Mechanics of Sheet Metal Forming

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Week- 05
Lecture- 12
Sheet Formability

So, let us continue our discussion on the Formability Testing of Sheet Metals. So, today we are going to discuss about two important you know rather three important sub topics relevant to formability testing of sheet metals ok. One is you know how certain important properties or parameters affect forming limit curve in general ok, independent of any material chosen or something like that. And the second important one is we are going to discuss briefly about some theoretical models ok, available for evaluating the forming limit curve of any sheet. And then we are going to briefly discuss about how to detect or identify necking ok. What are the what are some standard methods or procedures to evaluate the you know or to identify necking of a of any sheet.

This is what we are going to these three sub topics we are going to discuss in formability testing of sheet metal. So, in the previous module we discussed elaborately about what are the different formability testing methods available. So, it could be your stretchability or deep drawability or stretching cum drawing or it could be your in plane strain tensile test or tensile test ok. So, limit dome height test ok, Marciniak stretch test.

So, different varieties of you know formability test we have studied. So, now all this formability test is going to give you forming limit curve which we know what is it ok. And forming limit diagram we know these two ok, what are the you know what is the definition of forming limit curve forming limit diagram we know that. Now like your stress strain behaviour ok, like a materials stress strain behaviour or forming limit curve ok, will also be affected by certain important properties. We will discuss one after another, but in general ok, in general we are going to discuss.

This is first one which is we know that it is a strain hardening exponent we say n is not it. So, $\sigma = K\varepsilon^n$ and this n is what we call it as a strain hardening exponent. And we have already derived equation relating n to instability is not it. We have also derived before that $\varepsilon_u = n$ which tells the fact that ok your uniform elongation will be affected by strain hardening exponent and larger the n value larger would be the uniform elongation ok. So, which in a way tells the fact that your ductility can improve ok.

So, similarly if you want to know the effect of n value on the forming limit curve, theoretically it can be estimated and in general one can say that with increase in forming limit with increase in n value or forming limit improves. That means, so this is the forming limit curve it is plotted between ε_1 versus ε_2 and you will see that for larger n value let us say 0.3 you will have forming limit curve like this, but for lower n value like 0.15 you have a forming limit curve like this ok. So, forming limit curve improves with increase in n value the, but of course, the effect is very clearly seen in your FLD_0 this point know we said FLD_0 this is very clearly seen in the in the case of FLD_0 and in the stretching side generally ok.

So, what does it mean? That means, if a forming limit has improved means there is a large region in which you can accommodate safe deformation ok which means your formability is actually improving ok. FLC is moving up means your forming limit is actually improving because you can put more safe deformation to the material before it reaches the final you know stage or the final component is made ok. But the prior processing may affect the quality of sheet ok for example, cold working ok. So, by cold working you strengthen material that can there are lot of chances that your n value can reduce which means that it is difficult to form the component which means that it is difficult to form the component. And moreover we also know that this plane strain mode of deformation.

So, when you go along Y axis you reach the forming limit go at this location is not it. So, this particular location is actually conservative window for you know your forming limit ok. So, in such type of materials like cold work materials better to deform it in the stretching strain path. So, that you have more extra window for deforming the sheet to make the component ok. So, this is one important parameter and the effect of n can be understood just by relating your uniform elongation with you know n ok.

So, generally with increase in n value your forming limit is going to improve which means that formability will improve. So, the next important one is called as rate sensitivity ok m. So, we say the strain rate sensitivity index m ok. This comes into picture when you go for let us say $\sigma = C\dot{\varepsilon}^m$ and we have also studied that this m actually controls the post necking behaviour that is $\varepsilon_t - \varepsilon_u$ that particular region in the stress strain behaviour it is going to affect. So, which means that this m is going to control the growth of neck.

So ok once instability is started how quickly it can grow or how slowly it can grow will be controlled by this m value. So, the rate sensitivity effect you know it affects the rate of growth of neck ok, but generally you will see that this m value you know does not affect ok will not have much effect on the tension at which its maximum value is reached ok. So, it is going to have effect after that ok. So, now if these two diagrams if you see it will tell you in general about the effect of m value and you will see that higher the m value you will have better forming limit as compared to material with lower m value. This is where we were discussing about super plastic materials ok super plastic materials or super plasticity is a property ok which will have ductility of you know several 100% as compared to the conventional materials right.

So, higher the m value you will see that generally your forming limit improve mainly because you can delay the growth of neck. But it may so happen that for materials which has got higher m value your plane strain may not have be closer to n value it will be larger than n value. You can see that lower m I am saying n here which is what we studied which we discussed in the previous section FLD_0 ok, but for higher m value it may not be n actually it will be better than n. So, you have better window for in plane strain also ok. So, you have a material with one particular n value, but the same material has got larger m value then you may have little bit more window for safe deformation in even in plane strain.

And this diagram is a well known diagram for us you are monitoring ε_1 versus ε_2 in the A region and in the B region. So, again we are going back to the A region B region we know meaning of A region and B region right. So, A region and B region are nothing, but you are A neck region and the outside region neck region outside region ok. A region being outside region ok. So, we were discussing that you will have a thickness imperfection or the region where necking happens right.

So, this region is actually you can call it as B this region is you can call it as A ok. And then we said that we can give σ_1 versus σ_2 and see how strain evolves in these two locations. We have discussed in the previous to previous section how necking occurs in biaxial stretching. So, ok. So, we consider the same geometry and we will see when necking happens at B ok.

What happens to this ε_1 versus ε_2 in A and B region for two materials one with the higher m and one with the lower m one with higher m and one with the lower m. So, there are four curves here ok. Two curves for A and B and two for two materials higher m and lower m. And you will see that actually you have to monitor ε_1 versus time and ε_2 versus time ok. These two have to be monitored and you have to plot at a common time how ε_1 and ε_2 varies at two different locations one is A and B ok.

So, you will see that for higher m ok this dotted line is for B region that is where necking is going to happen. And you will see that it is for A region and you will see that when compared to A region B region will have unstable growth in you know your strain ok, unstable growth in strain especially ε_1 ok. Sudden growth unstable growth after a particular stage. You will see that this your A region will try to grow in the same direction whereas, you will see in the B region there is unstable growth in the limit strain ok. And so, naturally this indicates that B is the source of neck.

At the same time you will see a lower m value material. So, lower m value material means this one B and you will see this A ok. So, you will see that in the A region there is not much difference A region there is not much difference, but in the B region you will see that this unstable increase will happen bit early. So, which also means that you may reach limit strain little early as compared to higher m value ok. So, that is the main difference between a material with the higher m value and lower m value.

