monotonic process. Monotonic process we already discussed monotonic process means keep on increasing strain keeps on increasing. So, in two different processes if the work done in each is the same ok suppose you have uniaxial tensile test you have another plane stress process we can say that these two are two different process if work done in each is the same ok then flow stress are going to be same at these two process at that particular stage that is what is the hypothesis about ok.

You can describe these two schematically ok let us pick up an element deforming in the axial tensile deformation in this way this is a typical $\sigma$ versus $\varepsilon$ graph and you can see red colour curve is the stress strain behaviour and I am picking up a point here up to one particular $\varepsilon$ let us say may be $\varepsilon_1$ ok or $\varepsilon$. So, the area under this I am indicated it as work done in uniaxial tensile test. Suppose you have an element principle element which is deforming in plane stress ok like work done we have shown before then you will have two $\sigma$ one is $\sigma_1$ varying with respect to let us say $\varepsilon_1$ and there is a $\sigma_2$ varying with respect to $\varepsilon_2$ you can say in general X axis is $\varepsilon$ then you will have two curves separately $\sigma_1$ going in this way and $\sigma_2$ going in this way and they can be related $\sigma_2$ and $\sigma_1$ can be related like we have discussed before ok. So, if this is the case we can say that according to work hardening hypothesis the flow stress at the end of the process this plane stress process we do not know what is it end of the process is given by tensile test curve this curve when an equal amount of plastic work has been done ok. That means, when the sum of the areas in the above figure in this figure is equal to the area under the tensile test curve ok then you can get a you know the work done being same in both the cases you can get a flow stress at the end of this plane stress process when compared to uniaxial this is what is called as your work hardening hypothesis ok.

Anyway these are two small sections which we want to discuss here, but then let us go to an important section just to complete this module and let us do some problems ok. This is actually called as effective strain and effective stress. So, effective stress is generally referred as $\bar{\sigma}$ and effective strain is generally referred as $\bar{\varepsilon}$ this is the way we write this ok. And this $\bar{\sigma}$ effective stress is also called as equivalent stress effective strain is also called as equivalent strain different names for that ok. And for any yield function you choose ok let us say for example, Von Mises or any other equation that may derive later on there is going to be one equation for $\bar{\sigma}$ we can derive another equation for $\bar{\varepsilon}$ .

And as we know that for Tresca and Von Mises these are meant for isotropic materials ok that means $R = 1$. So, we do not have any $R$ values in this equation ok then we can derive $\bar{\sigma}$ for Von Mises material and $\bar{\varepsilon}$ for Von Mises material they will have different equations ok. And the $\bar{\sigma}$ and $\bar{\varepsilon}$ can also be related by any strain hardening law ok. But actually if you want to derive $\bar{\sigma}$ ok let us say for Von Mises equation ok Von Mises material sorry ok we have already derived this equation in this way in one way which is we are going to report it ok in that way that is all ok. So, $\bar{\sigma}$ is have been already derived for Von Mises equation in the in the previous section and we are just going to name that equation for $\bar{\sigma}$ that is all we are going to do, but then we derive $\bar{\varepsilon}$ separately for Von Mises materials.

So, we know that when yielding occurs during plastic deformation during deformation we can write ok by Von Mises function this function is nothing but $\left(\sqrt{1 - \alpha + \alpha^2}\right)\sigma_1$ ok. So, this is for plane stress case correct this is for plane stress case I hope you understand this ok $\sigma_3$ is not there in this equation ok plane stress ok means it is a function of $\sigma_1$ and $\sigma_2$, $\sigma_1$ , $\sigma_2$ $\sigma_3$ is not there. So, naturally $\alpha$ comes into picture. So, we have this equation $\left(\sqrt{1 - \alpha + \alpha^2}\right)\sigma_1$ ok. This function itself is actually called as effective stress or equivalent stress $\bar{\sigma}$ and if the material is going to yield then this equation will be equal to flow stress then equation will be equal to flow stress ok.

So, that is all. So, $\bar{\sigma}$ you can see we have already derived it only thing is we are just naming it here as this equation as effective stress equation for Von Mises material and the $\bar{\sigma}$ when it becomes $\sigma_f$ we say that the material is going to yield ok. So, the same equation this equation we have seen in the previous slide we have seen in the previous slide the same equation here this equation ok we are just going to name it as $\bar{\sigma}$ that is all ok. So, $\bar{\sigma} = \sqrt{\frac{1}{2}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}} = \sigma_f$. We can just directly write this as $\bar{\sigma}$ . When the $\bar{\sigma} = \sigma_f$ we say material is going to yield ok.

In plane stress this can be written ok. Same equation we are just naming it as $\bar{\sigma} = \sqrt{{\sigma_1}^2 - \sigma_1\sigma_2 + {\sigma_2}^2} = \left(\sqrt{1 - \alpha + \alpha^2}\right)\sigma_1$ where $\alpha = \frac{\sigma_2}{\sigma_1}$ nothing but your stress ratio. If a material is at yield this function will have magnitude of flow uniaxial flow stress that is $\sigma_f$ or yield strength ok. So, which is known to us ok. So, there is no separate derivation for $\bar{\sigma}$ .

$\bar{\sigma}$ is already derived for Von Mises material and we are just naming it here. If somebody asks us what is the $\bar{\sigma}$ for effective stress for Von Mises or derived from Von Mises equation for Von Mises materials means then you can tell any one of this. In general state of stress but in principle coordinate system this is the thing. In plane stress this is the thing. We can directly write this two ok.

$\bar{\sigma}$ is equal to this. That means if you know $\alpha$ and $\sigma_1$ or if you know $\sigma_1$, $\sigma_2$ or $\sigma_3$ then you can just simply substitute here and you will get effective stress. When this effective stress if the magnitude is equal to the flow stress of the material or yield strength of the material you can say material is going to yield ok. So now let us come for effective strain ok equation or equivalent strain equation. You can also derive effective strain increment and of course you have to integrate it to get the absolute value.

