Mechanics of Sheet Metal Forming
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Week- 02
Lecture- 03
Sheet Deformation Processes

So, now we are going to start module number 2 in the third lecture which I have written as sheet deformation process in general it is called a sheet deformation process. We will see several small-small sections in this which will be useful for our analysis further ok. So, let us go to the basic introduction of this. The first one what we are going to say is in sheet deformation process any process you pick up ok we are going to say that it is a plane stress process ok. So, when you say plane stress problem or plane stress process here I have written ok it means that the stress perpendicular to the surface of the sheet is small ok. You have a sheet of 2 mm thickness let us say so, along the thickness or perpendicular to the sheet surface we are going to say that you know the stress is going to be very small as compared to the stresses in the plane of the sheet ok.

So, we are going to assume this and this assumption we are going to follow it throughout this particular subject unless otherwise it is specified that it is not so ok. So, my assumption here is the normal stress which you are going to call this as normal stress is zero. Normal stress here means along the sheet thickness or perpendicular to the sheet is zero ok and this can be seen as a plane stress problem in general we are saying plane stress problem. So, what do you mean by that? It means that the contact pressure between the sheet and the tool is lower as compared to the yield strength of the material and that is why we are basically assuming this.

The contact pressure so, you have a punch let us say ok or a deep drawing tool something any tool rigid body ok which is actually contacting the sheet and deforming it. That contact pressure between the sheet and the tool is lower as compared to the yield strength of the material itself. So, we can you know assume that it is a plane stress problem and we are going to specify the directions now ok in the next couple of slides and you will understand that the normal stress is zero means actually what ok which one we are going to keep it as zero and that is going to be same throughout this particular subject. So, let us go to the next section I am going to take again an example of uniaxial tension test ok which you already discussed in the previous chapter how to calculate all the properties you know K, n, m, what is the effect of m, plastic strain ratio everything. So, now in this schematic which I have drawn here you will see that this is the gauge region ok when you do uniaxial tensile test there is a sample dimension which you already mentioned.

Now we are going to focus only on the gauge region which is undergoing permanent deformation or plastic deformation ok. So, we are going to consider the initial gauge length as l ok it has got a width w let us say and it has got an initial thickness t ok. So, now we are going to deform it little bit ok and you will see the extended dimensions ok. There is a dl extension which is actually increasing in nature and accordingly there will be dw which means the width is actually reducing you can see the red colour one red colour one is the new dimensions of the sheet the width is reduced and thickness also is slightly reduced ok I have given dt here ok. So, dl is a change in length, dt is a change in thickness and dw is a change in width and this direction which I mentioned here are very important for us.

We are going to consider this 1 along the length of the sample ok along l and 2 which is along the width of the sample that is w ok and 3 is along the thickness of the sheet ok. And thickness of the sheet the what you are specifying is of the order of let us say maximum let us say 2 or 2.5 mm not more than that by now you might have have some idea what thickness we are speaking about from the problems we solved we are speaking of this order ok. So, one is generally along the length of the sample two is along the width and three is along the thickness direction ok. So, during deformation the faces of the element ok for example, the gauge region the uniaxial tensile test sample will remain perpendicular to each other which is called as principal element ok.

So, now what we are going to do is though we are consider this as a gauge region mostly in this course or in the subject what we are going to see is like you can assume that deformation is going to be in the principal element. So, that the calculations are actually little easy for us to understand ok. So, now in this also we are going to say that we are going to have something called as a principal element ok and you will see that if that is the case then faces of that particular element will remain perpendicular to the each other ok which also means that there is no shear part of deformation associated with the element there is no shear part of the deformation associated with the element. This makes the you know most of the calculations that we are going to do derivations you are going to do pretty simple in nature ok. So, in this context I want to suggest that one should look into basic you know strength of materials or solid mechanics course wherein you know one should know about say for example what is stress tensor ok what are its components ok what do you mean by normal stress what do you mean by shear stresses what do you mean by you know principal stresses, principal directions, how to find principal stress, how to find principal directions ok all those things I think you can read it from any strength of materials book basic book or any metal forming book I think you should be able to go through it.

Brief idea will be given here then and there, but otherwise we are entering into an element which is called principal element and there is no shear part of deformation associated with the element. Now having said that I am going to discuss something called as a principal strain increments ok principal strain increments. So, when I am speaking about principal element I am going to term this as a principal strain increments and there are three principal strain increments one along axis 1 that is along length the other 2 is along width and in the thickness direction ok. So, along the length which is defined by 1 so I am going to call $d\varepsilon_1$ that is $\frac{dl}{l}$ that is the original definition which you have seen in the previous section and width in the thickness direction in a similar way we can write $\frac{dw}{w}$ that is along 2 which is in the width direction that is $d\varepsilon_1$ and $d\varepsilon_3$ would be $\frac{dt}{t}$ ok. So, one is you know the strain principal strain increment along the length the other one is principal strain is along the width the other one is principal strain increment along the thickness and this thickness you know why is this strain principal strain increment later on we are going to call this as a thickness strain which plays a vital role in deciding whether the sheet has undergone you know significant deformation or not ok which is going to tell us ok whether is there any localized deformation all those things those details will come later.

