Mechanics of Sheet Metal Forming

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Week- 07
Lecture- 18
Stretching of Sheets

Okay, so let us start a new topic stretching of circular sheets and in this particular topic, we are going to develop some models for evaluating some you know output properties say for example, pressure or force like that during stretching operations and we have seen in the previous couple of classes about deep drawing and bending. Similarly, we are going to develop some models here also and this is going to be a small section okay and some theories which you already seen will be adopted here to develop such models and whenever we speak about stretching of circular sheets, we can do that actually practically speaking we can do that in two ways. Okay, one is by using punch contacting the sheet okay, so punch along with other tools in bracket I will write tools in general okay. So what you do here is there will be a sheet which will be grabbed on the die and there will be a blank holder and the punch will come and deform the sheet okay and here the interface friction between the punch on the sheet is going to play a vital role in stress distribution or strain distribution, fracture location okay, the depth of fracture, you know forming depth all these things will play a, your friction is going to play a big role in this. And though this is the case of course when you use punch you can also change the friction right, so depending on the lubricant you use friction you know coefficient of friction that is prevailing at the interface one can control such you know deformation quantities. Another way is basically with fluid pressure okay, so that means what? That means there is no rigid punch instead we are going to deform the sheet with the help of fluid pressure okay.

So the tools could be slightly different, so you can have a maybe a die or open die or a shaped die and there will be a chamber through which you are pressurizing the fluid and then that fluid is going to deform the sheet to take up the particular shape or a free shape okay. So this is what the main difference here of course when you use fluid pressure you can understand that there is no friction between the sheet and the fluid okay. So which is the main difference with respect to the you know when you use a conventional deformation of sheet with the help of punch okay specifically in stretching. This using punch deforming the sheet we already seen several examples though we have seen that in bending and you know deep drawing and in the initial part of the modules okay we have seen stamping operations where punch has been used to deform the sheet.

So we have seen that predominantly and all those theories are valid here also except that we have not seen anything about this fluid pressure based deformation which will be the first part in this particular module okay. So the first heading is bulging with fluid pressure okay bulging with fluid pressure and this can also be called as hydrostatic bulge test in short they call it as HBT like uniaxial tension test we have like UAT we say similarly we can say hydrostatic bulge test okay this is nothing but bulging of a sheet with the help of fluid pressure okay. So you know this bulging with fluid pressure if you use it as an experimental method to characterize the sheet properties then this becomes hydrostatic bulge test otherwise bulging with fluid pressure is a method to deform a sheet and you can make any component shape with that okay otherwise we can also characterize it, characterize the sheet with the help of hydrostatic bulge test like uniaxial tension test we have similarly we can use hydrostatic bulge test. So schematic is described here so you can see that at the bottom there is a let us say there is a die and there is a retainer or a blank holder here you can say and initially the sheet is let us say flat okay initially the sheet is flight like this and then in between you give a fluid pressure let us say P that is a pressure with which your sheet is going to deform that is the amount of pressure that you give and let us say ρ is the radius of curvature we have and let us say at one particular stage your deformation height your forming height is about h okay. So you can imagine like there is a pressure versus height you know curve okay you can draw a graph between pressure versus height so with increase in pressure what happens to height that again that kind of characterization one can do and you can say that a is basically this particular dimension from the axis to the clamping location here okay and r is basically any radius from the axis r is basically any radius from the axis okay.

So I have written here that HBT is a common experiment used to characterize the material or a sheet a stress strain response at higher strain levels okay so like in uniaxial tension factors we characterize the materials stress strain behavior but in that case basically you may get the sheets characteristics only up to a particular limited strain value let us say here you can get to a larger extent okay. So you can deform a material the same sheet let us say aluminum or steel if you do it with HBT you can get a strain response to a larger extent to larger strains that is the main advantage if you speak it in terms of characterization I think we have seen this briefly in the formability test okay in one particular chapter we have seen this anyway. So let us come back to this you can see that it is made up of a metal sheet here okay this is your sheet it is partially formed sheet that is inserted in a die with a circular aperture so die has got a circular aperture here that is at the center you can see there is a hole and through which the material is going to deform okay and it is secured and deformed using an internal pressure okay that is hydrostatic pressure P okay. So and of course our main focus is basically this a region okay on both the sides you can say a region and let us say sheet thickness is t okay so you can pick up this a region only and we can show this kind of diagram about your state of stress we know that already you know like σ_θ is basically your one of the principle stress and $\sigma_\emptyset$ is going to be another principle stress okay. So $\sigma_\emptyset$, σ_θ already known to us okay so your σ_θ is on the circumferential direction and $\sigma_\emptyset$ we said that in the deep drawing the $\sigma_\emptyset$ was represented as a cup wall stress, stress in the cup wall $\sigma_\emptyset$ we represented.

