Mechanics of Sheet Metal Forming
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Week- 07
Lecture- 16
Cup Deep Drawing

So, this particular module we are going to discuss about Cylindrical Cup Deep Drawing ok. So, this is a new model and then let us see how to develop some expressions equations for predicting certain things in deep drawing, Sheet Cup Deep Drawing. So, we are going to specifically discuss only about Cylindrical Cup Deep Drawing in this particular module. So, this Cylindrical Cup Deep Drawing we already discussed this briefly in the first chapter, this is how the process will look like, this is schematic. You will see that so initially we have a flat sheet, initially we have a flat sheet like this and on top of that you are going to rest the punch and before punch starts moving down we have to clap the sheet on the die ok that will be done with the help of a blank holder ok.

And we know that we have to give appropriate blank holding force, we know how to get blank holding force that we already discussed in the stamping analysis. So, we have to give appropriate blank holding force so that the inward movement of the sheet can happen with some restriction due to the contact friction between the sheet blank holder and the die ok. And when the punch moves down so you will see that basically the cup bottom is formed first, this is your cup bottom we know that, this is your cup bottom that is going to form first. And after that if we see the entire process is all about converting the flange region into cup wall.

So, this is your cup wall region that also we know already and this is your flange region of the cup. So, once a cup bottom is formed that can form just when a punch touches the sheet a little movement maybe one of a movement itself is sufficient to create the cup bottom and after that your cup wall will form and that is going to happen due to the inward movement of this flange. So, it is all about converting a flange into your cup wall in this way. So, it is going to move like that and the schematic shown here is a partially drawn cup that is why you have a cup wall and a flange region ok. If you make the punch to move down further then the entire flange region will become a cup wall ok.

So, and previously we have already discussed what is the mode of deformation or maybe state of stress in the cup wall and in the flange region we have discussed these two cases ok. I am going to divide this deep drawing of circular cup ok, deep drawing of circular cup can be viewed as two different process of course, these two happen continuously, but you can view this as a two different operations or process. One is stretching sheet over a circular punch that is what I was telling you initially that the cup bottom is getting just get stretched below the punch and drawing an annulus inwards. So, drawing an annulus inwards is basically like converting this flange region into cup wall. So, you are pushing the you are drawing the this annulus region annulus region means this entire this diametrical region in the inward direction ok and the cup wall is going to transmit force between these two regions, cup wall is going to transmit a force between these two regions ok.

So, this is schematic from the schematic and from this you know view we can in fact, say that the deep drawing is nothing, but drawing an annulus inwards and if you see a top view of this ok, if you see a top view of this that is what is given in the right side diagram is a drawing of an annular flange I have said ok. So, you will see that so, this is your outer radius of your that is R_0 this is you can say this is o this is R_0. So, R_0 is basically your initial radius of the sheet you can say ok and r_i is basically the inner radius ok or you can also say this equivalent to punch radius you can also say this as a punch radius or punch dimension you can say or you can also convert that into diameter and say punch diameter and outside diameter you can say sheet diameter. So, you can see that the punch is actually perpendicular to the plane of this diagram and it is moving downwards let us say ok the sheet is actually getting a inward movement. So, this part is actually going to move inwards ok, so that it forms a cup.

So, there has to be an appropriate diameter or radius of the sheet because of which you can form a successful cup ok. So, larger the blank or smaller the bank may not be sufficient ok to make the actual cup which we want actually ok. So, we have to keep that in mind actually. So, if you see that we can define this you know flange into maybe like three different locations I mentioned A, B and C, A at the edge of the flange, B somewhere in the middle and C in the die corner you can see here. So, this is your C this is your C and this is your C you can say ok.

So, I have located it here A, B, C locations these three locations are there in the entire flange region you can imagine ok. And the state of stress in the A region B region are mentioned here this you have already discussed in other chapters. So, a region there is no σ_r here. So, σ_r is basically radial stress along the radial direction along r direction along r direction ok. By the way r is basically any radius ok in between your r_i and R_0 ok.

So, you pick up an element let us say here this element blue color element which I mentioned it is at r radius from the axis ok. So, r can vary between r_i and R_0 you can imagine like that. And let us go to this A. So, you will see that along the radial direction you will have σ_r that σ_r is missing here because it is at the edge of the flange σ_r will not be there at the edge of the flange. If you move inward direction somewhere in the middle region you will see that you will have both σ_r acting in this direction and σ_θ is in the compression direction ok.

So, σ_θ is inward arrow mark I put which is basically compressing type and σ_r is going to be pulling type. And C would be at the your die corner region ok. So, these three locations are important for us and what I am going to do now is I am going to consider an element in the flange region and we are going to apply force equilibrium. So, let us pick up an element in the flange region ok and I have already plotted all the you know stresses available in this particular element. You can imagine that this element is basically this in this location in this location you have this element let us say.

So, now you will see that so this element has radius of dr which is at r distance from the from the mid and this element subtends an angle of $d\theta$ ok with respect to the the axis and you will see that there is one σ_r radial stress and there is one σ_θ which is circumferential stress. And we know that at at a gap of dr distance let us say from this to this there will be some change in σ_r which you are going to call it as σ_r plus d σ_r. And if you see from thickness direction ok, so the thickness direction though we say t_0 as a initial thickness ok during deep drawing you will see that in the flange region ok there will be slight thinning in the die corner somewhere here and there will be slight thickening in the edge of the flange $t + dt$ ok that is why I mentioned it $t + dt$. So, and here it is t. So, these are the stresses on an element at radius r ok.