So, this three principal strain increments are very important for us ok. So, now we are going to introduce the next two small section this all are connected to each other which is called as a constant volume condition in the previous problems you know calculations we already use this condition, but we are you know defining here you know very officially ok. So, now any metal forming process which involves only plastic deformation then this condition is valid ok. So, what we are going to say is with no change in volume during plastic deformation we are going to say that the differential of volume of the gauge region will remain as 0 which can be written as $d(lwt) = d(l_0 w_0 t_0) = 0$ where this is what we said as you know in the previous calculations we said as you know volume remains same before deformation at any point of deformation ok. So, this equation can be written and this equation can also be modified in this way and a small calculation will do that dividing by lwt ok you will get $\frac{dl}{l} + \frac{dw}{w} + \frac{dt}{t} = 0$ which is what we defined in the previous slide as

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

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So, which means that in any deformation you pick up ok in the previous calculations also some of the problems also if you really look into the strains they are actually additional in nature and if you add it, it should become equal to 0 and this condition is very important condition especially for those process which involves plastic deformation which has got no volume change ok. Thus for a constant volume of deformation the sum of principle strain increment is 0 we are doing this and this will be helpful for us to calculate several things in due course either for isotropic material or for anisotropic material ok. Thus for a constant volume deformation means plastic deformation the sum of principle strain increments $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$. So, here I am going to introduce ok you know something called as stress and strain ratios ok. Right now what we are discussing here I am specifically writing it as for isotropic materials mainly because later on we will briefly discuss about anisotropic sheets also ok.

Until then whatever discussion we are going to make is for isotropic materials only and which means that as per our previous you know discussion it is going to have identical properties are observed in all the directions ok. With respect to sheets we are going to say it in terms of rolling direction and we can say *R* is equal to 1. What is *R* here? R is nothing but your standard plastic strain ratio standard plastic strain ratio which we called as true width strain divided by true thickness strain as per our definition *R* is equal to true width strain divided by true thickness strain right. So, this will be equal to 1 we said for isotropic materials or not equal to 1 which means less than or greater than 1 will lead to anisotropic sheets or anisotropic materials ok. This is the meaning of isotropic sheets and here we are going to decide or define something called as stress ratio and strain ratio ok in the next slide probably.

And this stress ratio and strain ratio are very important for the subject and that will the definition remains same one should remember throughout this particular subject which will be helpful for us. So, what is it we will see but before that because these sheets are isotropic in nature we are saying that by assuming symmetry in deformation we can say that the strain in the width and thickness direction will be equal in magnitude. What does it mean? That means in a way I am saying that if $R = 1$ I can say that true width strain will be equal to true thickness strain ok that is the meaning here ok. So, if I want to write it in terms of in terms of principle strain increments my width direction and thickness direction are nothing but 2 and 3 direction. So, I am going to write $d\varepsilon_2 = d\varepsilon_3$ ok.

And we are going to say that if these 2 are equal it is equal to $-\frac{1}{2}d\varepsilon_1$. How do we get it? We get it from this particular equation that is $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$. So, how do you get it? Let us say for example, $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$ then you will see that you will have let us say $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_2 = 0$. So, $d\varepsilon_1 + 2d\varepsilon_2 = 0$ and this will give me $d\varepsilon_1 = -2d\varepsilon_2$ which is nothing but $-\frac{1}{2}d\varepsilon_1$ which is nothing but your $d\varepsilon_2$ correct. So, $d\varepsilon_2 = -\frac{1}{2}d\varepsilon_1$.

So, I can write $d\varepsilon_2 = d\varepsilon_3 = -\frac{1}{2}d\varepsilon_1$ ok. What does it mean here? What is the meaning of this? For an isotropic material isotropic sheet it could be aluminum or steel sheet whatever if you assume it to be isotropic in nature then you can say that the principle strain increments are related in this fashion ok if you have any axial type of deformation ok. So, you will see that if you know $d\varepsilon_1$ which is nothing but your as per previous definition it is $\frac{dl}{l}$ ok. So, nothing but $\frac{dl}{l}$. So, if you know just $\frac{dl}{l}$ ok you can divide it by $\frac{1}{2}$ and put minus here and these two strains can be calculated without knowing what is actually the new thickness and new width.

So, without that this relationship will be able to identify what changes we have in the principle strain increments with respect to one of the strain increments. Now you will see that now I am going to put this condition like for example, during a tensile test for an isotropic material ok suppose you have tensile test for an isotropic material which is done ok then the strain increments and stresses can be in a combined way you can write it like this. So, how are we going to write like this? Your $d\varepsilon_1 = \frac{dl}{l}$ that is the original definition there is no substitute for this ok so what is the meaning of dl? dl is a change in length divided by the original gauge length. If this is known then you can get $d\varepsilon_2 = -\frac{1}{2}d\varepsilon_1$ correct and is $d\varepsilon_3 = -\frac{1}{2}d\varepsilon_1$. I mean previously we wrote this in terms of width that is fine but now here we are rewriting it in terms of $d\varepsilon_1$.

This also was written in terms of new thickness the definition is there but with this particular relationship we are writing $d\varepsilon_1 = \frac{dl}{l}$ if you know this you can $d\varepsilon_2$ and $d\varepsilon_3$ in this way. This is with respect to principle strain increment. Now let us come to $\sigma_1, \sigma_2, \sigma_3$ what are these? These are actually principle stresses these are actually principle stresses ok one should not get confused here ok what is the meaning of principle stress, normal stress, shear stresses that is why I ask you to go back and refer your basic strength of materials course. So whenever we write $\sigma_1, \sigma_2, \sigma_3$ it means that these are all principle components ok. So if you want to have normal stresses and shear stresses then you need to use 11, 22, $\sigma_{11}, \sigma_{22}, \sigma_{33}$ or $\sigma_{12}, \sigma_{23}, \sigma_{31}$ ok.

