

Thermal Engineering: Basic and Applied
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Lecture - 39
Problems on Steam Turbine

I welcome you all to the session of thermal engineering basic and applied. Today we shall solve a few examples and through this exercise we shall try to illustrate the concept that we have learned from the theoretical analysis of steam turbines.

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Problem 1: An impulse turbine with single row wheel has mean diameter 100 cm and speed of rotation 3000 rpm. The nozzle angle is 20° and the ratio of blade velocity to steam velocity is 0.44. The ratio of relative velocity of steam at the blade outlet to that at the inlet is 0.86. It is given that the blade outlet angle is 3° less than the inlet blade angle and the mass flow rate of steam is 10 kg/s. calculate the followings:

- Draw the velocity diagram for the blades.
- Calculate the tangential, axial and resultant thrust on the blade.
- Power developed by the blades.
- Blading efficiency.

Solution

C_1 : Absolute velocity at the inlet
 w_1 : relative vel. at the inlet

Mean Velocity

$$u = \frac{\pi D_r N}{60}$$

Blade Velocity u or V_b
 $C_1 = C_2$
 $\Delta C_a = C_{a1} + C_{a2}$
 $\Delta C_a = C_{a1} - C_{a2}$

So, the first problem that we will solve today is from the impulse turbine. Let me first read out the problem statement. Then we shall start solving the problem following the theoretical understanding that we have.

It is given that an Impulse turbine with single row wheel has mean diameter 100 cm and speed of rotation 3000 RPM. The nozzle angle is 20° and the ratio of blade velocity to steam velocity is 0.44. The ratio of relative velocity of steam at the blade outlet to that at the blade Inlet is 0.86. It is given that the blade outlet angle is 3° less than the inlet blade angle and the mass flow rate of steam is 10 kg/s. We need to calculate a few parameters, but first we need to draw the velocity triangles. Then we need to calculate the tangential, axial and resultant thrust on the blades, power developed by the blades and lastly the blading efficiency.

So, let me tell you one thing that it would be much more convenient to solve the problem if we draw the velocity triangles properly. So, the first task should be to draw the velocity triangles and then we can calculate the parameters those are asked in this problem. So, if we solve the problem as I said that first it would be essential to draw the velocity triangles.

So, let us now draw the velocity triangles in fact we have discussed this part in the context of the derivation of blading efficiency or diagram efficiency of impulse turbine. So, let us consider the blade velocity as u . Sometimes it is also denoted by V_b . And let us complete the Inlet velocity triangle first so, that is the absolute velocity of steam coming out from the nozzle which is making an angle α_1 that is the nozzle angle.

And since the wheel is rotating, so the velocity of steam relative to the blade velocity is the relative velocity and we can draw the velocity triangle as shown in the slide with C_1 , the absolute velocity at the blade Inlet W_1 , the relative velocity at the inlet. Since I have also discussed that blade velocity is calculated based on the mean diameter and if you look at the problem statement it is given that the mean diameter of the wheel is 100 cm.

That means we can now superimpose the outlet velocity triangle on the common V_b or common blade velocity. So, now we draw the outlet velocity triangle with C_2 & W_2 . Then let us name the points of triangles A, B, C, D. So, triangle A B C is the velocity triangle which is at the inlet of the blade and triangle B C D is the velocity triangle at the exit of the blade. And we can see that angle β_1 is blade angle at the inlet and the angle β_2 is blade angle at the outlet and this angle is α_2 as shown in the slide. Most importantly the component of absolute velocity in the axial direction is C_{a1} that is the flow velocity at the inlet. Similarly component of absolute velocity in the axial direction is C_{a2} that is the flow velocity at the outlet and we can clearly see that $C_{a1} \neq C_{a2}$ from the velocity triangle.

And since there is no pressure drop of the steam when it is passing through the blades or moving blades, the relative velocity at the exit of the blade is not equal to the relative velocity at the inlet of the blade and that too we had seen that it is because of this roughness of the blade surface, there is a reduction in the relative velocity and that is why we could define blade velocity coefficient K_b or K .

So, we can see that $C_{a1} \neq C_{a2}$ and this component is ΔC_a that is responsible for the axial thrust that would be produced. Similarly we try to draw the component $W_{\theta 2}$ & $W_{\theta 1}$ or we can draw $C_{\theta 1}$ & $C_{\theta 2}$. So $C_{\theta 1}$ is basically component of absolute velocity in the tangential direction at the inlet and $C_{\theta 2}$ is the component of absolute velocity in the tangential direction at the exit.

$$\text{So } \Delta C_\theta = C_{\theta 1} + C_{\theta 2} \text{ and } \Delta C_a = C_{a1} - C_{a2}.$$

So, this ΔC_θ is responsible for the tangential thrust and it is because of this thrust we are getting work output and power and ΔC_a is responsible for the axial thrust and that thrust should be consumed by the bearing.

