Mechanics of Sheet Metal Forming
Prof. R Ganesh Narayanan
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Week- 06
Lecture- 14
Bending Sheets

So, this particular module that is going to be module number 6 ok, we are going to discuss about Bending of Sheets. So, bending of sheets if you take it is there in almost all sheet components that we see in day to day life ok. So, it could be as simple as you know a plate type of shape or it could be a complex shape that we use in automotive industries, components, aerospace industries ok or in any other you know sectors ok, bending is a inevitable ok. So, analysis of bending of sheets ok which we are going to you know discuss in this module is going to be maybe about 2 classes we are going to see ok and some of the assumptions that we made earlier are valid for bending of sheets also or we are going to introduce one more assumption then we go ahead with our discussion. So, bending along a straight line is a very common form of sheet forming operations that we see in day to day life and that is a simpler you know process to understand ok.

So, of course in for making components it is not going to be a straight line bending it is going to be in any other any form ok. So, it is going to be any form ok. Suppose if you take a cup a deep drawn cup suppose we are making ok you will see that it is bent in this locations, but you will see that the bending is not actually circular in nature bending is going to be circular in nature ok. So, nevertheless we are going to study mainly the bending of bending along a straight line which is easy for us to understand the theoretical part of the bending process.

And bending along a straight line we know it is can be done in several ways we can just bend it with the help of you know die part set up ok it could be done in the form of you know folding operation or flanging operation in a special purpose machines ok. Like in workshops we used to we have bending machines available ok. So, we can also bend the sheets just holding it in a bench vice and then giving some displacement one of the edges it can bend ok. So, there are several ways we can do bending operation, but of course the complexity of the machine depends on what kind of component you want to make. So, first of all what we are going to discuss we are going to introduce mainly the variables in bending a sheet ok.

Suppose if you take a sheet aluminum sheet or steel sheet what are the variables in bending that we discuss for that the schematic would be useful ok. So, this is a bend sheet ok. So, initially it could be as flat as like this ok and now it is bent like this is not it. So, it is bent through an angle of θ that is given here θ is called as bent angle and you need a tool set up for that let us say and it has to be bent with respect to one particular radius of curvature let us say ρ in the bend region. And then the right side diagram will tell you that this bending is possible in two ways one is only moment the other one is moment and tension both.

So, bending with stretching, bending with stretching would be moment and tension both are applied in the other case only moment can be applied ok both are possible ways of bending a sheet along a straight line ok. So, you will see that this T is called as a tension per unit width we are saying. So, we are picking up a unit width which is applied at the mid surface of the sheet generally ok you apply T at the mid surface of the sheet. Theoretically, so practically how do you give this tension is by grabbing the sheet ok you have to grab the sheet at this let us say locations at this locations ok. So, and then you are trying to push you know the sheet through a die cavity ok something like that you can imagine you are going to push the sheet through this die cavity you can imagine.

So, you have a flat sheet ok and the flat sheet is grabbed at this end and you are going to pull it with a punch let us say a V punch or something like that. So, that it becomes this kind of shape ok. So, this is actually clamped here and you are going to give displacement in this direction. So, something like that is possible practically ok, but theoretically you apply T at the mid thickness or mid surface of the sheet and then moment is given in this fashion. So, this is a tension per unit width and then we get moment per unit width M and T can be related ok.

So, these are the variables. So, you have a bend angle θ then ρ radius of curvature then tension and moment applied in the sheet. So, now as usual what we are going to evaluate first thing is strain like in the previous chapters modules also ok we first evaluated strain from the new dimension with respect to the original dimension and then we went ahead in calculating the other you know quantities when you deform a sheet. Similarly, here also we are going to first evaluate strain in bending. So, again I am going to take a simple case for example, this is your sheet this is your sheet you are taking with the thickness let us say t ok t or $\frac{t}{t_0}$ either way is fine.

So, it is going to be only thickness ok. So, this is a total thickness is t ok. So, let us say for example, you consider a middle fiber let us say C_0, D_0 ok a line which coincides with the mid thickness and you can pick up any other line or a fiber which is y distance you know away from the mid thickness or C_0, D_0 that is A_0, B_0. So, we are finding we are picking up two locations one is C_0, D_0 which is at the center middle ok and A_0, B_0 which is at y distance from the middle and you are picking up a section ok in the sheet with a length original length of l_0 which I mentioned here as l_0 ok. So, you can imagine that this is t which means that this is let us say $t/2$ and there is let us say this is a $t/2$ you can say.

So, now this sheet is bent here ok. So, the bend sheet is drawn like this and as usual you have a bend angle of θ and the radius of curvature is ρ and you will see that the C_0, D_0 becomes C D and you will see that A_0, B_0 is going to become A_0, B_0 become is going to become A,B. So, how do you calculate strain in a simpler fashion ok. Of course, here we cannot put circle grid and evaluate strain ok unlike in other deformation process where we put circle grids on the surface ok. Here we are discussing about sheet bending and we are looking at a section that is why thickness is given here that is why thickness is given here ok.

So, why we are showing a section when we have a you know bend ok, bending is done with respect to this axis is not it. So, why you are we showing section we will see in the next slide, but before that anyway we cannot put circle grids and evaluate because the sheet thickness would be about maybe 1 , 1.5, 2 mm. So, naturally we have to get it in a different way. So, now we will say that while bending this particular sheet to this particular form.

So, C_0, D_0 becomes C, D at the mid surface that is number 1. So, now this l_0 ok is going to become let us say l_s when the sheet is bent with stretching ok. So, we are going to consider a case ok where the sheet is bent ok not only with moment, but also with tension both the tension and moment are available for us. So, basically you are going to bend the sheet with stretching ok. So, that means, assuming the sheet is stretched during bending the original length l_0 is going to become let us say l_s which is given by $l_s = \rho\theta$.

