Mechanics of Sheet Metal Forming

Prof. R Ganesh Narayanan
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Week- 08
Lecture- 20
Yield Functions with Sheet Anisotropy

So, we are into lecture 20 which is also included in module 8. So, in this particular section we are going to discuss about some yield criteria or yield function in which anisotropy of sheets are actually considered ok. So, they are called as anisotropic yield function, anisotropy yield criteria you can say. So, but these yield functions will contain or yield criteria will contain anisotropic characteristics of sheets. So, this particular topic is pretty large topic a lot of you know scientific things one can you know derive and discuss, but considering the constraints of this particular course and time limit ok. So, we are going to discuss only about brief account of all this criteria. Specifically, we will look into some important details about 5 important yield criteria after that one has to look into you know other textbooks or reference books which I already mentioned for further yield. So, so when we speak about anisotropy of sheets we all know that we are particularly discussing about plastic anisotropy ok. So, we are going to discuss about how anisotropy affects the plastic deformation part ok, elasticity is considered very simple. So, plastic anisotropy indicates a plastic properties are direction dependent.

So, that is the first thing we briefly discuss how to quantify all these things in the first chapter ok, but then here we briefly discuss and then we go ahead. So, plastic anisotropy indicates that plastic properties are direction dependent ok and this shows that what do you mean by that it means that the stress strain curve let us say or the strain handling behavior will vary with directions will vary with direction. So, that means what suppose if you take a sheet just randomly you pick up a square sheet and you cut a tensile sample along this direction you do not know what is this direction and you cut a rectangular sample along this direction from this you make a dog bone type of standard sample and then you do tensile testing they may show different properties, they may show different stress strain behavior or you know strain handling behavior ok. So, the which is what we are going to call as plastic anisotropy. So, when we speak about sheets when you speak about sheets we know that all are basically rolled sheets and a sheet may be isotropic to start with, but it may become anisotropic due to subsequent plastic deformation specifically in the case of sheets it is going to be rolling of sheets, it is going to introduce subsequent plastic deformation which creates preferred orientation of grains, preferred orientation of grains means grains oriented in a particular direction.

So, then this rolling of sheet is basically going to control ok the directions that we are discussing about whether it is along the rolling direction or perpendicular to that or it creates at any angle ok. For example, you get this along this direction could be a rolling direction. So, this sample is along rolling direction whereas, this sample is a transverse to rolling direction ok. So, with respect to rolled sheet the direction of rolling basically decides the anisotropic characteristics and the origin is basically preferred orientation of grains. So, how to measure the preferred orientation of grains all such you know scientific things one can refer other you know courses or other books ok, but you will look into whatever is required for us.

So, this is the origin of plastic anisotropy and you want to quantify it with respect to some property or a parameter which we can estimate by experimental measures. So, in that way we can define this anisotropy in as a two different you know quantities which we are going to define the next slide they are actually called as normal anisotropy and planar anisotropy. We have also discussed about it ok I hope you remember what is R, what is $\bar{R}$ and what is ΔR which we discussed in the first chapter when we discussed about the tensile properties the same one we are going to briefly study here. So, what do you mean by normal anisotropy in sheets plastic properties may differ along thickness direction when compared to in plane properties how do you quantify it that is where the question comes ok. So, you take a sheet ok so you take a sheet like this and you will see that the in plane properties are different ok when compared to its thickness let us say this is your thickness these are all in plane directions this and this.

The plastic properties may differ along thickness direction so along this direction it may be different as compared to in plane directions ok in plane properties ok so you need to quantify it. For example this is just for example high flow stress in thickness direction suppose you can measure thickness direction flow stress ok how we will see later on briefly high flow stress in thickness direction when compared to in plane flow stress is good for deep drawing ok suppose you measure flow stress along the thickness direction and you measure flow stress along the in plane direction ok and high flow stress in thickness direction is preferable to have a good deep drawing ok. When we quantify this R value we will see that why briefly but it is good why because it shows good resistance to thinning and tearing it shows good resistance to thinning and tearing. So, you have you know good strength in the thickness direction when compared to in plane means in plane directions means in the thickness direction very difficult to deform the material it shows good resistance to thinning and tearing so you may have good deep drawability. On the other hand there is another parameter called as planar anisotropy ok planar anisotropy means in the plane itself they may vary the properties may vary ok.

So, in plane properties are different along different directions in a sheet in plane means in one plane other than thickness direction the other in the in one plane the properties are going to be different. And we are also going to quantify one a good property which is going to you know in a way describe that and you will see that is going to actually control earring which is a defect in deep drawing. What is earring? Earring is nothing but a wavy edge on a fully drawn cup suppose you have a cup which is fully drawn like this ok so you may have some waviness like this. So, let us say for example so this is actually called as earring. This earring is actually controlled by this planar anisotropy. So, one should be careful about these properties ok. So, one is good one is actually not good for sheet forming operation specifically when you speak about deep drawing. So, how are we going to quantify it? So, you want to measure sheet anisotropy then we introduce a parameter or maybe a material property called as plastic strain ratio generally referred as capital R sometimes small r also depending on the situation we use. It is also called as Lankford coefficient. It is typically used to represent the condition of anisotropy in sheets whose characteristics vary with direction.

Again in sheets it is going to be rolling direction. For example suppose as I told you just now so you have a sheet here ok this is nothing but your rolled sheet. So, this is a long rolling direction means so here in this direction you may have some properties as compared to this as compared to 45°. So, this is what is called as directional dependence. So, orientation of sheets for measuring R I have written. So, in this way you can measure R in different directions with respect to rolling direction. So, anyway so what do you mean by plastic strain ratio R or Lankford coefficient as you know already R is nothing but true width strain by true thickness strain ok. True width strain is defined as $\frac{\ln\left(\frac{w}{w_0}\right)}{\ln\left(\frac{t}{t_0}\right)}$ using volume constancy equation we can write $\ln\left(\frac{w}{w_0}\right)$ remains as it is, but the denominator can be written as $\ln\left(\frac{w_0 l_0}{wl}\right)$ because $\frac{t}{t_0} = \frac{w_0 l_0}{wl}$ ok. So, true width strain divided by true thickness strain and there are standards available for evaluating R value which we already discussed in the first chapter itself ok. So, this is one good definition of R true width strain by true thickness strain. So, now along with that to quantify R for practical some practical reasons we are going to define two more important properties which are actually functions of this R value ok. So, now what we are saying is if the measured R value deviates from unity ok. Suppose R is equal to true width strain divided by true thickness strain is equal to 1 let us say that means true width strain thickness strain are same which means we are going to call this as isotropic material. So, until now we have seen all the discussion whatever done is only for isotropic sheets ok. This is the first time we are introducing anisotropy.

So, suppose if it is deviating from unity so not equal to 1, this not equal to 1 ok indicates that the in-plane and true thickness properties or characteristics are going to be different. So, suppose you say $R = 2$ which means that true width strain would be equal to $2\varepsilon_t$ ok, $2\varepsilon_t$ which means ε_w would be double the time that of thickness strain which means that the material is stronger in thickness direction that is why you will be able to give lesser strain as compared to width direction strain that is why we are saying that if R value is larger means ok it is good for deep dryability why because the thickness direction material is strong and it will you know resist or defend thinning and tearing ok. So, now let us go back to our point here it indicates a difference in in-plane and true thickness direction which is often represented by a parameter or property called as $\bar{R}$ ok. And it is a measure of normal anisotropy, it is a measure of normal anisotropy. So, how do you consider how do you you know define $\bar{R}$ if you consider 3 different rolling directions let us say R_0 , R_{45} , R_{90} .