So, this r is nothing, but this r only ok. So, now what I am going to do is I am going to apply equilibrium for this element and if you do that you will get this particular equation you can look into it is $(r + d\sigma_r)(t + dt)(r + dr)d\theta = \sigma_r . t . r . d\theta + \sigma_\theta . t . dr . d\theta$ and there will be one component of σ_θ along r direction. So, that is going to be $2\sigma_\theta . t . dr . sin\, {}^{d\theta}/_2$ which can be written as $\sigma_\theta . t . dr . d\theta$ you can in a simplified way you can write this. So, basically $sin\, {}^{d\theta}/_2$ can be written as ${}^{d\theta}/_2$ and you can write it in this way $\sigma_\theta . t . dr . d\theta$ ok. And of course, you can simplify this further.

So, I have not done it. So, you can one can do this if you do this it will reduce to one simple equation $\frac{d\sigma_r}{dr} + \frac{\sigma_r}{t} \cdot \frac{dt}{dr} - \frac{\sigma_\theta - \sigma_r}{r} = 0$ ok. So, and this is for a non-strain hardening material ok you do not consider strain hardening in this equation and it is a frictionless case. So, no effect of a friction ok these two are not available in this equation. So, the first part basically tells it with radius how σ_r changes the next part is going to tell you how t changes with r you will come back to this ok.

And then your $\frac{\sigma_\theta - \sigma_r}{r} = 0$. So, we will come back to this equation now how to solve this how to get basically our idea here is to get a σ_r ok. So, our idea is basically to get a σ_r which is nothing but your radial stress that is the whole idea here ok. Because if you get σ_r by assuming any yield criterion we can get a σ_θ also ok because these two are principle stresses we can get σ_θ also. So, now before we come to this equation ok let us get into some details of this ABC.

This ABC point which we have shown here A is at the edge B is at the mid and C is at the die corner ok covers the entire blank you know flange region and these three points are actually shown in the yield locus in this way ok. So, A point which is at the outermost edge of the flange ok we have said that $\sigma_r = 0$ only σ_θ would be there σ_r radial stress will end there ok it will not be there at all it is going to be 0 ok. So, that is why you will see that $\sigma_r = 0$ means it will fall in the X axis ok and it will be on the second quadrant it is going to coincide with X axis this point A ok. And we have already seen that at this location sheet will thicken at this point sheet will thicken at this point ok. So, we have already seen this as an example ok and we also I think worked out your you know why sheet has thicken in fact, the problem also one problem numerical problem we solved in the previous chapter also tells that ok if you follow this particular mode of deformation the sheet will have thickened would have thickened when compared to the previous its original thickness.

So, you have to you should know that at the edge location at the outer edge that is why we have said $t + dt$ ok. So, you can write the state of stress as σ_θ is nothing, but minus $-\sigma_f$ ok which is known as uniaxial compression ok and the σ_f is nothing, but our current flow stress our current flow stress before reaching current flow stress it would be your initial flow stress or the yield stress. So, now when you go to point B ok. So, we have to pick up a point B in such a way that basically we are picking up a point B in such a way that the radial stress is equal to opposite to hoop stress ok. So, the radial stress your $\sigma_r = -\sigma_\theta$ that is what we have said in this point B.

So, they are equal and opposite and this will give your $\alpha = -1$, if $\alpha = -1$ then you will see that you can get a β and at this particular mode of deformation you will see that there is no change in thickness at this point ok. If $\alpha = -1$ you can get a β from Levi Mises flow rule and you will find out that there is no change in thickness at this particular point. So, now when you go to point C radial stress would be maximum at that location because that is basically a die corner region ok and it is also expected that sheath thinning is going to happen in this location ok. So, I mentioned the C point somewhere here ok. So, this ABC points in the flange region is going to fall on the second quadrant A which is going to be $\sigma_\theta = -\sigma_f$ it is going to coincide with the X axis and you have opposites and minus, B we are choosing in such a way that σ_r is equal and opposite to your σ_θ and at C location you will see that σ_r would be maximum, radius would be maximum and here sheath thinning will happen.

So, the edge you will have thickening and the somewhere in the mid location you will have no change in thickness. So, so that means thickness strain is going to be 0 and in the die corner region you will have sheet thinning. So, that is the way this ABC points are actually characterized and you will also see that we already mentioned this particular one in the in the previous chapter that ok. So, anything below this particular location, below this mode of deformation you will have thickening which is what is characterized here and beyond this in all other β values you can see that it is going to thin down. So, which is basically at C location there is chances of thinning.

So, at B basically is we are picking up B in such a way that these two are actually equal and opposite. So, there will be no change in thickness ok. So, this can be compared this particular characterization this thickening and thinning which is mentioned here can be compared with the previous discussion we had similar diagram we already discussed ok. So, all the stress states like this can be represented in terms of let us say for example, a Von Mises you know yield locus like this. So, now let us go back to this equation ok.

