

**Thermal Engineering: Basic and Applied**  
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**Lecture - 38**  
**Analysis of Reaction Steam Turbine**

I welcome you all to the session of thermal engineering basic and applied. Today we shall discuss about the reaction turbine. If we recall in our last few classes, we have discussed about the impulse turbines, then we have also discussed about the need of compounding steam turbines that is nothing but staging of steam turbines. So, today we shall discuss about the reaction turbine. We all know that these turbines are mechanical devices which an important part of the steam power plant. And when steam is allowed to flow through the turbines we obtain work. So, turbines are basically work producing device.

So, steam which is produced from the boiler is allowed to flow through the flow nozzles and then the steam jet which is coming out from the flow nozzle with high velocity, impinges upon the blades of the turbine and because of both impulsive and reaction effects turbine wheel rotates and we are getting work output.

So, at the cost of some input energy, we are trying to obtain some work output and that is why these turbines are very much essential for steam power plants. Now try to recall that in the context of impulse turbine, we have discussed about the blading or diagram efficiency. The sole purpose was to establish the efficiency that means the fraction of input energy that is getting converted into work output.

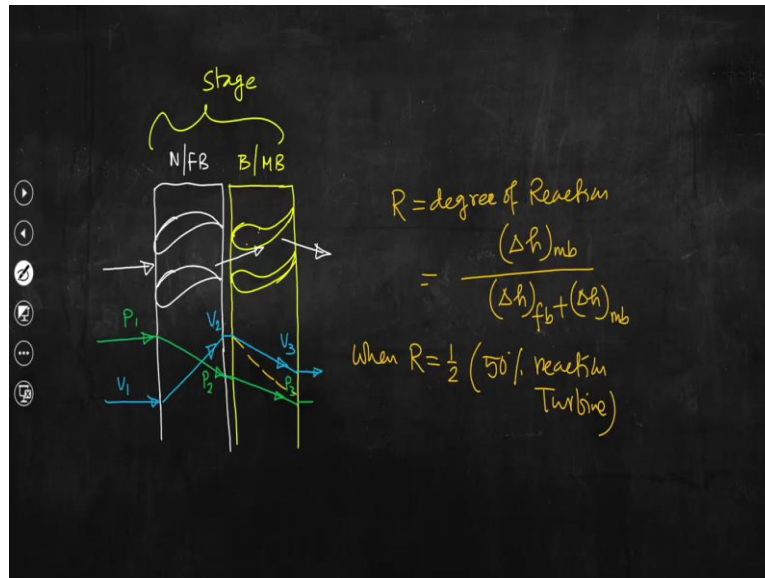
So, to know that even in the context of reaction turbine, we need draw the velocity triangles then we need to analyze velocity components and from there we shall try to quantify the efficiency which will give us an estimate about the work output from the input energy. We all know that the input energy is basically the kinetic energy of steam jets. So, the steam is having high enthalpy and at the cost of that enthalpy drop we will be getting some work output. So, if we can calculate the enthalpy of steam at the inlet to the turbine and enthalpy of steam at the exit of the turbine then from there we can calculate the enthalpy drop. At the cost of that enthalpy drop if we can somehow predict what would be the work output then that will give us an estimate about the turbine efficiency.

Before going to discuss about the blading efficiency or diagram efficiency of reaction turbine, let us briefly recall the basic difference that we also had discussed in one of the previous classes. An aspect for which reaction turbines are different from impulse turbines is the variation of pressures. We had seen that while steam is passing through the impulse turbines there is no pressure drop theoretically. Now again I am telling this is theoretical; in actual scenario there will be drop in pressure but theoretically there is no pressure drop when steam is passing through the blade passage. And we have also mentioned that flow nozzles together with the blades constitute the total steam turbine unit.