So, R_0 indicates means along the rolling direction R_{45} diagonal direction R_0 transpose direction which you already mentioned in the previous slide right. So, $\bar{R} = \frac{R_0+2R_{45}+R_{90}}{4}$ this is the way you define it ok, but it can also be defined and evaluated by numerical integration ok. So, $\bar{R} = \frac{\int_0^{\frac{\pi}{2}} R\mathrm{d}\theta}{\int_0^{\frac{\pi}{2}} \mathrm{d}\theta}$ ok. Suppose you can integrate it and find $\bar{R}$ may be like by numerical integration using let us say trapezoidal rule you will be able to find out $\bar{R}$ ok you will be able to find out $\bar{R}$. So, you have to discretize it basically depending on trapezoidal rule you can look into numerical integration part you will be able to get the same equation if you consider 3 different rolling direction. So, now you know R ok you know $\bar{R}$ so how do you get R you pick up a particular rolling direction and you follow standard method that we discussed already and then get $R = \frac{\varepsilon_w}{\varepsilon_t}$ you repeat that for 90° and 45° other than 0° then you get 3 values R_0, R_{45}, R_{90} you put it in this formula you will get $\bar{R}$ these 2 are done now. So, how do you define planar anisotropy which is going to define how this anisotropy characteristic change in the plane itself in plane it is written as ΔR you know third parameter is ΔR which is nothing but I also we already discussed about it, it is going to tell you how different is R_{45} as compared to R_0 *and* R_{90}. So, $\Delta R = \frac{R_0+R_{90}-2R_{45}}{2}$ok. So, $R_0 + R_{90} - 2R_{45}$ ok and this may be positive or negative ok depending on that earring characteristics are going to change ok. So, your ΔR could be positive or it could be negative depending on this your earring characteristics will change, earring will be there but characteristics will change ok.

So, one should look into it but although most of the steels they have generally positive ΔR value ok. So, since , $\bar{R}$ is going to control thinning resistance so we already said that for a good deep draw ability you need to have larger , $\bar{R}$ and lower ΔR, lower ΔR means earring behavior would be minimized, earring behavior would be minimized fine. So, now this R is basically one quantity that we are going to use in this entire chapter R or small *r* we are going to use it in this whole chapter and depending on the situation we are going to see maybe R_0, R_{45}, R_{90} and because you know R value changes you know with respect to rolling direction there are chances that real strength may also change during with respect to rolling direction that is why we said that stress strain behavior would be different ok. And the strain hardening will also be different with respect to rolling direction ok which means that we may see in several locations now depending on the situation, depending on the criteria we are going to use R_0, R_{45}, R_{90}, σ_0, σ_{90} like that ok. So, then and there we will define it and then we will go ahead and like in Von Mises you know yield function we have derived a pretty long you know discussion was made to some extent I mean we have made a long discussion you know what is Von Mises yield function, why do you need it,$\bar{\sigma}$ equation and then $\bar{\varepsilon}$ equation all these things we discussed.

But in this particular chapter we are not going to discuss all of them we are going to see mainly the yield function criterion directly and then some important characteristics of that

that is what we are going to do and then we go ahead ok. So, and whatever we discuss are very restricted only there are several other yield functions available beyond what we are going to discuss one should go ahead with further reading by following other you know textbooks or reference books. So, when you speak about an anisotropic yield function or criteria Von Mises himself has given 1 anisotropic yield function which is written here ok. It is a pretty long one and it is of quadratic nature ok. This is the first yield function of anisotropic sheet that is what it is claimed and which is a quadratic function ok.

So, f is nothing but a function and you can see that you have , $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}$ and interactions between them and interactions between them. So, you know what do you mean by $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}$ they are all normal and shear stresses which forms a stress tensor and all the h values h_{11}, h_{22} etcetera they are actually called as h_{ij} they are nothing but coefficient of anisotropic. If you find these values you plug in into this then yield function is actually defined and these values will of course change from one material to another material. So, how to get all these properties that is a big you know exercise but general you know framework of this yield function looks like this, function looks like this. So, you can refer this if somebody can get this particular reference long back mechanics of plastic deformation of crystals ok. So, one probably one should be able to get into this particular equation ok. Usage of this equation is restricted so there are better you know functions some of them we will see here. So, this is the first one so wherein your h_{ij} is actually going to describe the anisotropic characteristics of the sheet and you have to evaluate it by lots of mechanical tests. So, next one which is very important for us is Hill's 1948 yield function. This function is predominantly used of course there are some restrictions to use it we will see at the end of this particular yield function before we go ahead in the next yield function.

So, this Hill's 1948 yield criterion ok is also a quadratic function it is given by this particular expression ok you can look into it. So, there is a function which is nothing but $2f\left(\sigma_{\mathrm{ij}}\right) = F\left(\sigma_y - \sigma_z\right)^2 + G(\sigma_z - \sigma_x)^2 + H\left(\sigma_x - \sigma_y\right)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1$. And this is a generalized form of this Hill's 1948 yield function in general coordinate system that is why you have $\sigma_x, \sigma_y, \sigma_z$ which are all normal quantities and all τ s are basically shear quantities ok. So, just for a change we have written here you know $\sigma_x, \sigma_y, \sigma_z$ until now we were seeing $\sigma_1, \sigma_2, \sigma_3$ only right. So, and then just a small change here so I am using x y and z later on we will change it at the end of this particular section we will change it ok.

So now where is anisotropic coming into picture here this equation should have some symbols which is going to quantify or some parameter which is going to quantify anisotropy of sheet and they are nothing but this *FGHLMN* these 6 values ok *FGHLMN* are actually constants which is going to define your anisotropic state of the materials and you know that σ_{ij} as I told you all the stress components with respect to orthogonal coordinate system. So when you propose when we propose this particular yield function in this way it is assumed that anisotropy has 3 mutually orthogonal planes of symmetry at every point ok 3 mutually orthogonal planes of symmetry this is very important ok this is where Hill's 1948 yield function stand actually. So, the 3 planes of symmetry meet in 3 orthogonal directions which are actually nothing but principle axis of anisotropy ok. For instance in a rolled sheet the rolling direction is taken as one principle axis I am denoted it by X for a reason the other directions are in plane transverse direction Y and the thickness direction which is nothing but Z. So previously we have shown a diagram R_0, R_{45}, R_{90} it is same only but then 3 orthogonal you know we are saying principle axis of anisotropy would be there 3 orthogonal directions which may be called as principle axis of anisotropy one principle axis let us say X which is on the plane that is the rolling direction the other perpendicular one is the transverse direction in plane 1 and of course along the thickness direction that is let us say Z.

So this XYZ would be useful for us later on ok. So now this is a function ok and what we are going to do now is we are going to see some small-small derivations and we will see how to rewrite this yield function using known quantities that is what we are going to do. We are not going to do anything else how to rewrite this particular equation ok in different forms for that we are going to put some conditions some derives on small-small equations some case studies cases we are going to pick up and we see what is going to happen ok. So now I am going to rewrite this in terms of XYZ and RST ok what is XYZ what is RST it is available in this slide ok we can discuss. So what is X I am going to consider this X direction yield strength as X ok suppose I consider X direction ok yield strength let us say for example rolling along rolling direction so that is X direction just XYZ is just for convenience only so that is that the yield strength is X ok.

So now I am going to take *x* direction ok and I am going to do tensile test uniaxial tensile test then that yield strength $\sigma_x = X$ only and we know that $\sigma_y = \sigma_z = 0$ you put this in the previous equation put it in this previous equation ok. So what do you say y and z is going to go away and only X will remain so this fellow will remain this fellow will remain it is going to be X so I am going to take $(G + H)X^2 = 1$ right so our $X^2 = \frac{1}{G+H}$. So the yield strength in x direction can be written as $X^2 = \frac{1}{G+H}$. Similarly you can do same exercise with respect to y direction and z direction perpendicular to the sheet ok then you can get Y^2 and Z^2 so here y and z are y direction and z direction yield strengths you can call and $Y^2 = \frac{1}{H+F}$ and $Z^2 = \frac{1}{F+G}$ so this you have to exercise you have to do like this like this you have to do these two exercise it is for you to do. So no X^2, Y^2 and Z^2 are written in terms of F, G and H right so X,Y,Z are written in terms of F, G and H where F,G,H are nothing but in a way it is related to the plastic anisotropy for example R_0, R_{45}, R_{90} may come into picture in due course you will see that.

So this X^2, Y^2 and Z^2 can also be written in this format wherein on the left side you bring F, G, H some simple mathematical calculation will lead to $2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}$; $2G = \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}$; $2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$ ok. So you can rewrite this ok so now F,G,H in the previous equation Hill's 1948 yield function ok is going to be a function of all the yield strengths ok X is one yield strength, Y is one yield strength, Z is one yield strength ok. The only difficulty here is how to find Z right so basically yield strength in Z direction how are you going to find out that is only concerned here otherwise F,G,H can be calculated. So we know now what is X,Y,Z here similarly you can also get RST, RST is basically they are all basically shear yield stresses they are actually shear yield stresses and from Hill's 1948 yield function the previous one you can put the same conditions you know similar conditions here ok so $2L = \frac{1}{R^2}$ ok and $2M = \frac{1}{S^2}$ and $2N = \frac{1}{T^2}$ here we are going to pick up this part ok and we can get $2L = \frac{1}{R^2}$ ok and $2M = \frac{1}{S^2}$ and $2N = \frac{1}{T^2}$. So now the simplicity of this particular equation can be retained if you consider only principle coordinate system correct that is what we were doing right from the beginning even in Von Mises also ok.

