

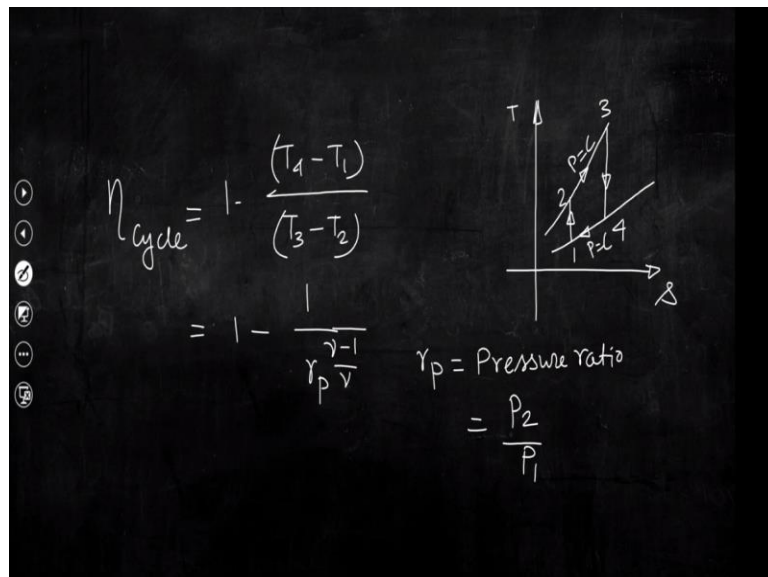
Thermal Engineering Basic and Applied
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Lecture - 63
Gas Compressor and Optimum Pressure Ratio

Very good afternoon, I welcome you all to the session of Thermal Engineering Basic and Applied and the topic of our today's discussion is gas compression and optimum pressure ratio. In the last class we have discussed about gas turbine units and to analyze the performance of gas turbine units we had tried to represent the processes those constitute together to form a cycle using an air standard cycle that is the Brayton cycle.

And we had seen that in a gas turbine unit whether the unit is based on open cycle or closed cycle compressor is an important part and the rise in pressure of the working fluid which is performed by a compressor you know is very important to dictate the overall efficiency of this cycle. So, today we shall discuss about the gas compression, but before going to discuss about gas compressor let us first focus on an important term that is called optimum pressure ratio.

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So, if we try to recall the cycle efficiency that

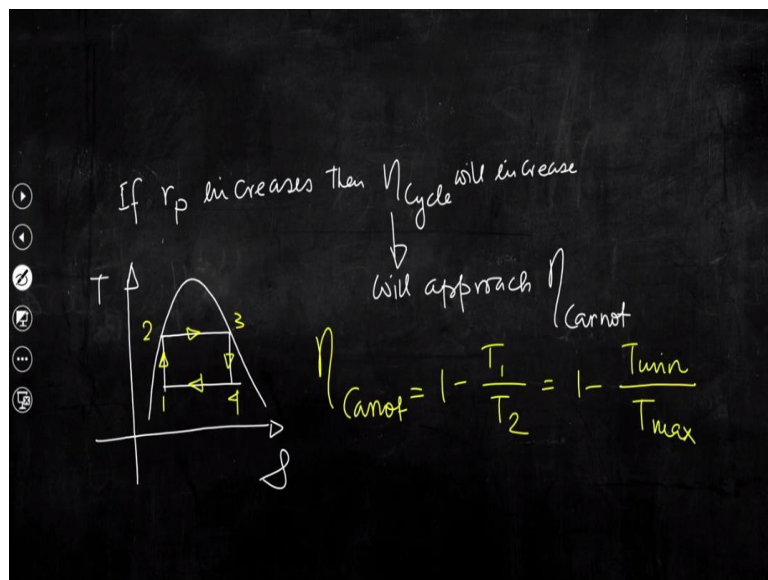
$$\eta_{cycle} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{r_p^{\gamma-1}}$$

So, if we try to draw the T s diagram then the processes if we represent here then this is 1, 2, 3, 4.

So, 1, 2 is the compression process then 2 to 3 is constant pressure heating. So, basically P equal to constant for this process and P equal to constant for this process that is 2 to 3 and 4 to 1 and we could represent the cycle efficiency by defining a term that is called pressure ratio and this r_p is the pressure ratio that equal to P_2 by P_1 . So, basically you can see the rise in pressure of the working fluid by this compression process that is pressure rise from P_1 to P_2 .

So, ratio of these two pressures is known as compression ratio. So, now if we increase r_p then we can see from this particular expression which is very trivial to understand that if we increase pressure ratio, cycle efficiency will increase and if we keep on increasing pressure ratio if it is possible at all then perhaps the cycle efficiency or efficiency of the Brayton cycle also can be compared to the efficiency of the Carnot cycle.

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So, that means if r_p increases then η_{cycle} will increase. So, what would be the r_p for which this η_{cycle} will approach η_{carnot} . So, that means if we keep on increasing pressure ratio there will be a pressure ratio for which this cycle efficiency will certainly approach the Carnot efficiency and what would be that. So, you know, that if we again draw the T s diagram for the Carnot cycle.

