Mechanics of Sheet Metal Forming
Prof. R Ganesh Narayanan
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Week- 04
Lecture- 10
Load Instability and Tearing(contd)

So, we are going to continue our discussion in this lecture load instability and tearing. So, in the last session we discussed about you know the meaning of imperfection that you can give in the sheet and then what is the use of that and then we derived an equation for $n - \varepsilon_u$ ok. It is a function of imperfection severity that is dA_0 and n strain hardening exponent right. So, we will continue our discussion in this session. So, next one what we are going to see is the effect of rate sensitivity or strain rate sensitivity ok. So, we understand this particular topic rate sensitivity means how sensitive is your is our deformation with respect to change in strain rate.

Slowly or quickly you do the test. So, how is going to affect the deformation ok or instability development or limit strains ok. So, that is what the main aim here is. But until now what we have assumed is basically the material actually strain hardens ok and mostly it is insensitive to strain rate mostly it is insensitive to strain rate that is what we were assuming.

But that can happen in room temperature ok but the problem is even necking starts then the effect of strain rate is going to become significant during that process during that localization or necking process. Even at room temperature when you assume the strain rate effect to be small ok but if you see that once necking starts then the effect of strain rate is going to be significant. So, we cannot actually neglect the effect of strain rate ok because when necking starts you will see that due course that there is going to be unstable increase in one of the strains and you will see that that is going to be a reason for material failure ok. So, now let us assume a material which is actually non strain hardening but it follows this particular strain rate you know flow stress model flow stress relationship $\sigma_1 = B\dot{\varepsilon_1}^m$ where m is called as strain rate sensitivity index. So, we are not bringing in n here strain hardening exponent n ok is not available in this equation which is a general equation that we have seen before generally we have seen that equation before but right now we are using $\sigma = B\dot{\varepsilon}^m$ that type of equation we are saying and here you know that m is called strain rate sensitivity index we know how to get this you know material property ok.

And here so B is actually a fit constant like what we see as K in case of Holloman power law equation. So, here you will see that $\dot{\varepsilon_1}$ is nothing but a true strain rate which can be defined as $\frac{d\varepsilon_1}{dt}$ ok that means rate of change of your major strain ok that is defined by $\frac{d\varepsilon_1}{dt} = \frac{dl/l}{dt} = \frac{v}{l}$ and dl/t is nothing but your cross at velocity that you use for tensile test and v is nothing but your cross at velocity and l is nothing but your instantaneous gauge length ok during tensile test. So, you can get true strain rate from these two values ok. So, you can see that $\dot{\varepsilon_1}$ depends on a cross head speed and the gauge length instantaneous gauge length l why because it is a true strain rate ok. So, now let us again go back to our this particular diagram which you have discussed in the previous class.

So, you have a sheet with imperfection at this particular position let us say ok and it is getting deformed. So, it is a uniform region is imperfect region. So, now the load can be written as $P = \sigma_1 A = (\sigma_1 + d\sigma_1)(A + dA)$ ok. So, this will give relationship $\frac{d\sigma_1}{\sigma_1} = -\frac{dA}{A}$ and you will see that when you have this particular equation you will see that the difference in stress in these two regions that is σ_1 is proportional to the magnitude of imperfection is proportional to the magnitude of imperfection defined by dA. So, larger the severity of imperfection you will have a large difference in the σ_1 values between the net region which is nothing but a imperfect region and the neighboring region which is nothing but a uniform region ok.

So, now what we are going to do is that is what I have discussed here difference in stress for the two regions is proportional to the ok your magnitude of imperfection which is nothing but dA. So, now as usual what I am going to do is I am going to use this particular material law ok and I am going to get a condition ok. So, now if I differentiate the material law ok and if I put differentiate the material law I will get a condition for my you know this particular change in strain rate with respect to the strain rate which is nothing but $\frac{d\dot{\varepsilon}_1}{\dot{\varepsilon}_1} = \frac{1}{m}\frac{d\sigma_1}{\sigma_1} = -\frac{1}{m}\frac{dA}{A}$ ok. So, you have to differentiate this ok and you can substitute here you will get this particular equation $\frac{d\dot{\varepsilon}_1}{\dot{\varepsilon}_1} = \frac{1}{m}\frac{d\sigma_1}{\sigma_1} = -\frac{1}{m}\frac{dA}{A}$ and minus $\frac{dA}{A}$ you can substitute here you will get this particular relationship and *m* is nothing but my strain rate sensitivity index ok. So, now this equation is going to tell you certain important observation for a smaller value of *m* ok you will see that the difference in strain rate ok the difference in strain rate will be large and the imperfection will go rapidly ok.

