# Mechanics of Sheet Metal Forming

**Prof. R Ganesh Narayanan**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Guwahati**

**Week- 05**
**Lecture- 13**
**Sheet Formability(contd)**

So, we will continue our discussion in this module instability and tearing of sheets and we have completed the theory part of this particular module and we will now enter into you know the problem solving part. So, today we are going to solve 4 problems in this particular module, question number 1, question number 2, question number 3 and question number 4. These are the 4 problems relevant to this particular chapter module we are going to solve 4 problems ok.

So, the first question is the following figure this particular one shows a 100 mm length of a tensile test piece ok total is 100 mm, 90+10 in which 10 mm that is this 10mm has a width of 12.4 mm ok width is reduced and the remaining is 12.5 mm ok. So, we have a 100 mm gauge length let us say 90 and 10 it is split into 2 parts 90 mm 10 mm the difference is in 90 mm you have the original width of 12.5 mm, but in 10 mm length that particular part you have 12.4 mm 0.1 mm is reduced actually. So, all dimensions are in mm you can say, but the entire sheet has got uniform thickness of 1.2 mm. So, $t_0 = 1.2\, mm$ .

So, this material obeys a stress strain law it is given here $\overline{\sigma} = 700(\overline{\varepsilon})^{0.22}$ MPa and so let us assume that these 2 regions deforming in the axial tension. Now the question is determine the maximum load the final length of 20 mm gauge length in the wider section. So, there will be one maximum load $P_{max}$ of the entire strip that we have to find out and then in this wider length that is your 12.5 mm this particular part you pick up a gauge length of 20 mm let us say at the center you have gauge length of 20 mm you are picking up ok.

So, what is the final length of that particular gauge length ok and the maximum strain in this section. So, in this 20 mm gauge length what is the strain that is going to be encountered ok during this tensile test that is a question here ok. So, this is like equivalent to what we have studied you know like we are defining a thickness heterogeneity right $\frac{t_A}{t_B}$. So, where $t_B$ is your neck thickness or a groove thickness and $t_A$ is outside region right. Similarly here also instead of a thickness heterogeneity you are having a reduction in width a similar situation we have solved one problem in the in the first lecture of this module if you remember it ok.

So, now when we are loading it in tensile test we have to find these 3 values $P_{max}$ the strain in the in this particular section 20 mm gauge length and what is the final length if that is the strain. So, directly we are going to write this particular one in a tensile strip ok. So, the load is given by $P = \sigma_1 A_1$ we know that and this already we know we already derived it $\sigma_1 = K{\varepsilon_1}^n$ and $A_1 = A_0 \frac{L_0}{L_1}$. So, $A_1 L_1 = A_0 L_0$. So, $A_1$ is nothing but so $A_0 L_0 = A_1 L_1$ .So, we want $A_1 = A_0 \frac{L_0}{L_1}$ ok that is what I have written here and $K{\varepsilon_1}^n$, and $A_0 = t_0 w_0$ , $\frac{L_0}{L_1} = \exp(-\varepsilon)$ ok. So, now what we are going to do is when you are going to do tensile test of this we know that this 12.4 mm width this particular region is actually the weaker region and hence in the narrow section I am going to call this as narrow section we can directly write the maximum load ok will be given by $\varepsilon = n$. So, I am going to write I am going to modify this particular equation for maximum load in narrow cross section in the narrow section. So, $P_{max} = K{\varepsilon_1}^n\, t_0 w_0 \exp(-n)$ ok.