So, now just for a change in this analysis we are going to use a Tresca yield function which is you have not used until now we have used only Von Mises yield function until now. If you use Tresca yield function this yield function is already derived in the previous chapter. So, $\sigma_\theta - \sigma_r = -(\sigma_f)_0$, $-\sigma_{f0}$ is nothing but the initial flow stress nothing but the initial flow stress ok. So, why we are saying that is because we are going to now replace this fellow by σ_{f0} to get into some details. So, now what I am going to do is I am going to make one important assumption that by considering $t = t_0$ ok.

So, what will happen now by considering $t = t_0$ ok. So, we will say that in this equation this part will vanish this part will go away ok. Why because we are saying there will be some change in thickness with respect to *r* value ok which is not the case here we are saying that $t = t_0$ ok which means change in thickness is actually neglected in this equation. So, you have differential equation is nothing but this first term and this second term first term and this second term ok. So, now what we are going to do is so there is a small change in this equation this equation is not valid here we are going to say that this equation , $\frac{d\sigma_r}{r}$.

So, $\frac{d\sigma_r}{r} - \frac{\sigma_\theta - \sigma_r}{r} = 0$ ok. So, if you want to integrate this equation ok so and by following this boundary condition what is the boundary condition we know that at R_0 that means at the edge of this flange region we say $\sigma_r = 0$ right. So, and another limit is basically this inner radius ok at $r = r_i$ we are going to say that there is some radius stress that is $\sigma_r = \sigma_{ri}$ ok. So, if you see σ_r how is it going to vary it will be σ_{ri} here and it will be 0 here it is going to vary between these two and let us integrate this particular equation ok by taking $t = t_0$ and by applying this boundary condition you will see that you will get a simple equation $\sigma_{ri} = -\sigma_{f0} \ln \frac{r_i}{R_0}$ ok. So, of course this can be written as $\sigma_{f0} \ln \frac{r_i}{R_0}$ ok and because this equation is known to as $\sigma_\theta - \sigma_{ri} = -\sigma_{f0}$.

So, now I am going to substitute this σ_{ri} in this equation and get a $\sigma_\theta = -[\sigma_{f0} - \sigma_{ri}]$ok. So, you have to integrate this equation and not this let us forget this equation ok this is not correct here. So, this equation we have to integrate by considering $t = t_0$. So, there is a second term which goes off ok and you will get $\sigma_\theta = -[\sigma_{f0} - \sigma_{ri}]$ ok. So, in this way for a non-stain hardening material for a non-stain hardening material of course without friction ok the radius stress is given by σ_{ri} equation derived here ok and radius stress will give you σ_θ by assuming one yield function here we are picking up Tresca yield function and we are getting it ok.

So, now in this equation you will see there is one important point which is what we are going to discuss in the next slide. We have already discussed briefly about drawing ratio, drawing ratio means basically its initial diameter of the sheet divided by diameter of punch ok. So, drawing ratio is nothing, but this called as DR we call ok. So, drawing ratio is nothing, but initial diameter of the sheet divided by diameter of the punch and this draw ratio will give limiting draw ratio which we call LDR, limiting drawing ratio which we call LDR considering this diameter of punch as a constant then what diameter initial diameter of the sheet you can keep ok to have a successful cup that will be decided by this LDR that will be decided by this LDR. So, which means in a way you see that it is $\frac{R_0}{r_i}$, R_0 is nothing, but your radius of the sheet which is nothing, but diameter of the sheet divided by r_i which is a inner radius which is nothing, but a punch diameter is going to give you some way it is going to define your LDR, is going to define your LDR.

So, we are going to put a condition now, we are going to put a condition now ok which will give us some value of this LDR, what is that? We are going to say that the greatest stress in the cup wall that it can sustain for a material being Tresca is actually σ_{f0}. So, in the cup wall if you see ok the greatest stress the material can withstand if you follow Tresca yield function is actually σ_{f0} itself. So, what we are going to say is in this equation if $\sigma_{ri} = \sigma_{f0}$ ok what will happen? Now, if $\sigma_{ri} = \sigma_{f0}$ we get the largest blank that can be drawn which is also given by. So, this equation I am picking up which is was introduced to you before $\sigma_{ri} = -\sigma_{f0} \ln \frac{r_i}{R_0}$ in this if I put σ_{f0} you will see that my $\frac{R_0}{r_i}$ is going to be equal to 2.72 which is nothing, but LDR is actually $exp\, 1$ ok.

So, I am going to put a σ_{f0} here and of course, this minus can be taken care automatically. So, you will get $\frac{R_0}{r_i}$. So, $\frac{R_0}{r_i} = exp\, 1 = 2.72$. So, $\frac{R_0}{r_i}$ which is nothing, but in a way to describe define LDR is nothing, but 2.72 if you pick up an extreme case of this particular one ok. The greatest stress the cup wall can withstand for a material it should not actually go to that level, but it is the greatest stress ok. If it goes to extreme then the LDR can be 2.72 that means, that means what if r_i is fixed suppose your punch radius or diameter of the punch is fixed you have to multiply it by 2.72 to get the initial diameter of the or initial radius of the sheet and only if you pick up that particular diameter you will get a successful cup ok.