So when you go for principle coordinate system this fellow goes off right this fellow goes off only the first three terms will come into picture this fellow goes off only first three terms will come into picture we will see that now ok. So before that if you want to rewrite that equation in terms of plane stress situation that is what we are generally looking for is not it when you form a thin sheet maybe of the order of let us say 2, 2.5 sheet thickness mm sheet thickness or less than that ok if you want to look into that situation which is what we are considering then in that situation you will see with respect to Hill's 1948 yield function which you have just now written we are going to put $\sigma_z = \tau_{zx} = \tau_{yz} = 0$ wherever z comes they will become 0 remaining terms σ s will remain ok $\sigma_x; \sigma_y; \sigma_{xy}$ will remain ok so then the previous yield function is going to become maybe you can look into it it is $\left(2f\left(\sigma_{\mathrm{ij}}\right) = (G+H)\sigma_x^2 - 2H\sigma_x\sigma_y + (H+F)\sigma_y^2 + 2N\tau_{xy}^2 = 1\right)$, $\sigma_z, \tau_{zx}, \tau_{yz}$ goes off right. So this will go this will go this entire thing will remain yz will go off zx will also go off this fellow will remain. So you will have Fσ_y^2 ok so that will be here you have to take σ_y^2 common so it will come out ok.

Then similarly you have minus $G\sigma_x^2$ that will have somewhere here so symbols the signs will be taken care automatically and then this has to be expanded $\left(\sigma_x - \sigma_y\right)^2$ you can expand it and you can combine with the other two ok and this since this is going to remain so you will get this particular equation. So in general coordinate system if you take plane stress then this is the equation the previous equation is general coordinate system considering all the quantities non-plane stress type you can imagine ok. So now you will see that here it becomes easy for us $G + H$ comes into picture your H comes into picture $H + F$ comes into picture and N comes into picture which already rewritten in terms of your yield strength ok your yield strength just before either X,Y,Z or RST ok which is already written of course there is only τ_{xy} here. So what is $G + H$, G plus H is nothing but $G + H = \frac{1}{X^2}$ correct so I am going to put $\frac{1}{X^2}$ here so $\frac{1}{X^2}\sigma_x^2 - 2H$ so what is 2H for me sorry $2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$ so plus minus will come X,Y,Z so plus minus X,Y,Z to$X^2Y^2Z^2$ so minus is written σ_x, σ_y plus what is $H + F = \frac{1}{Y^2}$ that is also substituted here. So $2N = \frac{1}{T^2}$ which already written this particular one and you can get this so this equation $\frac{1}{X^2}(\sigma_x^2) - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma_x\sigma_y + \frac{1}{Y^2}\sigma_y^2 + \frac{1}{T^2}\tau_{xy}^2 = 1$ you will see it is in general coordinate system this fellow but in plane stress.

Now in place of FGH we have written in terms of some material properties what are they? One is X yield strength along x direction and Y along y direction Z along thickness direction and then that is all only T is there T is one of the shear yield . So some known properties are inside that and σ_x, σ_y and then τ_{xy} I have usual definitions. Now comes further simplified part which is what we like suppose you want to write in terms of principle coordinate system then this fellow will go off from this equation so you can write $\frac{1}{X^2}$ is equal to I am going to change now $\sigma_x \sigma_y\ \sigma_z$, $\sigma_1, \sigma_2, \sigma_3$ anyway will not come here because σ_z goes off. So I am going to write σ_x become σ_1 and σ_y becomes σ_2 why because these two are $\sigma_1 \sigma_2$ are actually non-zero principle stresses they are actually non-zero principle. So how do you write? How do I write $\frac{1}{X^2}(\sigma_1^2) - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma_1\sigma_2 + \frac{1}{Y^2}\sigma_2^2 = 1$ and this fellow goes off ok so this is a much simpler form I have here.

So what I have done is basically a general a Hill's 1948 yield function I have related FGHLMN to the corresponding yield strength by applying this conditions and I rewrote it for plane stress first which is a first simplification then retaining plane stress I have rewritten that in terms of principle stresses principle coordinate system that is the latest one which we got here ok. So this is simple to use this particular one simple to use why because there are only few quantities that you need to measure that is nothing but XY and Z right. Now in this in all these equations there is no R value coming into picture right no r_0, no r_{90} nowhere it is coming so what are we going to do? The next stage is we are going to relate plastic anisotropic coefficients and Hill's coefficient. Plastic anisotropic coefficient means r_0, r_{45}, r_{90} depending on the situation and we are going to relate that to FGH and further we are going to modify this equation in a very known format which can be utilized easily by us ok. So now for this I am going to consider r_0, r_{45}, r_{90} ok as usual with the usual definitions just change here small r I am using does not matter it has got usual definition nothing but plastic strain ratio and I am going to use yield strength that is X becomes σ_0 , Y becomes σ_{90} that is all.

So XYZ we have seen before which I am going to write in terms of σ_0 and σ_{90} these are only changes now I have. So what I am going to do now is I have to find strain increments, I have to find strain increments and for that you know we are going to use a normal condition which we have defined in the second chapter $\mathrm{d}\varepsilon_{ij} = \mathrm{d}\lambda \frac{\partial f}{\partial \sigma_{ij}}$ where f is your yield function and for Hill's 1948 yield function we have to use this particular f which I have given here $f = F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2$ to get strain increment. So what do I need to do? I need to put f and then differentiate it with respect to $\sigma_1, \sigma_2, \sigma_3$ ok to get $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ respectively ok. So that one can do and find out and I will just go through it. This we have done it for Von Mises yield function before ok.

So when we derive $\bar{\varepsilon}$ for Von Mises yield function right now that you know your $\bar{\varepsilon} = \sqrt{\frac{4}{3}(1 + \beta + \beta^2)}\varepsilon_1$ we derived. So for that if you remember ok at initial stage itself we derived $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$. So similarly we are also deriving it here ok using this particular equation using this particular yield function of Hill's 1948 one particular form ok. So $d\varepsilon_1 = 2d\lambda[G(\sigma_1 - \sigma_3) + H(\sigma_1 - \sigma_2)]$. $d\varepsilon_2 = 2d\lambda[F(\sigma_2 - \sigma_3) - H(\sigma_1 - \sigma_2)]$, $d\varepsilon_3 = 2d\lambda[-F(\sigma_2 - \sigma_3) - G(\sigma_1 - \sigma_3)]$ So this yield function will be useful for a this strain increments are going to be useful in due course as well. Now what I am going to do is I am going to do some small calculation here to make things easy. So now I let us see I am going to consider this rolling direction ok. This is a sheet actually the blue one is actually sheet you can consider. Now this is along one direction this is along two direction. So along one is let us say rolling direction which is nothing but 0 let us say 0° and transpose direction you have already defined that is 90° rolling direction along 2 right in this framework.

So now I am going to consider RD along rolling direction. So I know that along rolling direction I can write $\sigma_1 = \sigma; \sigma_2 = \sigma_3 = 0$. So then what is r_0 plastic strain ratio in rolling direction r_0 or capital R_0 whatever you can write or RD both are same which is nothing but so if you cut a sample in this way and find R value then this becomes width direction correct and thickness is in this direction perpendicular. So $\frac{d\varepsilon_2}{d\varepsilon_3}$ correct. So $d\varepsilon_2$ is this $d\varepsilon_3$ is this you divide you get the ratio $\frac{d\varepsilon_2}{d\varepsilon_3}$ this fellow ok and then maybe you can just do some simple calculation you will see that you have to put this condition that is the main thing.