So when we mention $\sigma_1, \sigma_2, \sigma_3$ these are all principle stresses ok. So now when you speak about uniaxial tensile test, when you speak about uniaxial tensile test ok straight away we can say that σ_2, and σ_3 are 0 why because it is uniaxial in nature and it will have σ_1 only which is equal to $\frac{P}{A}$ which you have defined well before right. So and this when you say σ_1 exists and σ_2 and σ_3 does not exist it becomes 0 is only up to the uniform plastic deformation ok. So once you know your instability is started ok or UTS is reached then you have to be careful you cannot use this, you cannot use this ok. So $\sigma_1 = \frac{P}{A}$ will be there, $\sigma_2 = 0, \sigma_3 = 0$ when you have uniform plastic deformation until uniform plastic deformation ok.

So this will give some schematic ok if I got to tell this is basic schematic like you have a sheet with sheet thickness *w* thickness *t* and *w* as width and *l* is the you know your length let us say. So now we are going to the next one ok. So now you know meaning of principle strain increments ok we have a principle element and that is undergoing deformation like for example a gauge length in uniaxial tensile test ok and you can calculate the principle strain increments by $d\varepsilon_1, d\varepsilon_2$ and $d\varepsilon_3$ and if it is an isotropic material ok you can relate it in this fashion ok $d\varepsilon_2 = d\varepsilon_3 = -\frac{1}{2}d\varepsilon_1$ and with uniaxial tensile test you can summarize ok the your $\sigma_1, \sigma_2, \sigma_3$ and $d\varepsilon_1, d\varepsilon_2$ and $d\varepsilon_3$ in this fashion for an isotropic material ok. So now these are all incremental principle strains ok principle strain increments right. So now I want to integrate it to get $\varepsilon_1, \varepsilon_2$ and $\varepsilon_3$ok when I am doing it I will get true strains $\varepsilon_1, \varepsilon_2$ and ε_3 right.

So we already defined this in the previous one more from materials point of view from tensile test point of view ok. Here we are going to say that if you want to calculate true strains ok there are certain conditions that is applicable or you have to be little bit careful whenever you pose these conditions ok what are this. The first one is the principle strain increments increase monotonically in the same direction ok so it means suppose if you pick up $d\varepsilon_1$ ok it increases positively and does not reverse ok it increases positively and it does not reverse it is called as increasing monotonically continuously it has to increase ok that is one condition. The next one is the ratio of principle strain increments remains constant ok. So what is it exactly we will see in the next slide ok but right now we can say that the ratio of principle strain increments let us say $\frac{d\varepsilon_2}{d\varepsilon_1}$ or $\frac{\varepsilon_2}{\varepsilon_1}$ it is remaining constant and if you follow that then it is called as proportional loading it is called as proportional loading.

So this is monotonic it has to be proportional loading that means I have written when so this is what is important from the onset of yielding to the peak load or start of diffuse necking that means in the strain hardening region. So you remember that σ versus ε you have and you have a graph like this right. So you are picking up yield strength here let us say σ_{ys} and let us pick up a point here it is called UTS. So in this strain hardening region ok we are saying that from the onset of yielding to peak load or start of diffuse necking peak load means near to UTS ok diffuse necking is going to start because instability is started and that is why we said that after UTS you are going to have reduction in stress correct flow stress right. So in this case you are going to say that this proportional loading is valid only in this region after we know your diffuse necking is started necking started we do not know what is going to happen let us keep it open for you know right now so that is the meaning ok.

So now here this principle directions are fixed that is the third one principle directions are fixed what does it mean that means the material element which you are actually analyzing does not rotate with respect to principle directions ok that means what your direction one is always along the length of the gauge region in the sample that is why I am saying this direction is very important for us in the previous diagram we made direction is not it yeah. So this 1 is always along the length this 2 is along the width and 3 is along thickness ok. So there is no rotation in the material element so principle directions are fixed. So when you satisfy all these three conditions monotonic increase proportional process and this one the third condition ok in the gauge region we can integrate the strains ok and we can get $\varepsilon_1, \varepsilon_2$ and ε_3 in this fashion. So only when this condition apply the integrated strains can be calculated as this way in a very simple way and these are nothing but your $\varepsilon_1, \varepsilon_2$ and ε_3 and $\varepsilon_1 = ln\frac{l}{l_0}$ which you have already used for some calculations but right now we are defining it ok.

And $\varepsilon_2 = ln\frac{w}{w_0}$ right because this is where we are defining R know $R = \frac{\varepsilon_w}{\varepsilon_t}$ we said and this is where we calculated $ln\frac{w}{w_0}$, $ln\frac{t}{t_0}$ right same definitions are here but right now we are putting this condition in this way. So $\varepsilon_2 = ln\frac{w}{w_0} = -\frac{1}{2}\varepsilon_1$ as per the previous derivation and $\varepsilon_3 = ln\frac{t}{t_0} = -\frac{1}{2}\varepsilon_1$ which we have discussed in the previous slide ok. So under these conditions you can integrate it and you can find out this absolute strain values which we are going to call it as calculating true strains ok. So you have to be careful otherwise ok. So if there is no monotonic increase if there is no proportional loading one has to see probably piecewise type you can get this strains.

Now we have seen the uniaxial tensile test ok I am just writing UAT uniaxial tensile test as an example and we introduced some basic things right. So but the actual sheet deformation process are general in nature ok or general in nature ok. So I am going to pick up something called a general sheet process but though it is general it is not uniaxial we know but it can be considered as plane stress as per our first assumption made in this particular you know module right. So we are going to say that it is a plane stress. Now here since we have already you know discussed what is $\sigma_1, \sigma_2, \sigma_3$, 1, 2, 3 directions which direction does it belong to ok then we can directly write this particular one which will make us to be little bit clear about what is plane stress in this particular deformation.