So, directly I am going to write this and I am going to just substitute all the values $K$ is known which is 750 MPa ok and so here also you are going to have $n$ so $\varepsilon_1 = n$ so $n^n$ will be there ok. So, this is $n^n$ now $\varepsilon_1 = n$ ok so either $\varepsilon\ or\ \varepsilon_1$ either way you can call. So, $P_{max} = 750 \times 10^6 \times 1.2 \times 12.4 \times 10^6 (0.22)^{-0.22} \exp(-0.22) = 6.42 (kN)$ ok. So, this equation we are simply modifying it for situation in maximum load $\varepsilon$ is equal to n in narrow section we are picking a narrow section we are putting this condition ok. So, that you can convert this $\varepsilon_1$ into $n$ and this also as $n$ let us say so that you can write directly as $P_{max} = 750 \times 10^6 \times 1.2 \times 12.4 \times 10^6 (0.22)^{-0.22} \exp(-0.22) = 6.42\ kN$. Now let us come to wider section, wider section we will have the same equation $P = K{\varepsilon_{1A}}^n\, t_0 w_0 \exp(-\varepsilon_{1A})$ only thing is instead of $\varepsilon_1$ I am putting $\varepsilon_{1A}$ because we do not know that is what we need to find out ok. So, $P = K{\varepsilon_{1A}}^n\, t_0 w_0 \exp(-\varepsilon_{1A})$ ok. So, but whether it is this region wider region or a narrow region ok there will be only one $P_{max}$ so what I can do is I can directly equate this to $P_{max}$. So, at maximum load what I am going to do is I am going to directly equate this particular equation to 6.42 which I got from the previous narrow region.

To this narrow region what are I am getting here this I am going to equate it to the situation in this particular wide section because there is going to be only one $P_{max}$for the entire strip so I am going to say that $750 \times 10^6 \times 1.2 \times 12.4 \times 10^6 (\varepsilon_{1A})^{-0.22} \exp(-\varepsilon_{1A}) = 6.42 (\mathrm{kN})$, and what we can do is we can combine all the numerical values on one side and then keeping this you know the fact the terms having $(\varepsilon_{1A})^{-0.22} \exp(-\varepsilon_{1A})$ I am going to keep it as it is and I am going to combine all the numerical values it is going to be equal to 0.571. So, what I am going to do is I am going to iteratively change this $\varepsilon_{1A}$ value ok and see for which value is $\varepsilon_{1A}$ I am going to get 0.571 ok. So that I have just quickly took four different values and then plotted it here $\varepsilon_{1A}$ versus this entire thing function of $\varepsilon_{1A}$ this I need to get this.

So, if I use 0.2 ok here if I use 0.2 or 0.2 here so that I will get 0.5746, 0.18 I get 0.5728, 0.16, 0.5694 and 0.17 if I put you will see that I am getting 0.571 ok. So from this table I can directly say we get $\varepsilon_{1A} = 0.17$ so that is my maximum strain in this section ok. In the wider section I am taking a maximum strain of 0.17 that is the next question that is the answer for

next question. So $P_{max}$ has been found out already 6.42 and then in the wider you know section I am going to have $\varepsilon_{1A}$ as 0.17 ok and for this particular strain in that section what will be the value of 20 mm length or 20 mm length 20 mm gauge length in the wider section should become some value what is that value that you will get this particular strain. So it is very simple so we know the original definition of $\varepsilon_{1A}$ so I can directly write the new length would be equal to the original length 20 $exp\ \varepsilon_{1A}$. So this comes from the original definition of $\varepsilon_{1A} = ln\frac{l}{l_0}$ original definition is not it. So from that I can write this particular equation and you will see that for this particular value of $\varepsilon_{1A}$ I am going to get $l = 23.17$ mm. So that means when you are doing tensile test of this particular type of sample and $P_{max}$ will happen at 6.42 kilo Newton and during that particular situation you will see that this the strain attained in the wider region would be equal to $\varepsilon_{1A}$ that is along one direction would be equal to 0.17 and this when you have 0.17 strain the gauge length 20 mm would become 23.17 mm.
So at that particular stage you are going to have these three different values ok. So it is just a simple problem only thing is like we had two important points here one is for narrow section you are going to put this particular condition and get maximum load that maximum load you are going to equate it to the same situation similar situation in the wider region that is in the A region let us say ok. So and then finally we are going to get the strain and the corresponding gauge length new gauge length right ok.