But as I said this particular value $\sigma_{ri} = \sigma_{f0}$ is a too high and because of that what I am going to see is the instead of this 1, I may slightly reduce it depending on my you know requirement I am going to say let us say $LDR = exp\,(\eta)$ ok. And if you take $\eta = 0.7$ let us say or 0.6 to 0.7 0.62 let us say 0.7 I get LDR of 2 that means, exponential let us say instead of 1 you take exponential 0.69 or 0.7 or 0.6 you can take it will give you a value lesser than 2.72, 2.72 why because 2.72 is an extreme case. So, if you pick up an η value of 0.69 you will get LDR of 2 which means that your R_0 by your $\frac{R_0}{r_i} = 2$. That means, if you take r_i let us say radius of the punch or diameter of the punch let us say you take the diameter of the punch as 50 mm ok. So, then you multiply that with 2 the initial diameter of the sheet has to be about 100 mm or closer to that.

Generally 0.6 to 0.7 you can pick up which will give you a range of LDR which is going to give you a successful drawn cup. Through this derivation simple derivation though it is for non-strain hardening material and frictionless case we derived expressions for σ_r radial stress and σ_θ ok by assuming Tresca yield function this is the first time we are doing it in this in this course ok. And if you put a condition for σ_r ok which is nothing but if it is equal to σ_{f0} then you will get a limiting raw ratio as a particular value that is 2.72 and is an extreme case anything less than that would be better.

So, if you take η of 0.7 or 0.6 in between value so you may get a LDR of the order of 2 which is what is you can refer to. This is the simplest way to calculate your limit drawing ratio. So, now what we are going to do is so this equation has been obtained and from this equation we got σ_θ also. So, what we are going to do is we are going to modify the σ_r by considering a strain hardening effect and friction effect ok. So, modification of radial stress with the effect of strain hardening that is a next one.

So, now what is going to happen is because of strain hardening we know that the flow stress will increase right the flow stress will increase and if you pick up this flange region if you pick up this we will go to the diagram if you pick up this particular flange region ok. So, you will see that at different locations you may get different levels of flow stress ok and it will be non uniform it will be non uniform. So, you can imagine that you know like previously we discussed you know. So, you can maybe like you can put a lot of circle grids on the surface and or maybe you can get strains at you know different grids and from there you can get a $\bar{\sigma}$ from there you can get $\sigma_1, \sigma_2, \sigma_3$. So, all those things you can do in this flange also finally, you will see that the flow stress is going to be get distributed in a non uniform way in the flange region.

So, how are you going to quantify it is by taking an average flow stress in the entire flange region. So, which means you can imagine that from the strain evolution you can get maybe like $\bar{\varepsilon}$ and from there by using some flow stress model you can get some $\bar{\sigma}$ and you can just take an average value you will get some average value of your flow stress in the whole flange. Now, what I am going to do is this is my original equation I am going to just modify the σ_{f0} by $\left(\sigma_{f0}\right)_{avg}$ I am just saying average ok. So, which is what going to give $\sigma_{r_i} = \left(\sigma_f\right)_{\text{avg}} \ln\left(\frac{R}{r_i}\right)$ ok. Of course, this is inversed actually $\frac{r_i}{R_0}$ becomes $\frac{R_0}{r_i}$ and since I am going to update my σ_f ok by using this $\left(\sigma_{f0}\right)_{avg}$ I am going to use R instead of R_0 where R_0 is my initial you know outer radius right.

So, that will get updated depending on what average flow stress I am going to take. So, I am going to put R ok. So, that diagram is shown here you can see this is just a simple schematic this is one section of you know flange which is showing the movement flange movement in the drawing you will see that initially the flange region is up to this black part. So, which is actually R_0 ok and after some height let us say height h ok after some flange let us say cup height your so your cup wall height as let us say h ok the new radius is R. So, this radius I am going to update it here that is one small change we are having.

So, a simpler way to take strain hardening is to take an average flow stress value ok in the entire flange region and that is going to change my equation to $\sigma_{r_i} = (\sigma_f)_{\text{avg}} \ln\left(\frac{R}{r_i}\right)$ instead of R_0 I am going to put $\frac{R}{r_i}$ ok. So, now there are two things one is a strain hardening ok we expect that you know once a deep drawing happens once it started happening. So, flow stress will increase and hence you need a larger drawing force let us say or radial stress at the same time R is reducing at the same time you will see that the material required for drawing is also reducing because more and more you push punch down more flange region is getting converted into your cup wall region right your cup wall this is your cup wall region ok more flange region gets converted into cup wall region. So, naturally R becomes smaller. So, these two together is going to create one particular pattern of your evolution of σ_r which is what is given in this particular figure.

So, for a strain hardening sheet you will see that σ_r is of course plotted with height the height of cup you know it is plotted with σ_r you will see that initially there is an increment in the σ_r value in this way and it reaches a peak value here I represent it as a star and then in further punch movement will actually increment in height will actually reduce the radial stress. This is how a typical plot would be for a strain hardening material, for a strain hardening. Suppose the material is not strain hardening at all then this your radius your radius would be larger at the starting itself because the flow stress will not get updated after that so naturally it will be higher at the starting itself that is how it will be but for strain hardening material this is a typical you know nature of your radial stress it will increase for some time and once you reach a peak value it is going to come down. And these two factors your strain hardening and R becoming smaller together is going to determine this evolution. So, this equation $\sigma_{ri} = -\sigma_{f0} \ln\frac{r_i}{R_0}$ becomes this equation by considering an average flow stress in a way taking care of your strain hardening effect.