So, the growth of the evolution of A and B is going to remain same only thing is difference in lower m value is the unstable growth happens bit little bit early which indicates that its overall form it is going to be over because it is it has got lower m value and you will see that the saturation happens little early whereas, in the case of higher m value you will see the saturation happens little late and then there will be unstable growth ok. So, that is what is going to happen in A and B region with respect to m and we also studied this effect of m ok with respect to imperfection ok. We derived the relationship between the change in strain rate ok with respect to m. So, previously we derived it ok. So, larger the m value is better because it is going to control the post necking behaviour ok, it will delay the growth of neck.

This imperfection ok in homogeneity we say or f which is nothing, but you are that imperfection factor we said $f = \frac{t_B}{t_A}$ and of course, this is very theoretical in nature ok. So, practically we do not speak anything like your you know in homogeneity or $\frac{t_B}{t_A}$ or you have sudden change in thickness the way we have seen here ok sudden change in thickness ok. This is t_B this is going to be your t_A ok. This type of things we are not going to see practically, but theoretically if you want to assimilate what is happening in the actual material then you introduce this f right. So, we also discussed that this in homogeneity factor or f or which discuss about the imperfection which defines the imperfection in the material is equivalent to all the imperfections in the material including structural and geometrical heterogeneities.

So, we understand that intuitively like greater the imperfection greater the imperfection means there is large difference between thickness in B region as compared to A region ok. So, you can imagine that larger imperfection means let us say t_A is let us say 1 mm ok and t_B is let us say 0.999 mm there is another case where it is 0.95 and it is 1 ok. So, that means, this has got more imperfection right.

So, greater the imperfection you will see that lower will be the limit strain because the weaker region is actually very weak as compared to the other case ok. So, this what we know conceptually schematically I have drawn this is with respect to large imperfection ok which means that your forming limit is going to come down ok small imperfection means a better formability is obtained. So, with the large imperfections the plane strain limit strain may be less than the strain hardening index also ok. So, which means that theoretically when we say this $FLD_0 = n$ ok. So, as per our previous discussion it may not be in the case of materials with large imperfection ok.

That means, what it may not even reach n value it may fail before that ok. So, other than you know thickness imperfection we can also have you know practically you will see that inclusions are the local reductions in strength due to segregation of strengthening elements or texture variation these are all other situations ok which one has to take care surface roughness also plays a bigger role you know can be connected. So, how do you relate the surface roughness of a sheet to formability is through imperfection is through defining this kind of f or imperfection and homogeneity factor ok they can be related ok. So, but only concern here is surface roughness units ok the values are going to be very different as compared to the actual thickness of the sheet. So, this t_B, t_A we are all the t_B value is going to discuss will be of the of the order of sheet thickness 0.9 and 0.95 whereas, surface roughness you will see it is going to be a small value, but in a way surface roughness can be connected to this in homogeneity factor f ok. So, in that case other than thickness heterogeneity one can see practically there are other in homogeneity in the material like inclusions or local reduction strength due to you know segregation of strengthening elements surface roughness ok at one particular location it is very rough ok sheet is very rough ok. Then that means, at that location imperfection could be huge would be severe. So, which means that that type of materials may have you know less formability because of presence of large imperfections ok. So, but we should understand one particular fact that this type of changes this geometric change structural change are actually you know gradually available in the material rather in our discussion rather we actually take sudden change in thickness ok.

This is a sudden change in thickness that is what we discussed ok which is generally not seen in materials there will be gradual variation in thickness or other structural heterogeneities in the sheet. So, one should be careful with that. This anisotropy of the sheet which we generally quantify by r that is nothing, but plastic strain ratio small r or capital R either way is fine. So, $R = \frac{\varepsilon_w}{\varepsilon_t}$ and if it is equal to 1 we say isotropic if $R \neq 1$ we say the material is anisotropic in nature right. So, now, this effect of anisotropy or plastic strain ratio is a little it actually changes from the way we use the theories some theories ok.

It is not very straightforward I have shown you some examples here ok because these are all theoretical forming limit curves ok. We use some models and then we calculate the forming limit curve from this which means that. So, during such calculations we may assume some yield function like for example, Von Mises yield function we have studied right. So, like Von Mises yield function which is meant for isotropic materials there are several other yield functions meant for anisotropic materials we will see some of them later in the course in a very summarized way we are going to discuss. And if that yield functions are going to change in the theoretical estimation of this forming limit curve then that will have significant influence on studying the effect of influencing the forming limit curve ok because of change in R ok.

So, you have to choose this anisotropic yield function wisely to study the effect of R on the forming limit curve ok. So, in general if you see in general the existing data tells that this FLD_0 from experiments suppose you have FLD_0 means again I am telling ε_1 versus ε_2 you have this is your forming limit this particular point is FLD_0 that is your ε_1 , ε_2 in plane strain. It is a conservative location also we are taking it as a reference actually. So, if you take that in Y axis and r in X axis say let us say it varies from 0 1.53 let us say this r value ok. Then you will see that this theoretically ok you will see that this FLD_0 actually improves ok. Say for example, at about 1.5 it is about you know your FLD_0 is about you can say 0.2 is about it could be about 0.4 ok. Your major strain that means ε_1 is about 0.4 and if it is about 2 you can see it is about

0.5 and is about 3 it is closer to 0.6. So, increase in r value generally has got a positive influence on your FLD_0, but it also depends on the yield function as I said there are 2 examples given here ok.

So, of course if you want to study the effect of r value naturally you cannot use Von Mises yield function because it assumes $r = 1$ this case ok. So, naturally we have to go for other yield function which has got which is meant for an isotopic yield function. There are several yield such yield functions one famous one is called as Hill's 1948 yield criterion ok. One important yield criterion is actually called as Hill's 1948 yield criterion actually this Von Mises yield function is a case of Hill's 1948 yield function we will discuss about that later ok. And if you use this type of yield function ok then it will have some effect I have given that here from this particular book it is discussed more you can look into it.

And instead of this yield function Hill's 1948 if you use a higher exponent yield function ok we will see that higher exponent yield function then the effect is actually different on the forming limit curve that is given in this second diagram ok. So, you will see that forming limit curve for few r values I have given so r is say 2, 1.5, 1.5. So, this is isotopic you can say you reduce it and then you increase it ok.

You will see that how forming limit curve is going to change on the right side of the forming limit diagram mainly on the right side of forming limit diagram. You can see that ε_2 is only positive here of course ε_1 is always positive ok that is the way we draw it ok. So, if you see that r value is going to affect in this fashion ok. So, there is a large difference in you know your limit strain value when you go for the 45 ° line let us say balanced by axial stretching like this ok. Large difference 1, 2, 3, 4 this points has got large difference in forming limit but if you go closer to plane strain in this direction you will see the windows actually reduce the difference is actually little between this ok.

