

Thermal Engineering Basic and Applied
Prof: Pranab K Mondal
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture - 59
Numerical Problems on SI and CI Engines

I welcome you all to the session of Thermal Engineering Basic and Applied and in today's class we shall solve two problems one is from SI engine and second one is from CI engine.

(Refer Slide Time: 00:44)

Problem 1: A spark-ignition engine working on ideal Otto cycle has the compression ratio 8. The initial pressure and temperature of air are 1 bar and 35 °C. The maximum pressure in the cycle is 40 bar. For unit mass flow, calculate (i) p , V and T at various salient points of the cycle and (ii) the ratio of heat supplied to the heat rejected. Assume $\gamma = 1.4$ and $R = 8.314 \text{ kJ/kmol K}$.

Solution

$$\begin{cases} p_1 = 1 \text{ bar} \\ T_1 = 35^\circ\text{C} = 308 \text{ K} \end{cases}$$

$$\text{Number of moles } n = \frac{m \text{ (Unit mass flow rate)}}{M \rho \text{ (Molar mass of the substance)}} \sim 28.97$$

The diagram shows a p-v plot of an Otto cycle with points 1, 2, 3, and 4. Process 1-2 is isentropic compression, 2-3 is constant volume heat addition, 3-4 is isentropic expansion, and 4-1 is constant volume heat rejection.

So, this is the first problem let me read out the statement of the problem first. A spark ignition engine working on ideal Otto cycle has the compression ratio 8. So, it is telling that the engine is a SI engine and it is working on ideal Otto cycle because we have discussed that we need to compare several processes using an air standard cycle and for the SI engines we considered Otto cycle for this comparison.

Compression ratio is 8, the initial pressure and temperature of air are 1 bar and 35°C respectively. The maximum pressure in the cycle is 40 bar, per unit must flow calculate p , V and T at various salient points of the cycle that is pressure volume and temperature. Number two is the ratio of heat supplied to the heat rejected.

It is given that we can assume the value of gamma equal to 1.4 and R equal to 8.314 kilo joule per kilo mole Kelvin. So, if we start solving the problem now. So, you know, that the very beginning of this course I have discussed about the procedure of solving any numerical problem

particularly for this course that we need to know what are the processes and most importantly we need to map all those processes in any thermodynamic plane.

So, we have learned that if it is SI engine then we can have the P-v diagram and if we try to map the processes 1, 2, 3, 4; let me tell you once again you know we have not considered two processes that is the constant pressure intake and constant pressure exhaust. So, these two processes are thermodynamically same.

So, we have not considered these two processes in this plane and no need to consider this while solving this problem. So, what is given you know it is given $P_1 = 1 \text{ bar}$, $T_1 = 35^\circ\text{C} = 308\text{K}$, Now we need to know that it is given per unit mass flow rate.

So, we need to calculate pressure, volume and temperature at various salient points on the cycle per unit mass flow rate. So, now we have discussed that this is an air standard cycle which is used to compare the performance of this particular engine and we will be using ideal gas equation. Considering that the charge is like an ideal gas.

So, we need to consider moles. So, number of moles is unit mass flow rate divided by the molar mass that is $n = m/M$. So, this is molar mass of the substance and we are assuming this is as good as molar mass of the air and that is 28.97.

(Refer Slide Time: 07:48)

at Point 1
 $p_1 = 1 \text{ bar} = 10^5 \text{ N/m}^2$
 $T_1 = 308 \text{ K}$
 $V_1 = \frac{nRT_1}{p_1} = \frac{1 \times 8314 \times 308}{28.97 \times 10^5}$
 $= 0.889 \text{ m}^3$
at Point 2
 $p_2 = p_1 r_c^{1.4} = 10^5 \times 8^{1.4} = 18.37 \text{ bar}$
 $V_2 = \frac{V_1}{r_c} = 0.111 \text{ m}^3$
 $T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = 707.245 \text{ K} = 434.2^\circ\text{C}$

At State-1:

$$P_1 = 1 \text{ bar}$$

$$T_1 = 35^\circ\text{C} = 308\text{K}$$

$$V_1 = \frac{nRT_1}{P_1} = \frac{1 \times 8314 \times 308}{28.97 \times 10^5} = 0.889 \text{ m}^3$$

So, if we go back to the previous slide we have discussed this is the compression process 1 to 2 and that is modeled by an isentropic process.