So let us go to the next one, next one is also a similar you know geometry. So what is the question? The test piece geometry is used that it has got two parallel reduced lengths one is 10 mm width the other one is 9.8 mm width ok. So that is how the sample dimensions are. So width is 10 mm ok in another section you are going to have 9.8 mm width. So in the wider section a gauge length of 50 mm is marked ok. So let us mark it ok the strip is pulled to failure ok so you are deforming the material and it goes to failure and the gauge length measured to determine the true strain ok.

So this particular gauge in 50 mm is used to measure the strain now ok. So now the question is when you do that suppose if you say that ok you need to get that particular strain in the range of 0.05 to 0.2 what will be the change in strain hardening index or exponent strain hardening exponent that is the question ok. The strip is pulled to failure and the gauge length measured to determine the true strain $\varepsilon_a$ obtain a diagram relating true strain $\varepsilon_a$ in the range 0.05 to 0.2 to the strain hardening index ok. So that means how $n$ value is going to change depending on the strain that you get in the gauge region. So that is the question but the condition is the strip is not having uniform width it is going to have a slight change in width one is 10 mm width the broader region the other narrow region has got 9.8 mm width. So since you are going to speak about $n$ then $\varepsilon_a$ ok and then you are going to have some sort of area reduction between 10 mm width and 9.8 mm width ok directly we can relate all this you know parameters by this particular equation which you already $(n - \varepsilon_u) \approx \sqrt{-n\frac{dA_0}{A_0}}$. With this we already worked out one problem in this module so this equation can be used to direct. So now basically you need to get relationship between $n$ and $\varepsilon_u$ and that $\varepsilon_u$ you vary between 0.05 to 0.02 we need to see how $n$ is going to change that is all ok.

But for that you need to know this imperfection severity of imperfection so which is nothing but $\frac{dA_0}{A_0}$. So you can see that it is 10 mm width 9.8 mm width so the difference is you can take thickness, thickness is not given maybe you can take thickness of 1 mm let us say so if that is the case then it will $\frac{dA_0}{A_0} = \frac{-0.2}{10} = -0.02$. and as per this equation $(n - \varepsilon_u) \approx \sqrt{-n\frac{dA_0}{A_0}} = \sqrt{0.02\,n}$ so minus minus will become plus $0.02 \times n$. So $(n - \varepsilon_u)^2 = 0.02n$ ok. So now you can expand this $n^2 - 2n\varepsilon_u + {\varepsilon_u}^2 = 0.02n$ so and you will see that this will be a in the form of a quadratic equation. So you can see that $n^2 - (2n\varepsilon_u) + {\varepsilon_u}^2 = 0.02$ this is nothing but $n^2 - (2\varepsilon_u + 0.02)n + {\varepsilon_u}^2 = 0$. So $Ax^2 + Bx + C = 0$ it is of that form so you can find root of this equation as so this way so $n = \frac{(2\varepsilon_u + 0.02) \pm \sqrt{(2\varepsilon_u + 0.02)^2 - 4{\varepsilon_u}^2}}{2}$ ok.

So you can simplify this to get in a such a simpler format where $n$ is going to vary with respect to $\varepsilon_u$ in this way ok and it is said that if you change $\varepsilon_a$ or in this case $\varepsilon_u$ ok because you are saying that this pull due to fracture no so there is nothing wrong in you know keeping this $\varepsilon_a$ as $\varepsilon_u$ in this equation both are going to be same. So now if you change $\varepsilon_a$ let us say 4 different values you take between 0.05 and 0.05 let us say 0.1, 0.15 and 0.2 if you substitute here what is the value of n so you may get 2 you know you may get a range because there is a plus minus ok so you may get a range which is what I plotted here. So if you change $\varepsilon$ here 0.05 you may get one limit for you know 0.0 next limit for 0.1 then 0.05 you get another limit and 0.2 you get another limit. So obtain a diagram so this is the diagram we are going to get which basically tells you how n changes with respect to this $\varepsilon_a$ ok. So depending on $\varepsilon_a$, $n$ is not going to be a constant $n$ is going to vary in this fashion ok so this is your next problem.