For this we need to derive it ok. So, there is a small derivation here. So, for this what we are going to write the first equation as this equivalent of you know principle of equivalent work done ok. We are saying that the plastic work done in 1-D would be equal to plastic work done in a general state and this equation can be written. $\bar{\sigma}d\bar{\varepsilon}$ is nothing but plastic work done in effective terms that will be equal to this you already we have written work done during deformation is given as $\sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3$ .

In indical notation we have not discussed this, this is for you to understand this can be written in this way ok and this can also be written as $\sigma . d\varepsilon$ ok. So now what we are going to do is we are going to let us say pick up these two ok. We are going to pick up let us say the first and next part let us say ok your $\bar{\sigma}d\bar{\varepsilon}$ and this one we can simply say $d\bar{\varepsilon}$ ok. Why we need $d\bar{\varepsilon}$ on left hand side because that is the equation we have to get we have to actually get equation for $d\bar{\varepsilon}$. So, that you can get an equation for $\bar{\varepsilon}$ right like we have $\bar{\sigma}$ equation you need to get an equation for this only that is why what we are saying $d\bar{\varepsilon} = \frac{\sigma_{ij}}{\bar{\sigma}} d\varepsilon_{ij}$ .

$\frac{\sigma_{ij}}{\bar{\sigma}} d\varepsilon_{ij}$ will give me $d\bar{\varepsilon}$ and this $d\varepsilon_{ij}$ is nothing but plastic strain increment is given by your normal decondition which you have discussed in the previous section. So, $\frac{\sigma_{ij} \frac{\partial f}{\partial \sigma_{ij}} d\lambda}{\bar{\sigma}}$ and this is what $f$ is what I informed you as a yield function ok and $\sigma_{ij}$ is nothing but the individual element in the tensor and $d\lambda$ is an arbitrary constant ok. This equation can also be written as $d\bar{\varepsilon} = \frac{1}{\bar{\sigma}}[\sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3]$ equivalent work done and in this I can get this equation we will come to this little later in the next slide probably we will use this equation now. So,

$\frac{1}{\bar{\sigma}}[\sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3]$ and I want to put strain increments to get strain increments here ok. So, that I can rewrite $\sigma_1, \sigma_2, \sigma_3$ in terms of $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$.

So, that the entire equation becomes as a function of strain increments only ok. So, whenever I want to get relationship between principal stresses and strain increment I need to go for normality condition ok or I can get it from this part also normality equation also you can get but then here we are going to use your Levy-Mises equation, Levy-Mises flow rule both can be used ok. So, now my root is going to be this I need to get equation for $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ and rewrite that in terms of $\sigma_1, \sigma_2, \sigma_3$ ok I can substitute here and then ok everything will be in terms of strain increments. So, what I am going to do I am going to write $d\varepsilon_1 = (2\sigma_1 - \sigma_2 - \sigma_3)\frac{d\lambda}{3}$ right. So, this equation I think we already derived with respect to your Levy-Mises equation where the ratio of strain increment to deviatoric stresses is going to be a constant right.

So, my $d\varepsilon_1 = \sigma' \mathrm{d}\lambda$ right and then I can write if $d\varepsilon_1$ means $\sigma_1'$ , $\sigma_1'$ is nothing but $(2\sigma_1 - \sigma_2 - \sigma_3)\frac{d\lambda}{3}$ right. Similarly, $d\varepsilon_2 = (2\sigma_2 - \sigma_1 - \sigma_3)\frac{d\lambda}{3}$, $d\varepsilon_3 = (2\sigma_3 - \sigma_1 - \sigma_2)\frac{d\lambda}{3}$ right. So, this we can obtained from your Levy-Mises equation also ok which relates. So, $d\varepsilon_1 = \sigma_1' \mathrm{d}\lambda$ we can write $\sigma_1'$ yes and this can be changed depending on whether you want to find 1 2 or 3 you have $\sigma_2'$ and $\sigma_3'$ which can be written here ok. So, now what I am going to do is I am going to little bit adjust this 2 so $d\varepsilon_2 - d\varepsilon_3$ ok so 2 minus 3 ok then your 2 2 will 1 1 will go then you will have $(\sigma_2 - \sigma_3)\mathrm{d}\lambda$ and then $d\varepsilon_2 - d\varepsilon_3 = (\sigma_1 - \sigma_3)\mathrm{d}\lambda$ like small adjustments I am going to do here to get something which will be useful for me to rewrite it later on.

So, now what I am going to do in the next step is $d\bar{\varepsilon} = \frac{1}{\bar{\sigma}}[\sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2]$ in the previous equation I have written $\sigma_3 d\varepsilon_3$ now $d\varepsilon_3$ I am going to rewrite this in terms of 1 and 2 in terms of 1 and 2. So, for that I am going to write it in this way ok this will become $-d\varepsilon_3$ will be equal to $-(d\varepsilon_1 + d\varepsilon_2)$. So, I am going to get minus here these 2 is going to be added within the brackets right. So, this equation can be rewritten in this form this all will remain same now you have $d\varepsilon_1, d\varepsilon_1, d\varepsilon_2, d\varepsilon_2$ I can take $d\varepsilon_1, d\varepsilon_2$ outside and I can write like this only for this reason in the previous slide I have got $\sigma_1 - \sigma_3, \sigma_2 - \sigma_3$ ok which I got it in this way $\sigma_2 - \sigma_3$ is already there $\sigma_1 - \sigma_3$ is already there. So, I am going to substitute for these 2 ok.