$\sigma_2 = \sigma_3 = 0$ no. So if $d\varepsilon_2$ is what 2 and 3 are 0 so this fellow goes off 2 is 0 so what do you get $2d\lambda$ will anyway go off you are going to have $H/3$ is what this fellow goes off this also goes off this 2 fellow goes off 3 goes off so you will have $-G$ so it will have $\frac{H}{G}$ ok. So substitute this conditions into this equations and get it. So now we have related one of the R values let us say r_0 to H and G this is the way it turned out $r_0 = r_{RD} = \frac{d\varepsilon_2}{d\varepsilon_3} = \frac{H}{G}$ ok. Similarly I am going to consider transverse direction so I am going to cut a rectangular sample and let us find r_{90} ok. So in this going to put $\sigma_2 = \sigma; \sigma_1 = \sigma_3 = 0$ ok any axial situation let us consider ok.

And now if you want to get R value along 90° let us say r_{90} or r_{TD} ok then for this sample this becomes width so that means I am going to write $r_{90} = r_{TD} = \frac{d\varepsilon_1}{d\varepsilon_3} = \frac{H}{F}$. So it is very simple so $r_0 = \frac{H}{G}, r_{90} = \frac{H}{F}$ from this from this entire relationship I am going to write this 3 I am going to write this 3, $F = r_0$, $G = r_{90}$, $H = r_0 r_{90}$ how are we writing it we are writing it in combination with these two suppose you take substitute here $r_0 = \frac{H}{G}$ so $r_0 = \frac{r_0 r_{90}}{r_{90}} = r_0$, so fine satisfied so $r_{90} = \frac{r_0 r_{90}}{r_0} = r_{90}$ which is also satisfied. So it is going to be simple for us now FGH is becoming r_0, r_{90} and $r_0 r_{90}$ respectively ok so now you know what I am going to do we can directly write this equation in a simpler way F can be replaced G can be replaced and H can be replaced using this so this you have to get along with this ok. So let us see that I am going to do here yeah so before that this is one important result that we need to see we already discussed that $X^2 = \frac{1}{G+H}$ we derived in the previous to previous slide and $Y^2 = \frac{1}{H+F}$ this also we derived. So what is X^2? $X^2 = {\sigma_0}^2$ in the previous slide only I told you that $X = \sigma_0$, $Y = \sigma_{90}$ say definition remains same just a nomenclature is going to be different this fellow $Y^2 = {\sigma_{90}}^2$ correct.

So I am going to get a ratio of this $\frac{\sigma_0}{\sigma_{90}}$ that means your σ value ok your yield strength along $0°$ rolling direction yield strength along $90°$ rolling direction you want to get a ratio then it could be $\frac{\sigma_0}{\sigma_{90}} = \frac{\sqrt{\left(\frac{1}{\mathrm{G+H}}\right)}}{\sqrt{\left(\frac{1}{\mathrm{H+F}}\right)}} = \frac{\sqrt{\frac{1}{\mathrm{r_{90}+r_0r_{90}}}}}{\sqrt{\left(\frac{1}{\mathrm{r_{90}r_0+r_0}}\right)}} = \sqrt{\frac{\mathrm{r_0(1+r_{90})}}{\mathrm{r_{90}(1+r_0)}}}$. So everything is substituted here and you will get this simple equation you want to get a ratio of $\frac{\sigma_0}{\sigma_{90}}$ for Hill's 1948 yield function then you can relate it to r_0, r_{90} by the simple equation this is a very important result for us with respect to Heale's 1948 yield function this ratio is very important. So that means what that means if you know r_0, r_{90} and σ_{90} you can find σ_0 ok. So only three values are required to evaluate σ_0 or the other way if you want to find σ_{90} you should get r_0, r_{90} and σ_0 that way it is going to work. So now you will see that this equation implies that for $r_0 > r_{90}$ suppose you pick up a case where $r_0 >$ r_{90} , $\sigma_0 > \sigma_{90}$ and vice versa ok.

So you can take an example and find out let us say r_0 is for example let us say 2 and r_{90} is let us say I do not know maybe 1.5 you can say or you can take 1 also ok and you want you can check it here ok. So your $\sigma_0 > \sigma_{90}$ ok. So which means that if $\frac{r_0}{r_{90}} > 1$ then $\frac{\sigma_0}{\sigma_{90}} > 1$ that is another way. But the problem is some materials however it has been observed that some materials do not follow this particular pattern.

You will come back to this, this is very important result ok. So when you find $r_0 > r_{90}$ then your $\sigma_0 > \sigma_{90}$ but some materials is not going to follow this ok anyway so we will come to this later on. So now when the principle directions of stress coincide with principle anisotropic gases that means in principle coordinate system ok we have derived this right just before we have derived this. So now what I am going to do is I am going to rewrite this ok in terms of r_0, r_{90} I am going to replace all these things by r values. So what is $\frac{1}{X^2} = G + H$ right. So what is $G + H = \mathrm{r_{90} + r_{90}r_0}$. $G + H = \mathrm{r_{90} + r_{90}r_0}$ so which I have written here. This fellow $\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$ what is it? It is nothing but $2H$ which also we derived which is now becoming $H = \mathrm{r_0r_{90}}$ right that also I have replaced here. $\frac{1}{Y^2} = H + F = \mathrm{r_0r_{90} + r_0}$ which $\mathrm{(r_0r_{90} + r_{90})}\sigma_1^2 - 2\mathrm{(r_0r_{90})}\sigma_1\sigma_2 + \mathrm{(r_0r_{90} + r_0)}\sigma_2^2 = 1$ ok. So now this equation is becoming more relevant to us now in the Von Mises yield function we are going to bring we have already brought r_0, r_{90} into the equation and now what I am going to do is I am going to just rewrite this ok in this format. So the coefficients are basically divided ok let us say for example I am going to divide it by $\mathrm{r_0r_{90} + r_{90}}$ then $\sigma_1^2 - \frac{2r_0}{1+r_0}\sigma_1\sigma_2 + \frac{r_0(1+r_{90})}{r_{90}(1+r_0)}\sigma_2^2 = \frac{1}{r_{90}(1+r_0)} = X^2 = \sigma_0^2 = \frac{r_0(1+r_{90})}{r_{90}(1+r_0)}\sigma_{90}^2$.

So basically I am going to divide the entire equation by $\mathrm{r_0r_{90} + r_{90}}$ to get the first part of this equation until this part which is easy for me right. So $\frac{1}{r_{90}(1+r_0)} = X^2$ correct. So now I am going to rewrite this as $X^2 = \sigma_0^2 = \frac{r_0(1+r_{90})}{r_{90}(1+r_0)}\sigma_{90}^2$. So this entire equation now has been modified to the next form ok in which all the material properties are very relevant to us and you can see that this is the Hill's 1948 yield criterion when principle direction of stress coincide with principle anisotropic axis ok which means that in principle coordinate system we are writing this. So what are σ_1, σ_2 they are principle stresses and r_0 is nothing but plastic strain relation 0° rolling direction r_{90} along 90 ° rolling direction what else you want to know? Nothing else, of course σ_0 is yield strength and σ_{90} is yield strength.

So what are the properties you need to know? You need to know r_0 of course you need to know r_{90} and one of this σ_0 or σ_{90} because they are already related to each other ok. It is very simple to use this particular gradient is very simple to use ok. So r_0 you can you know how to find, r_{90} also you can find, σ_0 also you can find take a tensile test do tensile test along 0 ° rolling direction and find out all the values right. So now in the above equation you replace r_0, r_{90} by 1 if you replace what will happen? $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_0^2$ let us say. So this is what you get here know, this is what you get here right, this is the equation you get.

So this becomes square root of this fellow right. So $\sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \sigma_0$ ok. So you can convert this Hill's 1948 yield function in principle coordinate system again plane stress you can convert that into Von Mises equation which you derived long before by putting $r_0 = r_{90} = 1$ ok. So basically Von Mises yield function is a case of Hill's 1948 yield function ok. So now we have seen almost 3, 4 different forms of this equation ok and the one simplicity in this equation is r_0, r_{90} and σ_0 can be evaluated and you can use this for any modeling purpose ok. So now there are some cases in this let us say for example this particular sheet any sheet you pick up it is going to take only normal anisotropy, only normal anisotropy means what? Your $r_0 = r_{90} = r$ both are same basically they will have some value ok but they are equal but they are equal ok.

So if $r_0 = r_{90}$ then using this equation ok you can get $\sigma_0 = \sigma_{90}$ you can try this. This is another important result we are saying here if $r_0 = r_{90} = r$ ok. So any r value you can put ok and you will find out that $\sigma_0 = \sigma_{90}$ and what will happen to the previous equation? The previous equation is going to become very simple I am going to replace r_0 by r. So $\sigma_1^2 - \frac{2r}{1+r}\sigma_1\sigma_2 + \sigma_2^2 = \sigma_{ys}^2$ square both are same.