So, difference in strain rate means between these two region there is an imperfect region there is a perfect region ok let us see there is an imperfect region ok and there is a perfect region ok this is your neck let us say which is a imperfection we have and this is a neighboring region which is a uniform region ok. You will see that the difference in strain rate between these two ok is proportional to m in this fashion ok. So, smaller value of *m* which is generally seen in room temperature for most of the materials ok a room temperature if you evaluate *m* it is going to be pretty small you will see that there is going to be large difference in strain rate between this region and this region if you measure it there will be large difference in strain rate between this region this region which is actually one important reason for imperfection to grow rapidly. So, this neck will be further localized and you will have a fracture as quickly as possible if *m* is low which is what you see in a room temperature type of deformation ok. But there are some materials like super plastic materials where *m* is going to be very large of the all order of let us say 0.3 you will see that the extension could be several hundred percent as a growth of imperfection is very gradual. So, for the neck to have to become severe and you need to have a fracture in that particular failure in that particular location due to localization of neck you will see that *m* plays a larger role and for those materials which has got pretty large *m* value ok you will have several hundred percentage of elongation or extension why because the neck growth is actually going to be very gradual the imperfection growth is going to be very gradual which is just opposite to what you have seen in this particular cases for conventional material which are not super plastic in nature you will see that it will grow rapidly. So, that is why the super plastic materials are generally having very high ductility of the order of let us say 100% sometimes 300% why because it controls $\varepsilon_\mathrm{t} - \varepsilon_\mathrm{u}$ ok. So, $\varepsilon_\mathrm{t} - \varepsilon_\mathrm{u}$ is controlled by *m* that is why we were saying that it is going to control the post necking phenomenon which is actually going to control the growth of neck whether it is going to grow rapidly or slowly gradually. So, some metals you know people found out that like molten glass which has got m of the order of 1 ok $m = 1$ can deform almost indefinitely ok.

So, in materials with lower m value rate sensitivity will not greatly influence the maximum uniform strain ok, but it will affect post uniform elongation ok in which materials with higher rate sensitivity will show higher post uniform elongation. So, necking will be affected by rate sensitivity ok. So, though maximum uniform elongation is not getting affected, but around that particular stage you will see necking is going to start and that will be affected by rate sensitivity and it is found that the post uniform elongation we said that is $\varepsilon_{\mathrm{t}} - \varepsilon_{\mathrm{u}}$ is higher in which materials having greater rate sensitivity ok. So, through the simple analysis one can study the effect of strain rate sensitivity index on the growth of neck ok. So, now the point here is as we said that in the neck region your strain rate is going to be very large as compared to the outside region.

So, one can develop theories based on that to predict your limit strains or instability strains. So, now what we are going to discuss in this section is called as instability tensile instability in stretching a continuous sheet. So, when we say continuous sheet it means that for example a conventional sheet which is undergoing deformation through tools like punch ok. So, generally sheets are deformed by punch ok which will give some geometric constraint on the development of strain distribution in the sheet ok. So, like punch can change a strain distribution and the contact can change the strain distribution because there is some geometric constraint given to the sheets ok.

So, you will see that when we stretch a continuous sheet you know the standard way we will stretch we do it ok. So, you will see that a localized neck will develop in that location where diffuse neck was there before ok. So, like what we see in the case of tensile strip and in that situations generally it is found that width of local neck ok. Suppose if you measure width of local neck is almost equal to that of your sheet thickness ok and it is so localized that it generally does not affect the overall strain distribution outside that region ok. So, now what will happen because of such localization ok quick tearing can occur in that situations and your entire process will be over the overall formability will be lost in that.

So, if you want to analyze that we as usual we take a sheet with undergoing deformation in this state of stress you have $\sigma_1, \sigma_2, \sigma_3 = 0$ because of plane stress and σ_1 will give raise to T_1, σ_2 will give raise to T_2 for a particular thickness you can get this values and that is what set of stress and strain is actually represented here for a proportional process ok. Of course you have σ_1 and σ_2 is obtained by $\sigma_2 = \alpha\sigma_1$, $\sigma_3 = 0$ you have $\varepsilon_1, \varepsilon_2 = \beta\varepsilon_1$ and $\varepsilon_3 = -(1+\beta)\varepsilon_1$ which you have already derived and principle traction can be obtained by $T_1 = \sigma_1 t$ and $T_2 = \sigma_2 t = \alpha t$. These are all we already discussed and one can get data from this. So now what we are going to do is we are going to put a condition for local necking, we are going to put a condition for evaluate a condition for local necking and as usual we are going to use maximum tension for that. So we are saying that necking will to some extent start ok onset of necking ok is assumed to start when the major tension reaches a maximum value which is represented by this particular equation ok.

So we are saying that $\frac{\mathrm{d}T_1}{T_1} = \frac{\mathrm{d}\sigma_1}{\sigma_1} + \frac{\mathrm{d}t}{t} = \frac{\mathrm{d}\sigma_1}{\sigma_1} + \mathrm{d}\varepsilon_3 = \frac{\mathrm{d}\sigma_1}{\sigma_1} - (1+\beta)\mathrm{d}\varepsilon_1$. With this this entire equation can be framed. Now what happens when the tension reach maximum we are going to put this equation to be equal to 0 when we set it we can get $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1 + \beta$ is not it. So if you put this equation to be equal to 0 you will get this particular equation $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1 + \beta$ and this equation is actually known to us before ok in considering condition we also get this type of equation and we the left hand side we call it as a normalized strain hardening you know parameter ok $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1 + \beta$ ok. So here as usual your β is nothing but your strain ratio ok depending on β values one can get this condition and you will see that this equation is valid it is very important this equation is valid only for those process in which $\beta > -1$ why because during this process we can expect thinning we can expect thinning.