So, in Hill's 1948 yield function there are some values which you can fix say for example for this particular you know calculation you know like n value of 0.2, m value of 0 and this f value the inhomogeneity factor is kept as 0.98. So, when you do that you get this particular type of curve. Suppose now if you change the yield function to higher exponent yield function ok what is it we will see later then the effect of r value something like this given here you can see that.

Again this is also positive minus strain and of course positive major strain you will see that ok. So, r value has got no effect it is going to reach the same forming limit curve for different r values you can see $r = 0.5, r = 1, r = 2$, $r = 0.5$ is a circular filled circular one. So, these values are coming here somewhere ok then open square is also here ok which is $r = 1$ and your you know filled square is also here ok.

There is not much difference there is no difference between these forming limit strains ok. It comes with same forming limit curve it falls on the same forming limit curve and for this calculation in the deal function they have used let us say a is equal to 6 there is a parameter called a which is kept as 6 and n value as 0.2 same as this m as 0 same as this and f is little bit increased instead of 0.98 we have used 0.995 ok. So, the effect of r values little you know tricky and it depends on the yield function you choose ok because different yield functions have different accuracy of predicting the forming limit strain and two such examples are shown here ok. So, the next one there are two important parameters one is called as a temperature the other one is a strain rate ok. So, this of course, this is strain rate effect can be connected to m ok strain rate effect can be connected to m. So, that is why we have given here two or rather three graphs ok. This is effect of strain rate ok, this is effect of temperature.

So, these are all strain rate values these are temperature values given here. So, when you study the effect of strain rate temperature is kept constant let us say it is kept at some degree centigrade let us say maybe 150 ° C ok you are keeping and then you are doing forming you know using some form multi test and then you are getting forming limit curve ok. So, that means, the forming limit curve has been obtained at a higher temperature let us say not at room temperature maybe like warm forming or hot forming like that. So, in that situation if you see increasing the strain rate it is 0.1 per second, 1 per second, 10 per second ok. So, that means, you are increasing the forming speed actually ok you are increasing the forming speed actually ok. This is slowly deforming this is the highest you know speed and this is in between you will see that your forming limit is actually coming down forming limit is actually decreasing the difference between black red and blue. If you see that for higher strain rate you have a lesser forming limit and this depends on the m value of the material. And if you take the other effect suppose you keep it at one particular strain rate ok let us say strain rate is maybe 1 per second ok strain rate is fixed at 1 per second let us say and then you are changing the temperature ok. So, one forming limit is for 200 ° C which is better than the other case which is say for example, 125 ° C this is just a schematic I am drawing ok.

It may not be exactly like parallel in both the locations in both the regions one has to evaluate it and check it ok. So, you will see that generally increasing the temperature improves the forming limit in this fashion ok at a constant strain rate. But there are some examples which I have shown here you will see that practically ok they have evaluated the theoretical you know strain rate dependent forming limit diagram for this particular material one particular grade of copper if you see that what are the strain rates one for example, strain rate is 0.01 ok then 10 then 100 then 500 ok. So, it is more like a dynamic forming limit type ok dynamic testing type ok and you will see that so with respect to change in strain rate your forming limit actually improves ok.

So, probably one has to look into the materials of how is it behaving and probably the strain rate is too high ok strain rate is actually too high ok to show some better effect on the forming limit ok. So, you can see that the diamond one is here that is 10 per second ok square one is 100 per second and strain rate is this is 500 per second this is pretty high for you know your strain rate and forming limit is you know quite high in that case. So, it also depends on the material. So, one has to be little bit careful how to understand this effect ok, but otherwise generally this is the shown by effect of temperature and strain rate. So, UTS and total elongation total elongation effect we already know ok total elongation effect is already known.

So, quite naturally you know larger the total elongation it is expected that forming limit will also improve your FLD_0 again is considered as a reference for us because it is in plane strain ok. So, it has got a conservative window and lots of experimental data lots of experimental data has been taken from various you know data various you know literature ok. And if you plot everything together with respect to the materials total elongation that means, you need to know total elongation of variety of materials and for the same material you should have data for FLD_0 and if you plot it they have a exponential fit in this fashion. So, increase in total elongation will actually improve the FLD_0 you can see from here to here it has improved. Just schematically I have drawn and you can see that one can fit this type of you know equation for that $FLD_0 = A\exp(B\varepsilon_t) - 0.3$, ε_t is nothing, but total elongation FLD_0 is nothing, but limit strain in ε_1 in plane strain deformation. So, this A and B can change depending on the materials that you choose. Similarly UTS can also be quantified how it is going to be affect the you know FLD_0 and again here lots and lots of experimental data has been plotted in one graph and you will see that it is also exponential 1, but it is going to decay. So, increase in UTS will actually reduce the forming limit curve will actually reduce the forming limit curve ok. So, FLD_0 I just given a simple equation $FLD_0 = P + B\exp(-S\sigma_{UTS})$ *P,B,S* all are fitting constants.

They are actually fit constants one has to get after fitting after getting all the data let us say you have 100 data for this variety of materials or you know evaluator ok and you are putting all the data and then you are going to fit it then you will get this equation ok. So, you have to be little bit careful when you increase the UTS of a material in a way it is good, but generally otherwise you will see that it is FLD can come down ok following this particular it is just not a linear decrease it is exponential decay one has to be careful ok and total elongation if it improves it is good that FLD_0 there are chances that it may improve by following an exponential increase. So, *n, m* strain rate, temperature then R then you have ε_t, σ_{UTS} ok. These are all certain important properties that we get from tensile test ok some are actually working conditions say for example, strain rate and temperature are actually working conditions actually ok, but otherwise *n, m* or ε_t, σ_{UTS} these are all material properties which will affect the either the forming limit curve in both the regions positive minus 10, negative minus 10 or in FLD_0. Now this sheet thickness if you see it is actually not a material property rather it is more like a geometric you know property of a material and it is sheet thickness decided by the application ok.

So, and then whatever sheet thickness that we are discussing in general is of the order of let us say maximum say 2.5, 2.8 mm, 3 mm like that not more than that ok. So, and what is the effect of thickness that can be you know explained from this particular simple schematic ok. So, you can see that of course, there is some data also available for you we can check it.

So, X axis is n strain hardening exponent this diagram is done not only for thickness, but also for n. So, X axis is strain hardening exponent Y axis is as usual our FLD_0 ok and thickness. Thickness has been found to increase in this fashion you will see that is 0.5 mm, 1 mm, 1.5, 2, 2.5. It means that other than n and thickness all other properties of the material are kept constant or assume not going to vary.