So, that everything will come in the form of increment strain increment. So, what I am going to do $d\bar{\varepsilon}$ is equal to $\frac{1}{\bar{\sigma}}$ in place of $\sigma_1 - \sigma_3$ I am going to write $\frac{d\varepsilon_1 - d\varepsilon_3}{\mathrm{d}\lambda}$ and in place of this I am going to write this $\mathrm{d}\lambda$ will be common in the denominator ok. So, $d\bar{\varepsilon} = \frac{1}{\bar{\sigma}\mathrm{d}\lambda}$ this entire thing this entire thing can be written in this way you can just do a small calculation you should be able to get this $d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2$ ok. So, this is the equation actually I need ok this is a equation I need ok because $\bar{\sigma}$ can be found out dλ as an arbitrary value a constant and all this strain increments can be obtained from new and original dimensions but there is one problem that this dλ is actually undefined now I want to remove it when I am going to remove it I can also remove $\bar{\sigma}$ you know in one way so that this entire equation will be a function of strains only or strain increments only. So, what I am going to do is I am going to replace $\bar{\sigma}$dλ with $d\varepsilon$ term so for that what I am going to do is I am going to go back to my previous derivation that is $d\bar{\varepsilon} = \frac{1}{\bar{\sigma}}$ × ok. So, $\sigma_1 \times d\varepsilon_1$ is not it $\sigma_1 \times d\varepsilon_1$. So, in place of $d\varepsilon_1$ I am going to substitute here $d\varepsilon_2$ I am going to substitute here $d\varepsilon_3$ I am going to substitute here ok. So, that I am getting $d\bar{\varepsilon} = \frac{1}{\bar{\sigma}}\Big[\sigma_1(2\sigma_1 - \sigma_2 - \sigma_3)\frac{d\lambda}{3} + \sigma_2(2\sigma_2 - \sigma_1 - \sigma_3)\frac{d\lambda}{3} + \sigma_3(2\sigma_3 - \sigma_1 - \sigma_2)\frac{d\lambda}{3}\Big]$ then this one will this term will come then my this term will come ok. So, now what will happen $\frac{d\lambda}{3}$ is common so and then I am going to take it here and then what I can do $\bar{\sigma}$ and 3 is in denominator it will come in numerator here. So, my $\frac{3\bar{\sigma}d\bar{\varepsilon}}{d\lambda}$ can be written.

So, dλ when it comes to left hand side it will be denominator. So, I can write $\frac{3\bar{\sigma}d\bar{\varepsilon}}{d\lambda}$ ok. What is remaining? Remaining is going to be this ok this ok and my this is not it. So, which can if you rewrite it you will get it in a very useful form $(\sigma_1 - \sigma_2)^2$+$(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$ which is nothing but our $2\bar{\sigma}^2$ which is what we have seen in the previous section right this one. So, $2\bar{\sigma}^2 = (\sigma_1 - \sigma_2)^2$+$(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$ right.

So, which is what I am going to get here which is nothing but $2\bar{\sigma}^2$ . So, now what I am going to do I want to remove $\bar{\sigma}d\bar{\varepsilon}$ right. So, or in terms I want to remove dλ. So, dλ can be related to $\bar{\sigma}$ in this way. So, I can write $\mathrm{d}\lambda = \frac{3d\bar{\varepsilon}}{2\,\bar{\sigma}}$ .

$\bar{\sigma}$, $\sigma_1$ , $\bar{\sigma}$ will go away then $\frac{3d\bar{\varepsilon}}{2\bar{\sigma}}$ would be my dλ. So, I am going to substitute this into this equation ok so that we can do little bit more steps and then finally, we arrive at $d\bar{\varepsilon}$ equation. So, I have written here von Mises s effective sorry this is von Mises effective stress equation von Mises effective stress equation ok. So, now what I am going to do put dλ and dλ in $d\bar{\varepsilon}$ equation $d\bar{\varepsilon} = \frac{2}{3}\bar{\sigma} \times \bar{\sigma}d\bar{\varepsilon}$ into this entire equation I am going to write as it is. So, in case of dλ I am going to write $\frac{3d\bar{\varepsilon}}{2\bar{\sigma}}$

ok which will give me very nice this particular equation and you will see that $\bar{\sigma}$ $\bar{\sigma}$ will be cancelled $d\bar{\varepsilon}$ will go in the left hand side it will become square.

So, square root will come $\sqrt{\frac{2}{3}\left[d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2\right]}$ this is my useful equation for effective strain increment incremental effective strain equation for von Mises material ok. So, I can get $\bar{\varepsilon}$ from this directly I can get $\bar{\varepsilon}$ from this directly which is what is I have written here ok. So, you will see that it is nothing but $\sqrt{\frac{2}{3}\left[d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2\right]}$ ok it is integrated appropriately and I know $\beta$ now what is $\beta$ ? I can write it in terms of the same equation in terms of $\beta = \frac{\varepsilon_2}{\varepsilon_1}$ and though this is a plane stress process what we are discussing here 3 also exist inside these 2 when you speak about 1 and 2 there is a 3 inside ok there is 3 inside 1 and 2 ok because of volume constancy ok. So, now what I am going to do I am going to rewrite this entire equation in terms of $\beta$ but $\varepsilon_1$ will come into this equation outside the square root ok. So, what you need to do is basically you need to replace your $\varepsilon_1, \varepsilon_2, \varepsilon_3$ in terms of $\beta$.

So, you can substitute $\beta$ here and see whether you get the previous equation $\frac{\varepsilon_2}{\varepsilon_1}$ you can just check this so that whether you get this equation or not you can find out ok. So, what we got is basically this is one equation this is another equation which will be useful for us and again if you see that $d\bar{\varepsilon} = \sqrt{\frac{2}{3}\left[d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2\right]}$ if you want to rewrite this in terms of $\beta$ it is $\sqrt{\frac{4}{3}[1 + \beta + \beta^2]}\,\varepsilon_1$, $\varepsilon_1$ should come outside the square root you can just derive it and check ok. And you will see that all these values 1, 2, 1, 3 can be obtained from the original and new dimensions of the you know during deformation and also if you know what is $\beta$ ok how is it related you can get $\bar{\varepsilon}$ ok. The same equation can be written in general coordinate system like this, this is just for you to know ok. So, $d\bar{\varepsilon} = \left[\frac{2}{3}\left(d\varepsilon_{11}{}^2 + d\varepsilon_{22}{}^2 + d\varepsilon_{33}{}^2 + 2d\varepsilon_{12}{}^2 + 2d\varepsilon_{13}{}^2 + 2d\varepsilon_{23}{}^2\right)\right]^{\frac{1}{2}}$ that will come anyway.

