Mechanics of Sheet Metal Forming

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Week- 06
Lecture- 15
Bending Sheets(contd)

Ok. So, we will continue our discussion in this lecture that is we are still in bending of sheets and in the previous section what we discussed is basically summarized here. So, we evaluated strain distribution for bending of sheet with both moment and tension and then we evaluated ε_1. So, which is nothing $\varepsilon_a + \varepsilon_b$ and then ε_b can be approximated to $\frac{1}{\rho}$ which is actually $ln\left(1 + \frac{1}{\rho}\right)$ and can be approximated to $\frac{1}{\rho}$. So, this is what we have seen and then in that case bending without tension. So, we have evaluated two important things one is stress distribution other one is moment versus curvature diagram. We got both of this which means given a strain distribution ok that is bending without tension which means that is a specific case of what we derived for ε_1 and ε_1 .

So, in which basically ε_a would become 0 in this case bending without tension. So, we have ε_b only. So, for that we got stress distribution to get stress distribution then we have to relate your flow stress to strain. So, in that case we have discussed 3, 4 important models ok relating stress and strain and then we evaluated a moment for each case and then we drew a diagram moment versus $\frac{1}{\rho}$ diagram.

So, in that we saw actually 3 cases one is elastic bending rigid perfectly plastic other one is a strain hardening sheet. These 3 were discussed in the previous section. So, elastic bending means what would be the relationship between stress and strain then what will be a stress distribution what will be moment $\frac{1}{\rho}$ diagram and rigid perfectly plastic what will be the case and general one which is a strain hardening case what would be these 2 ok. This is what we discussed in the previous case there is one more case in this ok that we will see, but before that ok let us quickly discuss what we discussed as a last section which is nothing, but your strain hardening sheet ok. Strain hardening sheet we know $\sigma = K\varepsilon^n$ in general and with respect to bending we say $\sigma_1 = K\varepsilon_1^n$ and $\varepsilon_1 = \frac{y}{\rho}$ this way we can get and of course, we can directly draw this stress diagram across a section which is what is given here.

This is little different than what we gave in last class ok for the same situation there is little non-linearity here ok you can see that and it will drop to the negative values in the second bottom half of the sheet ok and this change has to be noted ok. And moment $\frac{1}{\rho}$ diagram is also shown here for the strain hardening sheet this is what we have discussed in the last part of previous section. So now, we will work out one problem later on using this bending of strain hardening sheet at that time you will again use this type of relationship ok. So now, let us go to the fourth case ok that is elastic perfectly plastic bending ok. So, whatever we have seen are basically elastic bending, rigid perfectly plastic bending and the strain hardening sheet ok.

So, now there is another case where we are going to see EPP it is called elastic perfectly plastic bending ok. This model we have already discussed ok we already discussed. So, it basically this elastic perfectly plastic bending it basically uses elastic perfectly plastic model which you already discussed ok and the model is actually shown in this figure. This is the same figure ok which is drawn between σ_1 versus ε_1 and you have the elastic part and then a constant flow stress plastic part which is nothing but *S.* So, when σ reaches S that is at this particular stage at this particular transit you will see that there will be change in your stress strain relationship which is nothing but $\sigma_1 = S$ less than that that particular stage you will see that $\sigma_1 = E'\varepsilon_1$ is what we have seen ok.

So, but this elastic perfectly plastic ok before that we have seen elastic bending ok. So, in bending we have seen only actually 3 cases right now in that first case was elastic bending in which we took only the first part of the curve. Now as the heading here suggests it is elastic perfectly plastic bending ok which means that it is suitable for curvatures beyond $\left(\frac{1}{\rho}\right)_e$ that is a limiting case of $\frac{1}{\rho}$ ok that is a limiting case of $\frac{1}{\rho}$ and $\left(\frac{1}{\rho}\right)_e$ we have already evaluated in the first case. So, if you go back to your previous one you can say that $\left(\frac{1}{\rho}\right)_e$ is this much a $2S/E^{'}t$ this is what we evaluated right. So, let us come back to this for the curvatures more than $\left(\frac{1}{\rho}\right)_e$ beyond $\left(\frac{1}{\rho}\right)_e$, but below where moment reaches M_p then this model can be applicable whatever we are going to discuss is applicable ok.

So, it is not fully M_p it is not gone to that state, but then it is suitable for curvatures beyond $\left(\frac{1}{\rho}\right)_e$. So, if that is a situation ok then this type of you know elastic plastic bending can be seen ok. So, for this a model is shown here which you already discussed. So, if you want to get a stress distribution for this it is it would be like a combination of your elastic bending and perfectly plastic rigid perfectly plastic bending right. So, in the elastic bending in the previous section we have seen this as a stress strain distribution right.

So, your this one this one is a your stress distribution right. So, in rigid perfectly plastic you will see this S there will be a constant S ok and then that we call it as rigid perfectly plastic. Now it is basically a combination of these two elastic perfectly plastic and you will see this would be a stress distribution that means up to a particular this is anyway $t/2, t/2$ up to a particular you know distance Y from the mid surface ok. So, you are going to have the elastic representation of elastic part and then it will become plastic part where the flow stress would be equal to plane strain flow stress which is nothing but S. So, when you see from upper part of the sheet when you start from here.

So, you will have of course this is the uppermost fiber will reach S first ok and then you will see that it will be a constant up to a particular you know you will have a flow stress of S up to a particular thickness and then below that you will see that it will be in elastic part it will reach 0 and on the bottom side will be just you know reflection of that and opposite to that ok. So, now what we are saying is this particular transition is what is called as Y_e, Y means in general any distance from the mid surface. So, we are calling it as Y_e here ok because this is a limit of your elastic part elastic representation this part ok. So, now for the case $Y > Y_e$ the material is plastic with the flow stress equal to S that is what we have written here ok. For a case $Y > Y_e$ ok so if you if you have a situation like this the material will become plastic with a flow stress of S that is what is given in this stress distribution ok.

So, similar distribution we got from in strain hardening sheet also only thing is it will become written non-linear ok in its variation that is the only difference. So, stress distribution of elastic perfectly plastic bending is this. So, now we can calculate this Y_e this because this transition is going to be important in the previous two cases elastic and RPP rigid perfectly plastic there is no such transition but here this transition happens Y_e ok. This Y_e can be calculated from simple equations which you already know ok. So, we know that this $\varepsilon_b = ln\left(1+\frac{y}{\rho}\right)$ ok.

