Thermal Engineering Basic and Applied
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Lecture - 66
Problems on Gas Turbine Cycle

Very good morning I welcome you all to the session of Thermal Engineering Basic and Applied and in this lecture we shall solve two numerical problems on the gas turbine cycle and I should say that these two problems are the last problems of this course.

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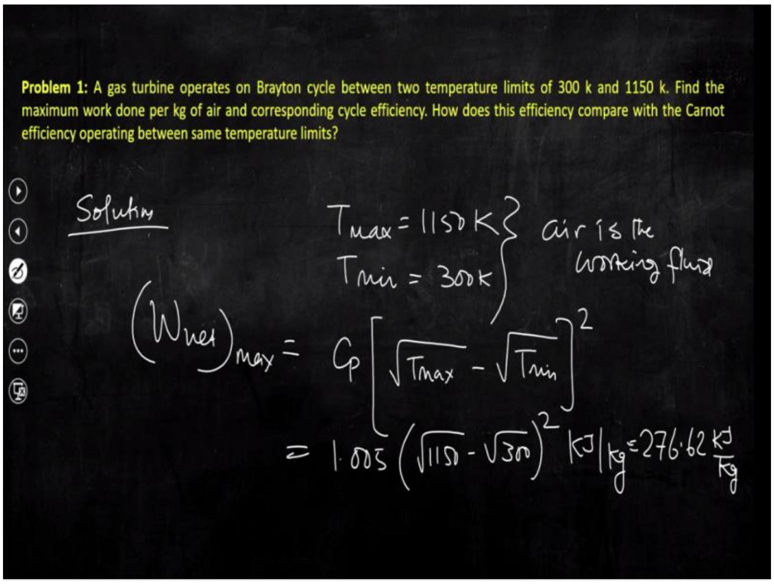

So, this is the first problem, let me read out the problem statement first. A gas turbine unit or cycle operates on Brayton cycle between two temperature limits of 300 K and 150 K. Find the maximum work done per kg of air and corresponding cycle efficiency. Then we need to show how does this efficiency compare with the Carnot efficiency operating between same temperature limits.

So, let us solve this problem. So, it is given

$$T_{max} = 1150\ K\ ;\ T_{min} = 300\ K$$

We need to calculate $(W_{net})_{max}$ that is work done per kg of air. So, air is the working fluid. If you try to recall in one of the previous classes we could establish the maximum work done corresponds to optimum pressure ratio because we never can achieve maximum pressure ratio and that is

$$(W_{net})_{max} = C_p\left[\sqrt{T_{max}} - \sqrt{T_{min}}\right]^2$$

$$\Rightarrow (W_{net})_{max} = 1.005\left[\sqrt{1150} - \sqrt{300}\right]^2 = 276.62\,\frac{\text{kJ}}{\text{kg}}$$

So, this is the maximum work corresponding to optimum pressure ratio because maximum pressure ratio is not easily achievable. So, this is maximum work done per kg of the working fluid and unit is kJ/kg. Next we have to calculate cycle efficiency.

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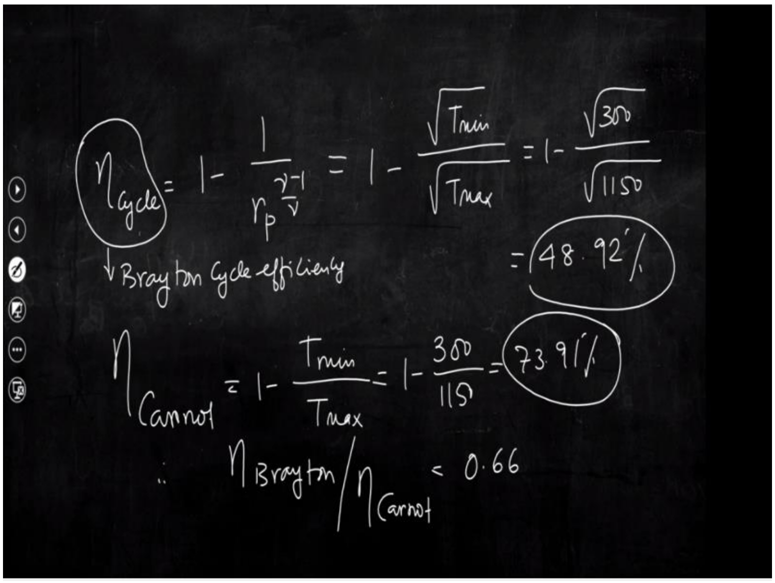