So, I am going to have this l_s ok l_0 is going to become let us say l_s and that will be given by $\rho\theta$ ok. This is with respect to the mid location C_0, D_0. Now, let us pick up a case which is a line or a fiber which is y distance away from C_0, D_0hat is A_0, B_0. Let us consider A_0, B_0 at a distance y from the mid thickness and what will happen to l for that that I am going to call it as l which is nothing, but I am going to add this is ρ. So, this point from here to here is ρ and from ρ I am going to add this y part to it assuming that this y remains same everywhere here also y here also y here also y here also y this is not going to change which is same as that of the earlier y which you have taken before bending ok.

So, I am just going to add $(\rho + y)\theta$ ok. So, $\theta(\rho + y)$ will give me my l which is basically A, B ok which is nothing, but you're A, B. So, this can be simplified as $\rho\theta\left(1 + \frac{y}{\rho}\right)$ and why I am doing it is because I can replace this $\rho\theta$ by l_s , $l_s\left(1 + \frac{y}{\rho}\right)$. So, this will give $\frac{l}{l_s} = 1 + \frac{y}{\rho}$ it will be useful for us now. So, you will see that now we need to find axial strain of the fiber AB ok axial strain of the fiber AB.

So, this AB is like any fiber ok you know with respect to your mid thickness mid fiber CD right. So, now we need to find strain at the fiber AB ok. So, which I am going to call it as $\varepsilon_1 = ln\left(\frac{l}{l_0}\right)$. So, AB is connected to l and original distance you know length is anyway l_0 for us. So, I am going to give I am going to write $ln\left(\frac{l}{l_0}\right)$ as per our original definition and this $ln\left(\frac{l}{l_0}\right) = ln\left(\frac{l_s}{l_0}\right) + ln\left(1 + \frac{y}{\rho}\right)$ ok.

So, my axial strain of a fiber AB at any distance let us say y ok. So, this AB could be not necessarily in this location it could be slightly above also does not matter.

So, which will be given by $\varepsilon_1 = ln\left(\frac{l_s}{l_0}\right)$ what is $l_s = \rho\theta, ln\left(\frac{\rho\theta}{l_0}\right) = 1 + ln\left(1 + \frac{y}{\rho}\right)$ which I am going to call it as $\varepsilon_a + \varepsilon_b$ and ε_a is nothing, but the strain in the mid surface whatever strain I have in the mid surface that is ε_a I am going to call and I am going to add a component to that that is called ε_a which is nothing, but bending strain. The bending strain is given $ln\left(1 + \frac{y}{\rho}\right)$ ok. So, this is ε_a and this fellow is ε_b. So, this $ln\left(1 + \frac{y}{\rho}\right)$ can be approximated to $\frac{y}{\rho}$ ok why because this y you see that it is going to be pretty small ok. So, we say this is t this is $t/2$ this $y < t/2$ ok.

Suppose if the thickness is let us say 2 mm ok then this y could be about 0.3 mm y could be about only 0.3 mm let us say ok. So, this 0.3 mm divided by this ρ could be about 50 mm this ρ could be the radius of curvature on which you are bending the sheet could be about 50 mm.

So, you can imagine that let us say this is 0.3 mm and this is 50 mm and then calculate $ln\left(1 + \frac{y}{\rho}\right)$ it will be almost same as it of your $\frac{y}{\rho}$ ok or you can expand this it is a series expansion $ln\left(1 + \frac{y}{\rho}\right)$ you can expand it maybe you can take the first term only which can be approximated to $\frac{y}{\rho}$. So, you can say axial strain $\varepsilon_1 = \varepsilon_a + \varepsilon_b$ as where $\varepsilon_a = ln\left(\frac{l_s}{l_0}\right)$ which will give you strain in the mid surface and you know ε_b which is nothing, but $\frac{y}{\rho}$ directly you can write as $\frac{y}{\rho}$ right. So, now if we plot this ε_1 ok in this thickness direction ok in the thickness direction if you want to plot this y this ε_1 it will look like this ok. So, you will see that your $\varepsilon_1 = \varepsilon_a + \varepsilon_b$ the ε_a is nothing, but what is $\varepsilon_a = ln\left(\frac{l_s}{l_0}\right) + ln\left(1 + \frac{y}{\rho}\right)$.

So, I am just keeping it as $\frac{y}{\rho}$ or $ln\left(1 + \frac{y}{\rho}\right)$ also you can write. So, you will see that this how the distribution distribution is shown in the blue colour ok. So, this basically a section in the sheet $t/2, t/2$ this is a mid surface let us say ok your middle surface mid thickness ok and this is a strain distribution I have given here ok and then your this blue colour line will tell you the distribution along its thickness and you will see that this blue colour line is actually crossing 0 at a distance little below the neutral axis little below the neutral axis. So, this also we need to know neutral axis is nothing, but this one the axis at the centre you can say ok. So, to start with ok it is at the centre now you will see that this neutral axis is shifted towards the bottom side where you generally expect some sort of compression ok.

And you will see that here it is going to be your 0 strain ok. So, which means in this equation if you put $y = 0$ you will see at the mid location ok there will be some strain available in the material that is what you are going to call it as a ε_a and along with ε_a you are going to add ε_b to get further strains above or below the section of the sheet. So, that is a meaning of ε_a and ε_b with respect to this particular strain distribution. So, if you have moment and tension this is how the strain distribution looks like ok with moment and tension is generally little bit complex to understand we will see that ok, but this is how strain distribution and bending can be calculated with respect to ε_1. So, ε_1 distribution in the thickness direction is given here there are two components ε_a which is which is nothing, but the middle surface where your strain and ε_b is going to be your bending strain which is additional with respect to ε_a and in this case at the mid location there is some strain ε_a provided here and $y = 0$ if you put this is what you will get as ε_a .

So, now this straight line bending can be taken as a plane strain bending process can be taken as a plane strain bending process plane stress it is still available with us along with that we are going to include something called as a plane strain bending. So, we are going to say that if there is no constraint on either side of the bend and hence no deformation the material in the bend deforms in plane strain ok. So, that means, the strain parallel to the bend will be 0 parallel to the bend would be 0. Suppose with respect to this if you take parallel to the bend means in this direction if you pick up ε it will not be there will not be any strain in this direction when compared to your thickness and across the bend when you compare thickness and across the bend bending along the strain along the parallel to the bend would be 0. So, for an isotropic sheet that is what we are discussing until now we can take it as ε_1 which is going to be available $\varepsilon_2 = 0$, but $\varepsilon_3 = -\varepsilon_1$ because $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$.