So uniaxial case you know what is it I just compare it here $\sigma_1 = \frac{P}{A}, \sigma_2 = 0, \sigma_3 = 0$ again until uniform plastic deformation ok when instability started is not going to work. Then now you go for general case in that case suppose you pick up plane stress ok plane stress means we are specifying mainly sheets that is why we are saying it as plane stress ok if it is a plate suppose 5 mm thick plate 6 mm thick plate means then you should not generally use plane stress one has to really look into it that is a different situation but then in general case if it is sheet we can say you are going to have σ_1, you are going to have σ_2 but σ_3 can be 0 why because it is plane stress ok. So we are simplifying the problem ok any sheet forming problem as this way ok you are going to have σ_1, you are going to have σ_2 but

$\sigma_3$can be 0 ok and σ_3 again you have to go back to your the first diagram which I have shown here. So σ_3 means the thickness direction, σ_3 is in thickness direction right. So one is along length and this fellow is along w right.

So you will see that now when you say plane stress means it is $\sigma_3 = 0$. So this particular one is very very important for us and we are going to use it throughout this particular course ok as long as it is the form of a sheet ok. So now the story is all about σ_1 and σ_2 but you have $\varepsilon_1, \varepsilon_2$ and ε_3 ok. Now all the relationship that we are going to have is between $\varepsilon_1, \varepsilon_2$ and $\varepsilon_3$and $\sigma_1, \sigma_2, \sigma_3$ will not come into picture because it is a 0. Out of this $\varepsilon_1, \varepsilon_2$ and ε_3, 1 will become 0 later on ok in certain process that we will see later ok.

So now if we assume the conditions of proportional and monotonic deformation applicable to the tensile test then we can develop a simple theory of plastic deformation of sheet that is reasonably accurate ok. So with this you know brief I am going to define now what do you mean by your stress ratio and strain ratio which will be useful for us. Whatever I have given in the previous slide right this particular one in the previous slide right $\varepsilon_1, \varepsilon_2$ and ε_3 for uniaxial plane stress. General case but plane stress $\sigma_1, \sigma_2,$ and σ_3 this can be represented schematically like this ok. Tensile test, so I have not drawn a rectangular strip or something like that I am just drawing in general only here you can see that ok this is a sheet ok.

So this is one direction so if it is tensile test I am going to have σ_1 and , σ_2 is going to be 0, and σ_3 is going to be 0 it is over but all 3 strains exist ε_1 will be there, ε_2 will be there, ε_3 will be there. ε_1 you have to get it from the original dimensions and new dimension new length. If that is known then ε_2 can be calculated in this way, ε_3 can be calculated in this way. How you got this we already derived it in the previous to previous slide.

So this is with respect to tensile test. The same diagram if you want utilize for plane stress there are some changes you have to look into it. What is that? σ_1 will remain, ε_1 also will remain right there is no question about it. Now σ_2 will remain, ε_2 will also be there what is it we will see now. σ_3 can be 0 why because it is plane stress and ε_3 will be existing ok. So $\sigma_1 = \frac{P}{A}$, $\sigma_3 = 0$ let us forget it ok.

Now you have σ_2. So how do you get σ_2? σ_2 can be related to σ_1 if I know σ_1 as $\sigma_2 = \alpha\sigma_1$ where this α is I am going to call it as stress ratio ok. So I am going to call α as stress ratio. So this definition is very important for us throughout this course ok. And ε_2 how do I get it for this if I know ε_1 is through β ok. So I am going to call this β as a strain ratio, β as a strain ratio ok.

So what is α? $\frac{\sigma_2}{\sigma_1}$ what is β? $\frac{\varepsilon_2}{\varepsilon_1}$ that is all. So what is stress ratio? It is $\alpha = \frac{\sigma_2}{\sigma_1}$ ok. What is β? That is $\beta = \frac{\varepsilon_2}{\varepsilon_1}$. 1, 2 directions are already decided in the previous to previous slide fine this is done now.

Here $\sigma_3 = 0$ fine. How do you get ε_3? So ε_3 can be obtained from these 2 ok. ε_3 can be obtained from these 2. $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ ok because now principle incremental strains can be integrated let us say ok. So now you will get $\varepsilon_1 + \beta\varepsilon_1 + \varepsilon_3 = 0$ ok. So now $\varepsilon_3 = -\varepsilon_1 - \beta\varepsilon_1$. So minus you will take it out will it come? So it will be $-(\varepsilon_1 + \beta\varepsilon_1)$. So $\varepsilon_3 = -(1+\beta)\varepsilon_1$. So see the differences σ_1 , ε_1 will remain same σ_1 , ε_1 will be there which has to be obtained from original you know relationship σ_2 will become 0 because it is uniaxial. Here σ_2 will be there and $\sigma_2 = \alpha\sigma_1$. Here $\varepsilon_3 = -(1+\beta)\varepsilon_1$. which you already derived in the previous one here we do not know ok.

So we are putting $\beta\varepsilon_1$ and thus that creates a definition of this $\sigma_3 = 0$ here also $\sigma_3 = 0$ ok because it is plane stress $\varepsilon_3 = -\frac{1}{2}\varepsilon_1$ here also we do not know what is that ok but from this relationship we are getting $-(1+\beta)\varepsilon_1$. So if you know β you will put substitute here and you can get ε_3 ok. So this is what is summarized ok here. So in general that is plane stress ok. So I can write ε_1 , $\varepsilon_2 = -\beta\varepsilon_1$ and $\varepsilon_3 = -(1+\beta)\varepsilon_1$ and of course σ_1 will be there $\sigma_2 = \alpha\sigma_1$ and $\sigma_3 = 0$ fine.