Similar one you can see a direction it is in this particular direction as compared to σ_θ and

σ_3 is always there perpendicular sheet thickness otherwise σ_t that also can be seen. So these are the stresses acting on at any element in this particular deformation zone that is a, a zone on both the sides okay. So the sheet is bulged to a spherical shape by a fluid pressure so you can imagine that this particular shape is approximately spherical shape you can imagine like that and if you look into the state of stress and strain okay so we are going to concentrate mainly in this pole region this topmost point in this deformation zone know this is called pole okay this is called pole region okay. So if you look into the state of stress and strain in this pole region that means you are going to take an element from this and you are going to see what is the state of stress and strain of course you will have σ the same thing will happen $\sigma_\emptyset$ will be there and σ_θ will be there okay corresponding strains also will come into picture and at this location if you see you are going to have $\sigma_\emptyset$ and then $\sigma_\theta = \alpha.\sigma_\emptyset$ okay and you have a $\sigma_t = \sigma_3$ which is going to be 0 because of plane stress assumption that we are doing right from the beginning. So you can get the nomenclature you know what do you mean by $\sigma_\emptyset$? $\sigma_\emptyset$ is like σ_1, σ_θ is like σ_2, σ_t is like σ_3 which you are regularly following until now there is a slight change here other than this everything is same.

So if you look into the pole okay this pole region it is characterized by $\alpha = 1$ what I am saying is in the pole region if you pick up an element here and see what is the relation between $\sigma_\emptyset$ and σ_θ they both are equal $\sigma_\emptyset$ is equal to σ_θ. So basically we say $\alpha = 1$ we say $\alpha = 1$ we say so when you say $\alpha = 1$ I am going to write σ_θ is equal to $\sigma_\emptyset$ directly I am going to write. So that is one important thing so now if you know if $\alpha = 1$ what is β that can be calculated again by using Levi Mises flow rule it is equal to 1 okay. So and I am going to write $\varepsilon_\emptyset = \beta.\varepsilon_\theta = \varepsilon_\emptyset$ and $\varepsilon_t = ln\frac{t}{t_0}$ new thickness divided by original thickness at any location okay and if $\varepsilon_\theta = \varepsilon_\emptyset$ then you can write $\varepsilon_t = -2\varepsilon_\emptyset$ we have already seen this already. So you can get $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ and in that 2 strains are same so ε_t or ε_3 okay nothing but ε_3, $\varepsilon_t = -2\varepsilon_\emptyset$ okay.