So, this also means that your $\beta = 0$ because it is plane strain bending now $\varepsilon_2 = 0$. So, $\beta = \frac{\varepsilon_2}{\varepsilon_1} = 0$ for plane strain bending ok. So, if $\beta = 0$, $\alpha = 1/2$ which we already derived using Levi Mises flow rule relationship between β and α we have already seen. So, $\beta = 0$ we already seen plane strain deformation process for which $\alpha = 1/2$ which means σ_1 will be available as usual and $\sigma_2 = \alpha\sigma_1 = \sigma_1/2$ and $\sigma_0 = 0$ because we are going to take it as a plane stress process ok.

So, now for this particular situation say $\beta = 0$, $\alpha = 1/2$ we can relate σ_1 to flow stress or effective stress and ε_1 to effective strain using our Von Mises effective stress and effective strain equations. So, I hope we know this $\bar{\sigma} = \sigma_1\sqrt{(1-\alpha+\alpha^2)}$. So, $\sigma_1 = \frac{\bar{\sigma}}{\sqrt{(1-\alpha+\alpha^2)}}$. So, in that if you put $\alpha = 1/2$ then you will get $\sigma_1 = \frac{2}{\sqrt{3}}\sigma_f$ which is nothing, but my plane strain flow stress which I am going to call it as S ok. So, my plane strain flow stress that means suppose if I am deforming a material in plane strain instead of uniaxial then that can be related to my uniaxial flow stress $\frac{2}{\sqrt{3}}\sigma_f$ which is nothing, but my plane strain flow stress.

Similarly, I can also get ε_1 as a function of $\bar{\varepsilon}$ ok using my effective strain equation which is nothing, but my you know if $\bar{\varepsilon} = \sqrt{\frac{4}{3}(1+\beta+\beta^2)}\varepsilon_1$ So, that equation 1 can refer and from there you can get $\varepsilon_1 = \frac{\sqrt{3}}{2}\bar{\varepsilon}$ by putting $\beta = 0$ in that equation. So, this is how you relate you evaluate σ_1 ok and your ε_1 in the plane strain bending process and we are going to use S here onwards rather than you know your you know σ_1 directly ok. So, where S is nothing, but your plane strain flow stress. So, we are going to say y your straight line bend can be assumed as a plane strain bending process and while assuming that we are going to put $\varepsilon_2 = 0$, is going to be there this ε_1 is nothing, but this ε_1 only ok.

We have got ε_1 here the same ε_1 and ε_3 which is basically along the thickness direction would be about $-\varepsilon_1$ ok. So, now we know how to calculate strain. So, now, we need to develop a general equation for evaluating tension and moment during bending for which I am going to use a simple equilibrium equation in this direction. This schematic will explain you that.

So, is the same schematic. So, I am going to have a bend sheet I am going to have a bend sheet with ρ as radius of curvature and you can see $t/2, t/2$ half thickness and you can see that I am going to take an element which is at y distance from the mid thickness the element is dy ok dimension ok. So, it is got a thickness of dy in the thickness direction ok and you will see that the force acting on that element is given by $\sigma_1 \times dy \times 1$ ok. So, this is basically for the unit section. So, force acting on a strip of thickness dy across a unit section is given by $\sigma_1 \times dy \times 1$ which is going to act in this particular section. So, in general I have given how tension is going to act in the sheet and moment is going to act in the sheet.

Now, this tension in the section is given by T is nothing but you have to integrate this particular fellow ok $\int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_1 dy$ ok. So, σ_1 that is the now next thing what we are going to discuss what will be σ_1 ok. So, for this tension we can get moment the same way $M = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_1 dy.1 \times y$ basically. So, we are going to multiply that with this distance from the center that is what is given here. So, in general you can write $\int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_1 y dy$.

So, if you know σ_1 then we can find out tension and moment at this particular section when you bend a sheet. So, now how do you get σ it depends on actually the material models that we are going to choose that is what the next one is ok. So, we are going to assume some material model which relates σ to ε one thing which you have already seen is power law that will be part of it other than that there are few other models one can assume ok. So, by knowing the stress strain law by knowing the stress strain law and by evaluating ε_1 ok. So, or ε you can say ok one can get the σ_1 and σ_1 can also be plotted like your ε_1 in the thickness direction ok and when we do that we have to define something called as ρ/t that is called bend ratio.

This ρ/t is going to be useful for us it is called bend ratio where ρ is your radius of curvature same thing which you define and t is the sheet thickness this ratio is called ρ/t. So, now there are few choices we have the first one is elastic perfectly plastic model ok. So, in this elastic perfectly plastic model the stress strain behavior is given in this diagram it will be like this.

So, elastic means you will have elastic part here and then perfectly plastic means there is no hardening ok. So, your flow stress is going to vary in horizontal way like this ok with respect to strain ok.

So, you have a linear first variation and then horizontal variation with respect to σ and ε ok. So, the initial part is defined by slope which is nothing but your E' the slope is actually E' we are saying and this height that are or otherwise this value is nothing but my plane strain flow stress my plane strain forces is nothing but probably this point is nothing but your yield strength you can say but in plane strain deformation process like bending which is what we are calling it as S. So, now with respect to this diagram there are two parts in this one is up to this strain corresponding to the end of elastic part and beyond that. So, there are two parts in this there are two parts in this. So, the stress is less than plane strain yield stress yes then ok this equation can be defined as $\sigma_1 = E'\varepsilon_1$ which is well known to us ok σ_1 in the initial part that is in the initial part in this particular part we can say $\sigma_1 = E'\varepsilon_1$.