So this diagram you have to refer ok so $\beta = -1$ ok in all this process you will get some sort of thinning in the sheet while deforming only in those process this equation is valid and if you go in this direction the one which is going to thicken ok below this in this zone if you pick up ok this equation is not valid why for simple reason that if it is strain hardening material the tension will never reach a maximum in this case tension will never reach a maximum and we are putting a condition here no maximum tension is reached ok maximum major tension is reached that is not going to happen in this type of situations where your $\beta < -1$ ok why because it is going to thicken on the other hand if it is a strain hardening material then further it is going to complicate the situation the tension will never reach a maximum. So you cannot put this condition this condition is valid in all this situations ok when you have $\beta > -1$ ok the form of the equation is already known only thing is in the previous equation we got the previous model we got $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1$ ok and then we applied $\sigma_1 = K\varepsilon_1^n$ we applied is not it and then we got $\varepsilon_u = n$ ok if you remember that ok but here only change is becomes a function of β now ok. So now as usual we say that for material obeying a power law like $\overline{\sigma} = K\overline{\varepsilon}^n$ and for von Mises material this can be written as a $\sigma_1 = K\varepsilon_1^n$ and here I just mention K' because K' could be a function of K and α and β , K could be a function of K strain hardening strength coefficient n strain hardening exponent stress ratio and strain ratio ok. So if you want to use original form of the equation still one can use now what our problem is now you have to if you differentiate this particular equation and put a condition you will see that $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = \frac{n}{\varepsilon_1}$. So it is going to be a similar derivation as we did before ok only thing is like on the right hand side you have $\frac{n}{\varepsilon_1}$.

So now both the you know left hand sides are equal $\frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1}$ and on the right hand side you have $1 + \beta$ and here you have $\frac{n}{\varepsilon_1}$. So we are saying that $1 + \beta$ and $\frac{n}{\varepsilon_1}$ ok. So $\frac{n}{\varepsilon_1} = 1 + \beta$ ok. So since this is a process happening at instability when necking is start so you can say $\varepsilon_1^* = \frac{n}{1+\beta}$ and $\varepsilon_2^* = \beta\varepsilon_1^* = \beta\frac{n}{1+\beta}$ ok. So assuming that assuming that your β is going to remain same even at maximum tension even at maximum tension or during necking ok during local necking during your local necking isn't it ok.

We are going to assume that β is going to remain same but then it may not be true we will see that ok. So still you will see that we are going to have this particular equation so $\varepsilon_1^* = \frac{n}{1+\beta}, \varepsilon_2^* = \beta\frac{n}{1+\beta}$ ok. So now if you add these 2 if you add these 2 ok $\varepsilon_1^* + \varepsilon_2^* = n$. So this is a simple condition for your local necking with the help of maximum tension we derived it ok. So with the help of maximum tension we derived it ok.

So now this ε_1^* and ε_2^* as we discussed before these are actually called as forming ok limit strains these are actually called as forming limit strain one is major strain other one is minor strain ok. So ε_1^* and ε_2^* are strains at maximum tension because it satisfy that maximum tension condition ok and that equation can be drawn in this diagram ok Y axis being ε_1 and X axis you have ε_2, 2 principle strains and if you plot that particular equation $\varepsilon_1^* + \varepsilon_2^* = n$ it will give you a line like this which is called as maximum tension line and it will meet Y axis at n it will meet Y axis at n why because $\varepsilon_2 = 0$ here $\varepsilon_1 = n$ in that equation. So it will meet at n and it is going to have an angle of 45 ° ok. So now so what is the physical meaning of this line this line tells that any data point you pick up any ε_1 data you have it here it is going to tell you that maximum tension is reached when you pick up that particular β when you pick up that particular β say for example when you put ok $\beta = -1/2$ ok when you put $\beta = -1/2$ in this equation ok $\beta = -1/2$ let us say ok here also $\beta = -1/2$ if you put what will happen here so your $\varepsilon_1^* = 2n$, $\varepsilon_2^* = -n$ ok. So it is $-n, 2n$ is a data point at which you are going to have localized snaking if you follow $\beta = -1/2$ if you follow $\beta = -1/2$ ok.

So that is the meaning of this particular line ok. Similarly if you follow any other you know β value you will get different place at which you are going to have localized necking ok. And it also tells that as long as you are below the line you are actually in the safe region as long as you are below the line you are actually in the safe region once you cross this line that means maximum tension is reached ok local necking is started that would useful deformation formability is lost so the material will be will not be will be unsafe ok above this so I am writing failed region above the line. So this actual maximum tension line actually separates safe strains from failed strains ok. So below the line you will have useful forming window so you can deform any sheet you can make any component made of any material ok.

So if you get this maximum tension line you have to maintain the deformation such that all the strain data points will be below this particular maximum tension line then there is no problem ok. But there is one point here so the important point here is when we do experiments when we do actual trials ok and find out this ε_1^* and ε_2^* there is some discrepancy as shown in this particular figure as shown in this particular figure. So I think we understand the fact that you know like this kind of you know sheets can be used for different you know strain paths we will discuss about it little later and you will see that all these sheets will have a circle grids on the on its surface of a particular dimension and then you deform the sheet ok say for example here all are deformed at different β values ok. So here probably $\beta = 1, 2$ this could be $\beta = 0$ plane strain 2 this could be β is equal to your uniaxial value whatever you have ok. So if you deform it and allow it to fail ok so the circle grids will be converted into ellipses ok and you can measure your limit strains ε_1 and ε_2 closer to the neck region and you can get ε_1^* and ε_2^* from this ok from experiments point of ε_1^* and ε_2^* you can get from this ok.

So and you can compare that all this data ok with respect to maximum tension line as discussed here ok. So both the quadrants you can get for first quadrant as well as second quadrant. So first quadrant ok and second quadrant one quadrant is $\beta > 0$ the other case is $\beta < 0$ ok. So $\beta > 0$ 0 other one $\beta < 0$. So this kind of data points can be compared from experiments as well as from maximum tension line and that will lead to something called as forming limit diagram or forming limit curve it is called forming limit curve FLC ok.