So this is what is main important state of stress and strain in hydrostatic bulging okay and just to extend this point we have you know the procedure of evaluating $\bar{\sigma}$ and $\bar{\varepsilon}$ also for any situation we have seen that several times in this case also we can get $\bar{\sigma}$ and $\bar{\varepsilon}$ so we can use Von Mises effective stress equation which we have derived before I just written here for your convenience so $\sigma_f = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \bar{\sigma}$ only so what will you get a $\bar{\sigma}$ is if you put $\alpha = 1$ here it is going to be equal to σ_1 which is nothing but my $\sigma_\emptyset$ okay you can directly write here and if you put $\beta = 1$, 3, 3 will be cancelled so you will get 2 ε_1 okay so which is nothing but $\bar{\varepsilon} = 2\varepsilon_\emptyset = -\varepsilon_t$ okay so you can get $-\varepsilon_t$ here. So in hydrostatic bulge test or when you stretch a sheet with a uniform fluid pressure P okay then we can write $\bar{\sigma} = \sigma_\emptyset$ only and $\bar{\varepsilon} = 2\varepsilon_\emptyset = -\varepsilon_t$ right. So of course one can you know see what is the comparison with respect to uniaxial tension test okay uniaxial tension test one can get what is α so $\alpha = \frac{\sigma_2}{\sigma_1}$ so $\alpha = 0$ is going to be equal to 0 in UAT it is going to be equal to 0 then what is β we can find out and we can also get such relationship for uniaxial tension test just for comparison right so that is one thing so this is one important result we have here. So now our main aim in this particular module is to get this P what is the can we derive an equation for P can we derive an equation for simple equation again okay there is no complexity there is simple equation for P okay so for that what I am going to do is I am going to use an equation which we already know we have not derived in this particular course we have not derived but I think we already known that you are this kind of you know stretching of or deformation of circular shells or circular sheets we can relate $\sigma_{\emptyset} = \sigma_{\theta} = \frac{P\rho}{2t}$ because $\alpha = 1$ okay I think we have already studied this probably in solid mechanics one can look into it so $\sigma_{\emptyset} = \sigma_{\theta} = \frac{P\rho}{2t}$ and all are usual definitions P is your pressure and ρ is indicated in the diagram and t is your sheet thickness. So this is one result we will come back to this okay there will be some small equation we are going to substitute here but before that we can also see how strain distribution is happening okay how strain distribution is happening when you see the deformed sheet from the pole to a region along radius okay pole means this is the pole to a region means the gripping region okay so from pole to this a from the gripping region this from a distance okay in the radial direction so along radius we are going to see how strain distribution is going to vary so naturally we have only two surface strains in plane strains one is you have $\sigma_{\emptyset}$ *and* σ_{θ} from these two we can get ε_t okay so we are not going to include that here so you will see that from pole which is at the center here to a which is a gripping region we have seen that you have of course same $\sigma_{\emptyset}$ *and* σ_{θ} at the pole region why because it is $\beta = 1$ okay the membrane strains or membrane strains or you can say the surface strains are equal at the pole okay where $r = 0$ if you put here so both the strains will be same why because $\beta = 1$.

Now at the edge of the sheet that means when you are coming here at the edge of the sheet this is your edge it is clamped no it is clamped so the circumverential strain at the edge is 0 the circumverential strain at the edge is 0 so you will see that your ε_{θ} will go to 0 but $\varepsilon_{\emptyset}$ will have some value $\varepsilon_{\emptyset}$ will have some value okay so ε_{θ} so the one corresponding to this will be 0 at the edge and this fellow will have some value which is what is given in this particular graph okay. And you will see that from the top two from the pole region to a both the strains are going to decrease both the strains are going to decrease okay which means when you do a fluid pressure based deformation of sheet or which is equivalent to saying a frictionless case totally frictionless case then there are chances that your surface strains in plane strains are going to be maximum at the pole region and it is going to diminish when you move towards when you move away from the pole okay so this region is of focus to that is why we always bother about pole okay. So your strain is going to be maximum at the pole region if it is a frictionless case for example if it is a hydrostatic bulge test then you can expect peak strain at the pole region and it is going to diminish in this way so ε_{θ} will be 0, $\varepsilon_{\emptyset}$ will have some value. So what is the importance of this particular test? So the primary justification for employing this test lies in its ability to induce substantial strain in materials with minimal strain hardening even before reaching failure okay so it means that suppose if you take a material with probably you know it has got minimal strain hardening suppose tectility itself is not that good so getting its characteristics stress strain characteristics using the actual test could be very restricted okay in such type of materials also you can use this hydrostatic bulge test okay and you get sufficient you know stress strain data before failure happens. And this particular FLD, FLD we are aware of that okay it is between you know ε versus ε_2 in this case basically $\varepsilon_\emptyset$ and ε_θ I have drawn and this is your typical forming limit curve we know that right this is your typical forming limit curve of that any material you can imagine okay.