So, in the second part for the strains greater than yield strain if you go beyond this we can simply say σ_1 is nothing but yes we will simply say σ_1 is nothing but yes. So, in this part it is nothing but $\sigma_1 = E'\varepsilon_1$ whereas in this part it is nothing but σ_1 is equal to yes we can say there are two parts here which will be useful for us ok. And what is this E' generally we say E but here we call it as E', E is nothing but your you know modulus of elasticity but E' here is nothing but modulus of elasticity in plane strain ok not in uniaxial but in plane strain mode of deformation ok. And this E' can be found out from this particular equation $E' = \frac{E}{1-\upsilon^2}$ here E is nothing but uniaxial Young's modulus and υ is your Poisson's ratio. So, both are given we can get E', E' can be substituted here and ε_1 can be evaluated from the previous distribution at any distance you know let us say y from the mid surface you can get σ_1 there you can get σ_1 in that location.

So, E is known and new Poisson's ratio is known you can get E' put it in this equation ε_1 can be found out from the previous distribution that we already shown and you will get the σ_1 at any distance from the mid surface for example at y distance you can get σ_1 ok. So, now how is that going to look like we will see in due course but since these are some material models we are going to see another model which is called as rigid perfectly plastic model, rigid perfectly plastic model. So, in rigid perfectly plastic model as the name suggests since it we say it is rigid we do not bother about elastic part which means it is neglected ok and strain hardening is also neglected in this and strain hardening is also neglected in this. So, you get a stress strain graph in this fashion. So, this S is nothing but your plane strain flow stress and this is how you define your stress strain behaviour for a rigid perfectly plastic model.

Strain hardening model is a conventional one that we know we have already known about this, this strain hardening model is a perfect one that we look into it which will take care of the hardening that you have in the bending process and this is given by σ_1 versus ε_1 is nothing but like this. It is going to have a very small elastic part and then a large strain hardening part which is modelled by $\sigma_1 = K'\varepsilon_1^n$ this K' is also actually like K only ok only thing is we are writing K' because it is a plane strain you know forming process ok. So, $\sigma_1 = K'\varepsilon_1^n$ and n has a usual definition nothing but the strain hardening exponent. So, we can have several other models we already seen like you know you can have with pre strain ok you can also have σ_0 into the equation ok. So, all are possible, but then we will restrict to only these three equations.

So, now we are going to pick up a one important case called bending without tension and we are going to do some analysis that is the first thing. So, whatever we are going to discuss is basically bending without tension. So, only moment ok. So, only moment is available for us. So, we can say that sheet is bent without tension, but with moment only ok.

So, what will happen the neutral axis will be at the mid thickness the neutral axis will be at the mid thickness. So, when we say there is no tension and only moment ok what will happen to ε_1 in this case $\varepsilon_1 = \varepsilon_a + \varepsilon_b$ is not it. So, we say that in this case since the now the neutral axis will be at the mid thickness ok this fellow will go off $\varepsilon_a = 0$ this fellow will go off. So, ε_1 will be approximated to ε_b directly ε_1 will be approximated to ε_b directly. So, that when you put a $\varepsilon_b = \frac{y}{\rho}$ if you put $y = 0$,then $\varepsilon_1 = 0$ which means.

So, you are strain at the mid location is nothing but 0. So, that is what we are going to have for this strain distribution now what will be the stress distribution for that how we are going to assume some material model and we get that that is the whole objective of this particular section. So, we are going to take the first case is nothing but elastic bending and for this case. So, the stress strain behavior will look like this ok. So, you will see that Y axis is σ_1, X axis ε_1 and the initial part is E' and then we are going to have a non strain hardening region with a yield strength of let us say plane strain yield strength of ok S. And for this case it is very straight forward you get a strain distribution in this fashion ok.

So, $t/2 = 0$ at the mid location why because a neutral axis will stay at the mid thickness ok and that is because $\varepsilon_a = 0$ and that is because $\varepsilon_a = 0$. So, for this you have to get stress distribution that is what our aim here is. So, here very simple is basically $\sigma_1 = E'\varepsilon_1$ because bending is only elastic. So, you are going to use equation relevant to the first part we already discussed just now that $\sigma_1 = E'\varepsilon_1$ where E' nothing but your plane strain elastic modulus ok. And since ε_1 distribution is known to you you can get σ_1 distribution as this particular figure.

The strain distribution the corresponding stress distribution is given in these two figures if you assume this particular model to describe the bending process $\sigma_1 = E'\varepsilon_1$ ok. So, now if you want to get a stress at any distance let us say y any distance let us say y from the neutral axis let us say then what is that then $\sigma_1 = E'\varepsilon_1$ this ε_1 has got two parts $\varepsilon_a + \varepsilon_b$ we said this fellow will go off ok then $\sigma_1 = E'\varepsilon_b$, $E'\frac{y}{\rho}$ I can write $\frac{E'}{\rho} = \frac{\sigma_1}{y}$. This is an important equation for us $\frac{E'}{\rho} = \frac{\sigma_1}{y}$. We will use this you know relationship in due course. So, now if this is the case how do we get moment in the section ok because we need to have moment no this moment is what is responsible for bending.

So, we need to get that moment and general equation we already discussed $M = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_1 y dy$ will remain I was hinting you that what will happen to the σ_1 in this particular case $\int_{-\frac{t}{2}}^{\frac{t}{2}} E' \frac{y}{\rho} y dy$ which we just now discussed ok and you will see that $\frac{E'}{\rho}$ will come out because they are constants ok and $-\frac{t}{2}, \frac{t}{2}$ you can call it you can take it as 0 to $\frac{t}{2} \times 2$ because it is symmetric in nature ok. So, $2 \times \frac{E'}{\rho} \int_0^{\frac{t}{2}} y^2 dy$ so you can integrate it ok put the limits finally you will see that you will get $\frac{E'}{\rho} \cdot \frac{t^3}{12}$ which can be written as this $\frac{t^3}{12}$ is nothing but my I ok with unit width considering unit width that is the thing ok. So, I can write $\frac{M}{I} = \frac{\sigma_1}{y} = \frac{E'}{\rho}$ which is obtained from this equation where $I = \frac{t^3}{12}$ for unit width it is second moment of area for unit width ok and $\frac{1}{\rho}$ we call it as a radius of curvature. So, we need to note down this $\frac{1}{\rho}$ is the curvature here. So, $\frac{M}{I} = \frac{\sigma_1}{y} = \frac{E'}{\rho}$ is the equation that we get and moment for this particular case is given by $\frac{E'}{\rho} \cdot \frac{t^3}{12}$.