And you will see that with increase in n value 0, 0.05, 0.1, 0.15, 0.2, 0.25 you will see that there is a significant increase in FLD_0 at any thickness this is what we have seen before. But of course, after a particular value let us say it is going to become saturates not going to show big effect ok which is we have seen before. And the effect of thickness which is a main purpose of this graph is also coinciding with the effect of n value.

So, larger the thickness you can say 0.5, 1, 1.5, 2, 2.5 you compare 0.5 and 2.5, 2.5 is got very high ok FLD_0 as compared to 0.5 at any n value you pick up. You take 0.1 you go along this line you will see that it has got this fellow has got about 0.2728 whereas, this fellow has got only 0.1. If you pick up 0.2 here you have got only 0.22 maybe, but here you have got about almost 0.55 ok. So, increase in thickness actually enhances the improves the FLD_0 which means that even in plane strain condition you can have a better formability if you replace 1 mm thick sheet by 2 mm thick sheet for a particular application ok. Assuming that n is not going to change if n is also going to be increasing then it is a favorable for us ok. So, instead of reaching this particular point in 2 mm thickness you may reach this particular point in 2 mm thickness right. So, that is effect. So, increasing the sheet thickness what happens actually why it is because increasing the sheet thickness you will see that the neck becomes more diffused and localization would be delayed to reach necking stage and to reach the critical depth.

So, it will take some time it will delay the larger the thickness the extra thickness that you provide actually can delay the you know the necking stage and then a critical depth to reach a particular depth you know the neck has to cross the entire thickness know. So, that will be little delayed. So, there are chances that your FLD_0 or FLC_0 can improve. So, we also have something called pre straining the sheet. Pre straining the sheet means I hope you remember an equation where we introduced ε_0 for describing the flow stress.

So,$\sigma = K{\varepsilon_0}^n$. We might have seen that this in 1 or 2 problems we worked out here is not it. So, which means that if you put $K = 0$ here still you will have some oh sorry. So, if you say $\sigma = K(\varepsilon + \varepsilon_0)^n$. So, if you put your strain as 0 here. So, you will get some strength in the material which means that this equation will take care of any pre straining that you have in the sheet.

So, now, this pre straining also may have some effect on forming limit curve ok. So, that means, you have to purposefully pre strain it and then see how it is going to affect and that is what I have given here in this schematic. So, what does that mean? That means, there are 3 forming limit curves here ok. So, one is no pre strain the black colour one is no pre strain ok which means that the material is actually coming to you and then you are using a forming limit or you know testing method like hemispherical dome test and you are getting this forming limit curve without any pre strain. But you can purposefully pre strain it in 2 different ways. Let us say for example, you are using uniaxial pre strain uniaxial pre strain means you take that material and you pre strain it in the strain path coinciding with uniaxial type of deformation up to a particular strain up to a particular $\bar{\varepsilon}$ let us say.

So, up to a particular $\bar{\varepsilon}$ you are deforming the material and then you are evaluating the forming limit curve using any test procedure ok. Then you will see that the uniaxial pre strain ok the forming limit curve is going to change in this fashion. The black colour one to red colour one you will see that your plane strain location is actually moved ok and then it is it looks like there is some slight upward moment of forming limit curve. And the same thing if you do it or totally opposite to that suppose if you are pre straining in equi biaxial tension equi biaxial plane strain that means on the extreme right of your forming limit diagram ok. Then you will see that the green colour one is going to be the one your black colour forming limit curve is going to become green colour where you will see that specifically your plane strain has moved towards the right side ok.

So, this also you can strain up to let us say one particular $\bar{\varepsilon}$ value some $\bar{\varepsilon}$ value ok. But you have to pre strain in such a way that some formability exists in the material ok. So, you should not exhaust the forming limit fully ok. So, before going for actual evaluation of forming limit curve. So, one has to do that appropriately and then get the forming limit curve one can get like this, but one can bring in this equation into forming limit estimation and you can virtually change this ε_0 to get the forming limit curve.

So, that is another way to get it ok. So, you are pre straining the sheet also has got effect on forming limit curve. So, these are some important you know your material properties and you know geometric properties of the sheet and testing you know conditions like pre strain, temperature, your strain rate ok all are going to affect the forming limit. And of course, there is one more important parameter that is nothing, but your coefficient of friction μ ok, coefficient of friction ok Coulomb's coefficient of friction is not it coefficient of friction or practically speaking whether you lubricate it or not ok. So, you use lubricant.

So, lubricated condition or dry condition ok lubricated condition or dry condition. So, this I think we have studied in the previous section that generally this whether you put dry situation or a lubricated situation when you evaluate forming limit curve it is not going to affect the forming limit rather it is actually going to change the strain path only ok. So, that means, suppose if you have a forming limit curve like this. So, ε_1 and ε_2 you have and this forming limit curve is a material property right. So, if you get this particular forming limit and you will see that when you change the friction coefficient it will rather have an effect like this ok. So, suppose you are using dry condition you will reach the forming limit here, you will reach the forming limit here let us say dry condition I am just putting or you put any lubricant you want and you change the lubricant to some other lubricant ok.

So, we do not know the characteristics of the lubricant we do not know μ. So, that if that is the situation then still one can study the effect. So, for one particular case you will reach a forming limit in this case if you change the lubrication ok. Let us say lubrication is better then you may get a higher limit strain ok you may get a higher coordinate, but at the same forming limit, but at the same forming limit ok. You may get a better $\varepsilon_1^*, \varepsilon_2^*$ value as compared to the previous one, but it will reach the same forming limit. So, basically it is actually changing the strain path from one β to another β rather than forming limit itself ok.

So, your ε_1 change will get adjusted with respect to change in ε_2 ok accordingly you will get the limit strain. So, individually you will see limit strain may change, but then it will finally lead to the same forming limit that is why we always say that the forming limit is actually a material property. So, that is the effect of μ the lot of discussion in the subject about effect of μ, but in general this is the way it is going to work. So, after studying all these parameters let us quickly in a very summarized way let us discuss some important theoretical estimation of forming limit curve ok theoretical models ok. We will there are several theoretical models you know simple once we have already discussed one or two before two complex once ok which will give you a forming limit curve theoretically ok.

Of course, all have to be validated by doing some formality test at lab scale to get the forming limit and compare it you have to validate it. If you go for a theoretical estimation of forming limit curve you will see that this forming limit models can be divided into two parts one is your analytical model other one is semi empirical models. So, analytical models are based on you know some physics ok you know like you put lot of equations we already derived one is not it you put some conditions and then you know apply some boundary conditions and then see finally, you will get the same $\varepsilon_1^*, \varepsilon_2^*$ and then change the β values or α values in that equations to get the forming limit curve for the entire list of strain paths you study. So, that is analytical models there are semi empirical models semi empirical models means basically models based on some fitting ok basically models based on some curve fitting ok so like the way we studied the effect of UTS and total uniform elongation total elongation right. So, similarly you can also estimate you can also estimate forming limit curve or data points in that using semi empirical models.