So your of course we know that in this you know hydrostatic bulge test where the strains are equal $\beta = 1$ then your strain path is going to be orange color 1 the bulge test will follow this particular strain path or mode of deformation and it will reach forming limit curve at this particular location okay and there you are mentioning these two limit strains as $\varepsilon_\emptyset^* \; and \; \varepsilon_\theta^*$ like $\varepsilon_1^* \; and \; \varepsilon_2^*$ we are aware of this nomenclature what is it from our instability chapter okay. So you will see that it is going to reach your forming limit curve along this. Now on the other hand when you speak about uniaxial tension test which is on the left most you know on the second quadrant on the left side you will see that the material is going to follow this particular black strain path and it is going to reach your forming limit curve here okay but we also know that tensile instability occurs at let us say $\varepsilon_u = n$ okay which is going to be somewhere here making in the tensile test may start here itself okay and you will see that this value is going to be larger than this value okay you know your bulge test value is going to be slightly larger than this fellow. So in a way you can say that okay the membrane strains at the point of failure under biaxial stress that is in bulge test denoted as $\varepsilon_\emptyset^* \; and \; \varepsilon_\theta^*$ in the FLC figure surpasses the strain observed at necking in the traditional tension test okay. In this way one can understand that when you do hydrostatic bulge test there is a little advantage even for low ductile material that you can have a stress strain behaviour comfortably which is not possible in your conventional uniaxial tension test and this I already mentioned okay so it becomes apparent that effective strain equals twice the membrane strain experienced during biaxial tension which is not the case in uniaxial tension test you can find out what is the value we already done that before okay.

So because of this small advantage other than uniaxial tension test if you want to have good fit for your stress strain curve okay beyond your tensile fracture okay then one can go for this kind of hydrostatic bulge test fine. So now let us try to get an equation for pressure a simple equation for pressure okay which is not again very accurate it is going to be an approximate model but it will give you a quick feel for what would be the pressure and how to calculate it. So here again we are going to consider bulging of a circular diaphragm it is like a sheet only thin sheet you can imagine okay and we are going to develop an approximate model for that. So with respect to the previous figure this particular figure you can imagine this particular figure okay we can write this area of deformed surface is $2\pi\rho h$ so ρ is the your bend radius and h is the deformed height if you multiply that 2π times then you will get area of the deformed surface that is one. So you want to equate volumes before and after deformation we can do that so initial one is up to clamping region so naturally $\pi a^2 t_0 = 2\pi\rho h t$ if you get it so you will get t , $t = t_0 \frac{a^2}{2\rho h}$ okay.

So a is a constant we know this a is also fixed ρ may change with respect to a and h okay and your h will anyway change height is forming height will anyway change. So now again as usual we are going to consider $\bar{\sigma} = K\bar{\varepsilon}^n$ let us say and we can write in a simple equation $\bar{\sigma} = \sigma_\emptyset$ you know, $\bar{\sigma} = \sigma_\emptyset$ which you have already derived. So $\sigma_\emptyset = \bar{\sigma} = K{\varepsilon_t}^n = K\left(ln\frac{t}{t_0}\right)^n$ here. This will be useful for us we are going to simply put this equation $\sigma_\emptyset$ into the previous equation for pressure.

So now pressure to bulge diaphragm is given by we are going back to this equation so $P = \frac{2\sigma_\emptyset t}{\rho}$ right and what I am going to do is very simple so in place of t which is a new thickness I am going to put my previous equation $t = t_0\frac{a^2}{2\rho h}$ I will put and finally I will get this equation and instead of that I can write $\frac{\bar{\sigma}a^2t_0}{h\rho^2}$. So finally I will get this equation for pressure $t = t_0\frac{a^2}{2\rho h}$. So a^2t_0 will remain here and fine. So and you will get this particular equation and the only important points that you should you know like some couple of important points that you should note here is like in the previous equations which we did not write in the previous chapter your $\sigma_\emptyset$ either you can take a constant flow stress or you can make it as a function of strain.

So in a way you can consider strain hardening if not you can have an average flow stress also like what we have seen in deep drawing and the only concern is this ρ. So this ρ of course can be if you can relate it to a and h then it becomes all known values all known values. So that one can get it otherwise from this you can get a graph between P and h one can get a typical characteristics of P and h. So now as I said when the sheet is bulging the pressure may peak at $dP = 0$ this is what I was telling you. So if you get P versus h there will be one peak let us say for example something like this I am just randomly drawing.