So now if you see that I can directly compare this fellow and this fellow and say that for uniaxial tension test $\beta = -\frac{1}{2}$ correct $\varepsilon_2 = -\frac{1}{2}\varepsilon_1$, $\varepsilon_2 = -\beta\varepsilon_1$ so $\beta = -\frac{1}{2}$ for uniaxial tensile test directly I am writing ok. And here also if I put $\beta = -\frac{1}{2}$ it is $\frac{1}{2} - \frac{\varepsilon_1}{2}$ that I will get it here not a problem and $\alpha = 0$ why because as per definition of $\alpha = \frac{\sigma_2}{\sigma_1}$ so σ_2 here it is 0 so 0 by σ_1 is 0 so α is $\alpha = 0$. So this is the first important one ok. So what is α and β for uniaxial tensile test means $\alpha = 0$, $\beta = -\frac{1}{2}$. So if you deform a material ok in uniaxial tensile test like this what we have discussed until now for this $\alpha = 0$ and if you get $\beta = -\frac{1}{2}$ ok.

Again we are keeping an assumption that the material is isotropic in nature ok. This is for uniaxial now like this there are 4, 5 other modes of deformation we will see in due course in other chapters wherein we are going to properly define what is β and α for each one how do you relate that ok that we will see in due course but right now one answer is given that is $\alpha = 0$, $\beta = -\frac{1}{2}$ for uniaxial tensile test. Now so I am going to the next one that is called yielding in plane stress. So with this the brief you know you know summary of what we can discuss about in general about plastic deformation and how to get the difference between uniaxial tensile test and in comparison that with plane stress or general deformation assuming plane stress how do get you know principle you know strains $\varepsilon_1, \varepsilon_2$ and ε_3 and what is the relationship and then what is going to be $\sigma_1, \sigma_2,$ and σ_3 in uniaxial and in plane stress what is the case we have seen. So now we are going to introduce something called as yielding in plane stress ok yielding in plane stress basically here in this chapter in this part we are going to see small small sections which are going to finally lead us to something called as yield functions or yield theories ok.

So what is the context now so I have given here some terms called yield surface and yield locus one after another we will know what is the meaning of that but the context is this ok so you have uniaxial tensile test for example and a stress strain behavior is drawn here. I just drawn a simple you know schematic so that is why you know it is clear the elastic part is so clearly seen and then you have a plastic deformation part ok and then UTS happens then you have the necking is going to start here let us say and then it is going to fail. So typical process and here we are mentioning something called as a σ_{ys} ok. So how to find this yield strength is known to us how to find this yield strength is known to us right. So you you can use a proof stress method which we discussed before and you can get this yield strength.

So once you get yield strength it means that ok for this material if it crosses yield strength then it may reach a plastic deformation which will start from here onwards and before that the material is going to be in the elasticity part ok. So in that way generally we decide that is why yield strength is going to be very very important. The first time you know when it crosses the yield strength that is very very important for us and after that of course you can update that you can update that then it becomes a general flow stress but when it is this transition is going to happen is important for us. Now the same concept in uniaxial tensile test if you ask in general deformation say for example if it is in plane stress general shear deformation but in plane stress ok where σ_1, σ_2 exist σ_3 is not there ok then how do you quantify this yielding that is a question ok. So for this just you know very theoretically we are explaining that suppose you have a square sheet ok let us say this is a square sheet you have and you are deforming it in both the directions you are pulling it in this direction you have σ_1, as well as σ_2 ok.

In this case you have only σ_1 right in this case you have σ_1, and σ_2 which is nothing but a plane stress process σ_3 does not exist let us say ok. So now depending on the proportion 2 and 1 you may have yielding that is going to happen at one particular stage of deformation correct. So what I am going to do is very theoretically I am saying loading and unloading a square sheet of material is done in two directions in any proportion that is what the schematic tells you here this particular one ok this one direction two direction. So I am going to load unload it I will see whether there is permanent deformation or not I am going to load and unload it there is going to permanent deformation or not I am going to check it so that I know where is this elastic to plastic transition or onset of yielding that will be there ok. So for that I will do some this kind of exercise say for example ok let us take I will pick up σ_2 I will increase it to σ_1 let us say some value ok and generate let us say σ_1 I am going to change.

So I am going to take σ_2 to σ_2' and I am going to you know calculate σ_1 such that I am going to have elastic to plastic transition elastic to plastic transition ok. So like this for example I am just checking at point number 1 no it is elastic 2 elastic 3 yes 4 5 but between 5 and 6 at one particular point I will see that the elastic to plastic transition happens. So I am just drawing this side is e and when it comes on this side it is P that means from here onwards I am going to start having plastic deformation. So I can pick up this as my one yield point or yield strength ok. The same one I am going to now repeat for several other values let us say I am going to take then σ_2' now I am going to convert that to σ_2'' and I am going to increase σ_1 such that I am going to find one elastic to plastic transition that will also happen let us say between 5 and 6 or 6 and 7 I will get one more point ok that also is schematically drawn like this if it is between σ_1 and σ_2 I am picking up the σ_2 to this particular value and I am increasing σ_1 and I am increasing σ_1 .