So, we are generally we see in principle coordinate system these two forms these two forms will be more useful for us to solve problems ok. So, now given von Mises your yield function given von Mises yield function we derived a $\bar{\sigma}$ equation and $\bar{\varepsilon}$ equation similarly if you want to go for any other yield function you have such equations ok. So, now some small note points ok so what will happen let us see ok for tensile test ok we know in uniaxial tensile test we know that 2 and 3 would be 0 and you have $\sigma_1 \neq 0$. So, if you see if you substitute that in this equation in your $\bar{\sigma}$ equation ok where is that equation let us say for example here ok or this equation you can directly substitute it ok 2 will go off 3 will go off then this fellow will go this fellow will go you know 2 $\sigma$ 2 2 will be cancel $\bar{\sigma} = \sigma_1$ ok. For uniaxial tensile test if you put the condition as $\sigma_1$ exist 2 and 3 does not exist then this will go this part will go this small part will go then you will have ${\sigma_1}^2$ here also ${\sigma_1}^2$ 2 times 2 2 will be cancelled so $\bar{\sigma} = \sigma_1$ ok.

So, this is one important result so effective stress is nothing but your uniaxial principle you know first principle stress $\sigma_1$ when you go for uniaxial tensile test. So, $\bar{\sigma} = \sigma_1$ this can be asked as a proof proof that $\bar{\sigma} = \sigma_1$ uniaxial tensile test means you should be able to derive this wherein you can say the material is considered isotropic and follow one message equation then you put this conditions in $\bar{\sigma}$ equation you should be able to get this. The same thing if you want to get $d\bar{\varepsilon}$ or $\bar{\varepsilon}$ equation that also can be done. So, in a tensile test you have all three strains $\varepsilon_1$ will be there 2 and 3 will also be there but they are not equal they are not equal how they are related they are related by this and we also said that $\beta = -\frac{1}{2}$ for uniaxial tensile test is not it $\beta = -\frac{1}{2}$ I think that we derived in the previous section. So, you can see that $\frac{d\varepsilon_2}{d\varepsilon_1} = -\frac{1}{2}$ so $d\varepsilon_1 = -2d\varepsilon_2$ if $d\varepsilon_1 = -2d\varepsilon_2$ then this will be equal to $-2d\varepsilon_3$ which you can get from your volume constancy equation.

If this is the case then you can substitute this in the $d\bar{\varepsilon}$ equation in this fashion suppose you go to this equation in place of ${d\varepsilon_1}^2$ you substitute $d\varepsilon_2$ you can substitute $d\varepsilon_3$ you can substitute you can you know all these values so you will get this $d\bar{\varepsilon} = \left[\frac{2}{3}\left(d\varepsilon_1^2 + \frac{d\varepsilon_1^2}{4} + \frac{d\varepsilon_1^2}{4}\right)\right]^{\frac{1}{2}}$ , $d\varepsilon_1^2$ will remain same in place of $d\varepsilon_2^2$ I am going to put this $\frac{d\varepsilon_1^2}{4}$ in place of $d\varepsilon_3^2$ I am going to put this. So, what will I get $\frac{d\varepsilon_1^2}{4}$ square root so this is 1 , $\frac{1}{4}$ this is $\frac{1}{2}$ ; $\frac{1}{2} + 1 = \frac{3}{2}$ so $\frac{2}{3} \times \frac{3}{2} d\varepsilon_1^2$ ; $\frac{3}{2}, \frac{2}{3}$ will be cancelled so square root will get cancelled out so $d\bar{\varepsilon} = d\varepsilon_1$. So, I can write this equation $\sigma = K\varepsilon^n \approx \bar{\sigma} = K\bar{\varepsilon}^n$ by using these two important results. $\sigma = K\varepsilon^n$ is a standard equation that we know that we can describe in describe the strain handling behavior of any metallic material power law using power law type that can be written in the effective quantities the effective quantities in effective terms as $\bar{\sigma} = K\bar{\varepsilon}^n$ by considering uniaxial deformation so by these two important results. And for other yield functions there will be a different equation for $\bar{\varepsilon}$ but again there also you can relate it to $\varepsilon_1, \varepsilon_2, \varepsilon_3$ your $\bar{\sigma}$ can be related to $\sigma_1, \sigma_2, \sigma_3$ it could be any yield function then you can find $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and substitute here and by knowing $K$ and $n$

you can get $\bar{\sigma}$ by knowing $\bar{\sigma}$ and by knowing $\alpha$ you can get $\sigma_1, \sigma_2, \sigma_3$ .

If it is a plain stress it becomes more easy for you because by knowing $\bar{\sigma}$ you have some equation relating $\bar{\sigma}$ to $\sigma_1, \sigma_2$ that is what your real function is going to do. So then by knowing $\bar{\sigma}$ by knowing $\alpha$ you can get $\sigma_1, \sigma_2$. Say for example if you want to get a $\bar{\varepsilon}$ for von Mises equation so $\bar{\varepsilon}$ for von Mises equation you can use this by knowing $\beta$ you can substitute here get $\bar{\varepsilon}$ you substitute in this equation $K$ and $n$ you know how to find out for uniaxial tensile test you can get $\bar{\sigma}$. If you know $\bar{\sigma}$ there is one $\sigma_1, \sigma_2$ which causes the $\bar{\sigma}$ that can also be found out by knowing $\alpha$ by knowing $\alpha$ because we already related a $\bar{\sigma}$ to $\sigma_1$ and $\alpha$ is not it we already related these two.

So from this we can get $\bar{\sigma}$. So that is how the entire thing is going to get related. So one last information last slide which you will see before solving this problem. So now when we say effective stress what do you mean by effective stress and how are they related to yield locus is what is given in this diagram. So whenever you draw graph between $\sigma_1, \sigma_2$ it means that we are going to draw yield locus in the subject. So the black one is the initial yield locus you can imagine and then this blue and red ones are basically updated yield locus because of strain hardening they are going to evolve in this way.

Only thing is we are saying the shape is same size is increasing simple this is the simplest way to model shape is same it is ellipse and size is increasing because of strain hardening. So flow stress keeps on increasing in this way. And this one measures yield function effective stress you know this. So $2\bar{\sigma}^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$ this is what we derived for effective stress for von Mises equation. And this $2\bar{\sigma}^2$ in general you can say as $K$ or you can leave it also this $2\bar{\sigma}^2$ signifies something called size of the yield locus and the right side of this equation tells something called the shape of the yield locus.