$$\eta_{cycle} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$$

So, this also can be written in terms of the maximum and minimum temperature of the cycle. Also we could establish the cycle efficiency corresponding to optimum value of r_p is

$$\eta_{cycle} = 1 - \frac{\sqrt{T_{min}}}{\sqrt{T_{max}}} = 1 - \frac{\sqrt{300}}{\sqrt{1150}} = 48.92\%$$

So, you can understand that cycle efficiency is less than even 50% that is what we have discussed several times. So, this is cycle efficiency and of course this cycle efficiency is Brayton cycle efficiency. Now we also could establish the Carnot efficiency. That means we now consider that the Brayton cycle is operating between two temperature limits and considering the same temperature limits if we consider the Carnot cycle then

$$\eta_{carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{300}{1150} = 73.91\%$$

So, you can understand that Carnot cycle is basically is one of the ideal cycles and efficiency following the Carnot cycle should be certainly higher than the Brayton cycle efficiency and that is what we can see from this.

Now let us compare Brayton efficiency with the Carnot efficiency operating between two same temperature limits. That means we need to consider

$$\frac{\eta_{Brayton}}{\eta_{Carnot}} = 0.66$$

So it is certainly less than 1. So Brayton cycle efficiency is 0.66 times of the Carnot cycle efficiency, so we can understand that gas turbine units which run on the Brayton cycle has efficiency significantly smaller than the Carnot cycle.

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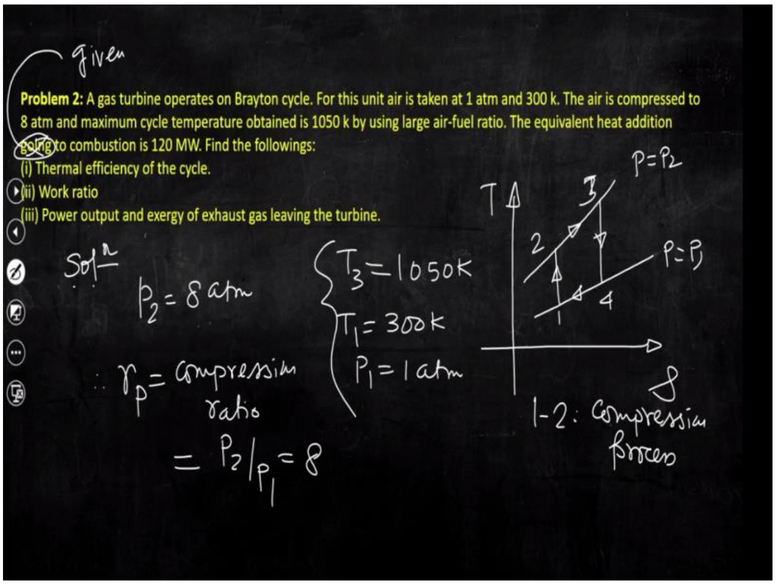

Now let us move to the second problem. The problem statement is that a gas turbine operates on Brayton cycle, for this unit air is taken at 1 atm pressure and 300 K. The air is compressed to 8 atm pressure and maximum cycle temperature obtained is 1050 K by using large air fuel ratio. So, it is already given that the air flow ratio is very large which in a way indicates that we can assume that the mass flow rate of the working fluid is $\dot{m}_a$. The equivalent heat addition given to the combustion is 120 megawatt. We need to calculate the followings, thermal efficiency of the cycle, work ratio, power output and exergy of the exhaust gas leaving the turbine. So, here it should be equivalent heat addition given to; so, this is not going, this should be given to.

So, let us solve the problem. First as I had discussed many times we need to tabulate the data given in the problem statement and draw the T-s diagram. In the T-s diagram there are 2 pressure lines $P = P_1$ & $P = P_2$. Then Process 1-2 & 3-4 has been drawn in the T-s diagram.

$$T_3 = \text{maximum temperature} = 1050 \text{ K}$$

$$T_1 = 300\ K$$

$$P_1 = 1\ atm$$

$$P_2 = 8\ atm$$

So, 1-2 is the compression process that we have discussed so many times. So, therefore

$$r_p = \text{compression ratio} = \frac{P_2}{P_1} = 8$$

So, we can write all these from the problem statement. Now what would be the cycle efficiency?