So, E' can be evaluated from E and υ and ρ is given t can t is given or sheet thickness is given then you can get a moment for that particular bending operation. So, we can also evaluate this limit of elastic bending we can also evaluate this limit of elastic bending by putting some conditions what we see during bending. So, the limit of elastic bending that means, limit of elastic bending means how long are you going to be there within this zone how long are you going to be there within this zone ok can be obtained by this particular discussion. The limit of elastic bending is when the outer fiber at $y = t/2$ is this particular one y is equal to this is number our y no this is our y, $y = t/2$ means this particular surface this particular surface ok $y = t/2$ if you put that $y = t/2$ you will reach this particular surface the upper surface. The upper surface is generally undergoing tension as compared to the lower surface that we understand while we bend ok.

So, reaches a plane strain yield stress correct. So, whenever you are bending it ok the upper most fiber which is at $y = t/2$ will reach a plane strain yield stress right if that is a limiting case. So, it should be as long as it is within this region no problem the moment it reaches yes where at $y = t/2$ ok at the upper surface we call it as a limit of elastic bending. So, what do we do now this moment equation is $\frac{\sigma_1}{y} \cdot \frac{t^3}{12}$ which already derived $\frac{M}{I} = \frac{\sigma_1}{y}$, I which is nothing, but $\frac{t^3}{12}$ right. So, moment $M = I . \frac{\sigma_1}{y}$.

So, $\frac{t^3}{12}.\frac{\sigma_1}{y}$ correct which is what I have given here. So, what I am going to do is I am going to substitute σ_1 when it becomes S, $y = t/2$ that means, when $y = t/2$, σ_1 becomes S for limiting case. So, I am going to say that substitute this. So, 2 will come in the numerator. So, $\frac{2S}{t}.\frac{t^3}{12} = \frac{St^2}{6}$. Since it is a limiting elastic moment I am going to call this as M_e this will be called as M_e .

So, $M_e = \frac{St^2}{6}$. So, the limiting elastic moment ok. So, given a sheet thickness given a yield strength of the material you can get plane strain yield stress or flow stress you will get M_e limiting elastic moment. When you reach this particular moment what will be the radius of what will be the curvature $\frac{1}{\rho}$ that will be given by this equation. So, $\frac{1}{\rho}$ is nothing, but again the same thing $\frac{1}{\rho} = \frac{\sigma_1}{E'y}$. So, this $y = t/2$ this fellow will become S.

So, it is nothing, but $\frac{2S}{E't}$. So, when M becomes M_e, $\frac{1}{\rho} = \frac{2S}{E't}$ ok which can be represented in a moment curvature diagram. Like we have a stress strain diagram we can also have moment $\frac{1}{\rho}$. If you plot it you will get a straight line and you will see that this particular height this particular height is nothing, but your elastic moment height $M_e = \frac{St^2}{6}$ which you already evaluated and this $\frac{2S}{E't} = \frac{1}{\rho}$ ok when you reach M_e that is $\frac{2S}{E't}$ both are represented in this particular diagram. So, this is moment curvature diagram for this particular case. So, this results will be useful for us in the analysis that will discuss in the next section.

Now we will go to rigid perfectly plastic bending. So, where we the model is very simple your $\sigma_1 = S$ ok. So, there is no elastic part there is no hardening also and it is pretty straight forward the $\sigma_1 = S$ will give you a stress distribution like this ok. So, you will have like step type of thing ok. So, this would be your S value ok and there will be a transit at the middle and you will have a another variation on the on this side of the bend region ok. So, now the point here is since there is no elastic part here the moment itself is actually fully plastic moment like in the previous case we say M_e here we are going to call it as M_p which is nothing, but fully plastic moment because there is only one case that is your plastic moment only because there is only because it is rigid perfectly plastic know.

So, there is no elastic part. So, directly we will write M_p, $M_p = 2\int_0^{\frac{t}{2}} Sydy = 2S\left[\frac{y^2}{2}\right]_0^{\frac{t}{2}} = 2S\left[\frac{t^2}{8}\right] = \frac{St^2}{4}$. This $\frac{St^2}{4}$ can be drawn in a moment curvature diagram as this way it is going to be horizontal line similar to what you see in stress strain graph and this height is $\frac{St^2}{4}$ ok. So,

M_e the previous one is nothing, but $\frac{St^2}{6}$ and M_p here what we are going to discuss nothing, but $\frac{St^2}{4}$ these 2 are important ones we need to remember. The next case is a general case which is what is important for us you are bending a strain hardening sheet ok which means somehow you have to bring in your strain hardening you know part of you know into the equation.

So, that is why we are saying it is $\sigma_1 = K\varepsilon_1^n$ same power law only thing is $\sigma_1 = K'\varepsilon_1^n$ we are keeping ok. And this is approximated to ok $\sigma_1 = K'\varepsilon_1^n$ is approximated to K' will remain as it is your ε_1 there is no ε_a . So, it is ε_b only. So, $\varepsilon_b = \frac{y}{\rho}$, $\sigma_1 = K'\left(\frac{y}{\rho}\right)^n$ would be your stress distribution that you are going to get and the stress distribution to some extent schematically it can be drawn in this fashion.

So, $t/2, t/2$ this will not come ok. So, you are going to say that it is going to vary in this fashion and it is going to come down like this. This would be your σ_1 distribution for a general you know material model like $\sigma_1 = K\varepsilon_1^n$ ok. So, now if you want to evaluate a moment for this ok. So, it is the same thing ok. So, you are you will see that your moment is nothing, but I am going to write $M = 2\int_0^{t/2} \sigma_1 y dy$ right where $\sigma_1 = K'\left(\frac{y}{\rho}\right)^n$ ok.