So at this location you will see $dP = 0$ we can put after which bulging will continue with the decreasing pressure gradient because it indicates something is happening in the material it could be instability just due to pressure without any friction. So which means that when the strain at the pole approaches forming limit as shown in forming limit curve earlier rupture will happen. So we have fixed that fracture is going to happen at the pole region and fracture is going to initiate there and if you put instability condition like $dP = 0$. So you can compare these two cases like when is your strain going to reach your forming limit curve like we have shown here this particular point and when $dP = 0$ these two can be compared to quantify your instability but it may so happen that your limit strain may reach FLC little after $dP = 0$. But if you consider this diaphragm or sheet as a load carrying member then you have to be very careful that maximum pressure point constitutes instability and fracture.

For maximum pressure point this particular point is actually should be the deciding criterion for your if it is a load carrying structure or if you are deforming it one should see the difference between $dP = 0$ and when ε_1^* and ε_2^* is getting reached. So one has to be careful in these two otherwise for hydrostatic bulge test a simple equation for P is obtained by $\bar{\sigma}$ which can be related to your ε as a function of ε you can say $\frac{a^2 t_0}{h\rho^2}$. So now with this simple analysis let us go to stretching over hemispherical punch. Now fluid pressure based deformation is done let us say same material is actually stretched over a hemispherical punch like what we have seen in limiting dome height test which we have discussed in the formability chapter. So similar situation is shown here you can see that the sheet is you know held in this particular location.

So and there is a hemispherical you know punch the punch has got hemispherical end at the this particular location you can see and all the values are given here. So you will see that the sheet is actually getting deformed and one thing which you should know here when you deform it using a punch is interface friction what is this fellow going to do ok what is this fellow going to do in this interface that is very important for us to know. And you pick up an element as usual like the way we say we discussed generally you pick up an element here in the deforming zone anywhere maybe here or here anywhere here but little away from the gripping and the pole region and you plot the you know tension here ok. So your normal pressure P is going to act on the sheet element and your $T_\emptyset$ is going to act in this way and $T_\emptyset + dT_\emptyset$ is going to act in this way this is similar to what we have seen in simplified stamping analysis. A similar diagram we drew similar one here ok and because of P you have μP here which is nothing but your you know effect of friction comes into picture ok.

So there is some change in tension which is on one side it is $T_\emptyset$ the other side it is $T_\emptyset + dT_\emptyset$ ok. So we know that the sheet tension escalates proportionally with the displacement of punch right because it is gripped in this two regions so naturally when you make punch to get displaced in this direction the tension will actually escalate it will increase right. So now when you pick up a frictionless case when you pick up a frictionless case in scenarios where there is negligible friction between the sheet and the punch the highest strain emerges at the pole as shown in this particular figure ok. The same figure which I have used before same figure which I have used before the higher highest strain emerges at the pole as shown in this figure so this is where you have highest strain. So because it is going to have a larger strain we expect a failure also to happen in the pole region through like you know maybe tearing or splitting like that the material will tear apart in this particular location F.

So it is very simple so strain distribution for frictionless cases already we know that so failure is going to get initiated at the pole because there it is strain is going to be larger as compared to other locations. So now if you do not put lubricant this is a frictionless case no frictionless case means you put lot of lubricant ok lubricant lubricated let us say I am writing well lubricated let us say ok frictionless case means well lubricated ok. So now there is another case where there is no lubricant ok or a dry friction maybe you can say the case with friction. So now this frictionless case is little impractical actually there is going to be friction whatever lubricant you can put ok then there is going to be it is going to be challenging to keep it frictionless otherwise one has to go for hydrostatic bulge type. So presence of friction ok it may change certain strain patterns in the in the sheet strain distribution in the sheet which is what very crucial for us to know what is it.

The strain distribution $\varepsilon_{\emptyset}$ ok is plotted with respect to r ok from pole to A let us say in this two are compared one is without friction blue colour other one is with friction red colour ok. So that is what I have shown in the second point due to the influence of friction context as that is μP just now we introduced this μP component here ok because of this μP it is going to this μP this is going to actually pull the point of maximum tension ok away from the pole region ok. So this μP what is going to do is because ok it is available somewhere in the deforming region ok if you see this because of the presence of μP ok in the case with friction this is going to actually pull the maximum strain from the pole you know towards some other location away from the pole ok. So that is the main job of this μP here the point of maximum tension and strain is located away from the pole region. So which means if you draw $\varepsilon_{\emptyset}$ from pole to A then it will follow this red colour pattern.