So, this is 1, 2, 3, 4, the Carnot efficiency is $\eta_{carnot} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{min}}{T_{max}}$ that means if we now compare the cycle efficiency of the Brayton cycle what we can do if we increase the pressure

ratio then certainly P2 will increase and this process in a way is attempting to increase the mean temperature of heat addition.

2 to 3 that is the constant pressure heat addition and that is basically mimicking the combustion process. So, if we increase r_p in a way we are trying to increase the mean temperature of heat addition and also an increase in r_p will result in a decrease in the mean temperature of heat reduction.

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Handwritten notes on a blackboard:

- mean temp of heat addition $T_{m,i}$
- heat rejection $T_{m,o}$
- $\eta_{cycle} = 1 - \frac{T_1}{T_3} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{1}{r_{p,max}^{\frac{\gamma-1}{\gamma}}}$
- $r_{p,max} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

So, what we can do if mean temperature of heat addition so that is $T_{m,i}$ and if mean temperature of heat rejection is $T_{m,o}$. So, if this is the case then what we can do? We can write this

$$\eta_{cycle} = 1 - \frac{T_1}{T_3} = 1 - \frac{T_{min}}{T_{max}}$$

So, that means by increasing the pressure ratio we are trying to increase the mean temperature of heat addition as if T_2 equal to T_3 .

And we are also trying to reduce the mean temperature of heat rejection that means T_4 equal to T_1 . So, that is the mean temperature of you know heat rejection and heat addition and that would be cycle efficiency and this cycle efficiency will be maximum if we can increase the r_p and then it should be $1 - \frac{1}{r_{p,max}^{\frac{\gamma}{\gamma-1}}}$.

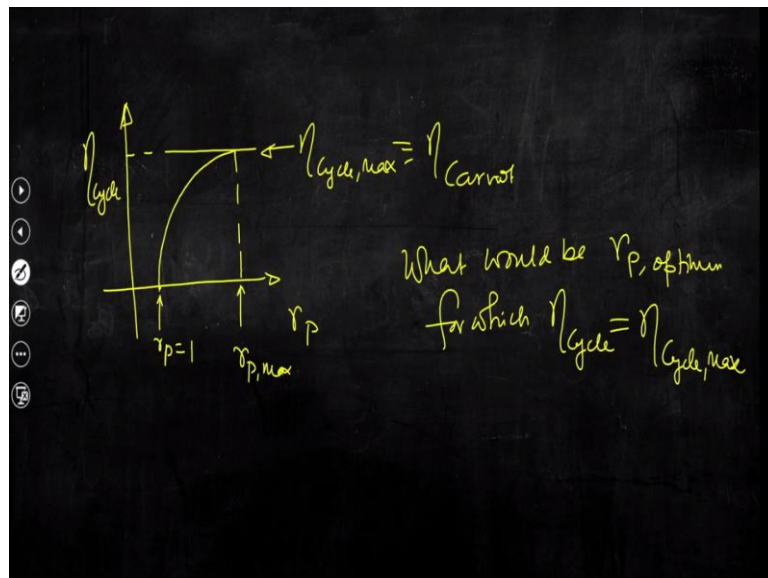
So, basically what we are trying to do we are trying to increase the r_p when r_p will be $r_{p,max}$ then cycle efficiency will be certainly maximum and that would be as if we are assuming that at that

cycle efficiency is equal to the Carnot efficiency and that is we are increasing the mean temperature of heat rejection from $T_4 - T_1$ to T_1 and mean temperature of heat addition from T_2 to $T_2 - T_3$ that is T_3 .

So, essentially this is the expression. Now then what we can write from this? Then we can write

$$r_{p,max} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

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Now question is if we try to plot so if we consider r_p here if it is η_{cycle} then say this is the Carnot efficiency and if we look at this particular expression when $r_{p,max} = 1$ then efficiency will be equal to 0 and when $r_{p,max}$ will be $r_{p,max}$ only then efficiency will be maximum and that efficiency you are assuming that would be equal to Carnot efficiency.

So, if we go up to $r_{p,max}$ then cycle efficiency will increase. Now it is really difficult to achieve the Brayton cycle efficiency which should be equal to Carnot efficiency because Carnot cycle is one of the most important ideal cycle.

So, next objective should be it is not possible to get $r_{p,max}$ so what would be the optimum value of pressure ratio for which this cycle efficiency will be maximum. So, this is basically a theoretical efficiency that we are assuming to achieve by the compression process then expansion process that is the cycle efficiency, but it will not be the case. So, now next objective should be what would be $r_{p,opt}$ for which cycle efficiency would be equal to $\eta_{cycle,max}$.

So, next objective should be to look at the expression of r_p for which cycle efficiency maximum.

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$$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1) = C_p(T_3 - T_2 - T_4 + T_1)$$

$$Q_{in} = C_p(T_3 - T_2)$$

$$\eta_{cycle} = \frac{W_{net}}{Q_{in}} \quad \text{for } \eta_{cycle,max} \quad W_{net} = W_{net,max}$$

So, let us now work on this particular aspect so you know what is W_{net} and what is Q_{in} .

$$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1) = C_p(T_3 - T_2 - T_4 + T_1)$$

$$Q_{in} = C_p(T_3 - T_2)$$

So, now for a given heat input that is certainly the addition of energy to the cycle through the combustion process we are getting that is W_{net} . So, now can we figure out what would be the value of $r_{p,opt}$ for which this W_{net} would be maximum. If W_{net} is maximum, you know, now $\eta_{cycle} = W_{net}/Q_{in}$. So, for a given heat input what would be the pressure ratio for which we can maximize the W_{net} if we can ensure that cycle efficiency will be maximum. So, that means for $\eta_{cycle,max}$, W_{net} would be $W_{net,max}$.