At state-2:

$$P_2 = P_1 r_c^\gamma \left(\text{given } R_c = \frac{V_1}{V_2} = 8 \right) \rightarrow P_2 = 10^5 \times 8^{1.4} = 18.37 \text{ bar}$$

$$V_2 = \frac{V_1}{r_c} = 0.111 \text{ m}^3$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow T_2 = \frac{P_2 V_2}{P_1 V_1} T_1 = 707.245\text{K} = 434.2^\circ\text{C}$$

So, remaining points are 3 and 4. Now, you can understand that at point 2 spark plug switches made to be on and combustion occurs and it is because of this process temperature as well as pressure increases. So, this P 3 and T 3 are the maximum pressure and temperature of the cycle. So, P3 equal to P max and T3 equal to T max.

(Refer Slide Time: 14:08)

The image shows handwritten calculations on a blackboard, divided into two columns by a vertical line. The left column is titled 'at Point 3' and the right column is titled 'at Point 4'.

at Point 3

$$V_3 = V_2 = 0.111 \text{ m}^3$$

$$P_3 = 40 \text{ bar} = 40 \times 10^5 \text{ N/m}^2$$

$$T_3 = \frac{P_3 T_2}{P_2} = \frac{40 \times 10^5}{18.37 \times 10^5} \times 707.245 = 1540 \text{ K} = 1267^\circ\text{C}$$

at Point 4

$$P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma$$

$$= 40 \times 10^5 \times \frac{1}{r_c^{1.4}}$$

$$= 2.17 \times 10^5 \text{ N}$$

$$V_4 = V_1 = 0.889 \text{ m}^3$$

$$T_4 = T_1 \frac{P_4}{P_1} = 308 \times \frac{2.17}{1}$$

$$= 670.32 \text{ K}$$

State Point 3:

$$V_3 = V_2 = V_{TDC} = 0.111 \text{ m}^3$$

$$P_3 = 40 \text{ bar}$$

So, process 2 to 3 is a constant volume process. Let me tell you once again air standard cycle we have considered to analyze this engine, performance of the engine and we are using ideal gas relationship.

$$T_3 = \frac{P_3 T_2}{P_2} = \frac{40 \times 10^5}{18.37 \times 10^5} \times 707.245 = 1540K = 1267^\circ C$$

So, process 3 to 4 is modeled by an isentropic expansion process.

State point-4:

$$P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma = 40 \times 10^5 \times \frac{1}{r_c^\gamma} \text{ (as } V_4 = V_1 \text{ \& } V_3 = V_2) = 2.17 \text{ bar}$$

$$V_4 = V_1 = 0.889m^3$$

$$T_4 = \frac{T_1 P_4}{P_1} = 308 \times \frac{2.17}{1} = 670.32 K$$

Now we have to calculate heat supplied and heat rejection. So, let us go back to the problem statement. So, now questioning we have to calculate the ratio of heat supplied to the cycle to the heat rejection. Now if we would like to calculate it because you know this is the heat addition. So, this is Q out or Q rejection and this is Q in or Q addition. So, these two processes are constant volume processes so we need to calculate Cv. What is Cv?

(Refer Slide Time: 20:50)

Heat addition & rejection

Thermal efficiency η_{th}

$$C_v = \frac{R}{M(\gamma-1)} = \frac{8.314}{28.97 \times 0.4} = 0.717 \text{ kJ/kg K} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

Per unit mass

$$\left\{ \begin{array}{l} \text{Heat rejection } (Q_{out}): C_v (T_4 - T_1) = 0.717 (670.32 - 308) \text{ kJ/kg} \\ \text{Heat addition } (Q_{in}): C_v (T_3 - T_2) = 0.717 (1267 - 734.24) \text{ kJ/kg} \end{array} \right. \quad (56.41\%)$$