It will tell you the onset of local necking it will tell you onset of local necking ok which means as I told in the previous diagram it will separate safe and failed strains in a sheet ok. But when you compare experimental data and maximum tension line as I mentioned ok this is your maximum tension line and your experimental data is given and maximum tension line is given ok and it continues here ok both are shown here. Maximum tension line and experimental data are put in one graph and they are compared here you will see that or when you say $\beta < 0$ ok that means $-1/2, 0$ or in between if you see the experimental data and maximum tension line to some extent can coincide well ok. So when you go for $\beta < 0$ experimental FLC coincides with maximum tension line mostly there is no issue. But on the right hand side ok in this particular stage the right hand side $\beta > 0$ if you see there is significant difference between your experimental data and your maximum tension line ok.

So you are forming limit curve ok you are forming limit curve from experiment does not coincide with maximum tension line that you get from the previous theory ok. Why something is happening to the growth of neck ok which is not observed in other β values until now but if you pick up on the right hand side quadrant when $\beta > 0$ ok. Let us say $\beta = 1$ ok $\beta = 1$ would be be forming limit curve would be somewhere here if you take $\beta > 1$ and if you take $\beta = 1$ for example ok. So there is something ok that stabilizes the or slows down a growth of neck ok once reaches once tension reaches a maximum value ok. So that stabilization actually delays you know whether you are forming limit is reached or not and there could be some more useful deformation safe region ok that can be attained by the material that is why you have large difference or some difference between your maximum tension line and experimental data and the pattern is also different you see the maximum tension line keep on decreasing but on the right hand side you will see there is some increment in your forming limit strain ok.

So there is some process that delays your or stabilizes or slows down the growth of neck ok once maximum tension is reached ok. So what is that ok so what is that that can be understood from this particular analysis. So this diagram is already known to you this diagram is already known to you already introduced this to you. So what we are considering here is a simple sheet a thin sheet ok and you are seeing that the principle stresses σ_1, σ_2 are mapped here and I know your 1 is along this 1 direction 2 is along 2 direction and it is undergoing deformation and a localized neck is formed here localized neck is nothing but this dotted region this region is nothing but your localized neck. The cross section is shown here you can see there is a local neck that is formed already formed here ok that region I am calling it as B and outside region I am calling it as A and that is also shown here ok this local region localized neck region is called as B and outside that region is called as A here and it has got a thickness let us assume that the sheet has got a thickness ok.

We are going to define something called as θ, θ is actually the angle between the neck direction and the major principle stress that is σ_1 ok. So now there are certain things that we need to understand in this first one is what we are saying is local neck as shown in this figure would occur along a line of pre-existing weakness at a limit strain ok in the uniform region that is approximately given by ε_1^* and ε_2^* which is described before ok. What does that mean? That means that suppose you take a sheet and you assume that there is an imperfection like we have done before ok then necking is going to happen in the pre-existing weakness let us say for example imperfection ok along the direction it is going to happen and let us imagine that necking is going to happen in that location then ε_1^* and ε_2^* uniform region is shown as a forming limit strain ok that is a physical meaning of this ε_1^* and ε_2^* and how is it related to this B region is what is given here ok. Local neck would occur along a line of pre-existing weakness at a limit strain in the uniform region ok why because the defect is already weaker it is going to fail first ok. So, we are going to have a conservative approach to choose ε_1^* and ε_2^* in the uniform region that is in the A region ok.

So now if we identify uniform region as A and imperfection as B and imperfection meaning that is where necking is going to happen then certain conditions have to be analyzed assumed when we do this necking process analysis. What are the conditions I have noted here the stress and strain ratios we call α and β must remain constant as assumed in the differentiation before ok before and during the necking process. Basically α and β should remain throughout the course of deformation ok even during necking also which is not actually true but let us say it is like this. Then the process is a localized one the necking process is a localized one it is going to be there in a particular very small location and the strain distribution outside the location is almost same something like that you have to maintain and the neck must take a form of narrow trough it is not you know that is why it is called localized one ok narrow trough in the shear rather than a patch or diffuse region that would influence the conditions away from the neck. So it is so localized that it should not affect anything happening outside that region ok which means that it is going to be a narrow trough ok.

So now what we are saying is once the necking process becomes severe ok so diffuse necking has happened and then now localization is severe let us say the uniform region A this region A ceases to strain ok it will not strain it will stop strain ok which means that the strain increment parallel to the neck in the Y direction in the figure ok the strain increment parallel to the neck that is along the Y direction ok will be 0. So I am going to say that along the neck direction that is Y ok I am going to say that $d\varepsilon_Y = 0$. So once the necking process becomes severe the uniform region A ceases to strain and moreover the strain increment parallel to neck ok gradient strain gradient will develop across the neck along the neck it is going to be strain increment is going to be 0. So I am going to write $d\varepsilon_Y = 0$. So once localized necking has done we can measure there are lot of grids on the seed surface so you can measure $d\varepsilon_Y$ ok practically you can measure it and that so the increment is going to be 0 ok.

So now let us go ahead to the next one there is something called a geometry constraint ok. Geometry constraint means these two regions are attached to each other is not it. So these two regions A and B are actually attached to each other right. So because of this geometry constraint requires that strain increment along the neck strain increment along the neck must be equal to that in the same direction outside it. That means if you want to measure $d\varepsilon$ along neck that is Y in B that should be equal to the strain increment along the same direction in A also Y because these two are actually constrained these two are actually connected these two are actually connected that means the strain increment in Y direction both regions A and B along the neck must be 0.