So this $\bar{\sigma}$ or $2\bar{\sigma}^2$ in general you just call it as $K$ or any other alphabet you want to put it means that it is going to define size of the yield locus. Size of the yield locus is defined by the magnitude of $\bar{\sigma}$ magnitude of $\bar{\sigma}$ that is why I have written here as the black one is connected to $\overline{\sigma_1}$ this blue one is connected to $\overline{\sigma_2}$ red one is connected to $\overline{\sigma_3}$ and another yield locus $\overline{\sigma_4}$ $\overline{\sigma_5}$ which means that $\overline{\sigma_1}\,\overline{\sigma_2}\,\overline{\sigma_3}$ are connected to each yield locus. And this $\overline{\sigma_1}\,\overline{\sigma_2}\,\overline{\sigma_3}$ as denoted by von Mises effective stress equation is going to be a function of $\sigma_1, \sigma_2, \sigma_3$ and the $\sigma_3 = 0$ in this diagram it is a function of $\sigma_1, \sigma_2$ or it is a function of $\alpha$ and $\sigma_1$ and further deformation $\sigma_1, \sigma_2$ may increase and you will get $\overline{\sigma_2}$ and then your $\overline{\sigma_3}$ that is the way it is going to work. And the right hand side of the equation which is going to tell the form of the equation will decide the shape of the yield locus. If this form changes then the shape of the yield locus will also change.

Suppose in place of 2 I am putting let us say *m*, *m* is a variable depending on the material *m* is going to be a variable let us say then I have chance of changing the shape of the yield locus when compared to the Von Mises yield locus. So, right now let us say this is 2 it is not a problem. So, in that case we are going to have one particular shape of yield locus. So, this equation has got that meaning. So, what are we going to say a single number $\bar{\sigma}$ one value in conjunction with form of the yield surface or yield locus form of the yield surface or yield locus means this right hand side provides all such combinations of one state.

That means if you take $\overline{\sigma_1}$ that is obtained because of one particular value of $\sigma_1, \sigma_2, \sigma_3$ and that is going to define one particular state that is a black color one. It may be initial yielding or it can be this blue one further you further deformation or red one further deformation like that is one information. Second one whereas all values for $\bar{\sigma}$ all values are $\bar{\sigma}$, $\overline{\sigma_1}\ \overline{\sigma_2}\ \overline{\sigma_3}$ represent set of allowable yield functions at various state. In the first stage this is allowed, in the next stage this is allowed, in the next stage 3 is allowed. So, one diagram one yield locus diagram with three curves for example black, blue and red will tell you this two sets of information.

First set is connected to one state $\overline{\sigma_1}$ that is given by $\sigma_1, \sigma_2, \sigma_3$ and if you have a combination of $\overline{\sigma_1}\ \overline{\sigma_2}\ \overline{\sigma_3}$ it is going to tell you different stages of deformation causing that particular you know deformation. So, size of the yield locus and shape of the yield locus is going to provide you this many information to you. A single number $\bar{\sigma}$ in conjunction with form of the yield surface or yield locus provide all such combinations in one state whereas all values of $\bar{\sigma}$ , $\sigma_1, \sigma_2, \sigma_3$ represent set of allowable yield functions at various state. So, one state and various state how do you define is by this equation that is the meaning of this. So, now what we will do is we will solve a couple of problems so that we understand what is you know application of whatever we discussed in this particular section.

So, let us pick up this particular problem I think this we already solved. So, problem 1, 2, 3 few problems we already solved in the previous chapter this all we solved. Let us go to this problem sheet deformation process. Correct let us go to sheet deformation process. So, what is the first question? The question is a square element 8 by 8 mm in an undeformed sheet of 0.8 mm thickness becomes a rectangle 6.5 into 9.4 mm after deformation. And the material stress strain law is given by this particular equation. $\bar{\sigma}$ is equal to see we started writing in effective format.

$\bar{\sigma} = 600(0.008 + \bar{\varepsilon})^{0.22}$(MPa). So, a square element we say 8 by 8 mm so 8 by 8 mm is going to become a rectangular element of this particular dimension. So, whenever you say rectangular element larger value will go to length is not it 9.4 and this will be given by 6.5. So, basically you are pulling it on one side say for example so it gets compressed on the other side.

So, 8 becomes 6.5, 8 becomes 9.4. Sheet thickness is already provided initial sheet thickness is provided to you. So, now the question is and the stresses normal to the sheet is 0. So, as usual we say $\sigma_3 = 0$ which is easy for us to understand now what type of equations to use. So, you have to find several parameters in this. You need to find membrane stresses which means principal stresses basically $\sigma_1, \sigma_2,$ and $\sigma_3 = 0$ .

So, you have to get a $\sigma_1, \sigma_2$ final thickness that is t, $t_0$ is given this is your $t_0$. t has to be found a principal strains we know $\varepsilon_1, \varepsilon_2, \varepsilon_3$ again I am cautioning here $\sigma_3 = 0$ does not mean that $\varepsilon_3 = 0$ unless otherwise something else is mentioned you have to be careful all 3 will exist. So, you should also get stress and strain ratios that means $\alpha$ and $\beta$ has to be found out and hydrostatic stress let us say $\sigma_h$, of course you know when you have $\sigma_h$ it will automatically come to deviatoric stresses that is maybe I will say $\sigma_1', \sigma_2', \sigma_3'$ and plastic work done. So, work done per unit volume or work done you need to get these are the things you have to get as output. So, now how are we going to proceed? So, whenever dimensions are given in the undeformed sheet and the first job is to find strains that is the way it has to go.

So, either if dimensions are not given strains are given then directly you can use strain otherwise if dimensions original new dimensions are given the main you know source point is to get the strains first. If you go by any other route you will not be able to solve this problem. So, now what I am going to do is I am slowly scrolling down and I am going to get $\varepsilon_1, \varepsilon_2, \varepsilon_3$. I am going to get $\varepsilon_1, \varepsilon_2, \varepsilon_3$. So, what is $\varepsilon_1$? As per original dimension $\varepsilon_1$ is 1 see 1 is along this 1 is this 2 is this 3 is perpendicular to the sheet.