So, this is the velocity triangles. In fact today we will be solving a few numerical problems and we will be using the similar velocity diagram at least for the impulse turbine and 2 important relations that I would like to now write from the velocity triangles that we have drawn. If you look at this as I said you this ΔC_θ and ΔC_a these 2 components are very important to calculate the tangential as well as axial thrust. So, we can see there are 2 ways by which we can really solve the problem one is through graphical representation. So, basically mean velocity can be calculated because RPM is given, diameter is given and we now use a suitable scale to represent that mean velocity. And knowing the nozzle angle we also can know what would be the absolute velocity. But today we shall try to solve the problems analytically just by using the trigonometric relations. So, we can now calculate that mean velocity that is very important quantity

$$\text{Mean velocity, } u = \frac{\pi D_m N}{60}; N = \text{RPM}$$

So, we can calculate mean velocity. If α_1 is given then next we can calculate C_1 . Now 2 relations will be used today one is

$$\Delta C_\theta = W_{\theta 1} + W_{\theta 2} = C_{\theta 1} + C_{\theta 2}$$

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$\Delta C_{\theta} = C_{\theta 1} + C_{\theta 2} = W_{\theta 1} + W_{\theta 2}$
 $C_{a1} = C_1 \sin \alpha_1$
 $C_{a1} = W_1 \sin \beta_1$
 $C_1 \cos \alpha_1 - u = W_1 \cos \beta_1$
 $(EC - BC = EB)$
 $C_1 \sin \alpha_1 = W_1 \sin \beta_1$
 $\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - u}$

So, we go to the next slide and we can write

$$\Delta C_{\theta} = W_{\theta 1} + W_{\theta 2} = C_{\theta 1} + C_{\theta 2}$$

That we will be using because essentially you have to calculate ΔC_{θ} for the tangential thrust.

Another thing is if you now look at the inlet velocity triangle ABC we can see that

$$C_{a1} = C_1 \sin \alpha_1$$

Here C_{a1} is flow velocity at the inlet that is the component of absolute velocity in the axial direction at the inlet. If we look at the velocity triangle and we give name AE as shown in the slide then from triangle AEC we could write that

$$AE = C_1 \sin \alpha_1$$

Similarly if you now look at the triangle AEB

$$C_{a1} = W_1 \sin \beta_1$$

Now we also can write another important relation that from ΔABC & ΔAEB

$$C_1 \cos \alpha_1 - u = W_1 \cos \beta_1$$

So, this indicates from the velocity triangles that

$$EC - BC = EB$$

And also we can write

$$C_1 \sin \alpha_1 = W_1 \sin \beta_1$$

So, from these 2 relations we can write

$$\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - u}$$

So, this expression will be used today to calculate β many times. So now let us go back to the problem statement. We have already drawn the velocity triangles for the blades, next we need to calculate the tangential, axial and resultant thrust.

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$\alpha_1 = 20^\circ$ (given) & $u = 157 \text{ m/s}$
 $\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - u} = 0.6731$
 $\Rightarrow \beta_1 = \tan^{-1}(0.6731) = 33.95^\circ$
 from the problem statement
 $\beta_2 = \beta_1 - 3^\circ = 30.95^\circ$
 $\Delta C_\theta = W_1 \sin \beta_1 + W_2 \sin(\beta_1 - 3^\circ)$
 $= \frac{C_1 \sin \beta_1}{\sin \beta_1} \cos \beta_1 + K_b W_1 \cos(\beta_1 - 3^\circ)$

Tangential thrust, $F_t = \dot{m}_s \Delta C_\theta$

Here, \dot{m}_s = mass flow rate of steam

If we go back to the previous slide, I have already written the expression of ΔC_θ

$$\Delta C_\theta = W_{\theta 1} + W_{\theta 2} = W_1 \cos \beta_1 + W_2 \cos \beta_2$$

Basically we know

$$\text{Blade velocity coefficient, } K_b = \frac{W_2}{W_1}$$

Now if we again read the problem statement then it is given that the ratio of relative velocity of steam at the blade outlet to that at the inlet is 0.86. That means this is the blade velocity coefficient

$$K_b = \frac{W_2}{W_1} = 0.86$$

So, you can understand if we can somehow calculate W_1 from the data given or using the blade velocity triangles, we will be able to calculate W_2 . Now it is also given that ratio of blade velocity to steam velocity is 0.44. So

$$\frac{u}{C_1} = 0.44 \Rightarrow C_1 = \frac{u}{0.44}$$

So, already we have calculated

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times 100}{100} \times \frac{2800}{60} = 157 \frac{\text{m}}{\text{s}}$$

$$C_1 = \frac{157}{0.44} = 356.82 \frac{\text{m}}{\text{s}}$$

So, we could calculate the absolute velocity of steam at the exit of the nozzle that is at inlet of the blade and also the blade velocity right. Now we need to find β_1 because that is very important.

$$\text{Given } \alpha_1 = 20^\circ \text{ \& } u = 157 \frac{m}{s}$$

$$\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - u} = 0.6731$$

$$\Rightarrow \beta_1 = \tan^{-1}(0.6731) = 33.95^\circ$$

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Problem 2: In a single impulse turbine, the mean diameter of the wheel is 100 cm and speed of rotation is 2800 rpm. The velocity of steam at the exit of the nozzle is 280 m/s and nozzle angle is 25° . The turbine blades are symmetrical and due to friction in the blade the relative velocity of steam at the blade outlet is 0.87 times the relative velocity of steam at the blade inlet. Calculate the power developed when axial thrust on the blades is 150 N.