So, now how are we going to modify this equation by including friction that is the next part. So, when you speak about friction so we are going to divide this further into two parts, one is effect on die radius you can see that effect on die radius I have mentioned this particular schematic which I showed you just before and this particular location we are going to concentrate and that is actually zoomed in here. So, you will say that in this the material is actually material is flowing this direction it is already given and your σ_{ri} is acting in this direction which we just now seen and the σ_{ri} can be converted into $\sigma_\emptyset$ which is actually stress in the cup wall ok. So, stress in the cup wall is this, this is going to be your $\sigma_\emptyset$. So, σ_θ circumferential σ_r is radial and $\sigma_\emptyset$ is nothing but a cup wall radius stress in the cup wall stress in the cup wall ok.

So, somehow this σ_{ri} and σ_θ can be connected and that can be done by this particular equation which we have already discussed. This was discussed in one of the previous chapters we have derived this $\frac{dT_1}{T_1} = \mu d\theta$ and in fact if you integrate it between two different points in between if you know the angular value then you can integrate it and you can get attention in the new location with respect to the original location ok. We derived an equation for this we worked out a problem also in this ok. So, now the same thing can be rewritten in this way $\frac{d\sigma_{\emptyset}}{\sigma_{\emptyset}} = \mu d\emptyset$ and of course you can integrate it finally you will see that $\sigma_{\emptyset}$ would be equal to your sorry this is $\emptyset$ actually this is $\emptyset$. So $\sigma_{\emptyset} = \sigma_{ri} exp\ (\mu\pi/2)$ ok.

So, the angle subtended angle is actually $\pi/2$ here so you will see that the coefficient of friction coulombs coefficient of friction will come into the brackets so it will be same thing only μ is going to coming here and $\pi/2$. So, we will see that your $\sigma_{\emptyset} = \sigma_{ri} exp\ (\mu\pi/2)$ ok. So, which can be obtained from the simple equation. So, this equation can be directly written ok without doing all these calculations also you can directly come out this equation based on your attention also which you already derived. So, if σ_{ri} is known from the previous equation so we can get $\sigma_{\emptyset}$ also using this particular equation ok.

So, σ_r can be found out and from this you can get a σ_θ which already done so from this you can also get a $\sigma_{\emptyset}$ radial stress circumferential stress now this is a cup wall stress, stress in the cup wall. So, that is this part ok. So, now we are going to this particular location between the blank holder and the die there is a flange region and that situation is shown here friction between the blank holder and the flange ok that is actually zoomed in in this particular schematic. So, this schematic basically tells you that so this is your part of the flange region you can imagine this part of your flange region if you see this location it is somewhere here this location this location is what is actually zoomed in here ok. So, now what we are going to say is though in this analysis we have taken $t = t_0$ so that there is no change in thickness, but originally in the schematic we have also shown that there is some change in the thickness at the edge ok at the edge of the flange you will see that at the edge of the flange here or at the edge of the flange here you will see that there is some thickening that is going to happen which is that is why we said $t + dt$ right.

So, because of that what we are going to say is we are going to tell that this blank holding force we are going to call it as B is distributed around the edge of the flange as a line force. So, we are going to say that practically your blank holder is going to come down and hold the entire flange region. So, blank holder will have perfect surface contact with the flange region and flange region will have full surface contact with the die surface that is how practically it works, but for modelling point of view we are saying that there is a blank holding force B that is going to act as a line force on the edge of the sheet, why because basically it is there is an inclined surface here as I shown here. So, this thickness here is larger than this thickness let us say this thickness is t, this thickness would be $t + dt$ here ok and because of that it is quite acceptable to consider this blank holding force as a line force, but of value $\frac{B}{2\pi R_0}$ per unit length. So, B is a blank holding force ok, so I am going to put a line force that is $\frac{B}{2\pi R_0}$ per unit length I am going to put that here.

So, that is why I plotted a downward arrow and then an upward arrow at both the surfaces ok. So, the material flow is going to happen like this, so naturally your friction force is going to be opposing nature, so I am going to write friction force as $\frac{\mu B}{2\pi R_0}$ here and $\frac{\mu B}{2\pi R_0}$ here, so combinedly we can write $\frac{2\mu B}{2\pi R_0}$ is a friction force, the friction force on the flange per unit length around the edge again is given by $\frac{2\mu B}{2\pi R_0}$ ok. The same situation can also be represented by this σ ok which is acting at the edge of the flange which is nothing but this $\sigma = \frac{2\mu B}{2\pi R_0 t}$ ok. So, the same thing the friction force can be termed can be written as stress acting on the edge of the flange that is nothing but your σ as this 2 2 will be cancelled anyway $\frac{\mu B}{\pi t R_0}$ ok. So, I am going to write where t is my actually it is $t + dt$ ok, but is understood that it is actually thickness at the edge of the flange that is all.