So I am going to get one particular point ok where between 5 and 6 I am going to have elastic to plastic transition like this. Similarly I am going to σ_2 I am going to take it to σ_2' let us say and I am going to change σ_1 I will get some other transition ok. So like this for different proportions of σ_1 and let us say σ_2 ok practically we may get that is a question but then theoretically one can get let us say at different data points and I am going to draw all the data points between σ_1 and σ_2 in this fashion ok. So I am going to draw it is like this suppose I get one transition here, here, here data point all are basically this elastic to plastic transition I do not know what to say these all are elastic to plastic data points randomly I am picking data points like this ok randomly I am picking also data points for that means what so I am going to deform the material with this proportion let us say $\frac{\sigma_2}{\sigma_1}$ is decided I am going to pick up this particular proportional path and I am going to get one elastic to plastic transition that is this point. Similarly this point, similarly this point, similarly this point, similarly this point like this ok.

So I am going to several other proportions I am going to get many such data points which I marked as into here ok randomly ok I am just drawing then I am going to use some sort of a smooth curve I am going to draw and it looks like this red colour one ok and this is what we are going to call it as a yield locus, this is what we are going to call it as yield locus. So I have just written here initial yield locus bracket indicates onset of plastic deformation ok. What does it mean? That means suppose if you pick up this particular stress path ok so if I pick up this particular data point I am in elastic deformation but when it reaches this particular locus it means plastic deformation is started why because I am going to reach the yield locus, I am going to use the yield locus ok if I follow this particular α, if I particular this α, α is what $\alpha = \frac{\sigma_2}{\sigma_1}$. But if I follow this α this also another α it will give another $\alpha = \frac{\sigma_2}{\sigma_1}$ may reach it here, if I follow this I may reach here ok. So it means that as long as σ_1, σ_2 are within this yield locus I am not going to have yielding in the material it will be in elastic part, moment I reach the locus it will be in plastic deformation.

So I am going to join all these data points in the form of locus and we are going to call it as yield locus. And this is called initial yield locus because that is a initial yielding that is going to happen ok. But we are going to deform it further for example I am going to this point is reach I am going to get this point, this point, this point, this point, this point up to UTS let us say for example which is equivalent to saying that this red color curve locus has become this blue color curve or black color one ok. So from here I am going to this particular one which means that this point is going to become this point, let us say this point and this point is going to become this point. Similarly several points are there which can give me the next yield locus and I am going to call this as a subsequent yield locus ok.

So will it expand like this ok or will it change its shape ok all those things which we will discuss to some extent in later chapters but otherwise let us assume that the shape of the yield locus remains same like this ellipse and it is going to increase its size due to strain hardening ok. So flow stress here is less as compared to this, similarly here as compared to this. So this is what is called as this term called as yield locus ok. If you see that in the form of three dimension it will become a surface which I will give you example later on ok. So now if you want to you know define you know or derive some equations for this then they are called as yield theories or yield functions.

It is called as yield theories ok or we call it as yield functions fine. So there are some introductory point here ok. So yielding in plane stress actually depends on current hardness or strength of the sheet ok and the stress ratio ok. So in this context again a stress strain graph would be useful for you. Suppose this is a stress strain graph you are having ok let us say like this ok and let us say this is your σ_{ys} any value here we can call it as σ_f let us say and let us say for example this is a particular strain ok this is point and let us say this point is also you can call this at particular one particular strain you are having this σ_f and this stress strain behavior you are getting because of one particular α, this α if it is 0 then it is uniaxial.

If it is some other value then you may have some other mode of deformation. Let us go ahead in this ok. So yielding in plane stress depends on the current strength correct ok because you need to know the current strength of the material when it is undergoing deformation only then you can say whether it is less than yield strength or above yield strength ok. So the current strength or flow stress σ_f is used to define the strength that is why I call it as a σ_f. So σ_f the flow stress is a stress at which material will yield in simple tension that is $\alpha = 0$. So when you keep $\alpha = 0$ ok whenever it is first yielding you can call it as σ_f ok.

And after that you are going to have increased flow stress ok. The yield strength is a minimum required stress after that if you are further deforming it you need larger flow stress to reach that particular strain. So now this is what I was telling you σ_f depends on the amount of deformation to which the element has been subjected and it will change during the process. So naturally so the amount of deformation means strain. So here you have one strain let us say it is reached after that you have another strain where there is different σ_f another strain another deformation. So continuously σ_f is going to change depending on the amount of deformation that is in that process.

Let us say consider only one instant let us say σ_f ok knowing α knowing α ok you need to be able to find out what is σ_1, σ_2 to reach that particular σ_f that is the question now. By knowing α ok and by knowing σ_f can we find out σ_1, σ_2 which causes that particular deformation ok. So or whether that σ_f is actually your reached you know you got yield strength or not ok or plastic flow will continue for a small deformation or not can be found out. So now there are some important points that we are going to discuss in small this one but then the basics of this I think you should understand more from material science point of view we will not go into details but these are all some basic things required for you know to understand the nature of plastic deformation. So if you want to define yield theories ok so then that the knowledge on nature of plastic deformation of metal is going to be important what is that these are all already you might have studied this but I am just summarizing here.

So most of the metals that we are speaking about are polycrystalline nature ok and here if you see the plastic flow occurs by slip on crystal lattice planes ok when the shear stresses reaches a critical value right. So, in a conventional you know polycrystalline materials that we are generally discussing like steel aluminum plastic flow or plastic deformation occurs by slip ok on lattice planes I think you understand the meaning of lattice planes ok when the shear stresses reaches a particular value as long as it is below the critical value plastic flow will not happen once it crosses plastic flow will happen ok. So now when you speak about slip more mechanisms on this you can look into material science book but then the slip which is associated with dislocations in the lattice is insensitive to normal stress on the slip planes ok. So, again here normal stress I want you to refresh it more from solid mechanics point of view you say normal stress and then shear stress ok so and this normal stresses are related to principal stresses ok when shear stresses are not there these are all certain things you should know. So now what we are going to say this this yielding is influenced by shear stresses on the element and is not likely to be influenced by average stress or pressure ok.