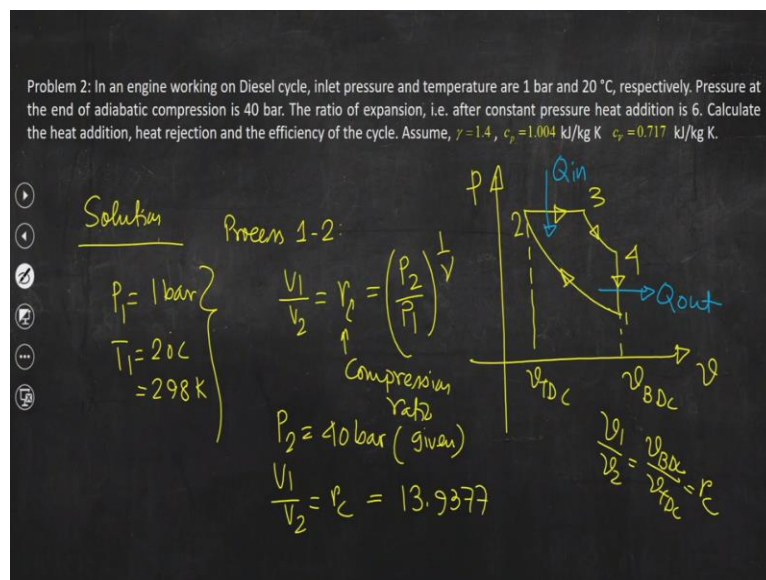
$$C_v = \frac{R}{M \times (\gamma - 1)} = \frac{8.314}{28.97 \times 0.4} = 0.717$$

Now we can calculate for unit mass heat rejection $Q_{out} = C_v(T_4 - T_1) = 0.717 (670.32 - 308) = 259.78 \text{ kJ/kg}$. Now, we can calculate heat addition $Q_{in} = C_v(T_3 - T_2) = 0.717 (1267 - 434.24) \frac{\text{kJ}}{\text{kg}}$.

You also can calculate efficiency $\eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}}$

So, another problem will be solving and I am telling you because you can calculate this efficiency would be coming around 56.4 percent.

(Refer Slide Time: 26:21)



So, then we have to move to the next problem that is this problem is taken from the CI engine. What is this problem statement? Let me read out once again in an engine working on diesel cycles you can understand this is the CI engine because we need to analyze the performance of this engine by considering an air standard cycle and that cycle is diesel cycle. Typically performance of compression ignition engines is analyzed based on the diesel cycle.

Inlet pressure and temperature are given 1 bar and 20 degree celsius respectively. Pressure at the end of the adiabatic compression is 40 bar, the ratio of expansion that is after constant pressure heat addition is 6. Calculate the heat addition, heat rejection and efficiency of the cycle. Assume gamma equal to 1.4, the value of cp and cv are given kilojoule per kg Kelvin.

So, we need to map the processes in P-v plane and processes are like this. So, this is 1, 2, 3, 4 you can see the difference between the SI and CI engines. Here the heat addition is at constant

pressure instead of constant volume. So, this is Q_{in} and this is Q_{out} . So, if we consider several processes now process 1 to 2. Process is isentropic compression process.

State -1:

$$P_1 = 1 \text{ bar}$$

$$T_1 = 20^\circ C = 393K$$

$$\frac{V_1}{V_2} = \frac{V_{BDC}}{V_{TDC}} = r_c = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = 13.9377 \text{ (Given } P_2 = 40 \text{ bar)}$$

Now we have discussed that pertaining to this particular type of cycle you can understand the combustion process is mimicked by a constant pressure heat addition process and you can see that there is a change in volume during this process and that change in volume is from V_2 to V_3 . So, the ratio of these two volumes is known as cut off ratio. So, the change in volume during the combustion process the ratio of this volume is known as the cut off ratio.

(Refer Slide Time: 32:16)

Handwritten calculations on a blackboard:

- Top right: $\frac{V_{BDC}}{V_{TDC}} = \frac{V_1}{V_2} = r_c$
- Left side: $\text{Cut-off ratio } \beta = \frac{V_3}{V_2} = \frac{V_3}{V_1} \times \left(\frac{V_1}{V_2}\right) \sim r_c$
- Middle: $\text{Ratio of expansion} = \frac{V_4}{V_3} = \frac{V_1}{V_3} = \frac{r_c}{\left(\frac{V_1}{V_3}\right)} = \frac{13.9377}{6} = 2.3229$
- Bottom: $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{40}{1}\right)^{0.286} = 2.872 \Rightarrow T_2 = T_1 \times 2.872 = 841.508K$
- Annotation: "Ratio of expansion = 6" with an arrow pointing to the denominator in the middle calculation.
- Annotation: "298k" with an arrow pointing to T_1 in the bottom calculation.