So, basically you have to use thickness at the edge of the flange here. So, I am just telling you that $t + dt$ for our explanation and t here that does not mean that you have to use that thickness. So, you can use thickness here. So, thickness at the edge of the flange. So, the edge of the flange ok can be used here.

So, $\frac{\mu B}{\pi t R_0}$. So, I am going to say there is stress acting on the edge of the flange that means, my $(\sigma_{ri})_{r=R_0} = \frac{\mu B}{\pi R_0 t}$ where t is a blank thickness. So, now what I am going to do is my initial σ_{ri} is actually modified to σ_{ri} ok by considering an average flow stress by considering an average flow stress right. So, this is σ_{ri}. So, again this is also σ_r. So, I can add this part to this part to get the effect of friction into that equation.

So, what I have done is here is we derived s$\sigma_{r_i} = \left(\sigma_f\right)_{\text{avg}} \ln\left(\frac{R}{r_i}\right)$ right. So, the above equation this equation I am going to modify like this. So, I am just going to add this part to this part. So, $\sigma_{r_i} = \left(\sigma_f\right)_{\text{avg}} \ln\left(\frac{R}{r_i}\right) + \frac{\mu B}{\pi R t}$ where R is actually a changing radius $\pi R t$ ok. So, now you will see that finally the σ_r ok which is nothing but the radial stress has got some effect because of strain hardening and then you are also bringing up your coefficient of friction in a way lubrication effect friction effect into this equation ok.

So, this is the first level of σ_{ri} and this is the second level of σ_r from this I am going to get next σ_{ri} by considering effect of μ that is a third one sorry μ plus $\left(\sigma_f\right)_{\text{avg}}$ both both this is the third one. So, three different ways we have done actually. So, now this equation this equation you say is radial stress for a strain hardening material with effect of friction that way you can tell and since σ_{ri} is known to you you can get $\sigma_\emptyset$ also how do you get you can just substitute it in this equation σ_{ri} you substitute here that means the entire σ_r is multiplied by

$exp\,(\mu\pi/2)$ to get the $\sigma_\emptyset$ which is what I have given as $\sigma_\emptyset = \frac{1}{\eta}\left[\left(\sigma_f\right)_{\text{avg}} \ln\left(\frac{R}{r_i}\right) + \frac{\mu B}{\pi R t}\right] exp\,(\mu\pi/2)$ ok in that way you can modify this equation ok. So, what are the things we got? We got σ_r then we got of course σ_θ the same σ_r can be used to get σ_θ also right you assume a Tesca yield function.

So, you substitute here. So, σ_r can be used this equation instead of this equation you use the later equation into this to get σ_θ. So, what do we have found out is σ_r we found out σ_θ we found out and then $\sigma_\emptyset$ also we found out from all these equations. With the effect of strain hardening and friction. So, these are expressions for radial stress and cup wall stress including strain hardening effect and friction effect ok. So, now there is one coefficient $\frac{1}{\eta}$ I have included ok multiply it $\frac{1}{\eta}$ just to take care of it is like an amplification factor.

So, some you know it is like an efficiency factor you can say to take care of any other approximation ok. Like for example, the sheet gets bent and un-bent right the sheet gets bent and un-bent now. So, that is not considered here for example, because of this bending ok. So, initially it is a flat then it is bent then it is again un-bent. So, it becomes straight right say for example, this is flat it is bent here then it is un-bent to become straight isn't it.

So, this type of effects will also lead to some change in your stress value. So, just to get you know some efficiency factor you can multiply it $\frac{1}{\eta}$ to take care of that changes ok. So, this value can be you know can be it is in our hand we can decide it to improve the accuracy of this cup wall prediction ok. So, now in this last subtraction but in this discussion there is going to be some relationship between this LDR we are say LDR we said right what is LDR we have said that it has $\frac{R_0}{r_i}$. So, there is some relationship between this LDR and the materials anisotropy that is sheet anisotropy. So, what is the connection? So, that is what we are going to see more in a very conceptual way we are not going to derive much rather what we are going to do is we are going to just discuss that in conceptual way.

So, from the previous formula equation we derived this LDR actually depends on the average flow stress we know that and the current thickness B and then maximum permissible value of wall stress which is nothing but my $\sigma_\emptyset$ ok. Average flow stress we said $\left(\sigma_f\right)_{\text{avg}}$ we brought it together current thickness is t this is your B ok and maximum permissible value of wall stress is $\sigma_\emptyset$ ok. So, there is a particular level of wall stress that is allowed when you do cup deep drawing that is your $\sigma_\emptyset$. $\sigma_\emptyset$ is a stress in the cup wall, but there has to be permissible limit ok. So, now let us discuss these points if we neglect strain hardening during drawing the maximum drawing stress will happen at the start of the drawing this is what I was discussing with you before if you do not consider strain hardening then the simplest choice we have is drawing stress being maximum can be maximum at the start itself because your flow stress is not going to increase beyond that ok.