So your third problem is also relevant to your imperfection related ok so this problem is also very important for us to understand what we studied before. So this diagram you can refer this diagram is known to us ok this was used before for some theoretical explanation. So you have a sheet which is pulled in both the directions let us say $\sigma_1$ and $\sigma_2$ principle stresses in one direction two direction and let us consider A region and B region, A region is actually is uniform region, B region is like a groove you can say or a neck region so where thickness is going to be $t_B$ and outside is going to be $t$. So this is where we defined $f = \frac{t_B}{t_A}$ if you remember this right. So this is the situation we have so what is it said in the question an element of material has an imperfection characterized by $f_0 = 0.995$ it is same as that of $f$ but we know that the $f$ is not going to remain same so we say $f_0$ as shown in this figure here this particular figure ok.

So that particular material has got this imperfection defined by 0.995 so small imperfection value ok so it is deformed in equi-biaxial tension that is given here ok. So $\sigma_{1A} = \sigma_{1B}$ means that means with respect to A location with respect to A region ok with respect to a region your $\alpha$ is known for biaxial tension $\alpha$ is nothing but one you can keep. So and the entire material is going to follow this particular stress strain law $\bar{\sigma} = 600(0.004 + \bar{\varepsilon})^{0.2}$ MPa and you know the fact that this is nothing but your pre-strain value such a small value 0.004 MPa ok so pre-strain value. So now what is the question? Determine the principal stresses, principal stresses means you have $\sigma_1$ and $\sigma_2$ and the stress ratio that is nothing but $\alpha$ in the groove region when the uniform region starts to deform, when the uniform region just starts to deform. So this A region is just going to deform start to deform at that particular situation you need to get principal stresses and stress ratio in the groove that is B. So I am going to say $\varepsilon_B, \varepsilon_{1B}, \varepsilon_{2B}$ these three values you have to evaluate.

So which means that one should understand the fact that the $\sigma_{1A}, \sigma_{2A}, \alpha_A$ are not going to be same as that of this that is why this question is actually asked right. So first of all we need to evaluate these three values in the B region that is in the groove region. This B is nothing but your groove region because your thickness is less a is a uniform region. So now when the uniform region starts to deform so that is a key point okay. We can say that for both the elements if this is the situation the material has zero plastic strain.

It is just going to deform okay. So if the plastic strain value is given you can directly use it. If it is not given then we can simply say material has zero plastic strain because we do not have any other reference of what is the strain when the uniform region starts to deform okay. It may have small value but then we do not have it in the question. So we can simply say that this $\bar{\varepsilon}$ which is nothing but plastic strain let us say this is going to be zero.

So that refractive stress $\bar{\sigma} = 600(0.004)^{0.2} = 198.9\ MPa$ okay. So this $\bar{\sigma}$ has been obtained now okay. So when the uniform region starts to deform suppose if plastic strain $\bar{\varepsilon}$ is some value is given then we have to use that value to get $\bar{\sigma}$.

So which is not going to be this it could be slightly some other value okay. So now given the situation we are saying that this fellow is going to be equal to 0 okay. So now let us come to A region which is a easy region for us to evaluate certain things. So in the A region now because I know $\bar{\sigma}$ because I know $\bar{\sigma}$ I have to relate this $\bar{\sigma}$ to one of the principle stresses. So directly we are going to use some yield function and well known in function right now for us is nothing but your Von Mises yield function.

So as per Von Mises yield function we can directly write $\frac{\bar{\sigma}}{\sqrt{(1-\alpha+\alpha^2)}} = \sigma_1$ we say since it is A region I am writing $\sigma_{1A}$. So suppose if you put $\alpha = 1$ because we say that it is the A region or the uniform region we say that it is balanced or equi, equi-biaxial tension. So $\alpha = 1$ okay. So this also is 1. So 1 minus 1 plus 1 square root goes off so, $\bar{\sigma} = \sigma_{1A}$.