This is $K'\left(\frac{y}{\rho}\right)^n$ you can keep ok. So, what will happen now? So, $2K'\left(\frac{y}{\rho}\right)^n$ will be there outside. So, 0 to t/2 will remain and then there will be a component of y^n. So, y is already there. So, $y^{1+n}dy$. So, you can integrate it apply your limits finally, you will see that it will be equation is going to be $\frac{M}{I_n} = \frac{\sigma_1}{y^n} = K'\left(\frac{1}{\rho}\right)^n$ ok.

Where I_n is same as that of your $I = \frac{t^3}{12}$ that we have seen, but the here it is very general in nature $I_n = \frac{t^{n+2}}{(n+2)2^{n+1}}$ ok. And if you want to draw graph between M and $\frac{1}{\rho}$ ok it will be something like this it will be a smooth curve it will be something like this M as $\frac{1}{\rho}$ would be similar to what you see in a stress strain graph ok. And this is a general equation taking strain hardening into consideration. And all our own values moment is moment, moment per unit with σ_1 is the is the principle stress and y means it is any distance ok from the neutral axis K' is nothing, but an equivalent of strength coefficient $\frac{1}{\rho}$ is a curvature and n is the strain hardening exponent that you get.

So, if you put two cases here ok for example, if you put $n = 1$. So, you will see that this $\sigma = K\varepsilon^n$ is not it. So, $n = 1$ it is like a K' will become E' ok like $n = 1$ if you put here. So, it is nothing, but $\sigma_1 = K'\varepsilon_1^n$, but which is nothing, but K' nothing, but K' only which you have defined before which you have defined before. So, that is what I have said here $K' = E'$ then we can directly write from this equation $\frac{M}{\mathrm{I}}=\frac{\sigma_1}{\mathrm{y}}=E'\left(\frac{1}{\rho}\right)$ from this you can directly write this when $n=1$ if you put you get this.

So, you can also put $n=1$. So, you can put $n=0$ it will go to the other case the other extreme case when you put $n=0$ here you will get $n=0$ you can put. So, $n=0$ here means it is $\frac{t^2}{4}$ you will get ok. And so your $n=0$ means yeah. So, your K' will be straight away it will be nothing, but S ok and then you can which means that you will have only M_p which means that you have only M_p ok. So, it is $K\varepsilon^n$, $n=0$ means straight away you can say your σ_1 is nothing, but K' here in this case S only S is nothing, but K' ok.

And it is going to be M_p only because it is a fully plastic moment and if you see that M_p can be calculated from the previous equation ok from the from this equation if you put M_p ok it is going to be $M=I_n.\frac{\sigma_1}{y^n}$. So, $I_n=\frac{t^2}{4}$. So, $\sigma_1=S$ in this case ok.

So, then y^0 it will vanish. So, $S.\frac{t^2}{4}$ will come ok. These two are two extreme cases when when you have n. So, this one will actually lead to the case which we already discussed. This one will when you put $n=0$ this one will lead to a case which we already discussed this will also lead to the previous case what we already discussed. So, this way one can get in a bending without tension ok one can get if you know the strain distribution the simpler case why because this fellow goes off ok. That is why we are picking up this particular case in this case if you know ε_1 you can get a σ_1 by assuming different material loss ok.

One is this first one next one is $\sigma_1=S$ next one is with strain hardening and in each case you are going to have moment curvature diagram that is going to be different here ok. Stress distribution is also different ok and you will see that your moment curvature diagram is also going to be different. So, now before we go ahead and discuss another small section in this particular module let us do one small problem with whatever we discussed until now. The question is given here a 2 mm thick aluminum sheet ok as a constant flow stress of 120 MPa ok. So, which means that it says constant flow stress which means that you have to take it as a in axial flow stress ok as 120 MPa it is not going to change with strain ok it is a constant one.

Now the question is determine the moment per unit width to bend the sheet to a limiting elastic state ok. That means, first question is you need to get M_e first one you need to get a moment per unit width to bend the sheet to limiting elastic state that is M_e. What is the radius of curvature $\frac{1}{\rho}$ at this particular stage? So, when M becomes M_e what will be the radius of curvature $\frac{1}{\rho}$. Determine the fully plastic moment the sheet is bent further ok. So, you are not stopping with the limiting case you are bending further ok what will be the your fully plastic moment that is M_p ok. Two cases two values are given one is Young's modulus other one is

Poisson's ratio these are known to us aluminum sheet has got Young's modulus of about 70 GPa and Poisson's ratio is about 0.3. So, we need to know M_e so it is understood it is $M_e = \frac{St^2}{6}$ we put a condition now. So, when $y = t/2$ your $\sigma_1 = S$ is not it. So, if you substitute in that this particular situation you will get $M_e = \frac{St^2}{6}$. Now thickness is given as 2 mm only thing is you have to get S, S is nothing but plane strain forces which you already got a relationship $S = \frac{2}{\sqrt{3}}\sigma_f$ the second section we have seen this particular one $\sigma_1 = \frac{2}{\sqrt{3}}\sigma_f$ right. The σ_f is actually given when you say simply flow stress we can take it as σ_f so $\frac{2}{\sqrt{3}} \times 120$ if you calculate it, it will give you about 138.6 MPa ok. So, S has been found out so what you can do is like you can substitute this S in this equation so 138.6 ok. So, you want to keep consistent units that you have to be careful ok. So, $M_e = \frac{St^2}{6} = \frac{138.6\times10^6(2\times10^{-3})^2}{6} = 92.3$Nm/m is a value that is given. So, this has been found out now.