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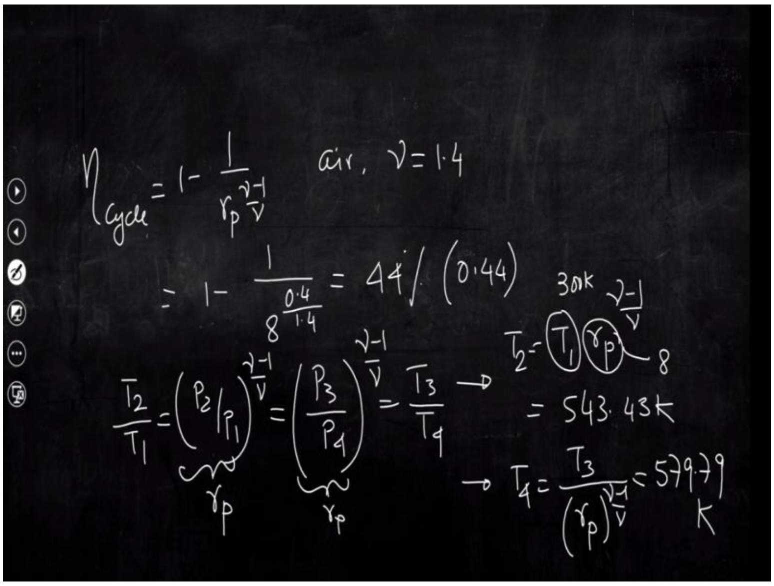

So, therefore

$$\eta_{cycle} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$$

For air we can consider $\gamma = 1.4$ and if we plug in the value here

$$\eta_{cycle} = 1 - \frac{1}{8^{\frac{1.4-1}{1.4}}} = 0.44 = 44\%$$

So, this is the cycle efficiency. So, this is solution of this first part of the problem that is thermal efficiency is 44%.

Now, from this T-s plane we can also calculate T_2/T_1 and T_3/T_4 because this two are the compression, expansion using air standard equation (ideal gas equation).

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$$

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = r_p$$

So, from there what we can calculate

$$T_2 = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 300(8)^{\frac{1.4-1}{1.4}} = 543.43\ K$$

So, we can calculate T 2. Similarly we also can calculate T_4

$$T_4 = T_3(r_p)^{\frac{\gamma-1}{\gamma}} = 579.79\ K$$

So, you can understand that the rise in temperature of the working fluid at the end of the compression is even less than the temperature of the exhaust gas. So, substantial amount of energy is getting lost because $T_4 > T_2$.

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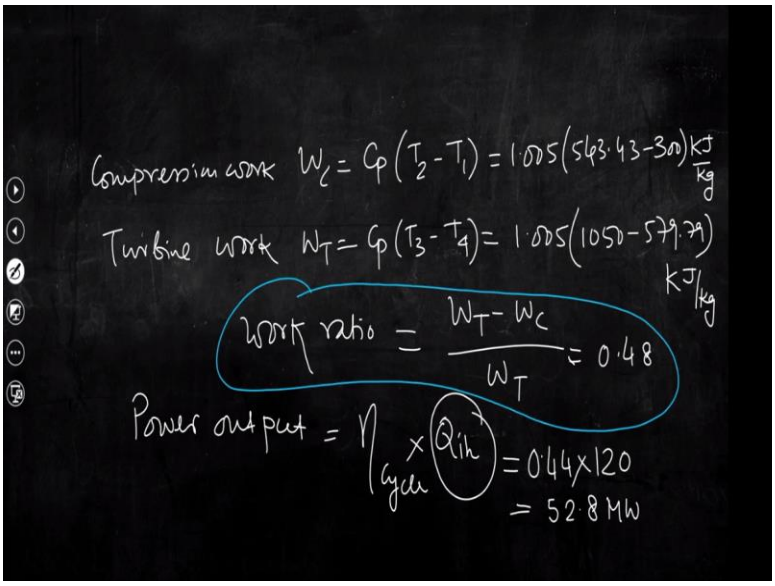