Solⁿ

$D_m = 100 \text{ cm}$ } $C_1 = 280 \text{ m/s}$
 $\alpha_1 = 25^\circ$ } Blades are symmetrical
 $\beta_1 = \beta_2$
 $W_2 = 0.87 W_1$
 \Rightarrow Blade velocity coeff $K_b = \frac{W_2}{W_1} = 0.87$

So, you can check again the numerical value, but the procedure is correct. So, this is β_1 . Now we need β_2 . If we go back to the problem statement it is given that the blade outlet angle is 3° less than the blade inlet angle. So, that means from the problem statement

$$\beta_2 = \beta_1 - 3^\circ = 30.95^\circ$$

So, basically now if we would like to calculate ΔC_θ then as already we have mentioned here

$$\Delta C_\theta = W_{\theta 1} + W_{\theta 2} = W_1 \cos \beta_1 + W_2 \cos(\beta_1 - 3^\circ)$$

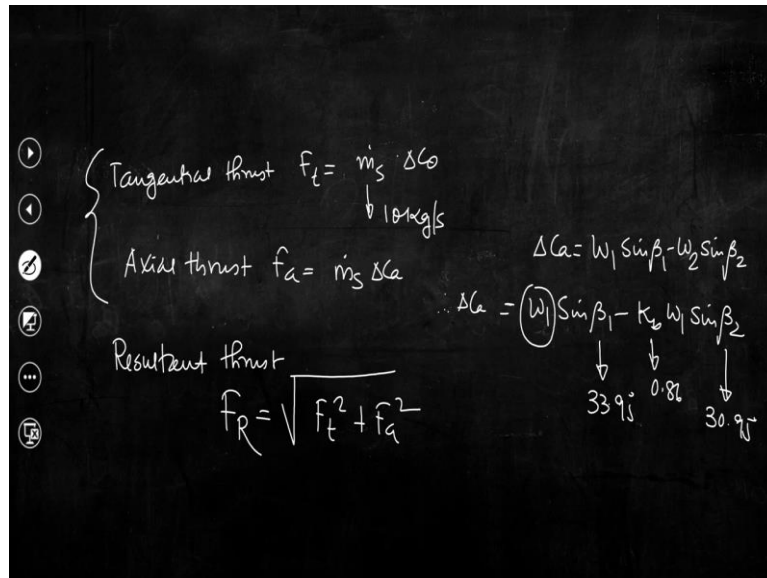
Now look at this expression this expression

$$C_1 \sin \alpha_1 = W_1 \sin \beta_1 \Rightarrow W_1 = \frac{C_1 \sin \alpha_1}{\sin \beta_1}$$

$$\text{Now } \Delta C_\theta = \frac{C_1 \sin \alpha_1}{\sin \beta_1} \cos \beta_1 + K_b W_1 \cos(\beta_1 - 3^\circ)$$

See in this expression all parameters like C_1, β_1, β_2 & K_b are known so we can calculate ΔC_θ .

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And if we can calculate ΔC_θ , then tangential thrust will be

$$\text{Tangential thrust, } F_t = \dot{m}_s \times \Delta C_\theta$$

I am not calculating this. Here \dot{m}_s is given as 10 kg/s. Similarly we can calculate axial thrust

$$F_a = \dot{m}_s \times \Delta C_a$$

So, now we need to calculate ΔC_a . So, if we look at the geometry

$$\Delta C_a = C_{a1} - C_{a2}$$

Already we know β_1 & W_1 so

$$\frac{C_{a1}}{W_1} = \sin \beta_1$$

And we already have calculated β_2 . We know W_2 from the blade friction coefficient and then we can easily calculate from these velocity triangles.

$$\Delta C_a = W_1 \sin \beta_1 - W_2 \sin \beta_2 = W_1 \sin \beta_1 - K_b W_1 \sin \beta_2$$

So, again you can calculate easily this ΔC_a because

$$W_1 = \frac{C_1 \sin \alpha_1}{\sin \beta_1}, \beta_1 = 33.95^\circ, \beta_2 = 30.95^\circ$$

So, you can calculate the axial thrust. Now calculating these 2 quantities F_t & F_a we can calculate resultant thrust

$$\text{Resultant thrust, } F_R = \sqrt{F_t^2 + F_a^2}$$

So, if we now move to another part of this problem that is power developed by the blades.