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$$W_{net} = C_p (T_3 - T_2 - T_4 + T_1)$$

$$W_{net} = C_p \left(T_3 - T_1 r_p^{\gamma-1} - T_3 r_p^{-\frac{\gamma}{\gamma-1}} + T_1 \right)$$

$$\frac{dW_{net}}{dr_p} = 0 \Rightarrow r_{p,opt} = \left(\frac{T_3}{T_1} \right)^{\frac{1}{2(\gamma-1)}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$$

So, now let us do here. So, $w_{net} = C_p(T_3 - T_2 - T_4 + T_1)$. Now we know if we go back to the slide 1 to 2 and 3 to 4 these two processes are reversible adiabatic processes. Reversible adiabatic compression 1 to 2 and 3 to 4 is reversible adiabatic expansion. So, now we, have also discussed in the last class that these two processes can be represented by you know $pv^\gamma = C; \gamma = C_p/C_v$.

And we can get the change or the property at the exit of the compression and at the exit of the expansion air standard equation ideal gas.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}}$$

$$w_{net} = C_p(T_3 - T_1(r_p)^{\frac{\gamma-1}{\gamma}} - T_3(r_p)^{-\frac{\gamma-1}{\gamma}} + T_1)$$

why you are doing so? Because you know that T_3 and T_1 . So, basically what we can do here you know that state point 1 is fixed and for a given T_3 that is after combustion. If we like to apply same amount of fuel, then certainly you know we can fix that T_3 , T_3 will increase provided we are trying to increase by having optimum pressure ratio.

But as I said you that I will be discussing today particularly for this cycle, you know the compression process is not only to rise the pressure of the working fluid, but also to increase the temperature as well. So, what we can do here we can fix T_3 , T_1 and that is why we have replaced here T_2 and T_4 in terms of T_1 and T_3 . So, now question is we could write the expression of W_{net} in terms of r_p you can see from here.

So, what we can do we can calculate $\frac{dW_{net}}{dr_p} = 0$. So, if we need to obtain r_p optimum for which W_{net} would be maximum because if W_{net} is maximum then only cycle efficiency will be maximum and to get it if we make $\frac{dW_{net}}{dr_p} = 0$ then from this particular expression we can figure out $r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$. So, this is the optimum value of pressure ratio for which cycle efficiency will be maximum.

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The image shows a chalkboard with the following handwritten equations:

$$r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\gamma_{p,max}\right)^{\frac{1}{2}}$$

$$\gamma_{p,max} = \left(\gamma_{p,opt}\right)^2$$

$$\gamma_{p,opt} = \sqrt{\gamma_{p,max}}$$

Now, if we write

$$r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(r_{p,max}\right)^{\frac{1}{2}}$$

$$r_{p,max} = r_{p,opt}^2$$

So, this is very important that we cannot really get r_p maximum because otherwise efficiency should be equal to Carnot efficiency. So, that is basically you know idealized situation, but what we can get we can really get an optimum value of r_p for a given T_1 and T_3 and for that cycle efficiency will be maximum. So, now $r_{p,opt} = \sqrt{r_{p,max}}$.

Now question is if we try to place or if we try to plug in the value of r_p optimum in this expression W_{net} and also in the expression of cycle efficiency let us see what would be the expression of cycle efficiency and W_{net} in terms of T_{max} and T_{min} .

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Handwritten derivation on a blackboard showing the cycle efficiency at optimum compression ratio:

$$\eta_{\text{cycle}}|_{\text{opt } r_p} = 1 - \frac{1}{\left(\gamma_{p,\text{opt}}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= 1 - \frac{1}{\left(\frac{T_{\text{max}}}{T_{\text{min}}}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$\eta_{\text{cycle}}|_{\text{opt } r_p} = 1 - \sqrt{\frac{T_{\text{min}}}{T_{\text{max}}}}$$

So, cycle efficiency for optimum r_p

$$\eta_{\text{cycle}}|_{\text{opt } r_p} = 1 - \frac{1}{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{\left(\sqrt{\frac{T_{\text{max}}}{T_{\text{min}}}}\right)^{\frac{\gamma}{\gamma-1}}} = 1 - \sqrt{\frac{T_{\text{min}}}{T_{\text{max}}}}$$

So, this is the expression of cycle efficiency at the optimum value of r_p . So, if we know the maximum and minimum temperature of the working fluid in the cycle, We can get what would be the maximum efficiency for that cycle because that would be the maximum efficiency because that is obtained at a optimum value of r_p .