It could be the effect of R becoming smaller only ok. Two opposing factors we discussed strain hardening and R becoming smaller now strain hardening is not there means the effect of R radius becoming smaller the material you know material required for drawing is reducing so R becoming smaller is the only thing which is going to determine so maximum drawing stress will be reached at the start itself. Now we are going to put a condition wall stress to initiate that drawing suppose if you want to initiate the wall stress ok what will you do the simplest choice we have is $\sigma_f = Y$ which is becoming a constant where Y is our initial flow stress or yield strength you can say so wall stress to initiate drawing or at the start of drawing ok the wall stress can be obtained by putting $\sigma_f = Y$ which is a constant in the previous equation. So from the previous equation for wall stress we can write now wall stress now wall stress becoming we are going to concentrate on cup wall now rather than flange. Now flange is being converted into a cup wall so now what is happening in the cup wall is the main thing now. So $\sigma_\emptyset = \frac{1}{\eta}\left[\left(\sigma_f\right)_{\text{avg}} \ln\left(\frac{R}{r_i}\right) + \frac{\mu B}{\pi R t}\right] exp\ (\mu\pi/2)$ right so what I am going to do is I am going to put this condition into this equation into this equation so this I am going to convert that into Y so I am going to say that general flow stress is going to become my yield stress and I am going to say that my $\sigma_\emptyset = \frac{1}{\eta}\left[Y \ln\frac{R_0}{r_i} + \frac{\mu B}{\pi R_0 t_0}\right] \exp\frac{\mu\pi}{2}$ only thing is like I am going back to my original dimensions R_0 and t_0 here or because you know it is a start of the drawing no so we are saying that the wall stress to initiate drawing so which means that I am going to modify this equation into this is R_0 and this is t_0 ok.

So this $\sigma_\emptyset$ so in this equation these are the modifications I am going to put so this becomes Y my $\sigma_\emptyset = Y$ everything else remains same and this related to original dimension ok. So now this wall stress has to be less than the load carrying capacity of the wall so wall has got a particular load carrying capacity you know you can imagine like that and this $\sigma_\emptyset$ has to be less than that. And we have also seen before the cup wall actually follows plane strain mode of deformation we have seen this example before in the when we discuss about stress contour strain contour like we have discussed OA, OB, OC, OD, OE mode of deformation ok. So like this diagram you can see that we saw OB, OA, OB there are 5 different you know we have seen 5 different you know stress paths we have seen. So in that we have mentioned this as an example that for plane strain mode of deformation the cup wall is a best example.

So we have earlier seen that the deformation in the cup wall follows in the cup wall follows plane strain mode ok and the stress at it would deform depends on the choice of yield criteria ok and the stress at which it would deform depends on the choice of yield criteria ok. So we know that this cup wall stress should be less than the load carrying capacity of the wall ok if that is the case ok if that is the situation then your cup wall should deform in plane strain mode of deformation ok and it has to reach that particular value of stress and that depends on what choice of yield function you have ok whether you choose Tresca or Von Mises or any anisotropic yield function yield locus so it depends on that yield locus which is going to tell you when that critical you know point is reached ok for this $\sigma_\emptyset$ ok. So in that case we have shown here three different choices for this what are the three different choices you can see that this is for representation of Tresca yield locus this is your Von Mises yield locus and this is some anisotropic yield locus ok. So in Tresca yield locus you pick up OP so plane strain mode of deformation is actually OP ok plane strain mode of deformation in the cup this is the loading path in the cup wall for various yield locus ok. This is the loading path so you are going in the flange region this is the way your stress is going to evolve so it will start from O it will reach P when yielding is going to start at Y and you will see that $\sigma_{\emptyset}$ would be of this value.

So same thing if you see from Von Mises yield function you start with O you will reach P ok and you will see that Y it is going to reach Y like this ok and your $\sigma_{\emptyset}$ would be this much value ok. And if you pick up an anisotropic yield locus for which we are taking one particular case $R > 1$. So true width strain divided by true thickness strain is nothing but your $R > 1$. $R > 1$ means what? So $R > 1$ means suppose like this ratio is 2 which means that your the material is stronger in the thickness direction than in the plane direction in the plane surface right so that is the meaning. So if you pick up $R = 1$ generally you will see that the yield locus gets elongated in the right hand diagonal ok you can see the elongation in the right hand diagonal in this way and other things are same you have OP which will reach it in this particular location your Y is here this much would be your $\sigma_{\emptyset}$ this much will be your $\sigma_{\emptyset}$ ok.

So this is what we were telling that the stress at which it would deform depends on the choice of yield criterion. So when it is going to reach this particular point this particular point this particular point actually is decided by the choice of yield function you choose. So now let us pick up all these three cases we will discuss something which will tell us how this sheet anisotropy or the choice of yield locus changes the LDR calculation. So Tresca yield locus it is pretty simple ok for Tresca yield locus the cup wall stress would be equal to Y so $\sigma_{\emptyset} = Y$ we have seen this here $\sigma_{\emptyset} = Y$ ok. So now this particular equation which you have just now we have shown this equation ok we said that you know to initiate your drawing ok just to initiate drawing ok then you have to put $\sigma_f = Y$ then we have got Y here is not it.