So I am going to say that $d\varepsilon_{YA} = d\varepsilon_{YB} = 0$ which also in a way tells that my $\beta = 0$ for necking ok. So $\beta = \frac{d\varepsilon_2}{d\varepsilon_1}$, so here you can say 2 is like you can say Y ok. So which also tells a fact that your $\beta = 0$ for localized necking. This indicates that necking neck can develop only along the direction of 0 extension. 0 extension means what? That is along your Y direction with respect to this figure it is along Y direction only in this particular region B you are going to have you know development of neck ok.

So with this entire thing from this figure what we are going to do is we are going to relate you can see that there are two coordinates X Y and 1 2 we can relate these two ok we can relate strain increments in these two you know axis ok 1 2 and X Y by this equation ok. So $d\varepsilon_Y = d\varepsilon_1 cos^2\theta + d\varepsilon_2 sin^2\theta = 0$ I am putting a condition now. So by putting this condition what I am going to do is I am going to show that when you take $\beta = 1$ that is when you pick up a β value on the right hand side of this diagram this particular diagram right hand side of this particular diagram I am going to say that this direction of 0 extension does not exist ok. Whereas if you put some other condition for example in this side tensile and plane strain ok the direction of 0 extension exists that means there is some θ there is some definite θ.

This θ actually defines the angle isn't it. So the B region being your neck region and the direction of that along Y is going to define the direction of 0 extension this θ is going to be something which you are going to calculate here ok. So this equation relates your strains in 1 2 with respect to your X Y and I am using this and this condition is already known to me $d\varepsilon_Y = 0$ ok. And now what I am going to do is for isotropic material which is what we are discussing until now and uniaxial tension ok this equation is already derived by us $d\varepsilon_2 = d\varepsilon_3 = -\frac{d\varepsilon_1}{2}$ you already derived this ok. So if you put this condition in this equation you will see that this equation will give you $\beta = -1/2$ and if you put it in this equation it will give $\theta = 55°$ ok. So you can get this I mean somehow you have to bring in β into this so $\frac{d\varepsilon_1}{d\varepsilon_1}$ here also $\frac{d\varepsilon_2}{d\varepsilon_1}$ you can put ok.

So this will become β ok that β value if you put it here you will get $\theta = 54.74°$ ok. So then which means that there is some definite θ ok that means the angle between your direction of 0 extension and the principal stress exists in uniaxial ok. Now when you go for plane strain we know that $\beta = 0$ so if you put $\beta = 0$ here ok that means $\frac{d\varepsilon_2}{d\varepsilon_1} = 0$ here then you will see that $\theta = 90°$ which means the pre-existing that your 0 extension line is going to be perpendicular to σ_1 there also it is existing. Now if you pick up a one case along on the right hand side that is $\beta > 0$ ok let us say $\beta = 1$ I am picking up.

If you put $\beta = 1$ here so $cos^2\theta + 1 \times sin^2\theta = 0$ it is not going to happen ok that means here θ is actually does not exist which means that there is no direction in which extension is 0 when you go for right hand side for example $\beta = 1$ there is no direction which extension is 0. So just to conclude if there is no direction of 0 extension for example in the stretching process in which $\beta > 0$ we took an example of $\beta = 1$ the strain circuit tension is a maximum or still given by ε_1^* and ε_2^* but geometric constraints prevent the instantaneous growth of neck ok. So because of that reason because of no direction of 0 extension ok for example when you have $\beta = 1$ ok we say that ok so that actually delays the growth of neck or slows down the growth of neck and hence there are chances that you will have larger strains in actual experimental data ok when compared to your maximum tension line. So that is why you have large difference in this particular zone $\beta > 0$ ok. So this can be explained with this particular simple analysis ok.

So now let us go into the details of necking in the biaxial tension how it is going to happen the biaxial tension. So we pick up only one case that is necking in biaxial tension that is on the right hand side of your forming limit curve that is this side yeah this side of the forming limit curve ok. So then we are going to this particular region and we are going to say how necking is going to happen we are going to briefly discuss it. As discussed before in the first quadrant of strain diagram your forming limit diagram where both principal strains are positive ok. So there is no direction of 0 extension ok ε_1, ε_2 or let us say positive ok ε_1 will be always positive but ε_2 could be positive or negative.

So we are picking up this particular region where both are positive there is no direction of 0 extension because θ does not exist that is what we said. However we say that under biaxial tension experiments necking occurs actually ok when we do when you take $\beta = 0$ and deform material is actually going to neck but it happens at a strain level beyond the point of maximum tension that is why you have larger ε_1^* and ε_2^* in the previous figure ok. So now what we are going to do is just to understand little bit more of what is happening in that region we are going to use this particular schematic. This schematic tells about the biaxial stretching of sheet with imperfection uniform region. As usual we are saying that so this sheet is actually stretched ok due to σ_1 and σ_2, 2 direction 1 direction is defined and as usual we are defining a region A and a region B, B is a region which is a weaker region which has got a thickness of t_B when compared to A t_B is less ok.