Then next we can calculate the compression work & turbine work.

$$\text{Compression work} = W_c = C_p(T_2 - T_1) = 1.005(543.43 - 300)\frac{\text{kJ}}{\text{kg}}$$

$$\text{Turbine work} = W_T = C_p(T_3 - T_4) = 1.005(1050 - 579.79)\frac{\text{kJ}}{\text{kg}}$$

$$\text{Work ratio} = \frac{W_T - W_c}{W_T} = 0.48$$

So, work ratio is very less that is 0.48. And finally we have to calculate power output.

$$\text{Power output} = \eta_{cycle} \times Q_{in}$$

So, instead of work we are writing power output. So, what is Q_{in}?

So, we have to calculate Q_{in} otherwise we cannot obtain power output. So till now we got two answers that is work ratio & cycle efficiency. So, next we can calculate the heat addition to this unit.

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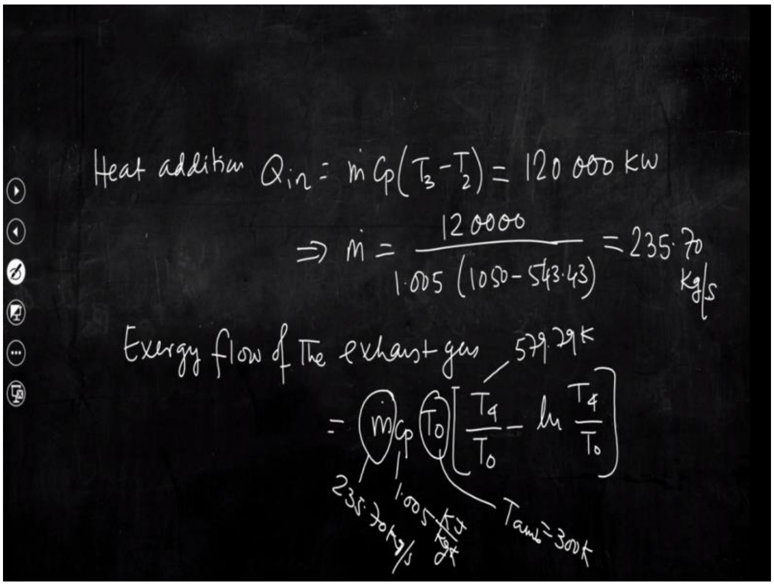

$$\text{Heat addition}, Q_{in} = \dot{m}C_p(T_3 - T_2)$$

And this is already given as 120 MW.

$$\text{Power output} = \eta_{cycle} \times Q_{in} = 0.44 \times 120 = 52.8\ MW$$

So we have no need to calculate Q_{in} that is already given 120MW because in the problem statement it is given that the heat addition because of this combustion is equivalent to 120MW.

$$Q_{in} = \dot{m}C_p(T_3 - T_2) = 120000\ KW$$

$$\Rightarrow \dot{m} = \frac{120000}{1.005(1050 - 543.43)} = 235.70\,\frac{\text{kg}}{\text{s}}$$

And the final part of this problem is exergy flow of the exhaust gas that is

$$\text{Exergy flow of the exhaust gas} = \dot{m}C_pT_0\left[\frac{T_4}{T_0} - ln\frac{T_4}{T_0}\right]$$

So, here we have already calculated mass flow rate $\dot{m} = 235.70\frac{kg}{s}$. We know $C_p = 1.005\frac{kJ}{kg.K}$ and we already have calculated $T_4 = 579.79\ K$ and this $T_0 = T_{ambient} = 300\ K$. So, we can calculate the exergy flow of the exhaust gas by plugging in all the values in the above expression.

So, this is the expression for the amount of exergy energy that will be released with the exhaust gas. I am very much sure that you know this expression because you have studied it in basic thermodynamics course. So, knowing the mass flow rate that you have calculated from here, we can easily calculate this quantity.

So, to summarize we have solved two different problems. And while solving these two different problems, we have tried to illustrate the concept that we have learned from the theoretical discussion of this particular module of this course. I hope you have learned the steps those are needed to follow while solving a particular problem. As I said that these two problems are the last problems of this particular course and I wish you all the best. Thank you.