So it will be having some value at the pole region and at a particular location let us say your r let us say it will be maximum and then again it will diminish again it will diminish so when compared to without friction case this is very different you will see that. So the peak value is going to happen not at the pole but little away from the pole this much distance away from the pole it is going to change ok. So whereas this is the pole region that is the main function of this friction. So this specific location becomes the anticipated failure point susceptible to splitting. So now here in this case you can see F here here in this case you can see F here ok.

So with friction failure is going to happen little away from the pole region maybe at r distance when compared to the case without friction where failure is going to happen at the pole. So you can also see that maybe this fracture this failure location could be circular in nature ok. So you can see you know from the top view you can see from the top view that this r location is going to be there at any radius in the circular diaphragm isn't it. So it may split you know depending on how instability develops it may split in the entire ring of the circular path ok between the peak and the gripping region somewhere in the distance of r it will face. This is the main difference between with friction without friction case ok.

So let us come to effect of punch shape. What is this punch going to do? So we are taking two extreme cases one is a flat bottomed punch. Flat bottomed punch means like this ok. So there is a vertical you know portion of this punch as usual and there is a small corner here and the bottom is flat ok bottom is flat like this. There is a small corner bottom is flat ok.

So in this diagram you will see that there are there is one upper figure and lower figure. Upper figure is going to tell you about thinning distribution ok with respect to r ok. So that means when you move from pole to that means let us say from the topmost point of the sheet let us say for example from here ok to this particular region how thinning is going to change ε_t thickness strain but we have put minus here so how thinning is going to change is what is shown in the upper figure and lower figure is going to tell you two different stages when you have you know with friction and without friction ok. Two things are shown here actually. So one this is a flat bottomed one ok with friction and without friction ok.

So and you will see that this thickness strain distribution is not same ok. In one case you will see that it is going to peak you know somewhere here in the other case peak is somewhere here ok. The peak value on the with friction case is going to be larger as compared to without friction if you go for flat bottomed case because your μ is going to play a big role only in the corner region nothing much will be there in the flat region. So I am saying the friction is limited to radius of the punch corner in the flat bottom punch. Radius of the punch corner means this radius.

This radius no. So it is limited to that particular location ok. This prevents the material from being stretched over the flat face of the punch as a result of increase in tension there. So naturally tension will be more in this particular location and hence your face region may not play a big role in stretching operation and what will happen here is since you know your friction is going to play a role only in the corner region ok in with friction case that means less lubricated with friction case you will see that thinning is going to dominate in this location as compared to the other case. So at the same stage ok you will say that thinning is going to dominate so naturally what you can do is so obviously in without friction case you can go for larger draw as larger stretching as compared to with friction case. As a result maximum depth or forming height that can be reached is lesser in the case of with friction.

That is why I just made this difference you can see this difference this difference comes into picture mainly because in the friction less case that is on the left hand side on the left hand side ok you may have reached a larger forming height ok maximum depth can be reached is lesser in the case of with friction case ok. That main reason is the corner region because there is a lot of friction the corner region thinning will be dominant in this location as compared to the other one. So now if you move to the other one pointed punch or hemispherical punch something like that you can imagine then the situation is shown in this diagram. Similar diagram thinning is plotted with respect to your radius and you have two locations one is left side ok and then the other one is right side. So right side you see that friction is available that is with friction left side is without friction in this case it is actually opposite.

In this case it is going to be opposite ok. So if it is with friction ok that means your μP is going to be there ok then we can imagine that your strain distribution thinning distribution is going to be something like this ok though we know that your peak is going to be away from the pole region and it is going to be lower as compared to the case in the left hand side that is the case with without friction ok. So this is going to have better forming height as compared to the other. So pointed punch has an opposing effect when there is no friction that means on the left hand side the stress is focused closer to the nose and depth before failure is constraint ok that we already studied. Friction has the effect of reducing the tension at the nose and dispersing the strain across a larger area allowing for deeper forming. So it will not allow ok your strain to concentrate in a localized region ok like in the pole rather what is going to do is it is going to distribute the strain uniformly when compared to you know without friction case and hence you will see that peak strain is going to be less as compared to this, this value you see it is going to be less so you may have a larger forming height.