So here it is t_A here it is t_B, $t_B < t_A$ ok and deformation is happening ok and B region is a pre-existing defect and that is where let us say that necking is going to start and we are going to have a simplest case where it is going to be oriented perpendicular to the principal you know stress that is σ_1 ok B ok. So now the point here is this belongs to same material A B both belong to same material only thing is their thickness is different which is nothing but an imperfection equivalent to all the imperfections in the material ok. So we are going to define something called as f ok. The imperfection is a groove denoted as B where the thickness is t_B it is slightly less than the uniform region t_A and it is characterized by a factor f that is called as inhomogeneity factor $\frac{t_B}{t_A}$. We are going to call $f = \frac{t_B}{t_A}$ is like for example t_A you take it as let us say 1 mm thick sheet and t_B would be let us say 0.9999 mm very small heterogeneity you are picking up, inhomogeneity factor you are picking up. So $1 - f_0 = 0.001$ you can say something like that you can imagine ok. So I am putting f_0 here for simple reason that it is at the start of the deformation it is at the start of the deformation initial one let us say which means that there are chances that during course of deformation during necking this f can slightly change ok. So as discussed in the previous section strain in the region B ok strain in the region B, B is your this region strain in the region B parallel to the groove would be constrained by the uniform region A and I can say $\varepsilon_{2B} = \varepsilon_{2A}$ ok.

So that is 2 ok ε_{2B} that is along this direction these 2 regions are going to have same strain along 2 direction $\varepsilon_{2B} = \varepsilon_{2A}$ ok. So and the process is going to be proportional process for the uniform region ok. So in the groove region neck region we do not know so for the uniform region we are saying that you are going to have σ_{1A}, $\sigma_{2A} = \alpha_0 \sigma_{1A}$ again I am using α_0 not α ok because it is initial one let us say on the uniform region and $\sigma_{3A} = 0$ plane stress ε_{1A} will be there ε_{2A} will be there α_0 will give you β_0 and $\varepsilon_{3A} = -(1 + \beta_0)\varepsilon_{1A}$ which we already know ok. So now this $T_1 = \sigma_{1A} t_A = \sigma_{1B} t_B$ correct it will be transmitted to both the regions A and B and this will lead to this particular equation we can say $\frac{t_B}{t_A}$ which is what we want $\frac{t_B}{t_A} = \frac{\sigma_{1A}}{\sigma_{1B}} = f = f_0$ also nothing wrong in it definition remains same ok. So, $\frac{t_B}{t_A} = \frac{\sigma_{1A}}{\sigma_{1B}} = f$ ok.

So, you can see the relationship ok where $t_B < t_A$. So, accordingly σ_{1A}, and σ_{1B} are related ok. So, now deformation is happening with these constraints and this is what will happen here ok the same figure can be referred. So, we are going to put one particular first stage for example. So, here is an yield locus is plotted σ_1 versus σ_2 red one is the initial yield locus you can say ok.

So, now consider initial yielding ok the entire material is going to yield the material has got a different yield strength let us say $(\sigma_f)_0$ ok. So, now you are going to deform beyond uniaxial yield strength ok what will happen is the groove region that is this B region will reach yield point first ok why because your $\sigma_{1B} > \sigma_{1A}$ correct. So, your $\sigma_{1B} > \sigma_{1A}$. So, because it has to it is a weaker region ok $\sigma_{1B} > \sigma_{1A}$ for $f < 1$ which is what we generally take for $f < 1$. So, the groove will reach or the imperfect region B will reach yield point first why because your $\sigma_{1B} > \sigma_{1A}$.

So, now the point is the material in the groove cannot deform because of the geometric constraint $\varepsilon_{2B} = \varepsilon_{2A}$ because of that what will happen is as a stress in A increases to reach the yield locus ok. Now A point will also start going in the same direction like that of B and it will also reach your you know yield locus by the time what will happen the point representing the region B must move around the yield locus to B_0 ok. So, you will see that when uniform region is deforming and reach yield locus by that time what will happen is the B would slightly rotate and will go to B_0 this itself indicates a fact that your α is not going to be same ok your it is going to be α, α_0 is going to be changed to your α ok. So, though we say it is a proportional process here itself you will see that ok the uniform region and the groove region are not going to have same α it is going to be slightly different why because your $\sigma_{1B} > \sigma_{1A}$ ok. So, this is the situation now what will happen is now what you are going to you are going to further deform the sheet considering some increment in deformation ok you are further deforming it ok for which increments parallel to the groove must be same correct.

So, $d\varepsilon_{2A} = d\varepsilon_{2B}$ right which is what we have seen before also ok $d\varepsilon_{2A} = d\varepsilon_{2B}$ ok. So, here here ok these two should be the increment should be same why because they are connected to each other. So, this situation what I am going to do I am going to draw it in a vector diagram like this. So, I am going to say that your $d\varepsilon_{2A}$ and $d\varepsilon_{2B}$ is represented by ok this length and my σ_{1A} is represented by this and σ_{1B} is represented by this ok and your A region is characterized by β_0 because of α_0 and your B region is characterized by β corresponding to α ok. And you can see that your $\varepsilon_{1B} > \varepsilon_{1A}$ quite naturally why because your $\sigma_{1B} > \sigma_{1A}$ as per the previous discussion ok.

Your $\sigma_{1B} > \sigma_{1A}$ ok and because of that ok your $d\varepsilon_{1B} > d\varepsilon_{1A}$ ok and all the four strains can be vectors can be increment can be represented in this particular diagram. So, the strain increments across the groove will be greater than that in the uniform region and uniformity will become greater. So, f will also change during the course of deformation ok and as shown in the figure above you will see that your $d\varepsilon_{1B} > d\varepsilon_{1A}$ ok. This can be directly interpreted from this relationship $\sigma_{1B} > \sigma_{1A}$ and that can be represented with this diagram right. So, now what will happen is so it is clear that in the B region strain in the groove ok or in the neck in the B region will increase ahead of that in the uniform region, but only slightly while the tension is increasing ok.