Suppose you are going to quantify something called an average stress or pressure the average of let us say principal normal stresses let us say principal stresses then ok we are going to say that yielding is influenced only by the shear stresses and not likely to be influenced by that average stress or pressure ok. So, it is not going to be influenced but what is this average stress we will see now. So, in a way we are going to say that if you want yielding in polycrystalline materials then shear stresses are the main one which are responsible for that ok. So, now if you want to speak about shear stresses then there is something called as maximum shear stress ok. Again we are going to discuss about principal element only ok and this is a principal element let us say we generally element means we take it in the form of a cube ok and we say that $\sigma_1, \sigma_2, \sigma_3$ are actually acting perpendicular to each phase and they are nothing but principal stresses.

Again 1, 2, 3 means principal stresses ok I have written it as principal element here ok. So, now on the phases of principal element there are no shear stresses that is why I have written $\sigma_1, \sigma_2, \sigma_3$ ok which are actually acting normally on the phases and then there are no shear part in that. But if you keep on looking at it there are some inclined phases at any angle where you will have both normal and shear stresses ok. But in the inclined phases also if you keep on searching ok there will be you know one particular phase ok on which you know locally the shear stresses will reach maximum ok and they are represented in this figure 1, 2 and 3 ok. So, you will have this plane where τ_1 is going to act you are going to have this plane τ_2 and τ_3 here and they are called as actually maximum shear stress planes and are shown in the figure.

So, these planes will have or called as maximum shear stress planes. So, you keep on searching and you will pick up this inclined phases on which the shear stresses are maximum and these planes are generally 45 degree to the principal directions ok. And maximum shear stresses can be found how τ_1 is generally written as it depends on which stresses it is going to cut this plane actually cuts 1 and 2. So, you can write $\tau_1 = \frac{\sigma_1 - \sigma_2}{2}$ and then τ_2 actually cuts 3 and 2. So, you can write $\tau_2 = \frac{\sigma_2 - \sigma_3}{2}$ and τ_3 actually cuts 1 and 3. So, you can write $\tau_3 = \frac{\sigma_3 - \sigma_1}{2}$ ok. So, out of this you know shear stresses there are τ_1, τ_2, τ_3 which are actually maximum ok shear stresses and 3 such values are found out in this way. So, now in a very summarized way we can say that the yielding would actually depend on the shear stresses in an element and the current value of flow stress ok. And we can say that *f* there is a function which depends on τ_1, τ_2, τ_3 and the flow stress. So, this entire statement can be summarized one form $f(\tau_1, \tau_2, \tau_3) = \sigma_f$.

So, now what I am going to do is I am going to this part ok. So, this part I am going to define something called as hydrostatic stress or mean stress ok. So, hydrostatic stress is represented as σ_h, $\sigma_h = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$, $\sigma_1, \sigma_2, \sigma_3$ are nothing but principle stresses, it is average of principle stresses ok. It is an average of principle stresses. What does that mean? It means 3 equal components acting in all the directions on the element ok, 3 equal components that is a σ_h acting in all the directions on the element ok. And it is nothing but σ_h is generally represented as $-p$ which is compressive in nature in general ok.

Now, the point is this is like an average quantity of $\sigma_1, \sigma_2, \sigma_3$ and we are saying that this will not contribute to deformation in a material that deforms at constant volume. For example, plastic deformation process ok. This σ_h which is like an average quantity of $\sigma_1, \sigma_2, \sigma_3$ principle stress, it will not play a any role ok or contribution to deformation in plastic deformation processes. Why it is so? You can explain it later probably in the next class ok.

So, I will try to explain later in the chapter. Because of this now what is going to happen is we can remove that part from the principle stresses and we are going to get one more part and this is called as a deviatoric stresses. So, we are going to say that the $\sigma_1, \sigma_2, \sigma_3$ can be divided into two components, one is only σ_h which is not going to play any role in plastic deformation plus there is another part $\sigma_1', \sigma_2', \sigma_3'$ which are actually called as a deviatoric part. So, the standard stress tensor which is defined by this has got two parts, one is hydrostatic part and the deviatoric part. So, which will lead to my next definition called as deviatoric stresses ok. So, what are deviatoric stresses? These are actually the stress component after removing or reducing σ_h from the principle stresses, they are called as deviatoric stresses that is what I have written it here.

So, what I have drawn here is this minus this ok, your $\sigma_3 - \sigma_h$, $\sigma_1 - \sigma_h$, $\sigma_2 - \sigma_h$ will give you $\sigma_1', \sigma_2', \sigma_3'$ ok. So, and these are actually called as a deviatoric part or deviatoric stresses $\sigma_1', \sigma_2', \sigma_3'$. $\sigma_1, \sigma_2, \sigma_3$ principle stresses, σ_h is nothing but hydrostatic stress, $\sigma_1', \sigma_2', \sigma_3'$ are deviatoric stresses ok. And deviatoric stresses can be obtained if you know principle stress and hydrostatic stress. Principle stresses you know how to find out from a given stress tensor ok that I want you to refresh from solid mechanics book again I am saying given stress tensor how to find principle stresses one should know ok that is a simple method to do that ok.