So, in this case only ε_b exists $1+\frac{y}{\rho} \approx \frac{y}{\rho}$ ok so this $\frac{y}{\rho} = \frac{S}{E'}$ ok. So, from this we can get this Y which is nothing but my Y_e if we put some condition to that if you put some condition to it. So, your $\frac{y}{\rho}$ you can see that this equation is already known to us is not it this equation is already known to us on the right hand side I have your $\frac{y}{\rho}$ suppose $\frac{y}{\rho} = \frac{\sigma_1}{E'}$. Ok $\sigma_1 = S$ actually $\frac{y}{\rho} = \frac{S}{E'}$.

So, we can directly write this equation $\frac{y}{\rho} = \frac{S}{E'}$ which is going to give you $Y_e = \frac{S}{E'} . \frac{1}{\left(\frac{1}{\rho}\right)}$ So, Y_e is nothing but I am going to take ρ on the right hand side and I am going to write $\frac{1}{\left(\frac{1}{\rho}\right)}$ because $\frac{1}{\rho}$ is what is generally we refer know so $\frac{1}{\left(\frac{y}{\rho}\right)}$ ok. So, this $\frac{S}{E'} = \frac{t}{2} . \frac{1}{\rho}$ is not it. So, $\frac{S}{E'} = \frac{t}{2} . \frac{1}{\rho}$ ok. So, into $\frac{1}{\rho}$. So, which if I put it in this equation in place of $\frac{S}{E'}$. So, you will see that it is going to be $\frac{t}{2} . \frac{1}{\rho}$ ok. Since I am going to put Y_e here I am going to say $\frac{\left(\frac{1}{\rho}\right)_e}{\frac{1}{\rho}}$ will come ok. So, this part is actually $\frac{S}{E'}$ how do we get from this. So, $\frac{t}{2} . \frac{1}{\rho}$.

So, I am putting a limiting case here Y_e. So, I am going to put a limiting case here $\left(\frac{1}{\rho}\right)_e$ ok. So, this entire thing this particular ratio this one is I am going to call it as m. So, I am going to say $m\frac{t}{2}$. So, $Y_e = m\frac{t}{2}$ of course, t you know is nothing but the original thickness of the sheet and m is this particular ratio $\frac{\left(\frac{1}{\rho}\right)_e}{\frac{1}{\rho}}$.

So, $\frac{1}{\rho}$ is actually reference for us $\left(\frac{1}{\rho}\right)_e$ is nothing but a limiting case. So, $\frac{1}{\rho}$ is nothing but the radius of curvature which should have been given to the sheet and $\left(\frac{1}{\rho}\right)_e$ is nothing but your limiting case $\left(\frac{1}{\rho}\right)_e$ this ratio I am going to call it as m this m has nothing to do with any other m which we have discussed before or let us not get confused with this. So, this m can vary between 0 ok. So, in this way your Y_e here Y_e here can be calculated just nothing but $m\frac{t}{2}$ and m is nothing but it is a ratio of basically 2 you know radius of curvature $\frac{\left(\frac{1}{\rho}\right)_e}{\frac{1}{\rho}}$ ok. So, a simple question can be asked like this evaluate Y_e ok Y_e that means the transition between this part and this part calculate Y_e for a material which follows elastic perfectly plastic bending means so we can say like we can go back to this original equation and from here basically we can get Y_e ok.

So now for this particular case you can also get moment so now the moment is basically the general equation is $M = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_1 y dy$ right. So, now here what we are going to do is we are going to say that if you want to get moment so we are going to divide that into 2 parts we are going to divide that into 2 parts one is from the mid to the transition happens that is Y_e and from Y_e to the uppermost layer that is your $t/2$ this is the way we are going to divide this. So, this is one part for integration this is another part for integration so the limit changes actually ok limit is going to change because situation is different so anyway I am putting 2 times here because it is symmetric so 0 to Y_e is one part ok. So, which is going to be my $\sigma_1 y dy$ so in this case σ_1 is nothing but this fellow ydy.

So, $E'\frac{y}{\rho}$ is not it so $\sigma_1 = E^{'}\varepsilon_b$ so just nothing but my $E'\frac{y}{\rho}$ so ydy will remain as it is and in the second part you will see that $\sigma_1 = S$ is nothing but S so I will say $\int_{Ye}^{t/2} Sydy$. So, you can integrate it appropriately and then put limits you will get $\frac{St^2}{12}(3 - m^2)$ so this is what you get generally so you can look into it and so like previous cases we can also have m versus $\frac{1}{\rho}$ diagram so here you will see that it is again a combination of your the elastic bending and perfectly plastic part of rigid plastic bending you can see that so this is your straight line then a small transition and then you are going to have the horizontal part ok. So this is moment $\frac{1}{\rho}$ diagram for EPP bending ok. So and this is going to be your $M_e = \frac{St^2}{6}$ we already derived this $M_e = \frac{St^2}{6}$ if you go back to previous one you can get it $M_e = \frac{St^2}{6}$ ok. Your limiting elastic moment ok is obtained $M_e = \frac{St^2}{6}$ similarly we got $M_p = \frac{St^2}{4}$ ok so these two can be combinedly used to draw this particular moment versus $\frac{1}{\rho}$ diagram $\frac{St^2}{4}$ ok and we also previously derived that $\frac{M_p}{M_e} = \frac{3}{2}$ so I can write this as $M_p = \frac{3}{2}\ M_e$, is nothing but $\frac{St^2}{4}$ right.

So in this way we can interpret it like it is understood from the figure this particular figure for a non-stain-horning material ok for a non-stain-horning material the moment still increases beyond M_e and reaches 1.5 M_e and becomes constant after that right. So the moment still reaches beyond M_e so M_e is this particular value that is a small transition and then it becomes 1.5 M_e so this height it becomes 11.5 M_e ok and then it becomes constant ok.