So what does it mean? That means we can directly write $\sigma_{1A} = 198.9\ MPa$. So we need to find $\sigma_{1B}$ and $\sigma_{2B}$. So now what we have done is we have found out $\sigma_{1A}$. $\sigma_{1A}$ has been found out okay. Then if you know $\sigma_{1A}$, $\sigma_{2A}$ can be found out but that may not be useful for us because we know $\alpha$ okay. So it may not be useful for us it will be same so but it will not be useful for us. So directly we are going to B region. So in B region okay if you want to go to B region then we have to use some already known value that is $\sigma_{1A}$ okay.

So if you want to find $\sigma_{1B}$, my aim is to find $\sigma_{1B}$. So $\sigma_{1B}$ has to be found out with a known value let us say $\sigma_{1A}$ then the best relationship that I can find out is nothing but my *f* value, my *f* value is not it. So we have already derived that $f = \frac{t_B}{t_A}$ from here we have derived the stress ratio also okay in connection with *f* okay and that is what is given here. So $\sigma_{1B}$if you want that will be obtained by $\frac{\sigma_{1A}}{f_0}$. This we already derived. $\sigma_{1B}$ if you want you can have $\frac{\sigma_{1A}}{f_0}$, $\sigma_{1A} = 198.9$ and $f_0 = 0.995$ and that will give you $\sigma_{1B} = 199.9\ MPa$ right. So it is okay like for example $\sigma_{1B} = 199.9\ MPa$ would be larger than $\sigma_{1A}$ that is what is given to us.

So now what I am going to do is I am going to now $\sigma_{1B}$ is found out. So my this value is now ready okay. So now this value is ready. So now I need to find out let us say $\sigma_{2B}$ or $\alpha_B$ okay. If I know $\alpha_B$ I can find $\sigma_{2B}$ or if I know $\sigma_{2B}$ I can find $\alpha_B$ but now what I am going to do is to find out $\alpha$ now.
e

So now the same thing. So if I know $\sigma_{1B}$ major principle stress is known. If I want to know $\alpha$ the simple relationship what I have is Von Mises yield function. So I can directly write for region B also $\frac{\bar{\sigma}}{\sqrt{(1-\alpha+\alpha^2)}} = \sigma_{1B} = 199.9$. If you simplify this you will get $\alpha = 0.99$ which is not actually 1. You will see that the with respect to A region $\sigma_{1A} = \sigma_{2A}$ which is given but actually if you calculate $\alpha$ in the groove region it is slightly less than 1. It is not equi-biaxial tension. It is biaxial tension but $\alpha \neq 1$ there is a change. So one has to be little bit careful and understand this particular thing and that is all. So if $\alpha$ is known to me so now I can find $\sigma_{2B}$ because $\sigma_{2B} = \alpha\sigma_{1B} = 0.99 \times 199.9 = 197.9\ MPa$. So I am going to use two important relationship. One is my Von Mises yield function $\frac{\bar{\sigma}}{\sqrt{(1-\alpha+\alpha^2)}} = \sigma_1$ and then this particular relation $\sigma_{1B} = \frac{\sigma_{1A}}{f_0}$ okay. These two relationship this one and this one these two expressions are actually used to evaluate the entire problem. So region A, region B okay are divided and we can apply these equations appropriately to get a required answer. But again I am telling uniform region starts to deform if some strain effective strain is given we have to use that instead of zero plastic strain.

So now here we do not have choice to assume it to be zero even at the start of the deformation okay. So this is the third problem. So now let us go to fourth one which is a general one okay. So now assuming that tensile necking begins at maximum load okay so that we already put this condition is not it. So we already said that your $df = f = dP = 0$