So this is another form of your Hill's 1940 yield function when the material has normal anisotropy which means $r_0 = r_{90} = r$ and if you use this equation this also means $\sigma_0 = \sigma_{90}$ ok. So now this equation can be used to check the effect of you know r and σ_{ys} ok on the locus and what is the effect of r you see that X and Y axis are actually normalized values σ_1/y and σ_2/y and you will see that if by increasing r value diagonally ok the yield locus is going to elongate that is one which is I think we have seen this before also. In the next one if you increase the yield strength it simply means that basically the yield locus is going to expand uniformly in the outward direction expand uniformly in the outward direction and the yield locus is defined by its form which is nothing but Hill's 1948 equation only ok which is nothing but your Hill's 1948 equation only. So the shape remains same and size increases with increase in yield strength let us say this is what is called as in a way isotropic hardening. You will see that later on so what is isotropic hardening means the material is anisotropic but hardening is isotropic what do you mean by hardening is isotropic it means that the yield locus is going to expand uniformly with increase in your flow stress but keeping the shape same because shape is decided by form of the yield function which is nothing but Hill's 1948.

So I am just written here that the second figure that is this fellow in this figure the yield locus expands uniformly with increase in yield strength this is called isotropic hardening. So we know that so now there are important things in Hill's 1948 deal function what are they anomalous behavior what is that so if we say that if $r < 1$ let us say in general using the Hill's 1948 deal function you will see that the yield locus predicted by Hill's 1948 yield criterion is located inside von Mises locus. Suppose $r = 1$ if you say σ_1, σ_2 and if $r = 1$ ok then if $r < 1$ your yield locus given by Hill's 1948 is inside this something like this ok and if $r > 1$ yield locus would be outside it goes out like this that is what is generally observed by Hill's 1948 yield function. But later on Woodthrope and Pearce these two scientists what they notice is some materials particularly aluminum alloys have yield locus outside von Mises though $r < 1$. Suppose if you pick up $r < 1$ instead of locating inside the von Mises yield locus it was outside von Mises yield locus which is what this peculiar behavior was unable to be explained by Hill's 1948 yield function and that behavior is called as anomalous behavior it is generally called as anomalous behavior first order.

There is some anomaly ok with respect to that material why because it is not following this particular requirement one of the requirement ok that if $r < 1$ which is what is seen in many aluminum alloys ok your yield locus is going to be outside the von Mises equation ok. This peculiar behavior was not explained by Hill's 1948 so we are going to call that as anomalous behavior of first order. This can also be defined in this way ok in this equation which we derived before just now for normal anisotropy if you put the equi-biaxial tension $\sigma_1 = \sigma_2 = \sigma_b$ where σ_b is equi-biaxial yield stress ok you are pulling the material in two directions equally like $\alpha = 1$ this will give you $\alpha = 1$ let us say ok $\alpha = 1$ then there will be one yield strength that is σ_b. If you put this condition in the previous equation you will get $\frac{\sigma_b}{\sigma_{ys}} = \sqrt{\frac{1+r}{2}}$ so in this equation if you see if $r > 1$ then $\sigma_b > \sigma_{ys}$ if $r < 1$ let us say 0.5 then let us say this 0.5 means $\sqrt{\frac{1+0.5}{2}}$ let us say then $\sigma_b < \sigma_{ys}$ ok. But if $\sigma_b < 1$ this may not be satisfactory ok when we look into this particular cases defined by this anomalous behavior ok. So this is the first anomaly that they found out. So now just a quick note of how do you find σ_b? σ_b means what? σ_b is means it is a yield strength in equi by axial tension that means this is your sheet you have to pull the sheet equally in the plane direction in plane direction and you have to monitor its deformation to get the yield strength. So there are machines that can apply tensile loads in two directions at the same time and these machines are meant for balanced by axial tension test that means you have to maintain $\alpha = 1$ let us say.

So balanced by axial tension let us say σ_b at which yielding occurs is yield strength in balanced by axial tension test therefore σ_b can be estimated from these machines however it is not so easy it is difficult. So you need to have a machine where you hold it and pull it all the sensors have to be properly designed to monitor the deformation load requirement all those things it is difficult. So instead of that what can be done it is generally like assume that the sheet is subjected to compressive stress let us say σ_c the thickness direction for example like this. So you have the same sheet element let us say you are compressing it with the σ_c this will produce same effect as it of applying tension in two directions simultaneously as depicted below instead of pulling equally in this directions which mentioned here what do you do is you compress it in this direction you provide c perpendicular to the slide instead of pulling it in the in plane direction. So when you compress it it will anyway expand which is what equi-axial tension is going to do.

So then I do not we have to see how accurate it is but then therefore σ_b can be equated to σ_c but σ_c is in the thickness direction. Your σ_c you have to compress basically sheet the thickness direction that is all. And we can say that this previous equation your $\frac{\sigma_b}{\sigma_{ys}} = \sqrt{\frac{1+r}{2}}$ this fellow can be replaced by σ_c. This fellow can be replaced by σ_c here which is what I have written $\frac{\sigma_c}{\sigma_{ys}} = \sqrt{\frac{1+r}{2}}$. So many times you know if it is difficult for us to get σ_b, σ_c can be obtained in this way. Okay so let us complete this part may out of the major demerits of Hill's 1948 yield criterion. The first one is it cannot address anomalous behavior of first order which we have just now discussed which means for $r < 1$, $\sigma_b > \sigma_{ys}$ were observed. Just now we discussed. There is one more thing which is called anomalous behavior second order also we discussed before. You remember this equation this we derived now this particular one $\frac{\sigma_0}{\sigma_{90}} = \sqrt{\frac{\mathrm{r}_0(1+\mathrm{r}_{90})}{\mathrm{r}_{90}(1+\mathrm{r}_0)}}$ now.

Now I we also said that if $r_0 > r_{90}$ then $\sigma_0 > \sigma_{90}$ but some materials do not agree with this. This is actually called as another anomalous behavior. Okay so this equation which you already derived what will happen if $\frac{r_0}{r_{90}} > 1$ but some materials will show $\frac{\sigma_0}{\sigma_{90}} < 1$. Okay so that is one problem with Hill's 1948. Moreover generally the criterion is applicable to sheet material which has got only 2 or 4 ears during deep drawing.

So like we said deep drawing no cup deep drawing earing is formed so only 2 or 4 ears will be formed not formed the deep drawing behavior can be predicted accurately using this yield function which has got 2 or 4 ears during deformation. Okay but practically you will speak there will be more ears that can form so which cannot be addressed by Hill's 1948. And moreover it is also said that in axial tension test the variation of yield strength with direction is poorly predicted by this criterion though the variation of r value it is described better. Suppose you do tensile test and get r_0, r_{90} and you also get the σ_0 σ_{90} and it is said that the variation of yield strength with direction is poorly obtained but r_0 or r_{45}prediction is or value prediction is acceptable. So these are the major demerits of this criterion specifically the first two are very important these two anomalous behavior.

Now before we go to next yield function I thought this particular small derivation is going to be useful for us. How do you relate stress ratio and strain ratio for an anisotropic sheet or sheet in which you are going to include anisotropic while modeling. Okay and of course when you say that the relationship is going to change with α and β a depending on the yield function you choose and at least it has choose Hill's 1948 yield function and how do you get α as a function of β or β as a function of α is what we will see here. So we know this this particular equation is known to us know we derived the following relationship between α and β considering sheet to be isotropy using Levi-Mises equation we already derived it you can look into it and using this only we solved a lot of problems correct $\alpha = \frac{2\beta+1}{2+\beta}$and $\beta = \frac{2\alpha-1}{2-\alpha}$ right. So we derived this equation before. So now for Hill's 1948 yield function in principle coordinate system we derived this 3 just now $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ we are going to use it how what is β here $\beta = \frac{d\varepsilon_2}{d\varepsilon_1}$ right. So this divided by this so $2d\lambda$, $2d\lambda$ will go off $\beta = \frac{d\varepsilon_2}{d\varepsilon_1} = \frac{F(\sigma_2-\sigma_3)-H(\sigma_1-\sigma_2)}{G(\sigma_1-\sigma_3)+H(\sigma_1-\sigma_2)}$ fair enough. Now how are we going to write it here for plane stress this fellow goes this fellow will go off right. So $\beta = \frac{d\varepsilon_2}{d\varepsilon_1} = \frac{F(\sigma_2)-H(\sigma_1-\sigma_2)}{G(\sigma_1)+H(\sigma_1-\sigma_2)} = \frac{\alpha F-H(1-\alpha)}{G+H(1-\alpha)} = \frac{\alpha r_0 - r_0 r_{90}(1-\alpha)}{r_{90}+r_0 r_{90}(1-\alpha)}$. right very simple equation ok. Now if you consider normal anisotropy just before we discussed where $\mathrm{r}_0 = \mathrm{r}_{90} = r$ so here if you in this equation if you put $\mathrm{r}_0 = \mathrm{r}_{90} = r$ so it becomes further simple case $\beta = \frac{\alpha r - r^2 + \alpha r^2}{r + r^2 - \alpha r^2} = \frac{\alpha - r + \alpha r}{1 + r - \alpha r}$ you will get and then you simplify it you get a simple equation here.