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Handwritten derivation on a blackboard showing the maximum net work for the cycle:

$$W_{\text{net}}|_{\text{opt } r_p} = C_p \left[T_3 - T_1 \gamma_{p,\text{opt}}^{\frac{\gamma-1}{\gamma}} - T_3 \gamma_{p,\text{opt}}^{-\frac{\gamma-1}{\gamma}} + T_1 \right]$$

$$\gamma_{p,\text{opt}} = \left(\frac{T_{\text{max}}}{T_{\text{min}}}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$W_{\text{net, max for the cycle}} = C_p \left[T_3 - T_1 \frac{\sqrt{T_3}}{\sqrt{T_1}} - T_3 \frac{\sqrt{T_1}}{\sqrt{T_3}} + T_1 \right]$$

$$= C_p \left[(\sqrt{T_3})^2 - 2\sqrt{T_3}\sqrt{T_1} + (\sqrt{T_1})^2 \right]$$

Similarly, what would be W_{net} at r_p optimum as such this is basically I should say so this is W_{net} maximum for the cycle. W_{net} maximum that we could write you know in one of the

previous slides is basically the Carnot efficiency that is that is not achievable in practice. So, this is basically, you know, W_{net} maximum corresponding to optimum value of r_p and if that is the case so what we can get here you know this is the expression.

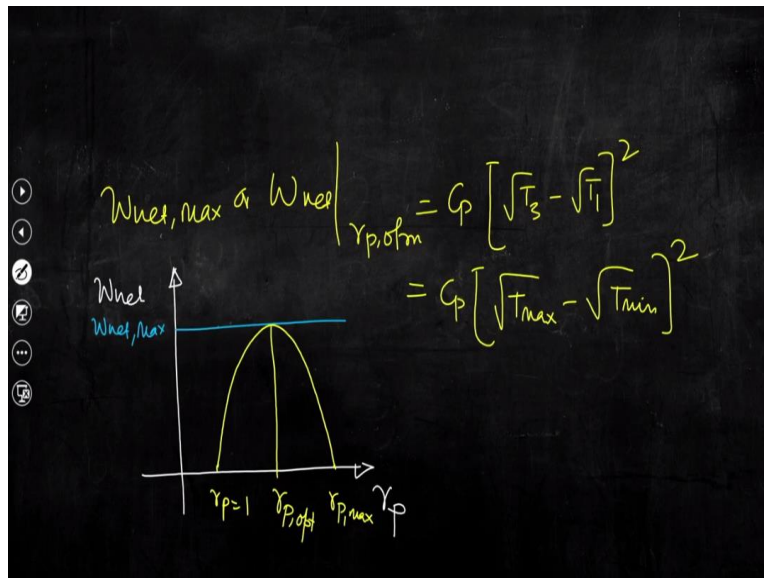
$$w_{net}|_{r_{p,opt}} = C_p(T_3 - T_1(r_{p,opt})^{\frac{\gamma-1}{\gamma}} - T_3(r_{p,opt})^{\frac{\gamma-1}{\gamma}} + T_1)$$

$$r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

So, if we write this then what we will be getting?

$$w_{net}|_{r_{p,opt}} = C_p \left(T_3 - T_1 \sqrt{\frac{T_3}{T_1}} - T_3 \sqrt{\frac{T_1}{T_3}} + T_1 \right) = C_p \left((\sqrt{T_3})^2 - 2\sqrt{T_3}\sqrt{T_1} + (\sqrt{T_1})^2 \right)$$

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So, that means we can write in the next slide that

$$w_{net}|_{r_{p,opt}} = C_p(\sqrt{T_3} - \sqrt{T_1})^2 = C_p(\sqrt{T_{max}} - \sqrt{T_{min}})^2$$

So, this is the maximum efficiency corresponding to the optimum value of pressure ratio. So, I should say this is the expression of the maximum efficiency obtainable from a gas power cycle or this is the efficiency corresponding to optimum value of pressure ratio that is

$$\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2.$$

So, with this if we try to draw the relation of W_{net} so this is W_{net} with r_p . So, this is W_{net} maximum and this is r_p optimum. So, what we can say that when r_p is equal to optimum we will be getting the maximum value of W_{net} because if we go back to the previous slide.

If it is r_p optimum we will be getting maximum W_{net} , so efficiently maximum. So, this is the variation of W_{net} versus r_p . So with this now we can start our discussion on the compression or a gas compressor to be precise. So, what we have understood from the discussion that we had until now that pressure ratio plays an important role.

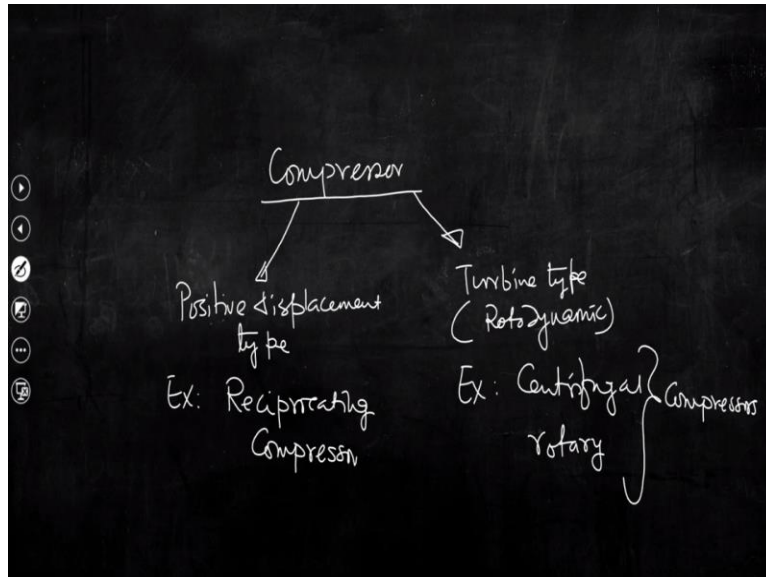
And the pressure of the working fluid is increased by an air compressor or gas compressor to be precise if the working fluid is not the air. So, what we can you know discuss now that compressor plays an important role on the overall cycle efficiency and typically we had seen that compressor is consuming work from the turbine.