So, first let us see few analytical models simple once we are going to discuss and we are not going to derive the entire equation here other we are going to discuss all these things in a very conceptual way ok. What is the basics behind it that way only instead of theoretical derivations. So, lot of derivations are available in text books and papers one can look into it or any plasticity book can be referred for that. So, when you come to analytical model the first one is basically we already studied for tensile instability that is a you consider a condition ok and we know that any material which undergoes plastic deformation there are actually two domains in that one is a stable plastic straining domain other one is a unstable plastic straining domain. Stable plastic straining domain means hardening is influenced ok on the tension force ok, but not the cross section reduction ok that is going to be actually your initial part of plastic deformation which we say it as a stable plastic deformation.

And then once you reach one particular stage after that if you see it is going to be unstable plastic straining if you see in that location in that region the material hardening ok cannot compensate the decrease in tensile force due to severe reduction in sample cross section area ok. So, your hardening is actually going to help the tensile force to improve in the first zone, but in the second zone you will see that the hardening actually is not going to compensate the decrease in load ok which is going to actually happening because of significant reduction area of cross section. This finally, will lead to a typical stress strain behavior which we have seen. So, now given that situation this transition is going to happen at one particular place ok from stable plastic response to unstable ok. And at that stage we can say that it can be said that onset of necking corresponds to maximum tension force and it can be written mathematically as $dF = 0$.

And simple derivation we already shown it can be shown that $\frac{1}{\sigma}\frac{d\sigma}{d\varepsilon}$ ok or $\frac{d\sigma}{d\varepsilon} = \sigma$. And if you put this particular power law equation and then we if you derive it we can show that $\varepsilon_u = n$ or we can say $\bar{\varepsilon}$. It means that as per this condition for a sheet obeying this particular law ok we found that the material starts to neck when the strain equals a strain hardening exponent of that particular material. And we have also seen that if you do not use this equation if you use pre strain equation this equation can be modified by including some factor of pre strain into it is not it some factor of pre straining into it. And if you will see that if you include some pre strain value it is going to be a subtraction that is going to play a role here and then you will see that your forming limit your instability strain or necking strain is going to actually decrease right that we already studied.

So, this is one important analytical model that people use to estimate your tensile instability. Now the next important model is called as a Swift model ok. This Swift model can be used to obtain limit strains $\varepsilon_1^*, \varepsilon_2^*$ by using consider a criterion in biaxial tension side ok. In instead of tensile instability you can apply this in biaxial tension by taking a sheet element loaded along two perpendicular directions. So, you take a sheet element ok and you load it along two perpendicular direction which means biaxial tension ok and you apply this consider a condition. So, after that what you do? Apply consider a criterion in each direction ok along one direction let us say along two direction you apply this particular condition and of course, one can put similar condition your $dF = 0$ you can say similarly you can put conditions on both the directions and if you assume the same power hardening law s $\sigma = K\varepsilon^n$ you will get two important expression one for $\varepsilon_1^*, \varepsilon_2^*$ right.

So, $\varepsilon_1^*, \varepsilon_2^*$ are nothing, but the data points on the forming limit curve ok. This is $\varepsilon_1^*, \varepsilon_2^*$ another ε_1^* another ε_1^* another ε_1^* right. So, this is a general equation one can get expression one can get for $\varepsilon_1^*, \varepsilon_2^*$ ok and here you will see that in this equation ok your σ_1 there is σ_2, σ_1 , σ_2 are there these are actually principal stresses and you will see that there is one more the two more one is your f this f is not your in homogeneity factor rather this f is actually yield function ok. This is the same f that we use in our multi condition $d\varepsilon_{ij} = \frac{\partial f}{\partial \sigma_{ij}} d\lambda$ that f is nothing, but this yield function and this n is nothing, but our own strain hardening exponent ok. So, you will see that now this limit strain $\varepsilon_1^*, \varepsilon_2^*$ that you get using this Swift model is simple to use, but it depends on the yield function only it depends on yield function only and of course, it depends on only one material property that is n that is because you are going to use this particular hardening law and it does not have any other you know forming parameter only n is available.

So, this becomes a function of n. So, now, what we can do is you can use a variety of yield functions in place of this f and you can you know derive this equations actual equations for $\varepsilon_1^*, \varepsilon_2^*$ and finally, you will see that $\varepsilon_1^*, \varepsilon_2^*$ could be a function of your n which is already there maybe strain rate sensitivity index and of course, plastic strain ratio also you can include because you may use an isotropic yield function and of course, it will be in a function of α. So, that you can get for different α s what are the forming limit strains ok. And if you use his 1948 yield function ok this is the famous you know yield function if you use that equation in place of f ok then you will get a nice $\varepsilon_1^*, \varepsilon_2^*$ expression ok and you will see that as I just now said it becomes a function of r and α and n.

So, of course, the strain holding law does not have m. So, m fellow this m fellow will not come here ok. So, this is a simple expression you will see that $\varepsilon_1^* = \frac{[1+r(1-\alpha)]\left(1-\frac{2r}{1+r}\alpha+\alpha^2\right)}{(1+r)(1+\alpha)\left[1-\frac{1+4r+2r^2}{(1+r)^2}\alpha+\alpha^2\right]} n$. Similarly ε_2 can be estimated from this ε_2^* can be estimated from this equation. So, what are the material properties you have there is only one quantity plastic strain ratio which quantifies the degree of an isotropy there is one parameter n strain on exponent other than that you have only α to change the stress ratio or to change β. So, now for a constant n and r ok let us say r I am going to fix as maybe let us say 1.5 ok and n I am going to change let us say 0.2, 0.25 and 0.3 then one can get variety of values of $\varepsilon_1^*, \varepsilon_2^*$ for different α s. So, α also is going to you know change from the leftmost side ok for the you know the negative minus strain to positive minus strain. So, I can get different values of α which already defined you already discussed 5 different α s right. So, all this α s can be substituted here and you can get a forming limit curve.

So, by calculating $\varepsilon_1^*, \varepsilon_2^*$ for different α we get the whole necking limit or forming limit curve ok. This equations are simple to use because it depends on only one material property 2 material property r and n and which means that if $r = 1$ if you put $r = 1$ in both the equations that it will turn out to be for forming limit is meant for isotropic sheets ok for the same n value.