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Power developed by the blades $P = \frac{\text{Tangential thrust} \times \text{Velocity}}{1000}$

$$P = \frac{\dot{m} \Delta C_\theta \times u}{1000} \text{ kW}$$

Blading efficiency $\eta_b = \frac{\text{Rate of work done on the blade}}{\text{Rate of energy input to the blade}} = \frac{\dot{m}_s \Delta C_\theta u}{\frac{1}{2} \dot{m}_s C_1^2} = \frac{2 \Delta C_\theta u}{C_1^2}$

So, let me tell you once again that you can easily calculate ΔC_a & ΔC_θ because all quantities are known. We know $C_1 = 356.82$, $\alpha_1 = 20^\circ$, $\beta_1 = 33.95^\circ$ so, you can easily calculate W_1 . So, now let us calculate power developed by blade that is

$$\text{Power developed by blade, } P = \frac{\text{Tangential thrust} \times \text{Velocity}}{1000}$$

We can divide this quantity by 1000 to write the quantity in KW unit.

$$P = \frac{\dot{m}_s \Delta C_\theta \times u}{1000} \text{ KW}$$

So, try to understand had we calculated tangential thrust correctly and we have already calculated blade velocity then we can easily calculate the power developed by the blades. And now finally we need to calculate blading efficiency. See let me tell you one thing that P is the power we are getting or power developed by the blades at the cost of some input energy. So, this much amount of power developed by the blades at the cost of the input energy that is supplied to the blade. So, we can write that

$$\text{Blading efficiency, } \eta_b = \frac{\text{Rate of work done on the blade}}{\text{Rate of energy input to the blade}}$$

So, rate of work done on the blade is the power developed by the blade. If we now try to simplify this quantity

$$\text{Blading efficiency, } \eta_b = \frac{\dot{m}_s \Delta C_\theta u}{\frac{1}{2} \dot{m}_s C_1^2}$$

So, basically the kinetic energy of the jet which is available at the inlet of the blade is the input energy and at the cost of this energy, P amount of power is developed. So, now we can write one step further that is

$$\eta_b = \frac{2\Delta C_{\theta}u}{C_1^2}$$

Now you calculate this quantity because already we have calculated ΔC_{θ} and we know blade velocity and we have already calculated C_1 that is absolute velocity of steam leaving the nozzle. So, we can calculate the blading efficiency.

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Problem 2: In a single impulse turbine, the mean diameter of the wheel is 100 cm and speed of rotation is 2800 rpm. The velocity of steam at the exit of the nozzle is 280 m/s and nozzle angle is 25° . The turbine blades are symmetrical and due to friction in the blade the relative velocity of steam at the blade outlet is 0.87 times the relative velocity of steam at the blade inlet. Calculate the power developed when axial thrust on the blades is 150 N.

Set:

$$D_m = 100 \text{ cm} \quad C_1 = 280 \text{ m/s}$$

$$\alpha_1 = 25^\circ \quad \text{Blades are symmetrical}$$

$$\beta_1 = \beta_2$$

$$w_2 = 0.87 w_1$$

So, with this next let us move to another problem. Let me read out the problem statement first and then we shall start solving the problem. In a single stage impulse turbine (It should be a single stage, there is a mistake in the problem statement itself it should be in a single stage impulse turbine) the mean diameter of the wheel is 100 cm, speed of rotation is 280 RPM, the velocity of steam at the exit of the nozzle is 280 m/s.

Let us first write the data given.

$$D_m = 100 \text{ cm}, \alpha_1 = 25^\circ, C_1 = 280 \frac{\text{m}}{\text{s}}$$

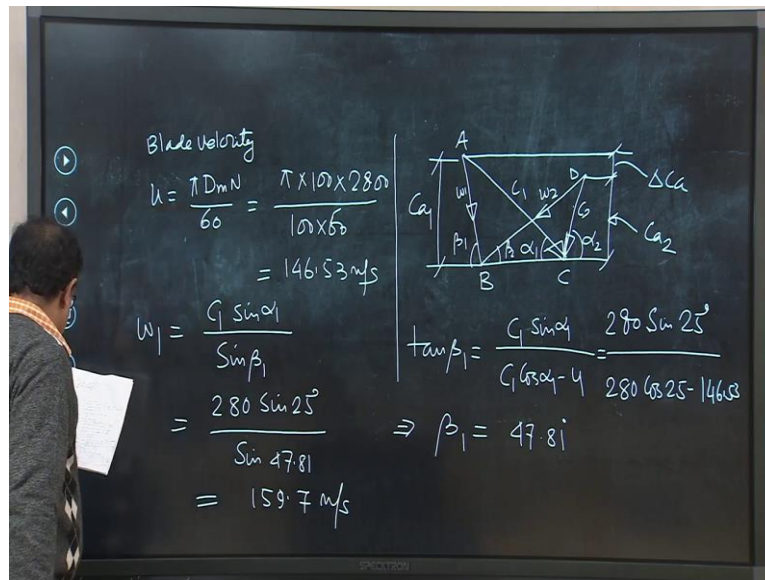
So try to recall the velocity triangles that we have drawn in the context of the previous problem. The turbine blades are symmetrical and due to friction in the blade the relative velocity of steam at the blade outlet is 0.87 times the relative velocity of steam at the inlet. That means the problem statement is telling us to consider the blade velocity coefficient. So, basically it is given the blades are symmetrical. What does it mean normally blades are symmetrical? I had mentioned that if it is not provided, we have to assume that the blades are symmetrical for the impulse turbine. Blades are fabricated from the same die so, basically the blades are geometrically similar. So $\beta_1 = \beta_2$. And it is given that the relative velocity of the steam at the blade outlet is 0.87 times of the relative velocity at the inlet that means relative velocity at the blade exit is less

than the blade relative velocity at the blade inlet and this is because of the friction. So, that means it is given that blade friction or blade velocity coefficient

$$K_b = \frac{W_2}{W_1} = 0.87$$

So, all this data are given, we have to calculate the power developed when axial thrust on the blade is 150 N.