And it consume a significant part of this turbine work output. So, today we shall now see the working principle of a compressor and then what are the different efficiencies of the compressor, what is compressor? A compressor is again a mechanical device in which work is done on the working fluid essentially to increase its pressure with an appreciable rise in the density.

So, let me tell you once again compressor is a mechanical device in which work is done on the working fluid. If the working fluid is air then work is done on the air, if the working fluid is gas then work is done on the gas, with an appreciable rise in you know density, but the main purpose is to increase the pressure. We had seen that temperature of the working fluid also increases because of this compression process.

But certainly the rise in temperature of the working fluid is little as compared to the rise in pressure.

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So, there are basically two different types of compressor. One is positive displacement type. Example is reciprocating compressor, another type is basically you know that turbine type that is roto dynamic. So, examples are centrifugal compressor, rotary compressor. So, these two are two different types of compressors. So, positive displacement type and the turbine type that is roto dynamic whirling motion will be there, so rotating part will be there.

We had seen perhaps you have studied in fluid machine course, compressor is also a fluid machine, but the fluid is you know either air or any other gas. So, in the hydraulic machines perhaps you have studied that there are different types of pumps available. Positive displacement type and also roto dynamic pumps similar to that classification here you also can classify compressors into two broad categories.

The first category is positive displacement type in which the main purpose is to increase the pressure significantly, reciprocating compressor and turbine type that is axial flow type that is you know centrifugal compressor, rotary compressor. So, basically you know that rotating components will be there. As I said you that the sole purpose of this particular component or the sole purpose of integrating this particular component in the gas turbine unit is to compress the air.

And the objective is to increase the pressure with an appreciable change in density certainly temperature may or may not increase. Now in most of the cases temperature also increases, but the rise in temperature is very little as compared to the rise in pressure. So, when the

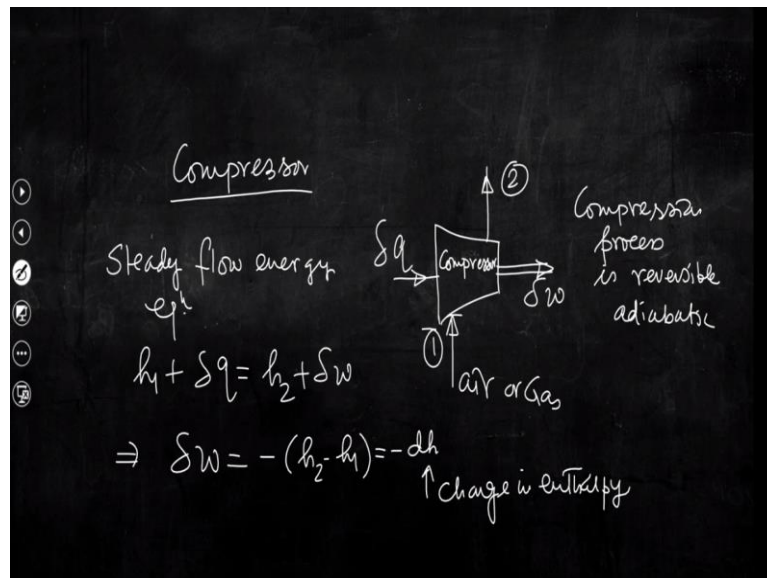
requirement of the compressor is prompt supply or we need to supply the working fluid which will be having high pressure say for example in a combustion chamber or in an IC engine.

In that case rise in pressure together with a little rise in temperature is important, but there are other areas in which what you need to do? We need to certainly increase the pressure of the working fluid and that compressed gas or compressed air will be kept in a cylinder or in a container for a while and that compressed air or gas which is having high pressure will be used later when need arises.

Now, when the compressed air or compressed gas is kept or stored in a cylinder you know it is very likely that though a little increase in temperature will be there, but that rise in temperature will certainly increase the temperature of the gas, but the gas temperature will be reduced because the compressed air will be kept in a cylinder. So basically there are two different applications in one application compressed air is promptly supplied to a combustion chamber or that compressed gas is needed for an IC engine.

In that case rise in pressure together with a rise in temperature is okay because we also need to get some more energy. So, basically the compressed air when will be supplied to the combustion chamber if that air stream or gas stream is having little more in energy in terms of enthalpy that would be good for the combustion process. On the other hand there, are applications in which compressed air or compressed gas will be kept or stored in a cylinder and that high pressure gas or air will be used later as and when need arises. In that case rise in temperature is not that much important.