So, you slowly deform the material tension is increased you will see that little bit ahead in B strains will be little bit ahead in B why because your σ_{1B} is going to be larger ok. But now this particular effect the difference is going to gradually accelerate after the tension reaches a maximum ok. So, the growth of ε_1 in the uniform region and in the neck region or the groove region ok if that difference is going to accelerate ok when it will accelerate and if it reaches a tension at maximum and continues till the groove reaches a state of plane strain $\beta = 0$ which is what is shown in this particular figure. So, it is a same figure you will see that it is a part between σ_1 and σ_2 ok and this situation is already known to us initial yield locus is there and this point ok is for A and it will slightly rotate for B and because of that there will be some change in α ok with respect to α_0 ok, but if you further deform it and once tension crosses maximum tension ok what will happen here is you will reach a state of plane strain which is what we have seen in the previous analysis also ok. So, like this which we have seen in the previous one also like for example, $\beta = 0$ I said for making will happen.

So, here also you will see that slowly the β value will tend to move towards this particular point where your $\alpha = 1/2$ which is nothing, but $\beta = 0$ ok. So, it will reach a state of plane strain which is shown in this particular figure ok, but in the case of A it is not so, in the case of A ok this is for your A_0 A point A location it is not so like that ok. So, when plane strain is reached at B_f this particular f basically says that it is going to fail ok B location is going to fail ok. The strain parallel to the groove ceases the strain parallel to the groove ceases the same thing has been represented in the strain diagram ε_1 versus ε_2 you will see that. So, the strain parallel to the groove means you are 2 ok, you are 2 if you see it is actually going to stop here ok like in the previous vector diagram I am saying $\varepsilon_{1A}^*, \varepsilon_{2B}^*$ this is ε_{2B}^* both are plotted here you will see that in the A region ok it will go up to a particular extent ok up to up to this particular stage ok both B and A are going to remain same almost same, but after some time you will see that there is a unstable increase in B that is why you will see that it is going to suddenly increase when compared to A, but A keeps on straining uniform region keeps on straining further here and when $\varepsilon_{2A}, \varepsilon_{2B}$ reaches star.

That means, when it reaches your limit strain you will see that these 2 points will also reach its star value that is limit strain is actually reached. When plane strain is reached at B_f the strain parallel to the groove ceases because that is actually plane strain ok. So, along the groove it is going to be 0, but in the thickness direction and perpendicular groove there will be some gradient ok. The groove will then continue until failure or tearing and the strain in the uniform region totally ceases ok. So, you will have more strain localization only in the groove region ok to have a localized necking and material is going to tear apart after that.

If the localization is not there it means that the strain is getting localized somewhere else also ok then you have to be little bit careful that out of these 2 whichever is weaker is going to dominate and material will fail here ok. So, now, you will this particular diagram you will see that your ε_2^* is going to be same ok. Why because it is along the groove region $\varepsilon_{1A} < \varepsilon_{1B}$ ok. So, now, this ε_{1B} if you look at and ε_{1A} if you see this ε_{1B} strain is going to be several values larger than this ok maybe like 4 times 10 times larger than with respect to A ok.

So, the strain state just outside the neck is of interest. So, that ε_{1A}^* and ε_{2A}^* when failure occurs can be estimated and can be called as forming limit strain for a particular α_0 and β_0 ok. So, which means that this B region will undergo unstable increase in your you know in your ε_1 ok, but you will see that this A region is not like that it is going to be very uniform strain and you will see that 2 A and B is almost same it is along the groove and this particular diagram tells a fact that when localized necking happens the reference is actually in the A region only the reference is actually in the A region only. So, that ε_{1A}^* and ε_{2A}^* becomes a forming limit strain ok becomes a forming limit strain ok. B region we do not choose because B region is already a necked zone. So, just to have a conservative approach you pick up a location just closer to the neck, but in the uniform region and call that as a forming limit strains theoretically you can predict like this.

So, in the biaxial you know deformation necking has got this many stages ok. So, initially you will see that it is going to the groove region or the B region is going to reach yield locus by when A reaches B will get rotated in this way. So, B_0 , A_0 are represented this it tells that α is going to be different and $\sigma_{1B} > \sigma_{1A}$ because $\sigma_{1B} > \sigma_{1A}$ then $d\varepsilon_{1B} > d\varepsilon_{1A}$ which can also be represented in this strain plot which tells that your strain increment in B is going to grow ahead of A ok. And once maximum tension is reached around that particular situation you will see that the B region the β is going to actually switch or tend towards pain strain type of deformation and which means that pain strain means the strain along the neck region parallel to the groove or along the neck region ceases ok which means you are going to put more strains in the other two directions ok. So, you further deform it the groove will or the neck region will further continue to deform it will fail it will tear ok and strain in the uniform region actually ceases ok. But we always refer uniform region ok when this situation is happening ok and that will lead to ε_{1A}^{*} and ε_{2A}^{*} to show at forming limit strains ok.