So though there is no hardening there will be slight increase in your $M = \frac{3}{2}\ M_e$, if you want to push it to a constant a moment so that much is required here so that is one interpretation one can get from this. So now let us go ahead in developing a simple model for theoretical model for spring back I think we discussed in the previous section itself that it is moment without tension again we are going to see so you are taking a bent sheet first and then you are taking an unbent sheet so since there is no tension ok only moment is given ok there will be a change in curvature and bend angle in this way θ will become $\theta + \Delta\theta$, ρ would become $\rho + \Delta\rho$ ok and the length of the mid-surface is given by $l = \rho\theta$ so this $l = \rho\theta$ ok this l length will remain same will be unchanged why because there is no tension given here this length will remain same ok and so it can be written as $\theta = l\left(\frac{1}{\rho}\right)$ ok and if you differentiate it by keeping l as constant we can say $\frac{\Delta\theta}{\theta} = \frac{\Delta(1/\rho)}{1/\rho}$. So this $\Delta\theta$ which is nothing but the angular change when you bend a sheet the angular change due to spring back is given by $\Delta\theta = \theta.\frac{\Delta(1/\rho)}{1/\rho}$ so either you calculate this or you calculate this we can calculate the spring back and calculate the spring back ok. So this we already discussed I just refreshing it here so now let us go to the spring back in an elastic perfectly plastic material let us go to this particular case ok we are going to see only one case and we see how a simple you know model theoretical model for spring back can be evaluated in this. So elastic perfectly plastic EPP we just now discussed it we just now discussed of you know how would be your stress distribution how would be your you know moment curvature diagram we just now discussed ok and for this you know spring back in elastic perfectly plastic material we are going to use a similar model which we have already discussed and a similar stress strain diagram is drawn for this particular case but with reverse loading ok both are shown here.

So you will see that so you are starting from here and there you are moving like this ok so then you know once you have a slope of E' then you reach this particular point before it becomes the flow stress becomes S that becomes a start of your plastic deformation and then it goes on but then when you unload it it comes like this and then you will see that you can further deform it to reach this particular stage to get S ok. So this can be drawn in this way the blue line and orange line blue one represents the tensile part and your orange one represents a compression part ok I mean the reverse loading part ok. So in this case you will see that this particular is you know height is S or the value is S flow stress to reach plastic state and in the opposite case you will get generally $-S$ let us keep it like that and these two are equal ok which means that we are not considering Bauschinger effect we are not considering something called as Bauschinger effect ok. So in the tensile part of deformation and in the negative part of deformation if this stress flow stress or yield stress S remain same then we say that the Bauschinger effect is not considered but if you consider some softening and the negative side suppose this is not S this is not $-S$ this is less than S let us say this is less than the first time flow stress that is your S then Bauschinger effect is considered but we are not going to consider it we take a simple case without considering Bauschinger effect we can say that the change in stress the change in stress is given by $-2S$ ok before it reaches the plastic state. So which means that you need a change in stress of $-2S$ to make that material to reach plastic state which can have flow stress plane strain flow stress of S that is the meaning of this particular diagram ok.

So now what we are going to do is we are going to use our previous equation which we already know ok so we are taking a case that unloading part is elastic in nature ok so this unloading part is basically elastic in nature that is why you we say that it is a parallel to the elastic part ok unloading part is elastic in nature the elastic bending equations which we wrote before can be written in this way also in this way also this equation is known to us $\frac{M}{\mathrm{I}} = \frac{\sigma_1}{\mathrm{y}} = \frac{E'}{\rho}$ which is known to us this can be written as $\frac{\Delta M}{\mathrm{I}} = \frac{\Delta\sigma_1}{\mathrm{y}} = E'\Delta\left(\frac{1}{\rho}\right)$. We are somehow bringing in this $\Delta\left(\frac{1}{\rho}\right)$ which is what we are going to calculate it you know to quantify spring back so this is the fellow which we need actually to be calculated ok. So, this same equation is modified as $\frac{\Delta M}{\mathrm{I}} = \frac{\Delta\sigma_1}{\mathrm{y}} = E'\Delta\left(\frac{1}{\rho}\right)$ and what is E' we know that it is plane strain Young's modulus. So, now if you want to get $\Delta\left(\frac{1}{\rho}\right)$ ok we need to know one more thing here now what we are going to do is unloading is an elastic process by considering that we are also going to assume one more thing like a case in which assuming that the sheet is bent to fully plastic moment ok it is already gone into fully plastic moment ok and unloading is done unloading will be parallel to the elastic loading line as shown in this particular figure. This particular figure is moment $\frac{1}{\rho}$ diagram which we have just now seen ok so for EPP bending plastic perfectly plastic bending you will see that the same diagram you have a I know elastic and then transition and then a fully plastic part with the height of M_p ok.

So, now you will see that we are going to unload it from this particular you know point ok unloading is done here and you will see that when you unload it actually we need to get this particular curvature $\left(\frac{1}{\rho}\right)_0$ is actually the reference that we we need to convert the sheet to ok. But this $\Delta\left(\frac{1}{\rho}\right)$ has happened because you are unloading it and which is what we are going to we just now encircled it $\Delta\left(\frac{1}{\rho}\right)$ this fellow know this fellow which we just now discussed is actually represented in this diagram which is what is responsible for our spring back ok. So, $\left(\frac{1}{\rho}\right)_0$ is a reference for us ok but it is not going to happen that way so there is going to be some change that is $\Delta\left(\frac{1}{\rho}\right)$ which is what we need to quantify and all other things are known to us this is $M_e = \frac{St^2}{6}$ and you are removing moment from fully plastic moment situation which is nothing but $M_p = \frac{St^2}{4} = \frac{3}{2}\, M_e$ ok. So, now from this figure I can directly write this $\frac{\Delta\left(\frac{1}{\rho}\right)}{\left(\frac{1}{\rho}\right)_e} = \frac{-M_p}{M_e}$ ok. So, similar triangle ok that particular concept we can use $\frac{\Delta\left(\frac{1}{\rho}\right)}{\left(\frac{1}{\rho}\right)_e} = \frac{-M_p}{M_e}$ up to this particular stage up to this ok.

So, now because I am unloading the material from fully plastic state ok which is nothing but my M_p is going to come I can simply say that this M_p is nothing but my ΔM the change in moment is nothing but my fully plastic moment only which is nothing but my M_p ok. So, this $-M_p = \Delta M$ is nothing but your ΔM directly we can write that because I am going to fully unload it unloading material from a plastic moment from a fully plastic moment. So, that the change in moment is nothing but my M_p itself ok. So, this equation conveys this many points for us ok.

And this is already known to us ok. So, my $\frac{M_p}{M_e} = \frac{3}{2}$ also given here $\frac{\frac{St^2}{4}}{\frac{St^2}{6}} = \frac{6}{4} = \frac{3}{2}$. Now it is very straight forward so I want this $\Delta\left(\frac{1}{\rho}\right)$. So, $\Delta\left(\frac{1}{\rho}\right) = \left(\frac{1}{\rho}\right)_e \times \frac{-M_p}{M_e} = \frac{2S}{E^{'}t} \cdot -\frac{3}{2} = \frac{-3S}{E^{'}t}$ which is nothing but my $\Delta\left(\frac{1}{\rho}\right)$. So, $\Delta\left(\frac{1}{\rho}\right) = \frac{-3S}{E^{'}t}$ which is a simple equation to calculate a change curvature because of spring back for a material which is following elastic perfectly plastic bending and unloading is done in fully plastic stage fully plastic moment.