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So, now question is if we discuss the compressor. So, what we can see from the schematic depiction that we had drawn in the last class that if we assume this is the compressor and we are supplying air or gas and then this is state point 2 and say this is state point 1 and so this is compressor and we had seen that we need to supply work. So, basically if we consider that convention wise that you have studied in thermodynamics, this is Q and this is W.

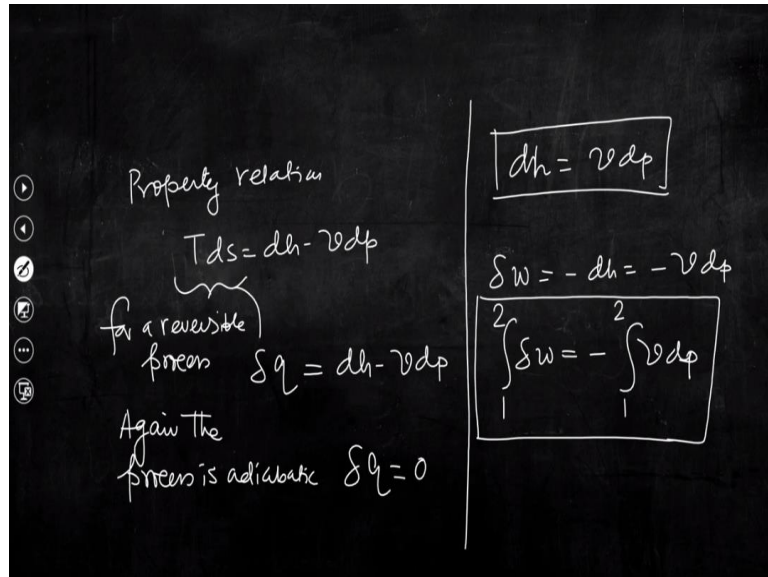
So, we are following the nomenclature that heat added to the system is positive and work which is you know extracted from the system is positive. So, this is W and this is Q and this is the working fluid. So, if we apply steady flow energy equation and we can write that

$$h_1 + \delta q = h_2 + \delta w$$

So, now we have assumed that the process is reversible adiabatic. So, the compression process is reversible adiabatic. So, if it is an adiabatic process, so we are not allowing any heat exchange to takes place from the working fluid to the surroundings or we are ensuring that there will be no heat flow from the working fluid into the surroundings.

So, that means $\delta w = -(h_2 - h_1) = -dh$. Change in enthalpy of the working fluid as the state changes from 1 to 2 that is the change in enthalpy. So, this is basically change in enthalpy.

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So, now let us use the property relation $Tds = dh - vdp$. We are using this relation because you know here it is a flow process. So, this is not a non flow system or process because there is a continuous flow of air from one end and other end and sole purpose is to rise the pressure of the working fluid and also the temperature. So, $Tds = dh - vdp$ and since the process is reversible adiabatic.

For a reversible process you have studied in thermodynamics this $Tds = \delta q = dh - vdp$. Now the process is you know adiabatic. So, that means $\delta q = 0$. So, $dh = vdp$. So, this is what we can write from this property relation and we could we can write the $\delta w = -dh$ from this steady flow energy equation.

So, what we can write that $\delta w = -dh = -vdp$. So, what would be the work needed for the compression process that is

$$\int_1^2 \delta w = - \int_1^2 v dp$$

So, this is basically you know that we are trying to compress the air by supplying certain amount of energy in the form of work and that energy will try to compress the working fluid and addition of that energy will raise the pressure of the working fluid along with its temperature.

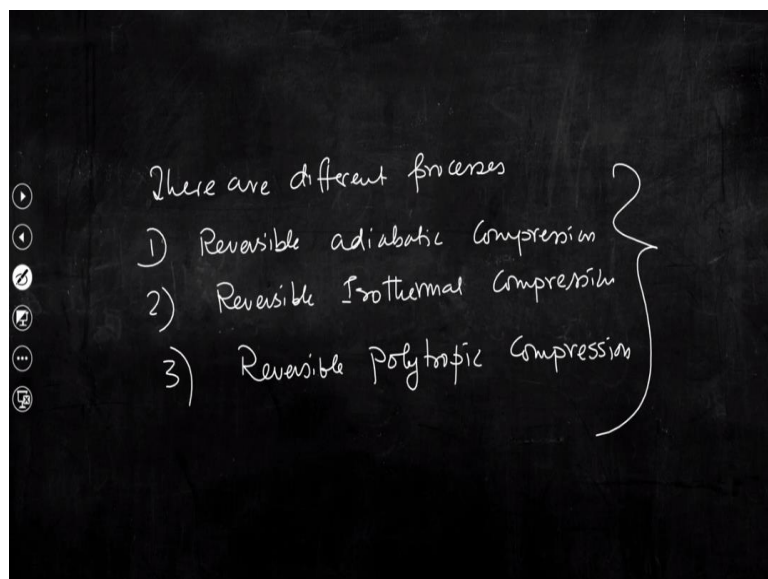
And that we could express by this equation and this is the work done. So, this much amount of work should be extracted from the turbine work output. As we have discussed in the last class that a significant part of the work output will be consumed by the compressor itself for its

operation and hence efficiency of the gas turbine units is very small. Now this is the expression this is not the $p dv$ work probably you have studied in thermodynamics.