So now what we are going to do is as per your Tresca yield function we are going to say that $\sigma_{\emptyset} = Y$. So this equation is modified to this form ok so you can say that this $\sigma_{\emptyset} = Y$ ok so then η can be taken here Y can come here Y can be taken inside so this Y vanishes and this Y will come in the denominator of the second part ok. So I am going to get $\eta = \left[\ln\frac{R_0}{r_i} + \frac{\mu B}{\pi R_0 t_0 Y}\right]\exp\frac{\mu\pi}{2}$. So this is the condition for maximum sheet size this is the condition for your maximum sheet size. So suppose if you use Von Mises yield locus ok then we already know this particular equation $\sigma_{\emptyset} = \frac{2}{\sqrt{3}}Y$ ok this is plane strain in bending also we derive this equation in bending also assuming it as a plane strain process we derive this equation $\sigma_{\emptyset} = \frac{2}{\sqrt{3}}Y$.

So now in the same way like what we have done in the Tresca yield function you can substitute $\sigma_{\emptyset} = \frac{2}{\sqrt{3}}Y$ ok. So here you can see $\frac{2}{\sqrt{3}}Y$ if you put and then you calculate it you will find out some η value and you will get you will see that we obtain a greater LDR as compared to Tresca ok. So I have not worked out here but you can just check it we obtain a greater LDR ok LDR would be slightly larger when you use Von Mises yield function for the same material as compared to Tresca. Now when it comes to anisotropic yield function ok that means $R > 1$ case which is the best case we have $R = 1$ indicates stronger in through thickness direction and the main effect would be to strengthen the cup wall ok. So if the cup wall ok you are going like this now this is your axis this is your sheet so cup wall you are speaking about is this ok this is a cup wall region and the wall region is actually stronger, wall region is actually stronger why because the thickness through thickness direction strength is going to be larger than the plane if you take $R > 1$ which means that LDR would be greater in this case as highest stresses are required to cause yielding in the sheet.

So LDR would be much larger if you use an anisotropic yield function for $R > 1$ material. So in these two cases does not matter actually R is equal to you know $R > 1$ in both the cases Tresca and Von Mises is for isotropic material you know. So if R greater than 1 in this case is picked then LDR would be larger why because there will be the material will be stronger in the thickness direction. So along with that we should also note that along with strain hardening cup wall strengthening is also important influencing LDR ok. So though we say that your strain hardening is an important thing ok when we discuss about flange getting converted into a cup wall during deep drawing in that analysis we consider strain hardening separately that is why we made σ_f converted into $\left(\sigma_f\right)_{\text{avg}}$ right but the point is along with that cup wall strengthening is also this strengthening, this strengthening is also important in determining the LDR ok. So the effect of anisotropy, sheet anisotropy if you want to study then we have to depend on the choice of yield function ok.

Tresca or Von Mises or any anisotropic yield locus for $R > 1$ indicates that if you have anisotropic sheet and if $R > 1$ then somehow if you bring that anisotropy into the model then LDR prediction would be accurate, LDR would be greater we say as compared to the other two ok when compared to Von Mises and Tresca and Von Mises if you compare with Tresca it is we obtain a greater LDR as compared to Tresca that is the way it is. So if you want to study the change in thickness of the sheet ok that is also possible. So this is just a brief note here when the sheet is bent and unbent under tension ok when the sheet is bent and unbent under tension which is what is going to happen in your cup drawing ok which is what is happening in the cup drawing right. So in bending we have seen two cases one is only moment and other one is stretching with you know with moment, moment with stretching right. So bending with stretching we have seen but in this case actually there is no other choice there is only under tension only because the sheet is gripped ok.

So because of this bending and unbending there will be reduction in thickness and that is given by this particular equation which we have not derived this is for our understanding only $\frac{\Delta t}{t} = -\frac{1}{2\left(\frac{\rho}{t}\right)}\frac{T}{T_Y}$ where Δt is nothing but your change in thickness and all other you know terms are known to us $\frac{\rho}{t}$ is nothing but your bend ratio which you already introduced T is your tension and T_Y is the yield tension which we introduced in the bending chapter. You will see that in from this equation a small bend ratio $\frac{\rho}{t}$ will increase the thickness reduction and that is going to reduce the load carrying capacity of the die wall and it is going to reduce the LDR ok. From this relationship we can find one important relationship on this equation we can find one important relationship between bend ratio and Δt. Δt is connected to LDR that is the way we are connecting. How is the relationship? For a small bend ratio ok that is in the denominator let us say it will increase thickness reduction because in the numerator by reducing the load carrying capacity ok of the die wall and it reduces the LDR further.

So, if load carrying capacity of the die wall is reduced which you have seen in this particular diagram your LDR will reduce ok. So, it is appropriate in a way to tell the largest blank that can be drawn is 2.72 that is a case ok it is an extreme case but it will be less than 2.72 generally it is between 2 to 2.2 which is what I told you take η value of 0.2 to 0.7 in that equation to get LDR that is the best option we have η value we discussed now is η. Ok just to summarize to have a better LDR we need to follow this particular you know thumb rule ok. So, if you want to have an improvement in LDR then $R > 1$ is preferred ok that means anisotropy of the sheet is existing means you have to consider that anisotropy ok. If we have $R > 1$ then LDR would be better and $\frac{\rho}{t}$ has to be larger flow by T is larger Δt would be less Δt would be less means you will have a better LDR ok and μ has to come down and your coefficient of friction has to come down that means you have to put appropriate lubrication you know to improve the LDR. So, these are some of the important thumb rules we can get ok. So, we stop here we will discuss it further. Thank you.