So, you can take $r = 1$ and you can get the effect of 0.2, 0.25, 0.3. So, then the entire value data forming limit curve is for isotropic sheets with different n value. So, this model is easy to use and then similarly there is another model called as the Hill's model ok and this Hill's model it is going to assume something which you already discussed. In uniaxial tension test we have also proved that the localized necking develops along a direction which has got some orientation with the loading direction correct that we have discussed when we try to prove that in biaxial stretching there is no definite θ and that is why your necking is not going to be rapid rather something is actually delay the necking process right. So, that is why it is creating difference between experiments and maximum tension line on the right hand side of forming limit diagram correct.

So, at that time we have shown that there is a definite θ existing for niaxial mode of deformation right. I think we have put you know α value as some value is one particular value and then we got θ. Similarly we did for plane strain also. So, localized necking develops along a direction which has got some orientation with the loading direction ok. So, but the Hill's model assumes that the necking direction coincides with the direction of 0 extension and straining in the neck is due to only sheet thinning ok. So, it is going to assume that there is one particular you know direction of 0 extension and moreover because of that in the neck region only sheet thinning is going to happen and further it is going to deform it is going to fracture ok.

And with this assumption one can evaluate you know $\varepsilon_1^*, \varepsilon_2^*$ and finally, you will get $\varepsilon_1^*, \varepsilon_2^*$ as this particular equation you will see $\varepsilon_1^* = \frac{\frac{\partial f}{\partial \sigma_1}}{\frac{\partial f}{\partial \sigma_1}+\frac{\partial f}{\partial \sigma_2}} n$. Similarly $\varepsilon_2^* = \frac{\frac{\partial f}{\partial \sigma_2}}{\frac{\partial f}{\partial \sigma_1}+\frac{\partial f}{\partial \sigma_2}} n$ whereas, f is our own ill function. And if you combine these two equations you will get this particular equation. This equation is already known to us is not it $\varepsilon_1^* + \varepsilon_2^* = n$. So, if we add these two it will lead to our own the previous equation which will give you maximum tension line which will give you maximum tension line ok.

So, now in this equation as per this equation you can simply say that your FLC estimated from the Hill's model above equation does not depend on ill function ok. So, though you are using ill function here finally, ok you will see it is going to depend only on the *n* value and nothing much is going to happen and this equation $\varepsilon_1^* + \varepsilon_2^* = n$ is nothing, but it is going to cut your Y axis at *n* value ok. $\varepsilon_2 = 0$ so, ε_1^* would be nothing, but your *n* value ok. So, as per this equation it is going to depend only on n value and not the ill function at all ok that is what Hill's model is going to tell you.

And this method is already discussed by us slightly before the concept is already introduced to you ok. Marciniak-Kuczynski model, this model is in short called as M-K model this is very famously used to predict the forming limit curve of any sheet material in this particular subject. And this particular model basically assumes that necking is generally initiated by geometrical or structural heterogeneity in the material that we already know that we already introduced this ok. You have geometrical you know defects ok like for example, sheet thickness change. Suppose you take a sheet of 2 mm thickness we say ok we take a sheet of let us say 2 mm thickness, but it is not going to be 2 mm everywhere.

So, it here it could be 1.98, 1.99, 1.985, here it could be 2, 2.01, 2.22, 2.05. So, it can vary within a small region we say 2 mm plus or minus some value it can change that sheet hetero thickness change itself is sufficient to create thinning severe thinning at one particular location which can lead to significant necking localized necking ok. So, that is the whole idea here of telling sheet thickness changes. Of course lattice defects are already there they are called as structural defects ok. So, as per this particular theory we are going to assume a sudden change in thickness ok to estimate limit strains this particular figure.

This we already introduced to you. However, you should know that the such type of changes is not actually true in practice it is going to be gradual change in practice that is why I given you different values here. So, it is not going to be a sudden change, but this is the simplest choice we have this is a simplest choice that is why we always say that the thickness difference is equivalent to the thickness difference that you are going to create the imperfection severity that you are going to create is equivalent to all the imperfections in the material ok. So, that is the meaning here. So, this is the simplest choice we have the simplest choice is given in this diagram.

So, this is geometrical assumption of M-K model in a sheet. So, A and B belongs to a same sheet ok, which has got same material properties, but only thing is your B region has got lesser thickness let us say t_B when compared to t_A ok. And your one direction and two direction is going to coincide with the principal directions as per this particular one to start with. So, σ_1 is going to act in this direction, σ_2 is going to act in this direction. So, if you know t_B and t_A you can create this f this f is nothing, but our inhomogeneity factor your non homogeneity factor which you already introduced the $\frac{t_B}{t_A}$. Sometimes in the biaxial stretching when we discussed about we also introduced $f_0 = \frac{t_{B0}}{t_{A0}}$ that means initial one.

So, your f can change during the course of deformation that is what we were discussing. So, we introduced this f ok. So, we introduced this f which is nothing, but $\frac{t_B}{t_A}$. So, now what we can do is you assume that let us take a case where you can you can you can you can do let us say modeling of this. So, you have some equations for all this ok we can put lot of conditions to it and then you know like you can find out some equations and in that equations or in any other way assume that we are going to monitor ok $\varepsilon_{1A}, \varepsilon_{1B}$ ok ε_1 that means, along this direction along one direction ok in A and in B you are going to monitor these two that means, your major strain in A and B can be monitored ok. So, now, we are going to say that when this particular ratio when this particular ratio reaches a critical value for example, 10 ok I am writing $\frac{\varepsilon_{1B}}{\varepsilon_{1A}} > 10$ then I am going to say that localization occurred in B ok which is an indication of onset of necking ok, which is indication of your necking of that particular sheet.

So, I am going to simply monitor eε_{1B} with respect to ε_{1A} ok once it reaches 10 or just crosses 10. So, I am going to say that the material is going to neck ok at that stage at that stage I am going to pick up $\varepsilon_1^*, \varepsilon_2^*$ closer to B probably in A ok closer to B probably in A ok to call it as a limit strains that you have to this exercise you have to do it for all the strain paths let us say all the β values ok. You will get a forming limit strains forming limit curve or neck curve ok. So, you will this is what I have depicted in this particular diagram you will see that you are monitoring $\varepsilon_{1A}, \varepsilon_{1B}$ ok. And you will see that in the A region the A region that is you are outside the neck region ok you are going to have some saturation whereas, in the B region see you see that it will keep on growing ok. And one particular location you will see that if you find out the ratio then it will reach 10 at that time you pick up ε_{1A}^* ok.