So, now if you try to get strain rate in these two regions ok let us say you know in the A region and in the B region if you compare strain rates because strain is larger for a common time you will see that strain rate in the B region is going to be you know several it is going to be a multiplication factor it is going to be large much much larger than the limit strains or the strain rates in the A region ok. Because strain is large for a common time you will see that the strain rate also would be larger in the neck region as compared to A ok. So, maybe like 4 times you know 8 times 10 times would be larger than in A. So, this can be used as a measure for you know localized necking ok. So, now let us assume that you are getting ε_{1A}^{*} and ε_{2A}^{*} ok which is going to denote forming limit strain ok by following α_0 and β_0 are the initial stress ratio and strain ratio that you have picked up and deformed the sheet.

Now you got forming limit strains now ok. So, now you have to repeat the same strategy for various various values of α s and β s ok. So, then you will get different data points in your forming limit curve ok different data points mainly on the right hand side of forming limit curve let us imagine. So, you will get this first star you will get the second star you will get this third star ok these 3 points let us say ε_1^{*} and ε_2^{*} will give you if you join it it will give you a locus that will give you the forming limit curve on the right hand side. On the left hand side we are still assuming that your maximum tension line is sufficient to get the the forming limit ok. So, on the left hand side is nothing, but a maximum tension line you know which you can get it from $\varepsilon_1^{*} + \varepsilon_2^{*} = n$.

So, only these 2 lines this maximum tension line and on the right hand side you repeat the same strategy for different values of α and β and you get ε_1^{*} and ε_2^{*} and you plot together this entire curve is called as forming limit curve in short it is called as FLC ok. So, FLC is going to tell you when localized necking is actually going to start. So, which also tells a fact that below this curve you are actually in the safe zone when you cross this you are going to be little bit careful the material can fail all is already it is already failed you have to be careful with this. So, when you as long as you are below this curve you will be safe ok. Sometimes you can also define band for this ok band this type of band can be defined which means that there is a transit zone ok.

So, below this lower line it is safe above this curve is definitely failure but you are in the transit zone ok which tells a fact that you have to be careful moment you enter into this particular zone at any time it may reach actual forming limit curve ok. So, forming limit curve essentially is nothing but it is actually what is it is a locus of all the limit strains in different α s and β s different α s and β s ok and like you are stress strain behavior it is a material property it is a material property ok. So, you change something in the material then it will change otherwise it is not going to change ok. Say for example you do some heat treatment your forming limit can change you do some material processing like you know friction stir welding then your forming limit can change otherwise it is not going to change ok.

So, what are the applications of forming limit curve ok. So, failure diagnosis of your you know sheet grades ok. So, when it can fail what type of fracture ok all those things you can understand from this quality of sheet estimation. So, whether the material has got is a good quality with respect to forming limit or not that is the main thing ok. So, maybe material is good in terms of corrosion but in terms of forming limit whether it is good or not we do not know.

So, that can be estimated and you can select particular sheet grade for a particular component ok. Suppose you want to make a sheet component used in let us say aerospace structures or automotive structures ok or even tube also ok. So, you want to select ok then ok it has to have a minimum forming limit to make that component that is the meaning. What does that mean? That means if you make a component out of a particular material let us say stainless steel then in none of the locations in none of the locations in that component it can cross the forming limit curve. The strains can cross the forming limit curve it is not allowed ok. It means that suppose if you want to make a you know a cup a little bit a complex cup which is used for some automotive application ok then you will you will have a big machine and then you have a die punch setup for that and you do it in shop floor you can get that part and you can locally check visually you can check ok whether there is any localized necking starts or there is severe thinning ok.

And if you measure ε_1 and ε_2 in that location by putting some circle grids of course before deformation you have put that and deform it and measure it in the location then that will give you some idea of what is the value of ε_1 and ε_2 with respect to forming limit curve of that particular material. Which means before going for component level you know your you know your stage ok you need to get forming limit of that particular stainless steel for example ok. So, that SS let us say SS you know stainless steel some grade is there of particular thickness 2 mm you need to have forming limit curve for that. Standard methods can be used and you have to evaluate and in none of the locations in that actual component strain can cross the forming limit curve ok. And you will see that in actual component you know at any location you pick up it is going to be one of the α s or β s or in between.

We already seen 5 different α s and β s right from you know your the least in the second quadrant to your first quadrant right. So, in the sheet component you will see so from this quadrant to this quadrant you already you have seen that. So, now in the actual component you will see that it will follow one of this strain or combination of all the strain pass that is why this forming limit curve is going to be important. It tells actually the necking strains or the forming limit strains at various strain pass which a component can follow during the course of manufacturing ok.

So, in none of the locations it can cross this particular forming limit curve. So, which means the selection of material can be done for a particular component using this forming limit curve ok. Suppose this is a forming limit curve of one material and you make a component and you will see that one particular critical location the strain has crossed this and it has reached here this particular point which means that the material is actually components already failed ok and you cannot make that component with that material. So, either you change the material or you change the process conditions ok. You are not you know it is not affordable for you to change the material let us say then you change the process conditions such that the same material you can make that component which also tells the fact that selection of process conditions such as lubrication forming temperature strain rate can be decided based on this particular forming limit curve ok. So, now here I made a note the shape of forming limit curve depends on number of material properties and on the initial inhomogeneity factor that is your *f* chosen theoretical theoretically ok so depends on you have to assume an initial *f* to get a forming limit curve theoretically.

So, it is said that your forming limit curve depends on f also. We will see that in due course how *f* is going to affect the forming limit curve ok by showing some schematics of forming limit curve you will see that ok. So, we stop here and then we will discuss in the next session.