So, $\varepsilon_1 = ln\left(\frac{9.4mm}{8mm}\right)$ because it is true strain $ln\left(\frac{9.4\text{mm}}{8\text{mm}}\right)$ you can check it will be 0.161 . $\varepsilon_2 = ln\left(\frac{6.5\text{mm}}{8\text{mm}}\right) = -0.208$ or 0.21 you can keep. Two strains are found out $\varepsilon_1, \varepsilon_2$; $\varepsilon_1$ is with respect to your 9.4 and $\varepsilon_2$ is with respect to 6.5 that you have to be little bit careful. Then $\varepsilon_3$ can be found out from $-(\varepsilon_1 + \varepsilon_2)$ and from this you can get the thickness new thickness.

From this you can get new thickness that means now $\varepsilon_1$ is found out $\varepsilon_2$ is found out $\varepsilon_3$ can be found out from this equation. So, 3 equations can be obtained if the $\varepsilon_3$ is known then $\varepsilon_3 = ln\frac{t}{t_0}$. So, that will give you t, $t_0$ is 0.8 so that will give you t that is one route to follow. The other route what you can follow is you can use this equation $lwt = l_0 w_0 t_0$ that will be equal to the same equation.

So, initial volume is this the initial volume is this of that element is not it. So, $8 \times 8 \times t$ of that is how much thickness of that is 0.8 is not it. So, $8 \times 8 \times 0.8 = 6.5 \times 9.4 \times t$ into new thickness *t* from this *t* you can you know from this equation you can get $t = 0.838\, mm$ .

If you use this then also you will get near about 0.838 mm or 0.84 mm either way you can use. You can find out it is not a problem. So, if you do not want to follow this then if you are following this from this you can get find $\varepsilon_t$ , $\varepsilon_t = ln\frac{0.838}{0.8}$ that will give you thickness strain or $\varepsilon_3$ or $\varepsilon_3$ either way you can find out. So, one important observation before we go ahead you will see the sheet thickness which is initially 0.8 has become 0.838. Initially sheet thickness is 0.8 let us say this is 0.8 mm that has become 0.838 mm which is actually little bit increased. But generally what we understand from whatever we discussed is generally thickness has to reduce because material deform when you pull it in one direction we expect width and thickness to decrease but here you should note down this thickness is slightly increased. We will see that later on but I am just giving an observation. So, now $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are obtained and thickness is also obtained. If $\varepsilon_1, \varepsilon_2$ are known you can get $\beta$ from this equation $\frac{\varepsilon_2}{\varepsilon_1}$ which is nothing but $-\frac{0.280}{0.161} = -1.29$. If $\beta$ is known to you $\beta$ is negative value here. So, $\beta$ is known to you can get $\alpha$ by $\frac{2\beta+1}{2+\beta}$ and I think you understand how when did we relate this $\alpha$ and $\beta$ we related with Levy-Mises flow rule. The same equation which you used previously to solve that effective strain equation to derive effective strain equation in that the end result one of the end result of Levy-Mises equation is to relate $\alpha$ and $\beta$. I am just reminding you though so you get $\alpha = \frac{2\beta+1}{2+\beta}$ this equation is going to be used predominantly in this course.

So, $\beta$ is known to you $-1.29$ you substitute here you will get $\alpha$ as 2.225 or you can say $-2.23$. So $\varepsilon_1, \varepsilon_2, \varepsilon_3$, new and new thickness $\alpha$ $\beta$ all are obtained now. So, what is the route? Route we followed is from original and new dimensions you get $\varepsilon_1, \varepsilon_2$ from that you can get $\varepsilon_3$ .

$\varepsilon_3$ is known you get $t$ sorry $\alpha$ is not found out $\alpha$ is not found out you got $t$. So, then from $t$ or from the initial original dimensions you can get $t$ and then you can get $\varepsilon_3$ either way is fine. So, by knowing $\varepsilon_1, \varepsilon_2$ you got $\beta$ and from $\beta$ actually you got $\alpha$ this is the best route to get any other route you will stop in between you will not be able to solve. So, anyway so now once your strains are found out the best choice for you is find out $\beta$ and from $\beta$ you got $\alpha$ now you know like what you can do is basically you need to get your membrane stresses because $\alpha$ is known if $\alpha$ is known you can get $\sigma_1, \sigma_2$, and $\sigma_3$ is anyway not there it is becoming 0 here. But there is one issue if you want to get $\sigma_1, \sigma_2$ at least you know you should have only any one of these values you should know only then you can use α to get another one or what you need to know is if you know α you should get $\bar{\sigma}$ you should get $\bar{\sigma}$. So, by knowing $\alpha$ and $\bar{\sigma}$ you can get $\sigma_1$ from von-Mises equation then you can get $\sigma_2$ because $\alpha$ is known.

The $\bar{\sigma}$ is also not known to us only thing is $\bar{\sigma}$ and equation is provided that is nothing but $\bar{\sigma} = 600(0.008 + \bar{\varepsilon})^{0.22}$. So, what we are going to do is this equation can be used to find out $\bar{\sigma}$ and from $\bar{\sigma}$ you can get $\sigma_1$ is known by knowing $\alpha$ you can get $\sigma_2$ that is the route we are going to follow and if you want to get $\bar{\varepsilon}$ here we are going to use $\varepsilon_1, \varepsilon_2, \varepsilon_3$ which we already know. And before we go for calculation I just want to focus on the form of this equation this equation we have not studied until now we have studied $600^{0.22}$ that much we know $\bar{\sigma} = K\bar{\varepsilon}^n$ but here we are going to add one more part to it 0.008 it is a small value which is called as a pre strain that is called as pre strain. What is the significance of that we will see in the next section but the flow stress model effective stress equation relating it to effective strain the form is different now. Anyway, so now what we are going to do is we are going to find effective strain this equation is known to you we derived just now $\sqrt{\frac{4}{3}[1+\beta+\beta^2]}\,\varepsilon_1$, $\varepsilon_1$ should be outside the square root then you substitute all the values $\beta$ is known to you, $\beta$ is known to you, $\varepsilon_1$ is known to you, you just now found out as 0.161 if you substitute here you should get 0.128. If $\bar{\varepsilon}$ is known then from the effective stress effective strain equation that power law which already given you you can substitute it here in place as $\bar{\varepsilon}$ you should get 0.218 substitute here you should get $\bar{\sigma}$ as 432.57 mega Pascal or 432.6 mega Pascal you can keep. So, $\bar{\sigma}$ in this now can be obtained by using this relationship for that $\bar{\varepsilon}$ has to be known and $\bar{\varepsilon}$ depends on $\varepsilon_1, \varepsilon_2, \varepsilon_3$ which actually depends on the new and original dimensions that is all.