So, that is why I have written clearly here this is M-$\frac{1}{\rho}$ EPP bending showing unloading from fully plastic moment ok and for that particular case you can use this particular equation. So, now it is all about slightly simplifying this ok we can also write this because my $\Delta\left(\frac{1}{\rho}\right)$ is known to me. So, from the previous equation $\Delta\left(\frac{1}{\rho}\right)$ is nothing but you take this way so your $\frac{\Delta\theta}{\theta}$ would be there and $\frac{1}{\rho}$ will go to the left hand side. So, I can directly write this as $\frac{\Delta\theta}{\theta} =$

$\frac{-3S\times\rho_0}{E't}$, ρ_0 I am keeping instead of just ρ_0 I am just keeping it as ρ_0. So, this way I can calculate my change in θ ok or change in $\frac{1}{\rho}$ that is actually responsible for spring back.

So, it is a very simple equation but not fully accurate one for simple reason that does not include strain hardening ok does not include strain hardening and I was previously telling you that though we are discussing spring back in terms of during bending but then spring back can happen during general sheet forming process also like for example cup deep drawing where bending is there ok and in that type of processes if you want to predict spring back then strain hardening consideration is going to be very important for us but here this particular model is not that accurate but it will give you a pretty good idea about what would be the spring back and we can understand certain things from this ok and the strain rate effect is also not considered ok and one simple way is you can this plane strain flow stress can be made as a function of strain ok. So, what we have done is basically we kept S as constant ok in the previous model also the stress strain diagram also you see that S was made as constant and Bauschinger effect was also not considered though so there is no softening because of bending ok reverse bending. So, in that case basically in one way you can consider you know somehow strain hardening ok is by relating this S to ε ok somehow if you relate S to ε then you can make it somehow a function of a strain hardening effect also ok. And it is also mentioned that the equation is valid or good only for small differences in angle or curvature and when the sheet has been bent to nearly fully plastic state this is what I was telling you ok. So, this equation is valid or good only in such situations.

So, now in this from this equation we can also say that a spring back is proportional to few things one is ratio of flow stress to elastic modulus $\frac{S}{E'}$ correct where $\frac{\Delta\theta}{\theta}=\frac{S}{E}$ I was telling you before that this ratio is very important this is nothing but your $\frac{\sigma_{YS}}{E}$ only ok yield strength elastic modulus only but for plane strain bending we are writing this as $\frac{S}{E'}$. So, this ratio becomes important and this because this both are related to somehow related to elastic deformation and this ratio is of the order 1/1000 that we should have some idea ok why because this S is generally said in MPa and E' is generally said in GPa. So, you can say that it is about 1/1000 in that order you will have and bend ratio $\frac{\rho}{t}$ we introduced only thing I am just changing the nomenclature it is $\frac{\rho'}{t}$ and of course bend angle of course this bend angle ok. So, from the simple equation one can understand that spring back is actually proportional to $\frac{S}{E'}$ and $\frac{\rho}{t}$ and the bend angle θ this many items can be understood from this. So, now spring back has been evaluated $\Delta\left(\frac{1}{\rho}\right)$ has been obtained or $\Delta\theta$ also can be obtained from the simple discussion ok.

So, now we are going to consider two cases one is what would be residual stress in that section seat section after unloading the next one is if you do reverse bending ok what are the details in case of reverse bending that you can evaluate that is what we are going to see in next two small substructures. So, now here we say when EPP sheet is unloaded elastic perfectly plastic sheet unloaded from fully plastic state ok the change in moment is nothing but $-M_p$ ok which you already discussed ok the change in moment would be $-M_p$. So, now this I am going to put it in this particular equation this equation is already known to you just now we introduced $\frac{\Delta M}{\mathrm{I}} = \frac{\Delta \sigma_1}{\mathrm{y}} = E'\Delta\left(\frac{1}{\rho}\right)$ is not it. So, just now we discussed about it this particular one $\frac{\Delta M}{\mathrm{I}} = E'\Delta\left(\frac{1}{\rho}\right)$ the same equation what I am going to do is I am going to substitute M_p here. So, $\frac{\Delta M}{\mathrm{I}} = \frac{\Delta \sigma_1}{\mathrm{y}} => \frac{-M_p}{(t^3/12)} = \frac{\Delta \sigma_1}{(t/2)}$, y would become my $y = t/2$ why because this when you enter into plastic moment which means that you are reaching the uppermost layer which is already reached a plastic deformation state and that uppermost layer is nothing but $t/2$ for me ok.

So, $\frac{-M_p}{(t^3/12)} = \frac{\Delta \sigma_1}{(t/2)}$ ok. So, my $M_p = \frac{St^2}{4}$, So $\frac{-M_p}{(t^3/12)} = \frac{\Delta \sigma_1}{(t/2)} => \frac{St^2/4}{t^3/12} = \frac{\Delta \sigma_1}{(t/2)} => \frac{-3S}{t} = \frac{2\Delta \sigma_1}{t}$. So, this $\Delta\sigma_1 = \frac{-3}{2}\, S$ when you unload it ok. What does it mean? This equation also indicates one important thing which is given here this equation indicates that unloading process is fully elastic why because it is 1.5 S which is less than my 2 S discussed before this $\Delta\sigma_1 < 2S$ which we discussed before 2 S is the $\Delta\sigma_1$ that is required this particular figure 2 S is the $\Delta\sigma_1$ that is required before reaching the plastic state that is a limit for. So, it is less than that.

So, it is 3/2 only. So, 1.5 times S only. So, which means that it be in unloading is fully elastic ok. So, now what I am going to do is I am going to add this part this $\Delta\sigma_1$ I am going to add with already existing one that is nothing but my S which is already existing one this can be represented schematically as adding an elastic stress distribution that is confirmed by now it is elastic stress distribution that is confirmed which is $\frac{-3}{2}\, S$ to a fully plastic moment as shown in this particular figure. So, this is already known to us ok. So, this value is a plane strain flow stress S ok. And we are going to add this $\frac{-3}{2}\, S$ at the top is not it.

So, we are putting t/2 now when you put 3/2 you are getting $\frac{-3}{2}\, S$. So, which means that it will be at the top which will be opposite to that at the bottom you are going to add these two ok which means you are giving moment and you are giving a negative moment also here. So, what will be the residual moment you have what will be the residual stress you have here. So, if you add these two you will get a distribution like this which has got $\frac{-S}{2}$ at the top ok. So, it is $S - \frac{3}{2}\, S$ ok which is nothing but half no $-1/2$.