So your friction and shape of punch is going to play a significant role in determining the forming height and strain pattern and localization of strain and fracture and failure ok one has to be careful ok in flat bottom punch how to manage μ ok how to manage lubrication and in pointed punch like hemispherical punch how to manage μ friction one has to be little bit be aware of what is the change. So before we complete this particular section let us see a small subsection stretching of a sheet with a hole, stretching of a sheet with a hole so ok we are not going to derive any equation rather we have seen some of this before and you have just picked up the details and let us try to understand some important parts here. Stretching of sheet with a hole means what suppose you have a sheet ok and then you are stretching it that you already seen let us assume that you have a sheet and there is a hole at the mid there is a central hole and that has been grabbed and stretched like this. A simple schematic I have drawn here let us say this is your you know let us say this is your punch is red colour one let us say this is your punch ok so let us say this is your sheet and there is a hole at the centre let us say like this. So hole radius let us say r_i and let us pick up r_0 as a reference radius for us ok at r_0 radius there is a tension $T_\emptyset$ that is acting on the material $T_\emptyset$ that is acting on the sheet so you are stretching it with the help of $T_\emptyset$ apply tension ok and hole will actually expand ok.

So this what we are studying is applicable for hole expansion test. So hole expansion test I think we already discussed in formability chapter basically it is a formability test which will tell you know the deformability of or formability of the hole of the entire sheet along with the hole. So when your hole is going to crack and formability will be stopped will be over is what you can find out from hole expansion test there are standards available but here we are just giving you a schematic so a circular bank with a hole stretched over a doomed punch is shown hole radius is r_i and $T_\emptyset$ tension is applied as shown ok. So now at the edge of the hole ok when you see this edge, this edge is what is going to crack fail it should fail ok with the deformation right. So if you see $T_\emptyset$ how is it moving how is changing from the this particular location r_0 to r_i ou will see that $T_\emptyset = 0$ ok at the edge because it is actually a hole itself ok and a state of uniaxial tension in the circumferential direction will exist as per Tresca yield locus.

Suppose if you follow Tresca yield function ok then your $T_\emptyset = 0$ which means that there is going to be only one stress which means that it could be uniaxial in the circumferential direction ok and this tensions can be plotted in Tresca in the first quadrant as shown in this particular right side figure ok. So I have drawn a tension locus, tension basically instead of you know stress based yield locus I am drawn a tension locus I think we have studied this at the end of your second chapter ok where T_1 and T_2 can be used to draw a locus like σ_1 *and* σ_2. Instead of σ_1 *and* σ_2 I have just drawn here T_1 and T_2 which is nothing but T_θ and $T_\emptyset$ ok and I have just drawn the first quadrant which is what is interesting to us ok. So that is why I have mentioned a part of second quadrant and a part of fourth quadrant here.

So this is your first quadrant. So the first quadrant of Fortresca is shown here and you will see that you know when r_i at $r_i$is here, r_i location that is at the whole edge is here where $T_\emptyset = 0$, that is what I mentioned is $T_\emptyset = 0$ at the edge ok. So this is nothing but your $\bar{T}$ ok or other words you can say T_Y yield tension or $\bar{T}$ ok $\bar{T}$ is what we have seen in the previous chapter, the redrawing chapter. So $\bar{T}$ could be your yield tension ok when this locus is actually getting satisfied. So when you move from r_i to r_0, so r_0 may not be equal to $\bar{T}$, so this height is $\bar{T}$ again here, this height is nothing but $\bar{T}$, it may not reach $\bar{T}$ ok, it will be here in this location and here you will see that there is going to be some $T_\emptyset$ and there is going to be some T_θ. So that will be the situation of your hole and away from the hole with respect to tension.