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So, we can again draw the velocity triangle with $C_1, W_1, \alpha_1, C_2, W_2, \beta_1, \beta_2, \alpha_2$. From the velocity triangle, we can find the component ΔC_a as we can see C_{a2}, C_{a1} . So, we name the velocity triangle A B C D. Now first we can calculate

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times 100 \times 2800}{100 \times 60} = 146.53 \frac{m}{s}$$

So, this is the velocity blade velocity. As I had mentioned, we can use the relation

$$\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - u} = \frac{280 \sin 25^\circ}{280 \cos 25^\circ - 146.53}$$

$$\Rightarrow \beta_1 = 47.81^\circ$$

Once again I am telling you that please check the numerical value but the procedure is correct.

Once we have calculated β_1 then quickly we can calculate W_1 .

$$W_1 = \frac{C_1 \sin \alpha_1}{\sin \beta_1} = \frac{280 \times \sin 25^\circ}{\sin 47.81^\circ} = 159.7 \frac{m}{s}$$

See I told you that we also can solve the problem graphically because we have calculated u and if we use suitable scale we can represent that u , we know α and C_1 is already given, so we can calculate C_{a1} and other quantities. But now we are trying to solve it analytically.

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Handwritten calculations on a chalkboard:

$W_2 = 0.87 W_1 = 0.87 \times 159.7 = 138.95 \frac{m}{s}$
 Power developed by the blade
 $= \frac{\dot{m}_s \Delta C_\theta \times u}{1000} \text{ kW}$
 $= 9.75 \frac{kg}{s}$

Axial thrust
 $F_a = \dot{m}_s \cdot \Delta C_a$
 $150 = \dot{m}_s (C_{a1} - C_{a2})$
 $\dot{m}_s = \frac{150}{C_{a1} - C_{a2}}$
 $= \frac{150}{280 \sin 25^\circ - 138.95 \sin 47.81^\circ}$
 $= 9.75 \frac{kg}{s}$

$$W_2 = 0.87 W_1 = 0.87 \times 159.7 = 138.95 \frac{m}{s}$$

If we go to the problem statement again then we need to calculate the power developed when axial thrust on the blades is 150 N. That means

$$\text{Axial Thrust, } F_a = \dot{m}_s (C_{a1} - C_{a2}) = 150 \text{ N}$$

$$\Rightarrow \dot{m}_s = \frac{150}{C_{a1} - C_{a2}}$$

Now if we go back to the geometry we can see

$$C_{a1} = C_1 \sin \beta_1 \quad \& \quad C_{a2} = W_2 \sin \beta_2$$

It is given that $\beta_1 = \beta_2$. So here C_1, α_1 is given and W_2 is not given directly but we could calculate W_2 . So

$$\dot{m}_s = \frac{150}{C_{a1} - C_{a2}} = \frac{150}{280 \sin 25^\circ - 138.95 \sin 47.81^\circ} = 9.75 \frac{kg}{s}$$

Now we have to calculate the power developed by blades because that is the quantity of interest for this particular problem.

$$\text{Power developed by blades} = \frac{\dot{m}_s \Delta C_\theta \times u}{1000} \text{ KW}$$

If we do so then we can write the unit KW provided this ΔC_θ and u is in m/s and \dot{m}_s is in kg/s.

So, in this expression we already know that

$$\dot{m}_s = 9.75 \frac{kg}{s} \quad \& \quad u = 146.53 \frac{m}{s}$$

So, we have to calculate ΔC_θ that is the tangential thrust.

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Tangential thrust $\dot{m}_s \Delta C_\theta$

Change in swirl velocities $\Delta C_\theta = W_{\theta 1} + W_{\theta 2}$

$$\begin{aligned}
 &= W_1 \cos \beta_1 + W_2 \cos \beta_2 \\
 &\left[\beta_2 = \beta_1 \right] = W_1 \cos \beta_1 + 0.87 \times W_1 \cos \beta_1 \\
 &= W_1 \cos \beta_1 \times 1.87 \\
 &= 159.7 \cos 47.81 \times 1.87
 \end{aligned}$$

So, now

$$\text{Tangential thrust} = \dot{m}_s \Delta C_\theta$$

That means this is due to the change in swirl velocity ΔC_θ . If we go back to the previous problem then

$$\Delta C_\theta = W_{\theta 1} + W_{\theta 2} = C_{\theta 1} + C_{\theta 2}$$

Here we are considering

$$\begin{aligned}
 \Delta C_\theta &= W_{\theta 1} + W_{\theta 2} \\
 &= W_1 \cos \beta_1 + W_2 \cos \beta_2
 \end{aligned}$$

We can verify whether we have written it correctly or not from the velocity triangles.