So, if a $\bar{\sigma}$ is known so I can write this von-Mises effective stress equation which is nothing but $\sigma_1 = \frac{\bar{\sigma}}{\sqrt{(1-\alpha+\alpha^2)}}$ that I already know I think we have used this before also is not it. So, in that power point side I think we have used it. So, you can use this equation so $\bar{\sigma}$ is known to you already this is the value. So, now what we are going to do is we can substitute this $\bar{\sigma}$ equation we can substitute this $\bar{\sigma}$ equation and we are going to use we are going to use this particular you know $\bar{\sigma}$ value is known to me divided by $\sqrt{(1-\alpha+\alpha^2)}$ is found out you can substitute it you can get the 151.3 the $\sigma_1$ is known $\alpha$ is known I can get $\sigma_2$ and $\sigma_1$ as minus 333.6 mega Pascal. So, $\alpha$ is known here $\sigma_1$ is also known to me I can find out. So, now there are two more steps to it so which we are going to derive now so how are we going to derive it we will see now. So, now your $\alpha$ and $\sigma_1$ or $\alpha$ $\sigma_1$ is known then $\sigma_2$ can be found out that is also done now we are going to the next one that is your hydrostatic part. So, your hydrostatic part so your hydrostatic stress is nothing, but $\sigma_h = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$ .

So, $\sigma_1$ is known to you $\sigma_2$ is known to you $\sigma_3$ is 0. So, which you have found out previously only $\sigma_1$ is 151.3 2 is minus 336.6 $\sigma_3$ is 0 divided by 3 should give you this one has to be very very careful here many times we do mistake that if $\sigma_3 = 0$ then $\frac{\sigma_1+\sigma_2}{2}$ is not correct the definition holds good $\sigma$ even if $\sigma_3 = 0$ $\frac{\sigma_1+\sigma_2+\sigma_3}{3}$ you can substitute the values appropriately and then you will get a -61.7 MPa if $\sigma_h$ is known then we can get deviatoric stresses $\sigma_1{}' = \sigma_1 - \sigma_h$, $\sigma_1$ is this minus of minus this you will get this $\sigma_2{}' = \sigma_2 - \sigma_h$ you should get this and $\sigma_3{}' = \sigma_3 - \sigma_h$ would be this ok and here also you will see that you can check the flow rule. So, the Levy-Mises flow rule is nothing, but your $\frac{\varepsilon_1}{\sigma_1'} = \frac{\varepsilon_2}{\sigma_2'} = \frac{\varepsilon_3}{\sigma_3'} = d\lambda$ that is what we studied know. So, you can calculate all these ratios and you can check all these things are almost same $\varepsilon_1$ is known $\sigma_1'$ is known here $\varepsilon_2$ is known $\sigma_2'$ is known here $\varepsilon_3$ is known $\sigma_3'$ is known here if you calculate it the ratio is going to remain same.

So, let me make a flow rule is checked. Plastic work done there is a last equation. So, again we know that ok so work done per unit volume is nothing, but I am going to write $\int \bar{\sigma}\, d\bar{\varepsilon}$ ok integral of that will give me my work done per unit volume is not it. So, now the limit is decided by this fellow $\bar{\varepsilon}$. So, actual equation is this equation is already given in the question for me correct $\bar{\sigma} = 600(0.008+\bar{\varepsilon})^{0.22}$ is already given I am just substituting it here and the limit is decided by this differential $\bar{\varepsilon}$ $\bar{\varepsilon}$ the limit is maximum limit is 0.218 which I calculated it here. So, as per the question when you convert a square element to rectangular element maximum effective strain that I am going to get is 0.218 up to that I have deformed. So, which is becoming my upper limit here 0.218 I am going to substitute here.

So, you can integrate it and finally you will be able to find out work done per unit volume is 78.77 $10^6\ J/m^3$ ok and volume can be found out instantaneous volume can be found out instantaneous volume is nothing but the previous one what is instantaneous volume $6.5 \times 9.4 \times 0.838\ mm^3$ right. So, if you multiply that ok with this with this ok. So, then you will get a work done which is nothing but 4.03

J ok. So, if this form of the equation is going to be different the natural different work done assuming that let us say 0.008 is not given $600\varepsilon^{0.22}$ then there will be some slight change in the work done ok. So, this is the route this route has to be carefully maintained ok this route has to be carefully maintained. So, you get all this from $\alpha$ you get this from $\alpha$ you want to get $\sigma_1$ $\sigma_2$ then you should know $\bar{\sigma}$ if $\bar{\sigma}$ is to be obtained then you should get $\bar{\varepsilon}$ the $\bar{\varepsilon}$ depends on these fellows $\varepsilon_1, \varepsilon_2, \varepsilon_3$ ok. Then once you get a $\bar{\sigma}$ you will get $\sigma_1$ using $\alpha$ if $\alpha$ is known you can get $\sigma_2$ if $\sigma_1$ $\sigma_2$ $\sigma_3$ all are known to me from this I can get $\sigma_h$ the $\sigma_h$ is known to me I can get $\sigma_1{}'\sigma_2{}'\sigma_3{}'$ dash ok and work done is separate work done per unit volume is separate which is given by $\int \bar{\sigma}\, d\bar{\varepsilon}$ ok. So, this is an important problem which is going to relate several things which you have studied ok and it is very practical in nature.