And once you know, you know $\sigma_1, \sigma_2, \sigma_3$ ok and if you know that you can get this and if you remove the σ_h part from each one $\sigma_1, \sigma_2, \sigma_3$ you will get $\sigma_1', \sigma_2', \sigma_3'$ these are called as deviatoric stress. Deviatoric stress is the difference between principle stress and the hydrostatic stress. So, we can write σ_1' each component $\sigma_1' = \sigma_1 - \sigma_h$, $\sigma_2' = \sigma_2 - \sigma_h$, $\sigma_3' = \sigma_3 - \sigma_h$. Now what I am going to do is I am know σ_h in terms of principle stresses just to simplify it I will get $\sigma_1' = \sigma_1 - \left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)$. So, you can take 3 here $\frac{3\sigma_1-(\sigma_1+\sigma_2+\sigma_3)}{3}$ that will become $\frac{2\sigma_1-\sigma_2-\sigma_3}{3}$.

Similarly σ_2' can be written as σ_2' is nothing but here I have written $\sigma_2' = \sigma_2 - \left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)$. So, if you calculate it you should get $\frac{2\sigma_2-\sigma_3-\sigma_1}{3}$ similarly σ_3' can be found out. So, this can be easily remember so σ_1' means start with 1, $2\sigma_1 - \sigma_2 - \sigma_3$ here σ_2' means start with 2, $2\sigma_2 - \sigma_3 - \sigma_1$, σ_3' means $2\sigma_3 - \sigma_1 - \sigma_2$. Fine so this will also help you to solve the problems given a stress tensor find principle stresses then you know how to get σ_h or you can directly use this formula to get $\sigma_1', \sigma_2', \sigma_3'$ otherwise you can use this formula to get $\sigma_1', \sigma_2', \sigma_3'$ which will give you the principle stresses, deviatoric stresses and the hydrostatic part. Stains you should get it from the original dimensions or the relationship between you know tensile test or any other form of deformation.

So, now I am going to further simplify this equations for plane stress process which is what we want in this course. For a plane stress process in terms of α I am going to what is $\alpha = \frac{\sigma_2}{\sigma_1}$ is known now for plane stress process I will put $\sigma_3 = 0$. So, what will happen now $\sigma_3 = 0$ means in σ_1' this fellow will go and σ_2' this fellow will go and σ_3' this entire fellow will go. So now $\sigma_1' = \frac{2\sigma_1-\sigma_2}{3}$, $\sigma_2' = \frac{2\sigma_2-\sigma_1}{3}$, $\sigma_3' = \frac{-\sigma_1-\sigma_2}{3}$ and now everything is in terms of σ_1, σ_2 and I can use this definition and I can write $\sigma_1' = \frac{2-\alpha}{3}\sigma_1$, $\sigma_2' = \frac{2\alpha-1}{3}\sigma_1$, $\sigma_3' = \frac{1+\alpha}{3}\sigma_1$.

I hope you are getting this point. So, for plane stress process σ_3 I put the 0 which means this will go, this part will go and I can rewrite remaining in terms of α. So, in fact you can also put $\frac{\sigma_2}{\sigma_1}$ here and check whether you get a σ_1'. $\frac{\sigma_2}{\sigma_1}$ and σ_1 goes above so you will get $\frac{2\sigma_1-\sigma_2}{3}$ here also you put $\frac{\sigma_2}{\sigma_1}$, $2\sigma_1$ and σ_1 will go, $\frac{2\sigma_2-\sigma_1}{3}$ and here also you put $\frac{\sigma_2}{\sigma_1}$ you will get this equation. So, what does it mean? That means how is the connection you see? Suppose if I know ε_1 I can get ε_2 of course I can also get ε_3 all 3 can be obtained from original dimensions but if I know ε_1 all 3 we can get from original and new dimensions but ε_1 is known let us say for uniaxial type of deformation then we can relate ε_2 to ε_1 and ε_3 to ε_1. So in that way I can get $\varepsilon_1, \varepsilon_2, \varepsilon_3$ so all can be obtained if I know $\varepsilon_1, \varepsilon_2$ of course from this I can get β .

Let us assume that if there is any relationship between β and α I can get α also we do not know what is the relationship but we can derive then I can get α also. If I know α and I can get if I know σ_1 let us say σ_1 from original definition if I know α I can get σ_2, σ_1 is known let us say and σ_3 is anyway 0 fine. So σ_1 is known, σ_2 is known from α I can get σ_2 if I know σ_1. So this is now understood so if I know σ_1 and, σ_2 or if I know α then I can get $\sigma_1{}', \sigma_2{}', \sigma_3{}'$ because α is known σ_1 is known, α is known, σ_1 is known, α is known, σ_1 is known.

So I can get $\sigma_1{}', \sigma_2{}', \sigma_3{}'$. So because I know $\sigma_1, \sigma_2, \sigma_3$ here, so I can get σ_h also from this. So if I know then σ_h then I can get $\sigma_1{}', \sigma_2{}', \sigma_3{}'$ this is another way to get right. So this is basically the connection ok when you deform a sheet ok. So these are all certain important terminologies and finally we can say now with all the discussion we can say that yield theory and plastic deformation can be described in terms of either maximum shear stresses or deviatoric stresses ok. So these are all certain minimum shear stresses which you have seen before and now we are saying that since hydrostatic part is not going to play in the onset of plastic deformation ok we can say the other part is nothing but deviatoric part can be used to develop yield theories or any theories of plastic deformation ok. So more details on this we will see in the next lecture. Thank you. .