So using Hill's 1948 yield function of course plane stress has to be there because you have to bring in α and β and if it is not the case of normal anisotropy then this is the simple equation you have you can relate it and you put a case here $\mathrm{r}_0 = \mathrm{r}_{90} = r$ then you get this particular equation. In this equation if you put $r = 1$ what will happen to β because it is isotropic case now $r = 1$ means $\beta = \frac{\alpha - 1 + \alpha\times 1}{1 + 1 - \alpha \times 1} = \frac{2\alpha-1}{2-\alpha}$ we got here right. So now depending on r value you see you know the relationship between β and r are actually changing until now we were assuming that $r = 1$ then it was simple now all our you know calculations whatever we have done in the same problem is going to change by looking at one r value. Suppose if you take r is equal to let us say any $r = 1.5$ then depending on α the β evaluation is going to be different. So now you can see for anisotropic sheets by considering this particular formula what will be the α β for uniaxial plane strain pure shear okay and you know other two cases balanced by axial stretching all these cases we can find out anyway so now 1948 is over now let us go to Hill's 1979 we will not discuss much in all these things we will quickly go ahead.

So Hill's has proposed another yield function Hill's 1979 which does not exhibit the anomalous behavior first order mentioned before okay the anomalous behavior first order which we mentioned before know that means your though $r < 1$ your yield locus is going to be outside Von Mises yield locus know for like aluminum alloys. Then for to avoid that particular problem then he proposed another yield function which is given here a little bit complex but values are

$$f|\sigma_2 - \sigma_3|^m + g|\sigma_3 - \sigma_1|^m + h|\sigma_1 - \sigma_2|^m + a|2\sigma_1 - \sigma_2 - \sigma_3|^m + b|2\sigma_2 - \sigma_1 - \sigma_3|^m + c|2\sigma_3 - \sigma_1 - \sigma_2|^m = \sigma_0^m$$

okay.

So now one advantage here is only principle stresses are allowed here where σ_i are principle stresses and $f\, g\, h\, a\, b\, c$ are for example anisotropic coefficients and σ_0 as usual is uniaxial yield stress. This is a modification Hill's has done and here there is one fellow called m this $m > 1$ and what value actually depends on the crystal structure of the material.

Let us say crystal structure you remember *BCC, HCP, FCC* depending on that crystal structure this m value is going to change okay and is generally greater than 1 sometimes people do some optimization to get this value for a particular material. What are these characteristics you can see in this criterion it is assumed that principle direction of stress sensor coincide with the principle direction on isotropy which means the principle coordinate system only we have written therefore the criterion does not include shear stress terms which is obvious from this equation. So it is restricted to loading around principle axis is what I was telling you. Moreover it is also observed I have not included any data in this but it is seen that the criterion does not always satisfy convexity condition which is a requirement for any yield function. So convexity condition you know we remember that convexity and normality we discussed two important points so your yield locus have to be convex at every point in the yield locus right.

So that condition is not satisfied by this criterion sometimes that is what is said this is two important characteristics of this particular yield function. So some cases we quickly discuss of course you can do this exercise I am not going to explain you this is very obvious and self-explanatory but this will lead to one important result. There are actually five special cases of Hill's 1948 function under plane stress under plane stress so which means that directly you can write $\sigma_3 = 0$. So if you substitute these conditions belonging to case 1, 2, 3, 4, 5 into this equation you will get some other forms of equation which is what is described here. So case 1 if I take $a = b = h = 0$ and $f = g$ if you put in this condition you will get this similarly you can take case 2 you can try it and check similarly you can take case 3 similarly you can take case 4 and similarly you can take case 5.

So basically the you know *f, g, h* and *a, b, c* if you put some conditions to it it will lead to five different cases. An important one it is said that this fifth case $a = b = f = g = 0$ and $f = g$ if you put it in this equation σ_3 will anyway go off this fellow goes off now, σ_3 will anyway go off $a = b = f = g = 0$, f this fellow goes off g goes off a also fully goes b also fully goes and $f = g$. So then you will get this particular equation. So $f = g$ means where is g for me here g is here, g is here now so this is going to become *f,* so $f|\sigma_2|^m + f|\sigma_1|^m + a|2\sigma_1 - \sigma_2|^m + a|2\sigma_2 - \sigma_1|^{mm} = \sigma_0^m$ ok and this is what is generally called as Hosford yield function. So this Hosford one form of Hosford yield function this Hosford- yield function is actually a case of Hill's 1979- yield function ok this Hill's 1979-yield function has got this particular characteristics it has got some restriction also which will lead to Hill's 1990 yield function.

This is just a general one I am describing we are not going to discuss much there is one more yield function called Hill's 1990 which is little bit complex $|\sigma_{11}+\sigma_{22}|^m + \frac{\sigma_b^m}{\tau_y^m}\left|(\sigma_{11}-\sigma_{22})^2 + 4\sigma_{12}^2\right|^{m/2} + \left|\sigma_{11}^2+\sigma_{22}^2+2\sigma_{12}^2\right|^{\frac{m}{2}-1}[-2a(\sigma_{11}^2-\sigma_{22}^2) + b(\sigma_{11}-\sigma_{22})^2] = (2\sigma_b)^m$ and you can understand that here there is only one small difference here that you have got τ_y, τ_y is nothing but yield stress in pure shear $\sigma_1 = -\sigma_2$ let us say that also comes into picture and a and b are material constants m has got usual definition like what we have discussed in the previous one. So we are not going to discuss much here only thing we should know that this type of Hill function exists and σ_b is again available here ok fine. Let us go to next one this condition this criterion is very important for us 1993 Hill's yield criterion described before generally shows the condition $\sigma_0 = \sigma_{90}$ enforcing $\mathrm{r}_0 = \mathrm{r}_{90}$ ok which you can check it by this particular equation correct which you already done. This equation is actually in connection with your anomalous behavior second one ok.

This is what is been said actually. However some material such as aluminum alloy and brass sheets they show almost equals yield stress ok they show almost equal yield stress but then *r* values are very different but there *r* values along rolling and transverse direction 0 ° is along rolling direction 90 is long they are very different 90 ° is along different. So some of the example you can see I have taken suppose this brass 70-30 sheet but this particular alloy you take it $\sigma_0 = 126\ MPa$ $\sigma_{90} = 125\ MPa$ they are almost same but you see $\mathrm{r}_0, \mathrm{r}_{90}$ they are very different which is actually breaking this particular requirement which is actually breaking this particular requirement provided by this particular equation. Similarly this particular alloy if you take σ_0, σ_{90} are almost same but $\mathrm{r}_0, \mathrm{r}_{90}$ are relatively different. These two examples show that ok we need to be careful using your Hill's 1948 yield function for some materials like this because this particular equation enforces these two conditions have to be there which is not the case in such materials.

This is referred as anomalous behavior second order. So now the question comes how Hill's 1993 is providing what is it providing. So in order to address the above concern Hill's 1993 yield criterion was proposed which has a generality of Hill's 1979 why because in Hill's 1979 it is addressing anomalous behavior number 1 which is what is said at the same time this Hill's 1983 also should address anomalous behavior 2 ok. So with that Hill's 1993 was first proposed in the biaxial tension zone. Biaxial tension zone means the first quadrant of your yield locus first quadrant of yield locus means this quadrant. σ_1 you can take σ_2 know this quadrant first quadrant of where is biaxial tension comes into picture know then this particular equation was proposed.