$$\text{Cut-off ratio } \beta = \frac{V_3}{V_2} = \frac{V_3}{V_1} \times \frac{V_1}{V_2} = \frac{V_3}{V_1} \times r_c$$

So, this is the compression ratio we have already calculated, but it is given that the ratio of expansion that is after constant pressure heat addition is 6. So, you know, that expansion is as good as V_4 by V_3 .

But note it this is V_4 equal to V_1 . So, the ratio of expansion that is V_4 by V_3 after the combustion process because after the combustion process we can see the process is 3 to 4 and that is the isentropic expansion process. So, there is a change in volume and that ratio of expansion, is V_4 by V_3 since V_4 equal to V_1 that we can see from this P-v diagram. So, V_4 by V_3 is V_1 by V_3 .

$$\beta = \frac{V_3}{V_1} \times r_c = \frac{r_c}{\frac{V_1}{V_3}} = \frac{13.9377}{6} = 2.3229$$

State-2:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{40}{1}\right)^{0.286} \rightarrow 293 \times 2.872 = 841.5K$$

You can understand that we are raising the temperature of the working substance that is fresh air and that temperature and pressure, these two properties are good enough to self ignite the fuel when fuel is spread into the combustion chamber. Now if we go to the process 2 to 3. So, process 2 to 3 is very important process that is the combustion process.

(Refer Slide Time: 37:24)

Process 2-3: Combustion Process [Constant Pressure heat addition]

$$T_3 = T_{max} \quad T_3 = T_2 \left(\frac{V_3}{V_2}\right) = 841.508 \times 2.322 = 1953.974 \text{ K}$$

Process 3-4: $T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = 1953.974 \left(\frac{V_3}{V_4}\right)^{0.4}$

$\frac{V_3}{V_4} = \frac{V_3}{V_2} \cdot \frac{V_2}{V_4} = \left(\frac{V_3}{V_2}\right) \left(\frac{V_2}{V_1}\right)$

So, I am writing here process 2 to 3 that is the constant pressure heat addition combustion process.

State-3:

$$T_3 = T_{max} = \frac{T_2 V_3}{V_2} = 841.508 \times 2.322 = 1953.974 \text{ K}$$

3 to 4 process is isentropic expansion process.

State-4:

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{\gamma-1}$$

$$\frac{V_3}{V_4} = \frac{V_3}{V_2} \times \frac{V_2}{V_4} = \beta \times \frac{1}{r_c}$$

(Refer Slide Time: 42:14)

Heat addition: $(Q_{in}) = C_p (T_3 - T_2) \frac{kJ}{kg}$

Heat rejection: $(Q_{out}) = C_p (T_4 - T_1) \frac{kJ}{kg}$

Per unit mass flow rate

$$\eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

So, next part is heat addition, see in the previous problem we could write heat addition per unit mass of flow of the working substance that is C_v multiplied with $T_3 - T_2$, but here it would be

$$Q_{in} = C_p (T_3 - T_2) kJ/kg$$

$$Q_{out} = C_p (T_4 - T_1) kJ/kg$$

So, we know all these things we can calculate what is the amount of heat being added to the cycle and what is the amount of heat being rejected from the cycle to make the processes to be executed in a cyclic manner.

Now unit should be as I told you kilo joule per kg. So, unit should be kilo joule per kg and we also can calculate thermal efficiency that is nothing, but $\eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}}$. So, to summarize today's class we have solved two problems because in the previous few classes we have discussed about the thermal efficiency of both SI and CI engines. Today we have taken two examples to illustrate our understanding on these two cycles. And we have solved these two problems and we could calculate their thermal efficiency not only thermal efficiency because to calculate thermal efficiency of both SI and CI engines we had seen that we need to calculate pressure and temperature at various salient points and for that we had to use you know ideal

gas equations. So, with this I stop here today and we shall continue our discussion in the next class. We shall move to the next module of this course. Thank you.