So, now next one is we need to get $\frac{1}{\rho}$ at this particular stage that is so you want to get this then we already derived this equation $\frac{E't}{2S}$ it is radius of curvature so this radius of curvature so it is ρ ok ρ_e because it is a limiting case know it is ρ_e. So, $\rho_e = \frac{E't}{2S}$ know so where is it yeah so $\rho_e = \frac{E't}{2S}$ this also we derived already ok. So, t is given S has been found out already E' have to be found out so $E' = \frac{E}{1-\upsilon^2}$, E is 70, $1-0.3$ keep consistent units here so you will get $E' = 76.9$ GPa it is slightly larger than what you see in your uniaxial Young's modulus 76.9 here you can see 70. So, that 76.9 you can substitute appropriate unit conversion you have to follow into thickness is $\rho_e = \frac{E't}{2s} = \frac{76.9\times10^9\times2\times10^{-3}}{2\times138.6\times10^6} = 0.56$m you can convert that into millimeter also if you want ok. So, second one is also found out ok. Next one is basically if you further deform it to fully plastic moment what will be the value so that is also found out $M_p = \frac{St^2}{4}$ which we already know So, either you substitute this S t and then get it otherwise what you can do is you can relate that to M_e ok.

So, we can also see $\frac{M_p}{M_e}$ is how much so $M_p = \frac{St^2}{4}$ divided by $M_e = \frac{St^2}{6}$ you will see that this is $\frac{M_p}{M_e} = \frac{3}{2}$ this fellow will go this fellow will go $\frac{6}{4}$ which is nothing but $\frac{3}{2}$. So, $M_p = \frac{3}{2}M_e$ that way I can get so M_e is already known to $M_e = 92.3$ you can substitute it here you will get $M_p = 138.5$ ok or you can directly get the $M_p = \frac{St^2}{4}$ you can substitute all the values here you will get the same value.

So, S is given here t is already given as 2 mm use consistent units finally you will get M_p either way it is fine. So, in this way we can find out moment per unit width for the limiting case or the radius of curvature at this stage and the plastic moment with whatever we have discussed until now these derivations are already done. So, now let us go to one important section we will not discuss this fully today but then rather we will discuss certain things conceptually. Elastic loading and spring back, this is spring back is going to be very important for us in sheet metal forming this is actually a defect is actually a defect ok. So, suppose if you consider thin sheet like this and you want to create a hat type of structure I want to create a channel like this hat type of structure you can imagine ok. So, once I unload the material it may so happen that my this fellow hat will become something like this I am just drawing schematic it can become something like this.

This is a your initial one ok this is a formed one ok and after unloading your sheet can become like this. This dimensional change which you are looking at know here, here, here and here this is actually called as a spring back. This dimensional change this angular change is called as a spring back. So, you need to have a flat punch but here you can see some angle is created this wall also can have some angular displacement which is what you are calling it as a spring back. Basically dimensional change in the sheet once you unload the material is what we refer as a spring back.

So, why this spring back happens can be understood from these three schematics ok. So, you will see the first one sheet is bent and you will see that this is your neutral axis of the mid region that is your pink color line and above that you will see that if you take any point or any element it will be pulled that is it will be in tensile mode of deformation and at the inside location you will see the elements will be compressed. Suppose this will be compressed this will be fully tensile mode of deformation ok and since we are discussing anyway bending without tension you will have a stress and stress distribution, strain and stress distribution to be 0 at the mid surface. And you are going to pick up let us say a point A ok point A here ok and the point A has got a tensile stress like this which can be represented in this stress strain diagram like this. Suppose a corresponding stress strain diagram of the material is given here you can see usual things like you have yield strength then you have UTS then you have F and this a point this a situation is measured here. This a situation is measured here that means you are bending a sheet at one particular location above the neutral axis you can see a tensile stress let us say A the A is in between yield strength and UTS which is mentioned here ok.

And that means when you say strain as unstressed as 0 at the mid surface it means you are starting from here it means you are starting from here and you are moving towards A and you are moving towards A let us say in this direction which means you are going to cross yield strength and you are going to reach A ok and then you are going to go further that is the meaning ok. So, moving from this point to A is nothing but you are going along this direction reach the yield strength and then cross that and go to A that is the meaning. What does it mean? That means that there is a small region above and below the neutral axis which is actually in the elastic part and above that and below that band there will be a plastically deforming zone which is described by the strain hardening part that is what I represented in this particular diagram. So, you will see that with respect to neutral axis there is one small red color hatched region this region is called as elastic zone which describes this particular part with respect to stress strain curve it describes the elastic part of the stress strain diagram and above that you will have a zone deforming plastically because of tension and below that this lower boundary you can have a region which is deformed plastically because of compression because of compression ok. That means above neutral axis below neutral axis there is a small band of elastic deformation and beyond that you will see that you will have plastic deformation because of tension and compression.

So, now this is the situation we have so we say spring back occurs because of variation in the bending stresses across the thickness from inner to neutral axis to now outer surface ok. From inner to neutral axis to outer surface ok there will be variation in bending stresses which you already calculated σ_1 and we are saying that is a reason for spring back and we are saying that the zone above neutral axis deform plastically because of tension and zone below neutral axis deform due to compression that also we know the stress at any point A in the tensile stress stone should be less than UTS. We are saying that this A point should be less than UTS otherwise what will happen if A reaches UTS means a crack can develop at this location at the upper surface a crack otherwise the outer surface will crack. So, now what we are saying is this particular one the metal region near the neutral axis is stressed below the elastic limit ok.

So, still it is elastic deformation ok that is near the neutral axis that is this particular zone this particular zone ok. This elastic deformation is a narrow band on both sides of the neutral axis that is what I have mentioned here. So, now what will happen ok so you will see that during stamping and after stamping. So, you want the sheet to be bent like this but after stamping after removing load it can become like this. So, this is what we said as a spring back this is what we said as a spring back. Upon load removal the elastic band ok which is just above and below the neutral axis tries to return to the original flat sheet correct that is the purpose of elastic deformation will try to recover right ok.