right. Begins at maximum load find the actual true strain $\varepsilon$ at necking for the following material loss after deriving a general equation okay. So basically you need to get the actual true strain at necking. So basically you can say $\varepsilon_u$ you want to get okay at necking. But only difference is there are three different stress strain loss that is given and some specific values are given to get some actual values for this $\varepsilon_u$ okay. One is $\sigma = \mathrm{K}(\varepsilon + \varepsilon_0)^{\mathrm{n}}$ this equation we already know where $\varepsilon_0$ is your pre strain that value is given 0.05 and $n$ is also given 0.25 and $K$ is given as 500 mega Pascal. There is next equation $\sigma = \sigma_0 + \mathrm{K}\varepsilon$ this is a very linear equation between true stress and true strain and $\sigma_0$ is nothing but 250 MPa and $K$ is about 350 MPa they have and another one is a trigonometrical function okay $\sigma = K\sin(B\varepsilon)$ and $K$ is given as 500 MPa and your $B$ is let us say $2\pi$. So since it is tensile necking begins as maximum load you want to get actual true strain we are going to put the same condition which we derived before that is $\frac{d\sigma}{d\varepsilon} = \sigma$ or $\frac{1}{\sigma}\frac{d\sigma}{d\varepsilon} = 1$ we said is not it the same equation can be used here. So $\sigma$ s are given so you have to differentiate and equate it to the same equation get a general form of the equation apply these values you will get $\varepsilon$ that would be the approach. So $\sigma = \mathrm{K}(\varepsilon + \varepsilon_0)^{\mathrm{n}}$ so $\frac{d\sigma}{d\varepsilon} = nK(\varepsilon + \varepsilon_0)^{\mathrm{n}-1} = K(\varepsilon + \varepsilon_0)^{\mathrm{n}} = \sigma$. So this $\sigma = K(\varepsilon + \varepsilon_0)^{\mathrm{n}}$ so you can equate like this from this you will get $n = \varepsilon_0 + \varepsilon_u$ and $\varepsilon_u = n - \varepsilon_0$, $n$ is already given as 0.25, $\varepsilon_0$ is 0.05 so $\varepsilon_u$ would be 0.2 okay.

So this is straight line fit $\sigma = \sigma_0 + \mathrm{K}\varepsilon$ so which is nothing but $Y = mX + C$ slope of this curve would be nothing but slope will be nothing but $m$ only so you will get $K$, $\frac{d\sigma}{d\varepsilon} = K = \sigma = \sigma_0 + \mathrm{K}\varepsilon$ is original equation so from this you can get $\varepsilon_u = \frac{350-250}{350} = 0$. Your $\sigma = K\sin(B\varepsilon)$ then $\frac{d\sigma}{d\varepsilon} = K\,cos(B\varepsilon) = K\sin(B\varepsilon) = \sigma$ so from this you can get $\varepsilon = \frac{1}{B}tan^{-1}(B)$ is nothing but you can substitute $2\pi$ here and you may get some value please check it this would be your $\varepsilon_u$. So three values of $\varepsilon_u$ all are different these are going to be different these are not going to be same so the condition remains same that is tensile instability condition at maximum load is being applied but if you change the material loss which the material is going to follow then accordingly your $\varepsilon_u$ prediction is also going to change it is not a small change it is a going to be a good change this could be 0.2 this is 0.29 this could be maybe 0.22 maybe 0.23 will come so all these values is going to tell you finally that depending on the material law the instability prediction is going to be different.

So we have seen four problems in this so four problems so here we have used the maximum load condition at the narrow section and got $P_{max}$ and use that $P_{max}$ to get to apply the same thing to the wider cross section wider section and in this from this you can get at that particular maximum load if that is the case what is the length in a small part of that wider region is what we found out. Next one is similar situation you have a narrow region and wider region if it is pulled up to failure now they are asking us how to find out the variation in n with respect to the strain values within a small range of strain. So in that way we looked into the problem the tone of the problem is basically to relate $n, \varepsilon_u$ and the imperfection so we started in this way. We started in this way third one is straight away that our balanced biaxial tension situation and then the situation in the uniform region that is in the A region is given to us now we wanted to find principal stresses and $\alpha$ in the B region so here also we have used two important expression one is the Von Mises yield function expression and the heterogeneity factor $f$ but not related to thickness we have already converted that into stress ratio that is what we have used here in the second equation and then we were found out all the values.

The fourth one is a very general one suppose if the material fails at the instability the maximum load then you want to find uniform true strain so then in that case how material law is going to change the value so that is what is given here. So with this we stop next module we are going to start a new section.