So now if you want to derive a simple equation for this tension $T_\emptyset$ I want to get this $T_\emptyset$ok we can apply force equilibrium and we can find out but what I would show here is we will use this differential equation which we have already derived in deep drawing process and we are just going to simply follow that ok. So in the deep drawing we already derived that you remember that we consider there is an element in the flange region ok and then we apply you know force equilibrium along the radial direction, so all these calculations we can get and you can see that $\frac{d\sigma_r}{dr} + \frac{\sigma_r}{t} \cdot \frac{dt}{dr} - \frac{\sigma_\theta - \sigma_r}{r} = 0$ ok and we also said that this term will be 0, why? Because we said t is not going to change, $t = t_0$ in that, this fellow will become 0. So what I am going to do is this part I am removing ok I am going to use this and this I am going to just rewrite it for this particular hole stretching problem and I am write in terms of tension I am writing, so $\frac{\mathrm{d}T_\phi}{\mathrm{d}r} - \frac{T_\theta - T_\phi}{r} = 0$, this part is removed everything else I am just keeping as it is which I am going to rewrite again as $\frac{\mathrm{d}T_\phi}{\mathrm{d}r} - \frac{\bar{T} - T_\phi}{r} = 0$ everything will remain same, this fellow is going to change why? Because this $T_\theta = \bar{T}$ ok at *r*, so I am going to write this, so I am just given a hint here tension in the sheet are $T_\emptyset$ and $T_\theta = \bar{T}$ ok and $T_\theta = \bar{T}$ ok. So now I am going to put a boundary condition and I am going to integrate it, so what is the boundary condition? At $r = r_i$, $T_\emptyset = 0$ right, and if I integrate it I will get a simple equation $T_\emptyset = \bar{T}\left(1 - \frac{r_i}{r}\right)$, what is r_i? This *r* is any radius away from the axis ok. So the above equation can be drawn schematically to show tension distribution during hole stretching which is what I have shown here, suppose if you plot *T* versus *r* you can see that so again as I said $T_\emptyset$, would be r_i, at r_i if you see it would be 0 that means at the hole edge and it will increase and will reach a point at r_0 ok and T_θ would remain almost constant at one particular value right, so this is one way to analyze it.

Now what is the similarity with respect to circular bank without a hole? Ok in circular bank without a hole we know that $T_\emptyset = T_\theta$ right, why? Because we already said $\sigma_\emptyset = \sigma_\theta$

why? Because it is $\alpha = 1$, same thing works out here $T_{\emptyset} = T_{\theta}$, so this can happen from the tension locus figure we can say that this can happen, this can occur only if these two are equal to $T_{\emptyset}$, only if these two are equal to $T_{\emptyset}$ otherwise this cannot happen right. And it is also obvious because when you say $\alpha = 1$ put it in that equation before ok you will see that this fellow will lead to $\sigma_{\emptyset} = \sigma_{\theta} = \bar{\sigma}$ is equal to we said instead of that we are writing $T_{\emptyset} = T_{\theta} = \bar{T}$ ok. So now this differential equation what will happen? So here this also will go off this will be removed ok you can simply say $\frac{dT_{\emptyset}}{dr} = 0$. In fact you can also try you know with respect to maybe you can use in this case we said α is equal to in axial know in this case we said $\alpha = 0$ you can say, say for example if we say at the edge ok $T_{\emptyset} = 0$ then your it may reach the other one $T_{\theta} = \bar{T}$ right so that can be obtained even if you take some α value if you assume uniaxial α and then you put it in the one mices and find out you may get this let us you can try that. Ok so now this equation we are getting so only difference here is only the first part of the previous equation comes into picture circular bank without a central hole which is nothing but the previous one a conventional you know stretching.

So this indicates that $T_{\emptyset}$ does not change with radius and hence your stress distribution is uniform which follows $T_{\emptyset} = T_{\theta} = \bar{T}$ right and this is what we also got before $\sigma_{\emptyset} = \sigma_{\theta} = \bar{\sigma} = \frac{P\rho}{2t}$. So with respect to your stress distribution and tensor distribution you can check the similarity between ok or you can make a case you know sheet with a central hole and sheet without the central hole you can make a case between these two you can compare these two and one can understand these things ok. So we stop here we again discuss in the new thing in the next chapter. Thank you.