And there will be a corresponding ε_{2A}^* and these 2 together will form one coordinate in the forming limit curve ok that is what I have written here and by changing the strain ratio from 0 to 1 ok. So, again we are looking at 0 to 1 ok let us say this is ε_1 this is ε_2 0 to 1 strain ratio 0 means in plane strain ok to 1 that is balanced by axial stretching you can get different $\varepsilon_1^*, \varepsilon_2^*$ and you can connect all these values ok let us say you have 4 different cases here and you will get the forming limit curve on the positive major strain of FLD like this 1 2 3 you are going to get this brown color line which we are going to call it as a forming limit curve ok. So, in this zone of FLD that is in biaxial stretching side in this side the orientation of the geometrical on homogeneity that is this thickness reduction this orientation ah, this orientation you see that this orientation with respect to principal directions assumed to be same during the entire deformation to start with it is same ok. That means, this orientation is like this, this orientation is exactly perpendicular to your one direction and that is going to remain same ok throughout the course of deformation that is what is given here ok. So, now, you have got a forming limit curve on the right hand side of your forming limit diagram by changing by monitoring your $\varepsilon_{1A}, \varepsilon_{1B}$ ok and you have got various values and you have got this particular forming limit curve.

Now, who will get a forming limit curve on the left hand side, left hand side maximum tension line if it is sufficient for us then that is sufficient which we already discussed, but if you want to use this M-K model then there has to be some slight change in the concept. So, here what the issue here is you are going to monitor the same strain pattern the same way by changing the orientation like this I just given a schematic here the same A region B region this also A region, but you will see that the orientation of imperfection is at some angle let us say $\emptyset$ with respect to the principal directions 1 and 2 that is why I have created 1' and 2' is 1' , 2' are actually local coordinates for this imperfection. The original coordinates actually 1 and 2 ok. So, this is what you are going to do it on the left hand side. So, M-K model can be extended to negative minus strain region of FLD that means on the left hand side by having some orientation for the imperfection with respect to principal directions.

So, in this figure this model is actually called as a Hutchinson and Neale model in short it is called as H-N model ok. It is M-K model, but it is modified 1 for the left hand side it is called as a Hutchinson and Neale model. So, I will not go into the equations and discussion only thing is I have written two important equations here 1 how this $\emptyset$ this angular orientation is going to change with strain the orientation of imperfection varies with strain as per this particular expression $\tan(\varphi + d\varphi) = \frac{1+d\varepsilon_{1A}}{1+d\varepsilon_{2A}} \tan\varphi$ and f which you are going to assume $\frac{t_B}{t_A}$ may also vary as per this particular law $\frac{f}{f_0} = d\varepsilon_{3B} - d\varepsilon_{3A}$ where $d\varepsilon_3$ is nothing, but you are incremental thickness strain incremental thickness strain in B and A region B and A region here also A region A region. So, this is A region ok. So, what is $\frac{f}{f_0}$? f is your current value of your inhomogeneity factor and f_0 is your initial value of inhomogeneity factor inhomogeneity factor.

So, this ratio actually depends on this particular one ok. So, of course, one can apply lot of theories you know different you know sets of equations in it finally, you may get one simple model to calculate the forming limit curve on the left hand side also. So, that one can refer this books or other papers to get into it. So, now, there are some semi empirical models. What are the semi empirical model? The semi empirical models are handy ok. The semi empirical models are actually handy in nature you can get a quick guess of what is the forming limit curve of these materials any sheet grade materials of course, they have some constraints and lot of validation is required to use that in actual practice, but this will be useful for us to quickly use in shop floor ok.

Instead of too many calculations and you know effect of parameters all those things on the theoretical estimation of forming limit curves one can use this simple empirical models to get the forming limit curve. So, I have introduced three important a semi empirical equations there are few more available in actual thing, but one can look into it if you are interested. So, one is the first one is called Keeler-BrazIer model Keeler-BrazIer model. This Keeler-BrazIer model again like the previous examples it is going to evaluate estimate FLD_0 it is going to find out FLD_0 in percentage.

The equation is $FLD_0 = \varepsilon_{1(PS)} = (23.3 + 14.13t)\frac{n}{0.21}(\%)$ where t is your our standard thickness of the sheet and n is the strain hardening exponent t is the sheet thickness and n is the strain hardening exponent ok. So, as per this equation your FLD_0 depends only on n and t only on n and t and this equation is valid only for thickness less than 3 mm thickness and that is what we are also discussing ok and once you get FLD_0. So, again you will see that ε_1 ε_2 .

So, you get FLD_0 from this simple equation you can put any t and n let us say n values may be like 0.15 and t as let us say 1.5 mm. So, you substitute it you will get FLD_0 and it will be a one point in this particular forming limit curve. So, now if you want to get the left hand side of the forming limit curve and the right hand side you can use these two equations ok. So, left hand side of the equation if you see that of the FLD if you want to get data points you can use $\varepsilon_1 = FLD_0 - \varepsilon_2$. So, whatever value you are getting here you subtract ε_2 virtually take different ε_2 values keep subtracting it you will get ε_1 .

So, you may get a curve like this and on the right hand side also same thing can be utilized

$\varepsilon_1 = (1 + FLD_0)(1 + \varepsilon_2)^{0.5} - 1$. So, ε_2 you can vary virtually ok on the right hand side of the FLD and you will get ε_1. So, you may get a curve like this. So, finally, you will see that you will get the entire forming limit curve using this. There are other models which are little bit more complex they try to bring in you know more material properties into it say for example, this particular model they evaluate they use they evaluate again ε_{10} means FLD_0 .

That means, ε_1 in plane strain mode of deformation you can calculate it from this equation. You will see that the equation is actually cubic form ok $a(\varepsilon_{10} - n)^3 + b(\varepsilon_{10} - n)^2 + c(\varepsilon_{10} - n) - 10 \cdot mt = 0$ ok. So, you will see that here you have *a, b, c* are actually fit constants for the material and the material properties are actually you have n strain one exponent you have brought in m also strain rate sensitivity index and of course, sheet thickness is also there ok. So, for a particular value of *n, m* and t ok, one can get *a, b, c* such that you will get ε_{10} to satisfy this particular equation ok.

So, from this you can get FLD_0. So, it is done ok. So, then a modification in this is also provided and here you will see that this is a cubic form of equation the same *a, b, c* are there you have a strain one exponent you have *m t* other than that they have brought in *r* value also they brought in *r* value. The form of the equation is almost same the cubic part is same square part is also same and this part is also same ok minus you have *m t* in place of before *m t* you have *r* ok. So, if you put $r = 1$ ok you may check whether you may lead to the same equation or similar equation where *r* is your plastic strain ratio you can check it. So, these are certain semi empirical models available for us ok.