No matter whether the process is reversible adiabatic or reversible you know isothermal work done is $- v dp$ for a flow process. So work done for a flow process whether the process is a reversible adiabatic process or the process is reversible isothermal process work done is $- v dp$. So, now this is very important to know when you will be solving numerical problems. So, work that should be supplied to compress the air you know that if you assume that this is v_1 so basically that is a specific volume.

If we assume that the specific volume of the air at the inlet or specific volume of the gas at the inlet is v_1 then we need to compress it from P_1 to P_2 . As I said you know that in the compression process, the objective is to raise the pressure with an appreciable change in density as well so specific volume also will change. So, knowing the specific volume at state points 1 and 2, We can calculate what would be the work done and that work that amount would be you know consumed by the compressor from the turbine work output. Now question is if we really do not know any explicit relation between specific volume and pressure we really cannot integrate this. So, we need to know what would be the explicit relation between the pressure and specific volume then only we can integrate it and we can quantify what would be the work done needed to compress the working fluid.

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So, now question is there are different processes. Number one is reversible adiabatic compression. Process here means the compression process. So, the process can be reversible

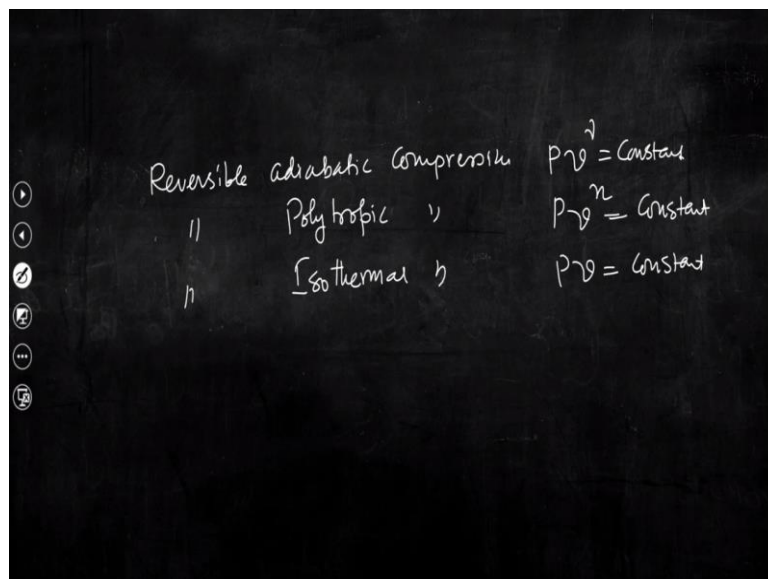
adiabatic compression, the process can be reversible isothermal compression or it can be reversible polytropic compression. So, these three are the possibilities. The compression process can be performed assuming the process is reversible adiabatic.

If compression process can be executed assuming the process is reversible isothermal, but an isothermal compression is very difficult to achieve though I said that there are certain applications in which you know that compressed gas or compressed air will be kept or stored in one cylinder or in one place and then that high pressure gas would be used. You had seen perhaps there are many practical applications to remove dust from different parts of a vehicle that mechanics or any other technical persons he or she uses high pressure compressed air.

So, that is basically compressed air is stored somewhere and that is used. So, now though isothermal compression is not of much use, but still we need to learn and then another could be reversible poly tropic compression. So, now let us first discuss about what would be the mathematical expression of work are needed for the compression be it a process which is equal to reversible adiabatic compression.

Be it a process which is you know equal to the reversible poly tropic compression or be it a reversible isothermal compression.

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So, now question is if we assume that you know that reversible adiabatic compression that is $pv^\gamma = C$. If it is reversible you know poly tropic compression then $pv^n = C$ or if it is reversible

isothermal compression then $pv = C$. So, now try to understand that if it is $pv^n = C$ if we replace n by this γ then will be getting reversible adiabatic compression.

If you assume n equal to 1 then will be getting reversible isothermal compression. So, now let us start with this general expression that is poly tropic expression $pv^n = C$ and we shall try to derive the expression of the work needed for the compression process.

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The image shows a chalkboard with the following handwritten equations:

$$pv^\gamma = \text{Constant}$$

$$v = \left(\frac{\text{Constant}}{p} \right)^{\frac{1}{\gamma}} = \left(\frac{p_1 v_1^\gamma}{p} \right)^{\frac{1}{\gamma}} = \left(\frac{p_2 v_2^\gamma}{p} \right)^{\frac{1}{\gamma}}$$

$${}_1W_2 \text{ or } \int_1^2 \delta w \text{ or } w_{1-2} = - \int_1^2 \left(\frac{p_1 v_1^\gamma}{p} \right)^{\frac{1}{\gamma}} dp$$

Today we shall try to get the expression of the work input to the compression if the compression process is reversible adiabatic. So, $pv^\gamma = C$.

$$v = \left(\frac{\text{Constant}}{p} \right)^{\frac{1}{\gamma}} = \left(\frac{p_1 v_1^\gamma}{p} \right)^{\frac{1}{\gamma}} = \left(\frac{p_2 v_2^\gamma}{p} \right)^{\frac{1}{\gamma}}$$

So, now if we consider then $\int_1^2 \delta w$ or w_{1-2} . This is inexact differential. So, we can write the work done needed for the compression of the air and air is changing or gas is changing its state from 1 to 2.

$$\int_1^2 \delta w = - \int_1^2 v dp = - \int_1^2 \left(\frac{p_1 v_1^\gamma}{p} \right)^{\frac{1}{\gamma}} dp$$

Now what we can do next? You can understand that P_1 is known so basically only the objective was to relate specific volume with P so that we can perform this integration.