So, $\frac{-3}{2}\, S$ would be there ok. And then you will get you can also add this part with this part ok. So, you will get that value here and it will be opposite at the bottom. So, this would be your residual stress distribution after unloading from fully plastic moment for EPP elastic perfective plastic bending. So, what interpretation you get from this? The stress distribution shows that after unloading the tension side of the bend ok let us say this particular one tension side of the bend your T I am saying would have a compressive residual stress at the surface $\frac{-S}{2}$ and in the inner surface ok there would be residual tensile stress inner surface is this I. This is outer surface where you have tensile in the inner surface you will see that there will be a residual stress which will be tensile in nature ok.

The tension side of the bend would have compressive residual stress and the inner surface or the bottom surface or the compression surface would have a residual tensile stress. This is how the distribution would be for EPP material ok which is unloaded from fully plastic moment. So, now this is one. So, now let us go to reverse bending ok. So, reverse bending means you are bending the material and then you are unloading it and reverse bend it ok.

So, this particular situation so your moment $\frac{1}{\rho}$ is like this. So, you are taking this part here and then it becomes a constant flow stress you unload it and then follow this part and then again you bend it and then you take it to the negative side. So, the question is we have seen that the change in stress required to have yielding in the outer layer of the sheet is of course $\Delta\sigma_1 = -2S$ ok. So, now what we are going to do is ok now this $\Delta\sigma_1 = -2S$ if you put it in this equation this equation we know already. So, we need to see this $\Delta M = \frac{\mathrm{I}.\Delta\sigma_1}{\mathrm{y}}$.

So, $I = t^3/12$ ok. So, $y = t/2$ why because again the outermost layer ok you know will reach yielding ok when $\Delta\sigma_1 = -2S$ happens ok and $\Delta\sigma_1 = -2S$ I am substituting here. So, if you do that $\Delta M = \frac{-\mathrm{S}t^2}{3}$. So, when you have $\Delta\sigma_1 = -2S$ then $\Delta M = \frac{-\mathrm{S}t^2}{3}$ ok and the moment for reverse yielding suppose you want to reverse bend and then you have to give some moment which will cause first yielding in the reverse side ok like we have a M_e here ok once you reach M_e ok. So, you are going to enter into the the plastic part similarly the bottom side what would be that value that can be obtained in this way. So, I am writing reverse bending $M_{rev} = \frac{\mathrm{S}t^2}{4}\frac{-\mathrm{S}t^2}{3} = \frac{-\mathrm{S}t^2}{12} = -\frac{M_e}{2}$.

So, which is what is represented here so this $M_e = \frac{St^2}{6}$, this $M_e = \frac{St^2}{12}$, which is just half of M_e ok. So, this figure shows that yielding at reverse bending occurs at half the value of initial yield moment ok that is your $\frac{St^2}{6}$ half the value of means $\frac{\frac{-St^2}{6}}{2} = \frac{-St^2}{12} = -\frac{M_e}{2}$. So, you need to in the reverse bending side you need to give M_e ok or M_{rev} bending which will be nothing but $\frac{M_e}{2}$ only half of the first M_e that you gave for the first bend. Even if you do not consider Bauschinger effect you need to give only this much ok. So, actual softening if you see would be greater than this if you consider Bauschinger effect also.

So, even without Bauschinger effect ok your $M_{rev} = -\frac{M_e}{2}$ only ok which is not actually M_e which is half of that only but if you consider Bauschinger effect then with further softening there would be a greater you know softening of the material so the estimated value would be different than this ok. So, these are the two important sub topics that we discussed for EPP material. So, what we have done is actually three parts considering EPP material and unloading from fully plastic moment we derived a simple equation for expression for $\Delta\left(\frac{1}{\rho}\right)$ or $\Delta\theta$ ok. So, which depends on $\frac{S}{E'}$ and $\frac{\rho}{t}$ and the bend angle θ. So, now if you want to calculate residual stress then we are going to add $\frac{-3}{2}$ S which is nothing but the elastic stress only ok.

This part will be added to your fully you know plastic state that is your S, S is nothing but your fully plastic that the plane strain flow stress right fully plastic state. So, if you add it finally you get a resistance stress distribution like this and then we also calculated the M required for reverse bending which will be just half of the M_e that is required in the first bending ok that is what we derived here even if you do not consider Bauschinger effect ok.

So, let us do this small problem ok a strip of sheet metal which is 2 mm thick and 200 mm wide ok. So, width of the sheet is given here we can see ok is to be bent in a die under conditions of zero friction ok and zero axial tension fine. So, that means there is no tension only moment is given, but you can see that here unit width is not considered. So, you have to be careful 200 mm wide is given and thickness is given. So, the radius of curvature of the die is 80 mm ok. The stress strain relationship is provided by this you know $\bar{\sigma}$ is equal to sorry M P a is here $\bar{\sigma} = 600\bar{\varepsilon}^{0.22}$ M P a ok. And the properties are basically E is equal to 200 G P a and Poisson's ratio is 0.3. So, this all are given what is actually wanted is radius after spring back. So, you need to get the radius of curvature after spring back ok. So, basically what we need to do is we need to get $\Delta\left(\frac{1}{\rho}\right)$ ok . So, $\Delta\left(\frac{1}{\rho}\right) = \frac{\Delta M}{\mathrm{I}E'}$ we know this we know this already.

So, what is E', one by one we will calculate it here. So, $E' = \frac{E}{1-\upsilon^2}$ right E is given υ is given you can get E' you can check calculation. So, now if we want to get ΔM which is nothing, but our M only ,is given by this equation $\frac{M}{I_n} = \frac{\sigma_1}{y^n} = K'\left(\frac{1}{\rho}\right)^n$ why this you are taking because the material is given $\bar{\sigma} = 600\bar{\varepsilon}^{0.22}$ which is a strain handling material ok. So, that is why we are using this equation and I_n is given like this ok. So, if you want to use this equation you will see that I_n has to be calculated to get ΔM you want ΔM ok you want to you need to calculate I_n and then K' should be known ok and then you have to calculate $\frac{1}{\rho}$ is already given for us ok.