Ok you can just look into this equation ok so of course you know that what is $\sigma_1, \sigma_2, \sigma_0, \sigma_{90}$ all are known to us except three things one is c then p and then q these three are bit different and what is c? c is also given here. c is actually a function of σ_0 σ_{90} which are nothing but yield strength in 0 ° and 90 ° with respect to rolling directions which is defined now ok. If you know σ_0 , σ_{90} you can get *c, c* can be substituted here but you need to know *p* and q. So this equation actually can be satisfactory in two cases that is what is actually I have shown here. Suppose you take a uniaxial tension case ok suppose like you take along rolling direction you take one yield strength let us say σ_0 since it is uniaxial $\sigma_2 = 0$ fine. So you put this condition in this equation and check the left hand side would be equal to right hand side ok because $\sigma_2 = 0$ know then this entire thing would be 0 this fellow also will become 0, $\sigma_2 = 0$ means this fellow also will become 0 so then what will happen is $\sigma_1 = \sigma_0$ so these two will be equal because here it is 1 it is satisfied. Similarly you can also get so when you take a transverse direction so when you call that yield strength as σ_{90} and you put $\sigma_1 = 0$ here similarly left side and right side would be equal so thus the criterion is satisfied identically when the state of stress is uniaxial. So when you become uniaxial type of deformation this equation is satisfactory. You can also get if it is biaxial $\sigma_1 = \sigma_2 = \sigma_b$ you put it in this equation ok you can find out that it is going to give you this result which is what we have written it here also.

It is going to give you the same result ok. So you put $\sigma_1 = \sigma_2 = \sigma_b$ in this equation. I think you do the small exercise ok so you will get this particular equation σ_1 is replaced by σ_b, ${\sigma_b}^2$ will come that is all. So ${\sigma_b}^2$ will come. So if you simplify this you should get this particular equation which also was shown before here for *c* this is been satisfied so it is suggested that the equation is same as the equation provided in the previous slide this indicates that the criterion is satisfied in the balanced biaxial or equi-biaxial mode also ok. So now what is *p* and *q* that is what we are going to discuss and then we stop here about this particular discussion. So now as I told you before this yield function has 5 different fellows one is your σ_0 σ_{90} σ_b and then *p* and *q* ok we said I said *c p* and *q* but *c* depends on σ_0 σ_{90} and σ_b so I am writing σ_0 σ_{90} σ_b *p* and *q* are there ok with respect to this equation right.

So this *p* and *q* can be evaluated so there is a big derivation available for this so I have not deriving it here so I mentioned very clearly not derived refer some reference book so please some reference book so I have also indicated a reference book here look into it and then finally you will see that you will get the simple *p* and *q* equation which are actually functions of already known values r_0, r_{90}. So we are going to just see the end result here if this is the yield function ok and one of that is *c* which is already known so then this yield function is easy to use why because all the properties are available with us σ_0 can be obtained σ_{90} can be obtained σ_b also can be obtained. Now *p* and *q* have you have to evaluate again *p* and *q* equations also available for us in which only unknown things are actually r_0, r_{90} which also can be obtained ok. So now finally we can say that in order to use Hill's 1993 the yield function only 5 properties are acquired σ_0 σ_{90} σ_b which are already there now *p* and *q* instead of that you need to get r_0, r_{90} which are easily available how do you get you can do tensile test you can do *r* value test and you can do equi-biaxial tension test equi-biaxial tension test. So you plug in all these things into this equation this equation can be used to model any deformation So now this is the case only if you have in biaxial this is how it is derived now.

So now you want to expand or extrapolate this for other locations 1, 2, 3 these 3 you know zones in the yield locus there is a small modification that is allowed here I have marked it in red color that small changes only there otherwise everything remains same. So σ_1 becomes $|\sigma_1|$, σ_2 becomes $|\sigma_2|$ and you can extend it to get other quadrants. The above yield function suitable for first quadrant can be extrapolate other quadrants by using this generalized function. So I think with this Hill's 1993 it is said that it can address the anomalous behavior number 1 and number 2 that is what is said and that is one of the merit of this one both first order and second order anomalous behavior are addressed here it depends on only 5 material properties just now we have seen right σ_0, σ_{90}, r_0, r_{90} and σ_b which can be evaluated easily. What are the demerits? Useful only if directions of principal sources coincide with orthotropic axis that is why you have only σ_1, σ_2 ok right.

It is also said that it does not allow variation of anisotropic coefficient in axial yield stress in the plane of the sheet. It is another demerit we say but then we have to accept it and go ahead because it is satisfying the first two problems which faced by 1948 ok. So before 1993 1979 also can be used but it can address only the first one anomalous behavior first one that is what is said ok. So but the second fellow addresses both, 1990 addresses both fine. So I am stopping here with respect to yield function discussion so there are lot of other yield functions ok we have seen only Hill's 1948, 79, 90 and 93, Hill's 1948 has been described little elaborately ok and other three cases are discussed minimally to what is actually required for us with respect to its characteristics and merits and demerits.

So this particular table I took it from this particular book. So we redrew this particular table and this particular table tells you different yield functions available plus what are all the data required if you want to use this yield function. For example, Hill's 1948 you need as we discussed you need σ_0 then $\mathrm{r}_0, \mathrm{r}_{90}$, σ_0 or σ_{90} any one you need $\mathrm{r}_0, \mathrm{r}_{90}$, 79 also we have seen σ_0 then of course σ_b was introduced then r_0 is required, σ_{90} there are lot of things required you can look into that equation, 93 also we said only 5 $\sigma_0, \sigma_{90}, \sigma_b, r_0, \mathrm{r}_{90}$ are there. There are several other yield functions you can look into it ok various Hill's 1978 and Barlat, Barlat himself has two three important yield functions ok you can see that 1, 2, 3, 4 and KB is also there ok there are 4 parameters in this 6 parameters, 6 parameters so there are several such yield functions available which can predict the forming behavior accurately. Again it is recommended just a recommendation only that Hill's 1948 is a useful yield greater than model forming of sheets and simple to implement because properties can be available easily but however avoid using it for aluminum alloy forming ok. But if it is aluminum alloy then one can use 1990 or Barlat's yield function any one or KB's or others also whatever is available better suitable for modeling aluminum alloys however several material properties are required so you have to be little bit careful which one to use to predict what.

Suppose you want to see the tube forming of aluminum alloy you want to model it then use one of this the bottom ones 1990, Barlat's or something but then there could be several parameters material parameters you have to characterize ok but it is generally it is to avoid it but if it is a steel cube and you want to study its bending behavior or sheet you have to study its deep drawing behavior or any stamping behavior Hill's 1948 would be more than sufficient ok because the properties can be easily calculated ok. So this is what we can discuss about anisotropic yield function so we have seen 4 important yield function predominantly Hill's's 1948 and Hill's 1948 also we rewrote that equation in few different forms that is the main thing. Next one is important one is we using Hill's 1948 yield function we derived relationship between α and β which was earlier for us was a simple one without considering R value this α β had R value into that equation which can be used for anisotropic sheets that is the main thing. So before we complete this we let us briefly discuss about the difference between isotropic hardening and kinematic hardening what is it isotropic hardening was just introduced to you just now it is the case in which yield surface or yield locus remains of same shape but expands with the deformation or increase in stress it is what is shown here for you this is well known to you σ_1 vs σ_2 yield locus we always draw first yield locus initial yield locus and then subsequent yield locus when you deform the material. So I have drawn here a red color line you can see from here to here you can see the elastic loading and after that it is plastic deformation or strain hardening and then you can unload the material that is the way it is going to work and for the yield function if you want to define which already define the function is a let us say $f(\sigma_{ij}) - K = 0$ in a simple way you can write which also tells a fact that the shape of yield locus specified by initial yield function that f and size changes with respect to hardening parameter K.