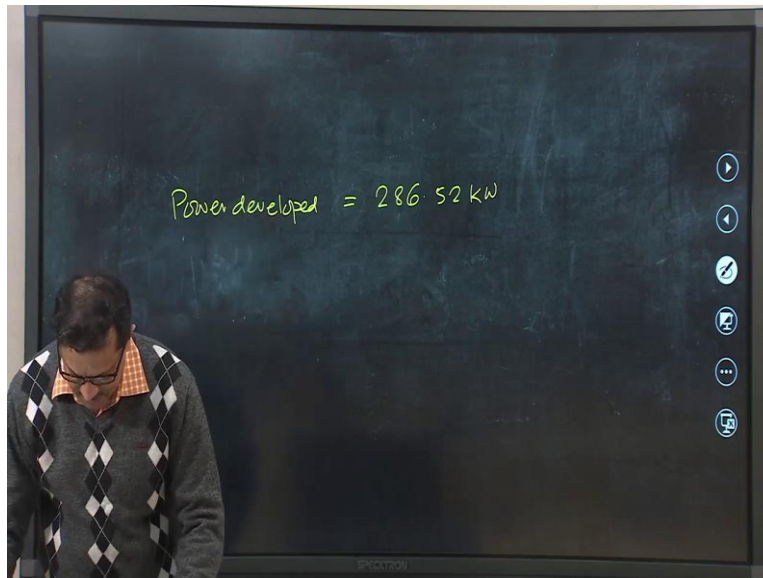
$$\Rightarrow \Delta C_\theta = W_1 \cos \beta_1 + 0.87 \times W_1 \cos \beta_1$$

$$\text{Because } \frac{W_2}{W_1} = 0.87 \text{ \& } \beta_1 = \beta_2$$

$$\Rightarrow \Delta C_\theta = W_1 \cos \beta_1 + 1.87$$

$$\Rightarrow \Delta C_\theta = 159.7 \cos 47.81 + 1.87$$

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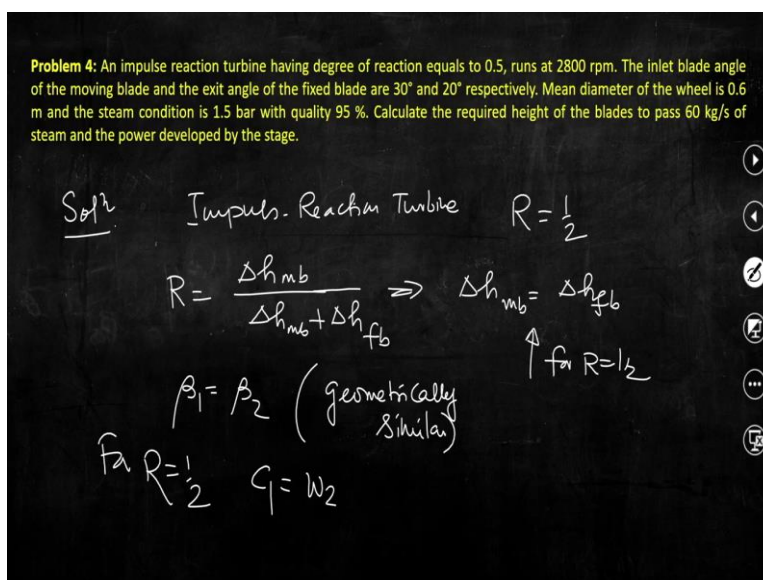
And if we calculate it and plug in the value of ΔC_θ in the expression of power developed here then we can get

$$\text{Power developed} = 286.56 \text{ KW}$$

So, you can check it whether it is correct or not. So, while solving these problems you can understand or recapitulate the theoretical part that we have learned.

And finally the last problem that I will solve today is from another type of turbine that is the impulse reaction turbine.

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So, if you try to recall we had established the expression of the blading efficiency for both impulse and reaction turbine. Today we have solved 2 problems from the impulse turbine. This problem is from the impulse reaction turbine. Now let us quickly read out the problem statement.

An Impulse reaction turbine having degree of reaction equals to 0.5; we know that this is sometimes called only reaction turbine to distinguish this type of turbine from the impulse turbine or it is also known as impulse reaction turbine. And degree of reaction is the enthalpy drop in the moving blades to the total enthalpy drop that we have discussed. The turbine runs at 280 RPM. The inlet blade angle of the moving blade and the exit angle of the fixed blade are 30° and 20° respectively, mean diameter of the wheel is 0.6 meter and the steam condition is 1.5 bar with equality 95%; of course that is the steam quality at the Inlet of the blade. We are asked to calculate the required height of the blade to pass 60 kg/s of steam and the power developed by this stage. So, let us quickly solve this problem.