So, this is one element in the sheet you can get the situation now ok. So, the question is a square element in an undeformed sheet ok an element let us say you take a sheet of let us say you know 50 mm by 50 mm ok in that ok or you can take 100 mm by 100 mm or a circular sheet of let us say 200 mm diameter in that there is a small element which is going this type of deformation and there are many such 100 different elements let us say rectangles each one will deform differently to make a component in all that location these all are going to change you can imagine like that ok. Let us solve one more problem ok in this which also you know is going to use all these equations that we derived. So, question is very simple compare two plane stress deformation process. So, it is set so you said plane stress deformation process $\sigma_3$ is 0 as we studied for a sample of high strength low alloy steel there is a steel like this ok of 1 mm sheet thickness ok. There is a steel sheet of 1 mm thickness which is undergoing plane stress deformation, but there are two cases a case is biaxial tension in which a strain ratio of 1 is maintained and in another one which is called as shear or drawing strain path where the strain ratio is minus 1 that is maintained ok.

One is a balanced biaxial tension where your $\beta$ that is strain ratio is maintained as 1 other one is your shear part where in $\beta$ is equal to $-1$ is maintained ok. I think now you can go back to that strain diagram which you have drawn in the previous section and see where this $\beta$ is going to fit ok where this $\beta$ is going to get fit ok. So, I do not have that diagram here I think you can go back and check it ok. Anyway so now the square element whose sides are aligned with the principal directions is initially 10 by 10 ok. So, again you take a square element which are aligned along 1 direction and 2 direction in this way and both are 10 by 10 ok and one side is extended to 12 mm ok and one side is extended to 12 mm ok.

And the material properties are described by this equation sorry this is $\bar{\sigma}$, $\bar{\sigma}$ is equal to $\sigma = 850\bar{\varepsilon}^{0.16}$ the same simple power law MPa ok. So, now what you need to do is you need to compare these two process ok. Same sheet that is the steel sheet of 1.2 mm is deformed in two different ways. One is biaxial tension ok where strain ratio is 1 the other one is shear or drawing where in you say $\beta = -1$ ok. You have to compare these two by finding out effective strains, effective stresses, principal strains and principal stresses and finally, sheer thickness.

This all has to be formed out ok. So, now standard route which you followed before dimensions are given ok. So, naturally you have to find these three first ok $\varepsilon_1, \varepsilon_2, \varepsilon_3$ . So, how am I going to find out? So, $\varepsilon_1 = \ln(12/10)$ is not it.

So, $12/10$ will give me my one direction strain that is $\varepsilon_1$ ok. So, which is going to be same 0.182 and 0.182 this is $\beta$=1 . This is a biaxial testing and this is a shear ok.

So, now if I want to find $\varepsilon_2$ I know $\beta\varepsilon_1$ . So, $\beta$ is going to be 1 here $-1$ here. So, this becomes 0.182 this becomes minus 0.182 right. So, if I know $\varepsilon_1, \varepsilon_2$ I can get $\varepsilon_3 = -(1+\beta)\varepsilon_1$ correct or I can get $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ either way I can find out.

So, I know $\beta$ here I know $\varepsilon_1$. So, I know $\varepsilon_3$ as $-0.365$ and it has 0. This point should be noted here ok. There is previous problem where thickness is increased in this problem if you see in shear $\beta = -1$ if you deform it thickness will remain same that is why your thickness strain is going to be 0. Thickness strain is going to be 0 means what thickness remains same $\varepsilon_3 = ln\left(\frac{t}{t_0}\right)$ ok and that is same means 0 means t and $t_0$ are going to be same ok.

So, that means thickness is not reducing. So, there is no thickness strain here that should be noted here. Here thickness strain is minus ok. So, that means thickness has gone down. Let us say now these 3 strains are found out now you got your you want to get $\bar{\varepsilon}$ ok. This $\bar{\varepsilon}$ derivation we already done today square root of $\sqrt{\frac{4}{3}[1+\beta+\beta^2]}\,\varepsilon_1$. So, I have written this in terms of $\beta$ and $\varepsilon_1$. So, you can also write the previous you know stage of this equation square root of $d\bar{\varepsilon} = \sqrt{\frac{2}{3}\left[d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2\right]}$ that also can be utilized.

So, everything is known to me $\beta$ I found out $\beta$ is given here 1 and $-1$ and then $\varepsilon_1$

is found out for both this and this. So, that will give me my $\bar{\varepsilon}$ which is 0.365 and 0.211. So, $\bar{\varepsilon}$ is known next to root is to find $\bar{\sigma}$, $\bar{\sigma}$ equation is provided otherwise you cannot get $\bar{\sigma}$.

$\bar{\sigma}$ equation should be given ok then just substitute $\bar{\varepsilon}$ here, $\bar{\varepsilon}$ here you will get $\bar{\sigma}$ ok. So, you can also understand that you know like because deformation is happening in constant thickness that means thickness strain is 0 you need a lesser effective stress to reach that level. Here you need larger effective stress ok. And one straight forward thing which you can find out $\beta$ if is known $\alpha$ can be found out it is 1 and $-1$ using the Levy-Mises flow rule is equation you can use correct. So, if $\bar{\sigma}$ is known α is known I can find $\sigma_1$ which is nothing but my major principle stress $\sigma_1 = \frac{\bar{\sigma}}{\sqrt{(1-\alpha+\alpha^2)}}$ where $\frac{723}{\sqrt{(1-1+1)}}$.

So, it will be 723 mega Pascal ok this all are in mega Pascal. Effective stress this is in mega Pascal this also in mega Pascal and for this fellow if you put $-1$ here $-1$ here and $\bar{\sigma} = 662$ then you will get minus stress again mega Pascal as 723 and $-382$ right. So, thickness you can find out directly you can find out. So, how do you find thickness $\varepsilon_3$ is known right.

So, you can find thickness ok from that directly this is what I was telling you. So, thickness is going to be 1.2 which is same as that of your initial thickness. So, you deform a sheet in pure shear or $\beta$=$-1$ or they call it as drawing strain path. So, you can deform the material ok up to a strain of $\bar{\varepsilon} = 0.211$ ok by keeping thickness as 1.2 itself ok without much change in thickness but here you see there are significant change in sheet thickness ok. So, these are two important problems in this section in this module ok and so we will start a new module in the next class. Thank you.