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$$\begin{aligned}
 {}_1W_2 &= - p_1^{1/\gamma} v_1 \int_1^2 \frac{dp}{p^{1/\gamma}} \\
 &= - p_1^{1/\gamma} v_1 \frac{\gamma-1}{\gamma} \left[p_2^{\frac{\gamma-1}{\gamma}} - p_1^{\frac{\gamma-1}{\gamma}} \right] \\
 &= - p_1^{1/\gamma} p_1^{\frac{\gamma-1}{\gamma}} v_1 \left(\frac{\gamma-1}{\gamma} \right) \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]
 \end{aligned}$$

So, what we can write that

$$\begin{aligned}
 w_{1-2} &= - p_1^{1/\gamma} v_1 \int_1^2 (dp/p^{1/\gamma}) = - p_1^{1/\gamma} v_1 \frac{\gamma-1}{\gamma} [p_2^{\frac{\gamma-1}{\gamma}} - p_1^{\frac{\gamma-1}{\gamma}}] \\
 &= - p_1^{1/\gamma} v_1 p_1^{\frac{\gamma-1}{\gamma}} \frac{\gamma-1}{\gamma} \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 = - \frac{\gamma-1}{\gamma} p_1 v_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1
 \end{aligned}$$

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$${}_1W_2 = - \frac{(\gamma-1)}{\gamma} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

Indicates that work is done on the working fluid
 For a reversible adiabatic compression

Try to understand few minutes back we could write the pressure ratio and what is this pressure ratio P_2 by P_1 that is r_p . So basically if the pressure ratio is more this negative sign indicates that work is done on the working fluid.

So, basically this is the expression of the work done and it depends on the pressure ratio. If the pressure ratio is more work done needed to be supplied to the compressor will be more. So, this is basically work done needed to run the compressor and that amount of work should be consumed by the compressor from the turbine itself would be more if pressure ratio is more.

If pressure ratio is more cycle efficiency will be more. So, now it is very contradicting you know that compressor work that would be you know absorbed from the turbine work output then certainly the pressure ratio that would be high. So, basically these two are contradicting I mean if you need to get more you know pressure ratio then certainly work input to the compressor will be more.

In other way if the pressure ratio is more, then work or energy input in the form of work that should be supplied to the compressor will be more and if it is more, then net work output will be less, but that means if we can increase the pressure ratio then cycle efficiency will be maximum, if we increase this pressure ratio then compression work that is the negative work because this is the work consumed will be more, so these two are very contradicting.

So, that is why we had to you know establish the optimum pressure ratio for which the cycle efficiency will be maximum. So, this is the expression of the work ratio for reversible adiabatic compression.

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The image shows two handwritten equations on a blackboard background. The first equation is for reversible polytropic compression, showing work W_2 as a function of pressure ratio $(P_2/P_1)^{n-1/n}$. The second equation is for reversible isothermal compression, showing work W_2 as an integral of $v dp$ which simplifies to $-P_1 v_1 \ln(P_2/P_1)$.

$$W_2 \Big|_{\text{Reversible Polytropic Compression}} = - \frac{(n-1)}{n} P_1 v_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_2 \Big|_{\text{Reversible Isothermal Compression}} = - \int_1^2 v dp = - \int_1^2 \frac{P_1 v_1 dp}{P} = - P_1 v_1 \ln \frac{P_2}{P_1}$$

Similarly, we can quickly write for reversible polytropic compression

$$w_{1-2} |_{\text{Reversible polytropic}} = -\frac{n-1}{n} p_1 v_1 \left(\frac{p_2}{p_1} \frac{n-1}{n} - 1 \right)$$

So, this is the expression of the work needed to run the compressor following the reversible poly tropic process. Now similarly for reversible isothermal compression it would be

$$w_{1-2} |_{\text{Reversible isothermal}} = -\int_1^2 v dp = -\int_1^2 \frac{p_1 v_1}{p} dp = p_1 v_1 \ln \left(\frac{p_2}{p_1} \right)$$

So, work that comes out from the system is taken positive, but in this case work is added to the systems hence negative sign is coming. So, to summarize today's discussion we have discussed about the optimum pressure ratio which is very important to ensure that the cycle efficiency will be maximum then we have talked about the compressor, the need of a compressor in a gas turbine unit.

And then we have tried to establish the expression of the work done on the gas or on the working fluid following several realistic processes those are reversible adiabatic compression, reversible polytropic compression and reversal isothermal compression. So, with this I stop here today and we shall continue our discussion in the next class. Thank you.