I hope you remember that we introduced this one when we discussed about at the end of Von Mises yield function where we are going to discuss about $\bar{\sigma}$ at that particular you know slide or you know lecture you can see that we wrote a similar one and then I was discussing with you that one part of the equation on the right hand side is going to decide the form of the yield equation that is going to tell you the shape of the yield locus on the left hand side which is nothing but your I think $2\bar{\sigma}^2 = K$ we wrote I think that K was going to decide the size. So now this equation is written now so now what we are going to do is this is also something new for us suppose if you want to write Von Mises yield function so how do you write so Von Mises yield function at yield can be written in this fashion so f is equal to this is known to you right $f(\sigma_{ij}) = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} - Y$, Y is nothing but let us say σ_0 or σ_f like that we might have used to σ_f is not it. So now in this if you put 3 is equal to 0 it will lead to Von Mises equation we just now discussed 4, 5 slides back we use discussed that now when we study a case of Hill's 1948 it resulted in Von Mises yield locus know the same equation this equation is well known to us which is also equal to we can write the same equation in two other forms one is using J_2 the other one is using deviatoric stress $f(\sigma_{ij}) = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} - Y = \sqrt{3J_2} - Y = \sqrt{\frac{3}{2}S_{ij}S_{ij}} - Y$ this S_{ij} is nothing what is S_{ij}? S_{ij} is nothing but stress deviator tensor this also we have seen in expanded form when we derive Von Mises you know equation there is one particular stage where we derived that equation as a function of σ_{ij}' right that is nothing but this S_{ij} only there we have used $\sigma_1', \sigma_2', \sigma_3'$ here we are just using S_{ij} only where S_{ij} is nothing but deviatoric stress but only difference is here we are writing in indical notation this is index notation we are writing there we wrote expanded form so how to expand it one has to understand from different book we are not going to discuss here. Y is known to you what is J_2? J_2 is a second invariant of deviatoric stress invariance you know there are invariance like I_1, I_2, I_3 and J_1, J_2, J_3 ok so I_1, I_2, I_3 are generally related to stress tensor and J_1, J_2, J_3 are related deviatoric tensor and J2 is a second invariant of deviatoric stress that also can be written in this way. So these are the different forms we use these two forms are already known to us only this one is something new I have written here this also you should know that $\bar{\sigma} = \sqrt{3J_2} = \sqrt{\frac{3}{2}S_{ij}S_{ij}}$ which is what you have written here.

So now with respect to the previous equation what I am going to do is I am going to just substitute this you know this function ok this function is nothing but f is becoming now this is a function so I am going to use this $f(\sigma_{ij}, k_i) = \sqrt{3J_2} - Y - K = 0$ ok this is how Von Mises yield function can be written ok. So now isotropic hardening simply implies that if the yield strength in tension compression are initially the same they remain equal even in further deformation with increase in plastic strength which means Bauschinger effect is not considered here which means Bauschinger effect is not considered here. So I think we have also seen studied this in bending no spring back if you take σ versus ε so you deform it like this and then you unload it and then you go this way ok you will see that there is no change in yield strength because of softening in the compression side they remain equal they remain equal. So which is what is a general assumption we are making here so we are saying that if the yield strength in tension compression are initially the same they remain same during the evolution also which means you cannot use you cannot define or model Bauschinger effect with respect to this which actually lead to kinematic hardening. So what is that to model Bauschinger effect where hardening in tension will lead to softening in compression will lead to softening in compression which means that in the compression side your yield strength will be less than what it was in the first cycle in the tension side.

Then if you want to include that then we can include we can do that by kinematic hardening and kinematic hardening if you want to describe with respect to yield locus it will look like this again blue one is the initial one and the red one is the current one because of kinematic hardening. So this is a elastic part and this is a plastic deformation part and then unloading part is written and you will see here that the shape is same your yield locus actually gets translated in the stress space. The yield surface remains same shape and size but translate in stress space. The shape and size remain same but it gets translated it moves in the stress space which actually is a good one to model Bauschinger effect. So how are we going to write it? We are going to write that yield function any yield function you pick up with respect to kinematic hardening if you want to write it then you can write a $f(\sigma_{ij}, \alpha_{ij}) = 0$ where α_{ij} is called as back stress or shift stress. So what do you mean by α_{ij}? α_{ij} means it is a back stress or shift stress which can be defined schematically like this. So let us say σ_1, σ_2 you are drawing let us say this is initial yield locus. So the locus is shifted by α_{ij}, locus is shifted by α_{ij} relative to the stress space as shown in this figure. So you are going to shift the entire yield locus by α_{ij} that is what meaning of this particular equation. So now what are we going to do is very simple we are going to rewrite this particular equation in this format.

Now in this what I am going to do is I am going to use this equation is okay so I am going to use this known format for me. The deviatoric stress tensor this particular form $\sqrt{\frac{3}{2}S_{ij}S_{ij}} - Y$ now what am I going to do is this S_{ij} is a deviatoric stress tensor or no and what am I going to do here is $\sigma_{ij} - \alpha_{ij}$ right. You imagine that this α has got one deviatoric part which I am going to remove from S_{ij} okay which is what I have written here. By considering Von Mises material again Von Mises is used but kinematic hardening which is the simplest case Von Mises isotropic hardening we have seen just before okay Von Mises kinematic hardening is what we are looking at here. Other yield function and then kinematic hardening is further complex by considering Von Mises material and using deviatoric part of $\sigma - \alpha$ okay deviatoric part of $\sigma - \alpha$ instead of deviatoric part of σ okay. So I am going to write that $\sqrt{\frac{3}{2}S_{ij}S_{ij}} - Y = \sqrt{\frac{3}{2}\left(S_{ij} - {\alpha_{ij}}^{d}\right)\left(S_{ij} - {\alpha_{ij}}^{d}\right)} - Y = 0$ where α^d is nothing but a deviatoric part of α.

Since we are speaking about a deviatoric stress then π plane or deviatoric plane will come into picture and what is it? It is what is given in this particular figure in π plane or deviatoric plane we know what is it deviatoric plane means on the deviatoric plane you know how Von Mises going to be it is going to be a circle right which we discussed long back on the deviatoric plane Von Mises going to be a circle in 3D actually it is a surface a cylinder right. So when you look on the deviatoric plane it will look like a circle okay. So now with respect to that what is this $S - \alpha^d$ is going to tell that is what is discussed in this particular figure. So in π plane the deviatoric part of α deviatoric part of α^d let us say denotes shift of Von Mises circle as shown in this figure okay what happens so this is your Von Mises circle ${\sigma_1}', {\sigma_2}', {\sigma_3}'$ you see this is a Von Mises circle okay on the deviatoric plane pi plane we have given here. Now this circle is shifted here okay let us say by α^d times α^d shift is seen here with on the π plane then $S - \alpha^d$ denotes this particular vector if this is the S what is S? S is nothing but S_{ij} which is a deviatoric part α^d is nothing but the deviatoric part of back stress and S minus α d is this which is what is actually put here to define your kinematic hardening Von Mises material using Von Mises yield function.

So it is very simple to tell $\sqrt{\frac{3}{2}S_{ij}S_{ij}} - Y = 0$ which is a the conventional Von Mises yield function is going to be modified as $\sqrt{\frac{3}{2}\left(S_{ij} - {\alpha_{ij}}^{d}\right)\left(S_{ij} - {\alpha_{ij}}^{d}\right)} - Y = 0$. So this part is the deviatoric part of the back stress that is what is written here of course one can write it with respect to σ_{ij} also that is another way people write okay that is also possible there is only one small thing that we are going to do now what is that we are going to see how this α is connected to your plastic strain increment or the direction of plastic strain increment. So this linear kinematic hardening model it defines actually how yield surface is translated with respect to direction of strain increment you can see that suppose this is a $d\alpha$ okay how is it actually connected to the direction of $d\varepsilon^p$ direction of $d\varepsilon^p$ is known it is actually perpendicular to the normal drawn at any point in the yield locus right. So in linear kinematic hardening model we are introducing here the back stress assumed to depend on plastic strain following this particular equation you can say that your $\alpha_{ij} = C\varepsilon_{ij}{}^p$ that means it is linearly related using C where C is actually a material parameter and it is constant when you speak about linear kinematic hardening model okay. So what does that mean it means that yield surface is translated in the same direction as a top plastic strain increment linear kinematic hardening model as shown in the schematic if this is the direction perpendicular to it your $d\alpha$ or α is going to be in the same direction but if C if your C changes deformation then you need a non-linear kinematic model which is not discussed here you can go through it okay.

So now if you want to replace α_{ij} by $C\varepsilon_{ij}{}^p$ we can do that in the previous equation and you can modify the kinematic hardening model. So thank you. Thank you.