So, I_n has to be calculated and K' have to be calculated right. So, K' how do you calculate is by using this relationship you will see that this is already derived $\sigma = \frac{2}{\sqrt{3}}\bar{\sigma}$ for plane strain bending already derived it $\frac{2}{\sqrt{3}}\varepsilon$ also we derived $\left(\frac{2}{\sqrt{3}}\varepsilon\right)^{0.22}$, 600 is same 0.22 is same instead of $\bar{\varepsilon}$ you are putting $\frac{2}{\sqrt{3}}\varepsilon$ this also we derived for plane strain bending if you calculate it it will be $715\varepsilon^{0.22}$ please check this is going to be your K' this 715 MPa is going to be your K'. In a normal sense $\sigma = K\varepsilon^n$ we use so now what we have done is we have converted that $\bar{\sigma} = K\bar{\varepsilon}^n$ this has been converted to plane strain bending plane strain bending situation as shown here by using this particular strain horning law so which means that this is not K this is actually K' .

So, K' has been found out not a problem so now what we need to get it is I_n so $I_n = \frac{t^{n+2}}{(n+2)2^{n+1}}$ only thing is in this it is considered as unit width but width is given as 200 mm so you need to have w also here which should be considered somehow and that is why I put a circle here you can see by substituting all the known values in this equation you will get this particular value so 0.2 which is nothing but my $I_n = \frac{0.2\times\left(2\times10^{3}\right)^{2.22}}{2^{1.22}\times2.22} = 3.94 \times 10^{-8}$.

$\mathrm{M} = I_n K'\left(\frac{1}{\rho}\right)^n = \frac{3.94\times10^{-8}\times715\times10^{6}}{0.08^{0.22}} = 49.1\ mm$, K' we keep unit consistent then $\left(\frac{1}{\rho}\right)^n = 0.08^{0.22}$ if we check it will be 49.1 mm we will get so that you can substitute in this equation so $\Delta\left(\frac{1}{\rho}\right) = \frac{\Delta M}{E'\mathrm{I}}$ so all are known to us now $E' = 219.8$ GPa I we can calculate which is nothing but $\frac{wt^3}{12}$ here you have to be careful it is $\frac{t^3}{12}$ for unit width but it will be $\frac{wt^3}{12}$ in this particular case because w is given ΔM is nothing but your M which you already know we can substitute all these things so you can see that $\frac{-49.1}{219.8\times10^{9}} \times \frac{12}{0.2\times\left(2\times10^{3}\right)^{3}} = 1.675\ m^{-1}$.

So this is only $\Delta\left(\frac{1}{\rho}\right)$ so now final curvature how do you get it you can get it by $\frac{1}{0.8}$ which is again the original radius of curvature this is what actually references okay we need to bend a sheet to this much of radius of curvature so but then it is not going to happen we are going to remove this particular part from this which already calculated which is about 10.8 m^{-1} and if you want to get radius of curvature okay so you can take $\frac{1}{10.8} = 0.093\ m$ just check the units consistently okay so you can also convert into millimeter if you want so just check the units consistently finally radius of curvature is obtained in this which is what has been asked from us okay. So this is a question number 3 because we worked out two problems before okay question number 1 and 2 we solved in between also okay we have done it so this way you can calculate spring back using simple equations so $\Delta\left(\frac{1}{\rho}\right)$ is what we need to get so all other things are calculations responsible for your ΔM , E' and I okay.

So instead of this test strain behavior I can give a different test strain behavior okay maybe I can include a temperature also in this and spring back the bending is done at a different temperature we can say okay and then the equation can change all the values can change and one can compare also okay. So now let us go to the last section in bending so we are going to now demonstrate if you include stretching also what will happen which is what we are going to call bending with stretching okay bending with stretching or bending with tension okay. But in this what we are actually going to do is we are going to consider a case similar to a stamping operation stamping operation means a forming operation okay suppose you are making a large curvature okay you want to make a large shed you know something like that okay. So similar to stamping operation where the sheet is first curved elastically to the shape of the die and then tension is applied okay so what you do is you actually wrap the sheet on the die okay by giving some elastic deformation and then you apply tension that is the way we are actually going to do so bending first and then stretching over a die with a larger or not the larger curvature and that situation is given here you can see this is your die let us say and this is your sheet so on that die you are actually clamping it and you are you know giving a curvature to that sheet okay. But you are first bending it without providing tension so that the shape is attained and then you are going to stretch the sheet to get a full shape and it is a frictionless case okay.

So, a moment is given and tension is given. So, in this situation your stress distribution can be drawn in five different stages okay from *a, b, c, d, e* you will see and in these cases *a* is actually without tension that is your initial stage *b* to *e* is basically the second stage but at different levels if you provide with tension because we first said that you are going to give only bending that means only moment is given okay then you are going to pull it to create a full shape okay. So, if you quickly observe what is going to happen here this is a stress distribution all are σ_1 only okay. So, the first *a* case is known to you because there is no tension only moment is given so it will be 0 at the center okay this we already discussed and then we evaluated this stress distribution it is done. So, now when you provide some tension little bit small tension is applied you will see that it is going to go down your neutral axis to go down this also we evaluated this contains ε_a *and* ε_b right. So, now *c, d, e* are further stages of providing tension okay *c* is little specific why because you are providing tension such that your plane strain yield stress is reached at the upper surface okay at the upper surface and if you further progress it the plastic deformation region will propagate throughout the section.

So, you see that *S* is here now *S* is up to this particular thickness now in the last case you will see the section is fully plastic so that your S flow stress is reached fully in the throughout the section this is *t* this is your sheet thickness this is your sheet thickness okay. So, only tension sorry only moment is *a* you just provide some tension *b* increase the tension so that the upper surface reaches S flow stress nothing but the yield stress then with further progress you will see that the S will propagate in the section this is a elastic to plastic transition and then here you will have fully plastic section these are the differences between *a* to *a, b, c and d and e* okay. So, which is what is we are going to provide details to each one of this you quickly do one after another initially when the moment is applied without any tension okay the stress distribution will be shown in *a* okay. So, now here the strain at any distance let us say y from the middle surface is given by $\varepsilon_1 = \frac{y}{\rho}$ actually $ln\left(1 + \frac{y}{\rho}\right)$ which is written as $\frac{y}{\rho}$ okay since the radius of curvature is ρ_0 I am going to write ρ_0 here $\varepsilon_1 = \frac{y}{\rho_0}$. So, for this stress distribution we can get stress distribution is nothing but $E'\varepsilon_1$. So in this case okay it is $E'\frac{y}{\rho_0}$ okay and if you want to get slope of this particular stress distribution we can get $\frac{\sigma_1}{dy}$ which is nothing but our $E'\frac{y}{\rho_0}$.