So, this is impulse reaction turbine. As I said you that this type of turbine is also known as reaction turbine because sometimes we need to distinguish them or to differentiate them from the impulse turbine, we call it reaction turbine otherwise the impulse reaction is also a common name of this type of turbine. The degree of reaction $R = 0.5$. And we know that degree of reaction is basically enthalpy drop in the moving blades to the total enthalpy drop.

The fundamental difference in a reaction turbine when steam passes through the blades or moving blades is that the pressure drops. And at the cost of the pressure drop the relative velocity is increased a little and steam turbine rotates because of both impulsive effect that is change in momentum due to the change of direction of the jet as well as the reaction force that is impressed on the blades in the opposite direction. So, for $R = 0.5$

$$R = \frac{\Delta h_{mb}}{\Delta h_{mb} + \Delta h_{fb}} = 0.5$$

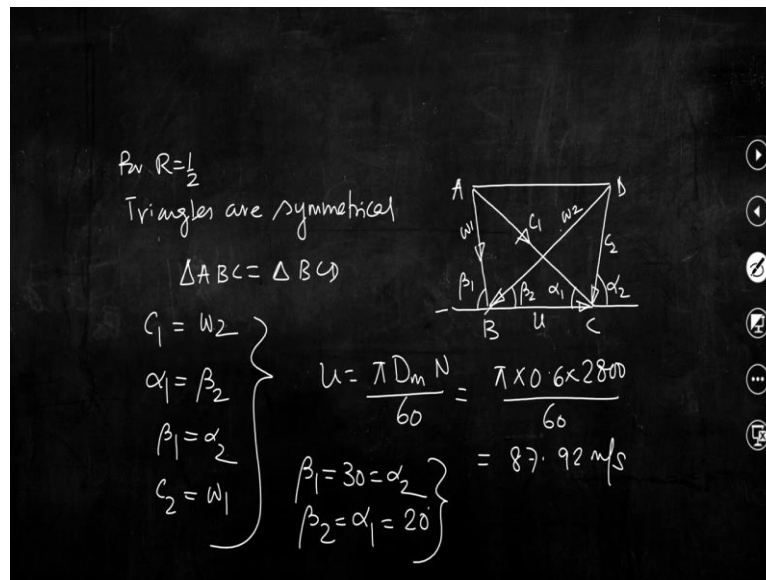
$$\Rightarrow \Delta h_{mb} = \Delta h_{fb}$$

As I told you that typically blades are symmetrical because fabricating blades of a turbine is not so easy. So, basically blades are fabricated from same die and $\beta_1 = \beta_2$ for the geometrical similarity.

So, basically blades are geometrically similar otherwise we have to use different dies for different blades and that would be again much more expensive. So for $R = 0.5$ we know that $C_1 = W_2$ that means absolute velocity of steam from the fixed blade or nozzle is equal to relative velocity of steam from the exit of the moving blades and we know that we are using a common V_b to superimpose the velocity triangles.

So, that means if we consider velocity triangles both at inlet and outlet of the blades then we can see that these 2 triangles are having a common side that is V_b ; $\beta_1 = \beta_2$; $C_1 = W_2$. So, the triangles are symmetrical. So, basically if we try to draw the velocity triangles, triangles will be symmetrical. Let us first draw the velocity angles and then we will discuss.

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So, first we have drawn the velocity triangles in the slide with parameters $\alpha_1, \beta_2, \beta_1, C_1, W_1, C_2, W_2$ & α_2 . Then we have named the vertices of triangles A, B, C, D. So, for $R = 0.5$, the triangles are symmetrical.

$$\text{For } R = \frac{1}{2}; \Delta ABC = \Delta BCD$$

$$C_1 = W_2; \alpha_1 = \beta_2; \beta_1 = \alpha_2; C_2 = W_1$$

So, try to understand we have a common side BC and blades are symmetrical. So, triangles are symmetrical. Now from the data given in this problem statement, we can easily calculate u . So,

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times 0.6 \times 2800}{60} = 87.92 \frac{m}{s}$$

So, that is the blade velocity. And it is given that the inlet blade angle of the moving blade is 30° and exit angle of the fixed blade is 20° . So,

$$\beta_1 = \alpha_2 = 30^\circ$$

$$\beta_2 = \alpha_1 = 20^\circ$$

So, these are the data given. Now we need to calculate the power developed.

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Power developed by the blade $P = \frac{\dot{m} \Delta C_\theta u}{1000} \text{ kW}$

$$\Delta C_\theta = C_{\theta 1} + C_{\theta 2}$$

$$= C_1 \cos \alpha_1 + C_2 \cos \alpha_2$$

$$= W_1 \cos \beta_1 + W_2 \cos \beta_2$$

$\alpha_1 = 20^\circ$
 $\alpha_2 = 30^\circ$

Power developed by the blade, $P = \frac{\dot{m} \Delta C_\theta u}{1000} \text{ kW}$

Now $\Delta C_\theta = C_{\theta 1} + C_{\theta 2}$

If we look at the velocity triangles

$$\Rightarrow \Delta C_\theta = C_1 \cos \alpha_1 + C_2 \cos \alpha_2$$

$$= W_1 \cos \beta_1 + W_2 \cos \beta_2$$

This we have discussed in the context of the solution of the previous problem that is $\Delta C_\theta = W_{\theta 1} + W_{\theta 2}$. Now we can easily calculate the value because we know everything. We know $\alpha_1 = 20^\circ$; $\alpha_2 = 30^\circ$. So, if we can calculate C_1, W_1 then we can calculate easily ΔC_θ . For that from triangle A B C, we can apply sine law because already we know u .