It will try to return to the original flat sheet but it cannot do that due to restriction given by the plastic deforming zone. So, what we are saying is this red color portion which is representing elastic part will try to recover ok which is equivalent to change in dimensions but the plastic part which is outside and below this neutral axis below this region elastic region will not allow it to come back fully ok. But there will be some small change in the dimension which is what we are going to represent as a spring back. But some return occurs as elastic and plastic zones reach an equilibrium condition this return is termed as a spring back. So, full recovery full dimension change is not possible because you already given some partial plastic deformation to it ok and that actually suppresses the release of elastic part ok.

But still ok some dimensional changes occur which is what we are called as spring back. And if you want to represent it using a stress strain diagram this we already discussed. So, this is a typical stress strain diagram of any material. So, you have yield strength then you have a strain hardening portion then after that you have a decrease in stress strain curve. Now, so in the loading part the loading part means your σ will keep on increasing let us say when you are trying to unload the material we know that the unloading curve is going to be this one which is going to be parallel to the elastic part and will reach this particular point ok. And from this to this if you measure this would be a cause of permanent deformation the remaining one is actually responsible for spring back this is a this part is actually responsible you are for spring back.

So, schematically this way we can explain what is spring back with respect to stress strain graph. So, there is a loading part in the stress strain graph there is a unloading part the remaining one which is what is responsible for spring back. So, now with respect to this diagram one can tell some important properties which can affect the spring back. For example strength ok strength of the material ok. So, for that I have given some simple example here suppose this is your σ versus ε graph ok. So, you will see that this black color portion is nothing but your high strength material and the red color one is nothing but your low strength material you will see that at the same strain you deform it and then try to unload it the red color one low strength will try to unload it in this path whereas, the blue one will be in this path and you will see this a change in the spring back.

So, high strength actually has more change in strain which is responsible for more spring back. So, larger the strength spring back would be larger. But at the same time if you change the elastic modulus here these two materials have called same elastic modulus suppose elastic modulus is like this ok and then you are deforming it to the same strain and then you are unloading it your unloading curve your unloading curve would be something like this which will be parallel to this. Now you will see that this change is going to be much much larger as compared to these two which means that if you change if you increase the elastic modulus there are chances that spring back will reduce. If you increase there that means if you decrease the slope ok you are you will see that your spring back is more that means larger elastic modulus will lead to lesser spring back. Often this $\frac{\sigma_{YS}}{E}$ this ratio becomes very important because the σ_{YS} can be controlled individually in this fashion and E can be controlled in this fashion to control spring back.

But both these values are in a way related to your elastic part of deformation which is what is responsible for spring back. Often this ratio is used we will also derive one equation in that you will see instead of of course we say $\frac{\sigma_{YS}}{E}$actually it is actually going to be $\frac{S}{E'}$. Now the corresponding one in plane strain bending know that is nothing but $\frac{S}{E'}$ this ratio becomes important for us. So one can control spring back by controlling the strength and young modulus of the material and moreover you can also control spring back by this particular ratio called as $\frac{R}{t}$ where R is your you know the radius of the tool at which you bend the sheet and t is a sheet thickness. Basically R determines whether it is sharp bend or you know blunt one or a blunt one.

So suppose if you take a sharp bend what will happen it will concentrate stress more in the gradual bend ok. So resulting in more plastic strain so smaller $\frac{R}{t}$ ratios will result in less spring back. So what we are saying is if the radius is small you put lot of plastic deformation to it plastic strain to it which can suppress the elastic part so there are chances that you will have lesser spring back. And if you increase you know your radius to a larger value then it is going to be opposite to that.

So other than this you know spring back can be controlled by tool design also. General you know rule is you over bend it ok. Suppose like in this particular case this particular case what we do is suppose this is a flat sheet and you want to bend it to this one actually so instead of that what you do is you little bit bend inwards so that when you release it can become straight. In that way one can this is basically called as over bending. That is one way to control it. There are a few other ways also people control.

So one can deform material at higher working temperatures ok in that way also one can see how spring back can be controlled ok. Die design can be modified to take care of spring back. There are several you know lots of work people have done to take care of your spring back. How to control spring back. So now if you want to evaluate some theoretical model ok let us say you want to develop some theoretical model for spring back ok.

So a simpler way to evaluate theoretical model for to estimate to derive a theoretical model for spring back will be discussed here. And then of course we are going to first take a case of moment without tension ok. So moment without tension we are not going to derive it now we will just introduce this here now. So you have a sheet as usual you can see I have just shown a thin line which is a sheet this is your let us say sheet no tension ok. So only moment is applied as shown in this figure and there is a θ bend angle ok and radius of curvature is ρ is known and this l is actually the length of the mid surface which is already known to us. So now this is a bend sheet ok there is a flat sheet and you are bending it this is a situation now you are releasing the load which is a unbend sheet this will be a situation ok.

This θ will become $\theta + \Delta\theta$ ok and ρ would become $\rho + \Delta\rho$. So since there is no tension only moment is given we can say that this length will remain same the length of the mid location the mid surface ok or the neutral axis will remain same ok it is not going to change. When the sheet is bend and released by removing moment there will be a change in curvature and bend angle θ will become $\theta + \Delta\theta$ ok and ρ would become $\rho + \Delta\rho$. The length of the mid surface $l = \rho\theta$ this we already discussed in the first slide. This length we are saying will remain unchanged during unloading as a stress and strain at the mid surface is 0 why because it is moment without tension ok.

So because of that the length will remain same and then from this equation we can directly write this $\theta = l\left(\frac{1}{\rho}\right)$ if you differentiate the equation you will get $\frac{\Delta\theta}{\theta} = \frac{\Delta(1/\rho)}{1/\rho}$ ok. So where

$\Delta\theta$ is a change in angle due to this dimensional change called spring back ok there will be a corresponding $\Delta(1/\rho)$ ok and if you can normalize it with respect to original θ and original $\frac{1}{\rho}$ you can this equation is valid ok. So it is like either you calculate $\Delta\theta$ or $\Delta(1/\rho)$ to quantify spring back either you calculate $\Delta\theta$ or $\Delta(1/\rho)$ in a way to quantify spring back or to estimate spring back when you have moment without tension when you have moment without tension ok. So we stop here we will continue this.