The stress distribution is given by $\sigma_1 = E'\frac{y}{\rho_0}$ okay material is in elastic state only which has got a slope of $\frac{d\sigma_1}{dy} = \frac{E'}{\rho_0}$ that is done. So, let us go to next stage figure *b* to *e* show stages when tension is applied and it decreased further and it increased further. So, we apply tension and then slowly increase it that is what is shown in figure *b* to *e* *b* to *e* this is what is shown. Now let us go to specific stages *b* and *c* *b* and *c* stages shows stages in which slope remains constant and tension increases correct. So, this slope will remain constant only thing is your tension will keep on increasing so that you are going to reach *S*.

So, we are going to say that the initial moment when $T = 0$ is given by $M = \frac{E' t^3}{12}\frac{1}{\rho_0} = M_0$ this we already know we derived it okay. So, if you want to know that you can go back and check initial moment that is $T = 0$. So, only moment this you already calculated okay. So, you can go back and check which will be nothing but my I am just denoting it as M_0this moment will be constant until stage *c* okay is reached or material starts to yield okay. This moment will be constant until stage *c* is reached or material starts to yield because at *c* only the material will starts yield why because at upper surface you will see it is going to reach S plane strain flow stress.

So, now we will use this equation later now you will see that at stage *c* the sheet will start deforming plastically when stress at the outer layer reaches the yield stress S. At stage *c* the sheet will start deforming plastically at the outermost fiber when your σ_1 reaches S that we already know. At stages *d* and *e* with further tension plastic deformation zone increases like you have shown here and then shown here also at *e* the whole section is fully plastic with S as flow stress okay which is what is represented here. Now at this stage okay you want to get a tension $T_y = St$. So, the tension applied okay to enter into the situation seen in *c* okay you want to cause some yielding for that so one tension has to be applied that is nothing but $T_y = St$ the original definition.

Actually it is $\sigma_1 t$ but then in this case it is S only St plane strain flow stress into t. So, now what we are going to do is let us get some details into the stress distribution okay. So, between *b* and *c* the stress distribution at any location okay that is my σ_1 okay will have two parts one is σ_{1a} and σ_{1b} this is something new which you are not introduced before we have introduced only two parts in strain distribution. Now we are saying that the principal stress σ_1 has got two parts one is σ_{1a} and σ_{1b} similar one to like previously we discussed the σ_{1a} is nothing but uniform stress at the mid surface and then there will be an addition of σ_{1b} which is meant for bending stress as in the case of section a okay that we stage a.

So, $\sigma_{1b} = E' \frac{y}{\rho_0}$. So, this we already discussed now this one $\sigma_1 = E' \frac{y}{\rho_0}$ only thing I am writing it as σ_{1b} here it is specifically σ_{1b} okay. So, in this case you will see that of course in the a part the a part of course σ_{1a} will not be there σ_{1b} will be there so that is why it is actually at 0 at the center okay. So, now let us go to a limiting case now these two are known to us okay let us go to limiting case for a limiting case c why c is limiting case because it is going to reach S at the upper surface the outer layer stress is S okay if that is the case you want to get stress at the middle surface here you want to get stress at the middle surface what would be the one okay. So, I am going to say that so your $(\sigma_1)_{y=0} = S - E' \frac{1}{\rho_0} \frac{t}{2}$ is not it.

So, my this S is already there I am going to remove this $E' \frac{1}{\rho_0} \frac{t}{2}$. So, $E' \frac{1}{\rho_0} \frac{t}{2}$ I am going to put $y = t/2$ here okay. So, I am going to remove that part from that okay so that I will get $(\sigma_1)_{y=0}$. So, at the same time I can get tension on that section okay $(T)_{y=0} = T_y \left(1 - \frac{\rho_e}{\rho_0}\right)$. So, what is this T is for the same thing see $T = St$, St means this into t. So, $(T)_{y=0} = St - E' \frac{1}{\rho_0} \frac{t^2}{2}$ this is what I should get okay.

So, and of course you can simplify it in this fashion okay and you will get $T_y \left(1 - \frac{\rho_e}{\rho_0}\right)$ where your $\rho_e = \frac{E' t}{2S}$, $T_y = St$ okay. So, you will see that the stress at the middle surface can be obtained by this and the tension on the section can be obtained by $T_y \left(1 - \frac{\rho_e}{\rho_0}\right)$ where $\rho_e = \frac{E' t}{2S}$, and ρ_0 is actually the original one and in this $T_y = St$ okay. So, what does this equation tell some interpretation we will get let us go to this tension in the section. This equation says that if $\rho_0 > \rho_e$ so ρ_0 is actually the original value which we need to reach and ρ_e is a limiting elastic case okay if this is larger than this then the applied tension will reach yield tension T_y. So, the applied tension is this and yield tension is T_y okay it will be almost the same if your ρ_0 is going to be very large as compared to ρ_e if that is the case this will automatically reach that okay.

So, now what will happen is if your applied tension reaches T_y then there is no need for moment the moment will actually get lower you do not need to give that much of moment which you gave before it will be actually lesser than that that is a you know main point the moment tension is given then moment is actually not required it will be lowered automatically okay. So, but now let us go to one important one that is in the d stage let us increase the plastic deformation okay and then you will get a interface elastic to plastic interface what is that that is this one this diagram is shown in this particular figure. At some instance okay an elastic plastic interface EP interface can be identified at a distance $\frac{qt}{2}$ from the mid-surface I am referring to this particular distance $\frac{qt}{2}$. So, what is this? This is a stress distribution and this is a strain distribution in an EPP sheet bend to a larger gentle curvature and then stretched.

So, ρ_0 is large and then you are actually stretching it. So, this is already known to us this particular stress distribution is already known to us from the middle surface you see that there will be a little bit of elastic part above that and then it becomes plastic and below thickness it is going to be fully elastic only because it has not reached plastic deformation so that is a situation. So, this line is actually EP interface. This line is actually EP interface. The corresponding strain distribution is shown here okay so of course this plastic portion this shader region and this shader region are same. So, here you will see that at the mid location there will be some ε_a and there will be an additional component of ε_b which is nothing but $\frac{y}{\rho}$ from you move from middle surface to upper surface when you from move from middle surface to upper surface along y there will be ε_a plus some part of ε_b when you reach the upper surface and once you cross a particular value let us say $\frac{qt}{2}$ you are in plastic part that is this part which is what is shown here okay.