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Applying Sine law $\frac{C_1}{\sin(180-\beta_1)} = \frac{W_1}{\sin \alpha_1} = \frac{u}{\sin A}$

$$\Rightarrow C_1 = \frac{u \sin(180-30)}{\sin 10}$$

$$= 253.15 \text{ m/s}$$

$$\Rightarrow W_1 = \frac{u \sin 20}{\sin 10} = 173.16 \text{ m/s}$$

$\angle A = 180 - (10 + 20)$
 $= 16^\circ$
 $\alpha_1 = 20^\circ$

So, applying sine law

$$\frac{C_1}{\sin(180 - \beta_1)} = \frac{W_1}{\sin \alpha_1} = \frac{u}{\sin A}$$

Now from this we can easily calculate C_1 & W_1 . From the velocity triangle,

$$\text{Angle sign } A = 180 - (150 + 20) = 10^\circ$$

$$\text{Now; } C_1 = \frac{u \sin(180 - 30)}{\sin 10} = 253.15 \frac{m}{s}$$

$$W_1 = \frac{u \sin 20}{\sin 10} = 173.16 \frac{m}{s}$$

I am telling you once again to check the numerical values. So, we have calculated C_1, W_1 . Next we can calculate ΔC_θ .

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$C_\theta = \text{flow velocity} = C_1 \sin \alpha_1$
 $\Delta C_\theta = C_1 \cos \alpha_1 + C_2 \cos \alpha_2$
 $= C_1 \cos \alpha_1 + W_1 \cos \beta_1$
 $= 387.84 \text{ m/s}$
 $P = \frac{\dot{m} \Delta C_\theta u}{1000} \text{ KW}$
 $= \frac{60 \times 387.84 \times 87.92}{1000} \text{ KW} \Rightarrow 60 \times 1.1024 = C_\theta \times \pi D \times b$
 $v = v_f + r \omega = 1.1024 \text{ m}^3/\text{kg}$
 $C_1 = 253.15 \text{ m/s}$
 $W_1 = C_2 = 173.16 \text{ m/s}$
 $\alpha_1 = 20^\circ$
 $\beta_1 = 30^\circ$
 $\dot{m} \times v = \text{flow vel} \times \text{flow area}$

$$\Delta C_\theta = C_1 \cos \alpha_1 + C_2 \cos \alpha_2$$

$$= C_1 \cos \alpha_1 + W_1 \cos \beta_1$$

$$\text{Data we have; } C_1 = 253.15 \frac{m}{s}; W_1 = C_2 = 173.16 \frac{m}{s}; \alpha_1 = 20^\circ; \beta_1 = 30^\circ$$

$$\text{Putting all the values; } \Delta C_\theta = 387.84 \frac{m}{s}$$

So, this is ΔC_θ . Now we can calculate the power developed.

$$\text{Power developed by the blade, } P = \frac{\dot{m} \Delta C_\theta u}{1000} \text{ KW}$$

In the problem statement it is given that mass flow rate $\dot{m} = 60 \frac{kg}{s}$.

$$P = \frac{60 \times 387.84 \times 87.92}{1000} \text{ KW}$$

Finally last part is that if we need to really have flow of steam 60 kg per second then at this rate we need to calculate the height of the blade (h). So, if we go back to the previous slide then one important part is that it is given that the steam condition is 1.5 bar with quality 95%. So, from here we can calculate the specific volume of steam at this condition.

$$\text{Specific volume of steam, } v = v_f + xv_{fg}$$

So, corresponding to that pressure 1.5 bar

$$\text{Specific volume of steam, } v = 0.001053 + 0.95 \times 1.1594 = 1.1024 \frac{\text{m}^3}{\text{kg}}$$

So, this is the specific volume of this steam. Now let me discuss here that steam will flow.

$$\dot{m} \times v = \text{flow velocity} \times \text{flow area}$$

$$\Rightarrow 60 \times 1.1024 = C_{a1} \times \pi D_m \times h$$

So, now try to understand in this expression we know D_m , we have to calculate h that is the blade height provided that we know C_{a1} .

$$C_{a1} = \text{flow of velocity} = C_1 \sin \alpha_1$$

We can easily calculate this as we have already calculated C_1 and α_1 is given. So, if we plug in the value of C_{a1} & D_m then we can calculate easily h . So, you can check the numerical value of h that should be in cm.

So, to summarize today's class, we have solved a few numerical problems from Steam turbine. We have covered both impulse as well as impulse reaction turbine essentially to illustrate the concept that we have learned from the theoretical discussion. So, with this I stop here today and we shall continue our discussion in the next class. Thank you.