So, now analytical models are done semi empirical models are done there are various other analytical models available. So, there are damage theories damage models which can be used to get sheet fracture strains ah. So, after necking one can go for fracture strains also ok. So, that fracture limit curve can also be obtained ok. Then one can go for this kind of semi empirical models which are very handy and useful in actual industry practice, but validation is important for us ok.

Then the last sub part in this particular module before we go into problem is basically necking identification methods ok. So, you are doing deformation and you are identifying necking, but there should be some standard ways of identifying that yes necking has happened or not ok. So, because I may say that you know like necking has happened somebody else may say that no no necking has not happened let us deform it by another point to mm or something to enter into the onset of necking. So, how do you you know identify neck ok you know practically ok. So, experimentally of course, of course, you can show we can see the sheet and then you can say that yes necking has happened or you can touch it and see, but of so which which not actually possible in theoretical estimation.

So, how do you identify necking. So, there are 3 4 methods available, but I am going to explain you only two methods which are simple to implement one is called a slope method or you can measure thickness reduction rate in this measuring thickness reduction rate in this. So, what do you do in this actually you have to follow this particular graph this graph is between ε in general strain with respect to time with respect to this is actually time not thickness of the sheet it is actually time here. So, basically you are going to monitor strain throughout the entire process duration that is the meaning here ok.

So, what do you do here is. So, let us say for example, at one particular sheet element ok we can monitor ε_1 and ε_2 and ε_3 ok. Let us assume that we can monitor ε_1 and ε_2. So, numerically you are doing you are getting ε_1 and ε_2 continuous change in ε_1 and ε_2 with respect to time ε_1 versus time we already discussed ε_2 versus time is obtained for the same time interval ε_1 and ε_2 will give you ε_3 for volume constancy you can get ε_3 also. So, now, if ε_3 is known then you can get right you can evaluate thickness reduction rate you can get thickness reduction rate ok. Thickness reduction means how much thickness is reduced with respect to a particular time.

So, that can be plotted with respect to time itself that is what I have shown here ok. So, there are 3 curves in this one is your ε_1 which is nothing, but your major strain other one is ε_2 which is nothing, but your minor strain and then you can see there is one plot for thickness strain, but from thickness strain you can get thickness reduction rate and it will look like this this red color curve this red color curve nothing, but thickness reduction rate curve. So, there are 3 thickness reduction rate. So, now, what you can do is like you can draw the slope lines ok you can draw this slope lines from the initial part of the thickness reduction curve and the unstable part of the thickness reduction curve and you can find out this intersection point here you can see this black color line and this black color line are actually slope lines for this part and this part and they meet here this intersection point is finding it is going to be very important for us ok. This intersection point is a point at which a strain localization happens and that is why there is a sudden increase in your thickness reduction rate and there will be a corresponding value of ε_1 and ε_2 which you are going to call it as $\varepsilon_1^*, \varepsilon_2^*$.

This $\varepsilon_1^*, \varepsilon_2^*$ is nothing, but you are forming limit strain at that particular β value strain path or a strain ratio. So, you have to repeat it for various β values or monitor the same thickness reduction rate ok and the corresponding ε_1 and ε_2 where this intersection point happens would be a $\varepsilon_1^*, \varepsilon_2^*$ you can get the forming limit curve for the entire material for the entire material ok. So, basically you take a sheet one particular strain path you have to monitor ε_1, ε_2 and ε_3 ok and then from ε_3 you can get thickness reduction rate and you will see that this slope if you draw for the stable and unstable region it will have an intersection point. The intersection point is going to tell you which ε_1 and ε_2 you have to pick up to call it as $\varepsilon_1^*, \varepsilon_2^*$ which is nothing, but your limit strain of that particular β value and you can change it change the β value to get different limit strain values.

This is one way. There is another method which is also easy that is called time dependent method. This is similar one, but slightly different ok. You will see that in this method you have to choose two locations. One location is a neck zone a neck region which has schematically drawn here this is a neck this is a neck portion. This entire constriction is actually called as your neck ok. So, it may not be like this, but I am just showing a case ok. So, you have to choose one element or a location where you can measure strain in the neck zone and immediate immediately just in the vicinity of the neck zone or just outside it you have to choose another location.

Let us call that this is N and this is O ok. N is a location or element in the neck region, O is a element or location just outside the neck region. Again what you need to do is you have to monitor strain in that location and strain rate in that two locations. So, I have drawn a graph between in Y axis you have ε_1 and $\dot{\varepsilon_1}$. $\dot{\varepsilon_1}$ means strain rate major strain rate and of course, this is your time process time ok.

So, you will get two curves actually these two curves are represented by ε_1 neck, ε_1 outside ok. You can call it as ε_{1N} this is ε_{1O} ok. That means, ε_{1N} is nothing, but how ε_1 changes with the time in this particular location neck zone. ε_{1O} means same ε_1 how is it going to vary just outside the neck zone that is this O region. So, this these two are monitored and we already discussed that the in the outside region it has to saturate actually after some time. You can see that both the regions are equally deforming and after some time you are going to have saturation in ε_{1O} location, but in the neck location it is going to unstably increase it is going to unstably increase. But only thing which you have to be cautious is which location you are going to pick up for this O that is very important because neck zone you can somehow identify because it is a small constriction it may develop or there will be you know some severe contour deformation contour may develop ok.

So, but this outside region which location you want to choose this region or this region or this region one has to be very very careful ok. So, now, these two curves are obtained. So, now, what you do is you have to get how this $d\varepsilon_{1O}$ changes with the time that will give you this red color curve that is why I have written $\frac{d\varepsilon}{dt}$ basically this $\frac{d\varepsilon}{dt}$ it is strain rate only ok strain rate only major strain outside location whatever major strain you have variation for this variation you are going to get the slope and you plot it with respect to time. So, ε_1 outside how is it changing with respect to time you will see that at one particular instant it will keep on increasing at one particular instant it will be it will be reaching a maximum or a peak and then it will decay actually and then it will decay. So, this peak value is what you have to pick up. So, the maximum point is identified as the stage of localized necking at this particular time the corresponding major and minor strains denote forming limit strain.

So, you can draw a vertical line to hit this portion which is nothing, but your ε_1 ok and then the corresponding ε_2 together will give you a forming limit strain ok. So, to keep it little bit more conservative you can also choose the outside region also as a reference and you can get $\varepsilon_1^*, \varepsilon_2^*$ ok. So, this diagram is going to give you only the variation major strain and major strain rate, but then if you know the stage at which you have to pick up ε_1 and call it as ε_1^* at the same stage you can get ε_2^* for that particular location to keep it more conservative better to refer the outside region as a location for evaluation ok. This is another way to to locate your necking stage ok. So, we stop here and then we will discuss further in the next topic.