So, now this EP interface is going to be very important for us because at this particular stage S is known to us it is $\sigma_1 = S$ okay. So, now what will happen if you provide more details into ε_1 when you have elastic to plastic transit this is your EP interface in terms of strain distribution. So, now what details can we provide in that so at some instance and EP interface can be identified at a distance $\frac{qt}{2}$ from the mid surface right. So, now in general strain can be written as $\varepsilon_1 = \varepsilon_a + \varepsilon_b$, $\varepsilon_b = ln\left(1+\frac{y}{\rho}\right) = \frac{y}{\rho}$ which we discussed before itself so which is one of the $\varepsilon_1 = \varepsilon_a + \frac{y}{\rho_0}$ I am just keeping okay. So, now what I am going to do is at $y = \frac{qt}{2}$ distance that means at the EP interface ε_1 is nothing but the yield strain $\frac{S}{E'}$.

So, I am going to this particular interface beyond that I am going to have plastic deformation means at this interface okay $\varepsilon_1 = \frac{S}{E'}$ okay basically $S = \varepsilon_1 E'$. So, my yield strain is nothing but now $\frac{S}{E'}$ okay. So, what I am going to do now is I am going to equate this $\varepsilon_1 = \frac{S}{E'}$ and then I am going to get ε_a and then I am going to get ε_a which is at the middle surface. So, what is the value $\varepsilon_a = \frac{S}{E'} - \frac{1}{\rho_0} y$ and this y is a specific case of $\frac{qt}{2}$ which is what I have given here okay. So, I am going to say that the we are saying $\varepsilon_a = \frac{S}{E'} - \frac{1}{\rho_0}\frac{qt}{2}$ okay that would be my $\frac{S}{E'}$ that will be my yield strain.

So, $-\frac{1}{\rho_0}$ in place of what is it my y I am writing $\frac{qt}{2}$ here. So, I am going to get ε_a this ε_a is actually plotted at the mid surface here. So, now what I am going to do I am going to substitute this ε_a into this equation and I am going to get in general ε_1 the strain at y distance from the mid surface okay in this situation that is your d this is d part d stage know this is d stage d

stage the strain at y distance is given by $\varepsilon_1 = \varepsilon_a + \varepsilon_b$, $\varepsilon_a = \frac{S}{E'} - \frac{1}{\rho_0}\frac{qt}{2}$, $\varepsilon_b = \frac{y}{c}$ this would be your ε_1 distribution for d stage okay. So, the stress distribution for this stage is actually very easy for us so $\varepsilon_1 = (\varepsilon_a + \varepsilon_b)E' = S - \frac{E'}{\rho_0}\left(\frac{qt}{2} - y\right)$ which I can get directly okay. So, this ε_a and ε_b are already known to you from the previous slide you can substitute it and you can find out okay which is nothing but $S - \frac{E'}{\rho_0}\left(\frac{qt}{2} - y\right)$will come. So, in this way the elastic state can be applied so the elastic part after this it is S that is known but how is it going to vary okay we can get.

So, now what we are going to do is basically moment and then tension these two we are going to evaluate how we will going to evaluate is not $M = \int \sigma_1 y dy$ is right so only thing is limit we have to be careful because something is changing. So, what I am going to do is this moment is integral so there are two locations this would be $\frac{-t}{2}$ to $\frac{qt}{2}$ this will be 1 okay and $\frac{qt}{2}$ to $\frac{t}{2}$would be my another one that is what I have given here. So, $\frac{-t}{2}$ to $\frac{qt}{2}$ is a limit within that $\sigma_1 y dy$ will come that σ_1 is nothing but this one so this entire thing is nothing but my σ_1, $\int_{\frac{-t}{2}}^{\frac{qt}{2}}\left[S - \frac{E'}{\rho_0}\left(\frac{qt}{2} - y\right)\right] y dy$ okay after that okay my $\frac{qt}{2}$ to $\frac{t}{2}$ is nothing but S only σ_1 is nothing but S so this is nothing but my S so $\int_{\frac{qt}{2}}^{\frac{t}{2}} S y dy$ this limits have to be careful so you can simplify this and finally you will see that your M is nothing but M_0 we already evaluated itwhere is that this one $M_0 = \frac{E' t^3}{12}\frac{1}{\rho_0}$, $M = M_0\left(\frac{2+3q+q^2}{4}\right)$ so then what do you get as a applied tension that is same thing $\frac{-t}{2}$ to $\frac{qt}{2}$ only thing y will not come here okay $S - \frac{E'}{\rho_0}\left(\frac{qt}{2} - y\right) = \sigma_1$ and here it is S only that is also σ_1 okay so which reduces to the simple equation $T = T_y\left[1 - \frac{1}{4}\frac{\rho_e}{\rho_0}(q+1)^2\right]$ that is why it is going to come you can check it what is T_y we already derived it T_y is nothing but your tension at yield which is nothing but $T_y = St$ you can check it okay. And in this case if you want to relate M and T okay because we are applying both right first actually moment is applied okay so when tension is 0 X axis is 0 only moment is applied how much M_0 okay how much that is your M_0 we already calculated that much amount of moment is applied okay just to give some shape to it and after that the full shape is given by providing tension and if you increase tension from 0 to T_y to T_y okay you will see that after the elastic part moment is not required at all moment will come down and will decay to 0 when T reaches T_y. So when T reaches tension reaches T_y you will see that moment is going to be 0 only and between this elastic to plastic this particular region is very important for us okay particular region is very important for us and in this actually is going to tell you something which is responsible for spring back control okay.

So we know that spring back is actually proportional to change in moment that we already discussed okay your spring back we already related to change in moment okay to your $\Delta\left(\frac{1}{\rho}\right)$

which we already discussed which means that spring back is proportional to change in moment actually. If the moment in loaded condition is reduced to 0 moment in loaded condition is reduced to 0 by applying yield tension let us say T_y this is your ty the spring back on unloading would be 0 the spring back on unloading would be 0 because the change in moment is not there at all so spring back proportionally will come down okay. It also means that applying yield tension will set the shape in the sheet to the top of the die. So if you provide some yield tension it will actually provide nice shape to the sheet as per what you have in the die okay. So in actual stamping what you can do is by providing small plastic strain throughout okay will ensure negligible spring back okay.

So you provide small plastic strain okay let us say it can be created by T_y okay you provide you convert tension into yield tension you push it forward in that way then small amount of plastic strain can be provided okay throughout the section and hence you will have negligible spring back. So this interpretation can be obtained from this. This is one way to control spring back how in actual stamping you need to provide small plastic strain okay with the help of tension okay so that you will have negligible spring back. So with this we stop here we look into next module later. Thank you.