So, basically for the impulse turbine when steam is passing through the flow nozzle (flow nozzles are also called as fixed blades), there is a pressure drop and at the cost of that pressure drop there will be an increase in velocity of steam at the exit of the nozzle. So, the steam jet which is coming out from the flow nozzle will be having high kinetic energy and that kinetic energy would be utilized to get the torque or work output. So, kinetic energy would be absorbed by the wheel of the turbines for the impulse turbine. But for the reaction turbine, we have discussed that pressure drop occurs when steam flows through the fixed blades or nozzles as well as through the moving blades or blades. So, this is an important difference between impulse and reaction turbines. So, basically in a reaction turbine when steam is passing through the blade passage pressure drop occurs.

Basically we had seen that for the impulse turbine, it is due to the impulsive effect that is due to change in momentum difference, steam jets will be deflected by the blade and that steam jet will suffer a loss of momentum and that momentum will be absorbed by the wheel. So, this is the impulsive effect that is difference in momentum. In a reaction turbine that effect will be there. Over and above there will be a pressure drop when steam is passing through the blade passage that is moving blades. You know steam will expand and there will be an increase in kinetic energy of steam jet and it is because of this increase in kinetic energy of the exiting jet, a reaction force would be there in the opposite direction according to Newton's third law of motion. So, that means the turbine wheel will rotate due to both impulsive effect as well as the reaction of the exiting jet which is impressed on the blades in the opposite direction. So, it is basically an impulsive reaction turbine but only to distinguish it from the impulse type turbine, we call it reaction turbine.

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So, let us now briefly draw the schematic diagram. So, there is the first row of nozzles or fixed blades and first row of blades or moving blades. So, basically the passage between 2 consecutive blades form a nozzle shape. So, the first row of nozzles or fixed blade and first row of blades or moving blades, these 2 rows together form a stage which is first stage. Now the steam which is coming out from the boiler is entering into the first row of nozzles or fixed blades and then that steam will come out from the first row of nozzles and will strike the first row of blades or moving blades and again it will come out from the first row of blades or moving blades. It will enter into the second row of nozzles or fixed blades and again second row of blades and moving blades. We have discussed about this particular aspect that is how many stages are required for the efficient operation of a turbine unit and it is totally dependent on the total enthalpy drop.

So now let us try to draw the pressure and velocity variation. So, let us first draw the velocity. The velocity of stream is  $V_1$  that is coming out from the boiler and entering into the nozzle and it is because of the pressure drop of steam inside the nozzle, there will be an increase in velocity of steam to  $V_2$ . See basically this is the first row blades are moving blades, so while steam is passing through the blade passage, the kinetic energy would be absorbed by the moving blades rows. Basically that kinetic energy would be utilized to produce torque. So, what we can understand is that the velocity will drop. So, while steam is passing through the first row of moving blades or blades or even second row of blades or moving blades, steam velocity will drop. So, the steam velocity is say  $V_3$ . If we consider only one stage then this would be the velocity of variation.

Now what about pressure? So, let us try to draw the pressure variation here. So, the initial pressure is  $P_1$ . We know that at the expense of pressure drop, stream velocity increases in the first row of fixed blades or nozzle. So, pressure will fall to  $P_2$ . Now what we had seen for the impulse turbines that pressure is remaining constant when steam is passing through the blade passage or moving blade passage. But in case of this reaction turbine, there will be a pressure drop when that steam is passing through the passage of the blades or moving blades. So, the pressure is now  $P_3$ . As I said that while steam is passing through the blades passage or first row of blades or moving blade, second row of blades, third row of blades, pressure will drop and because of this pressure drop, kinetic energy of steam will increase.

Now from this variation we can understand that the velocity decreases, so kinetic energy will also decrease. That is obvious because that energy would be absorbed by the wheel and that will produce torque. Now had you not allowed steam pressure to fall inside the moving blades or blades, velocity of steam inside the moving blade or at the exit of the moving blade first or moving blade would have been even more.

So, now we can see from this diagram that velocity is  $V_3$  and it is because of this pressure drop. Now let us consider the case when there is no pressure drop inside the first row of blades or moving blades. In such a case the steam velocity would have been as shown in the slide. So, that means it is because of this increase in velocity of steam as a result of drop in pressure that kinetic energy increase will give rise to reaction in the opposite direction. And that reaction of the exiting jet impressed on the blade in the opposite direction and that is why the wheel rotates because of both impulsive effect as well as this reaction effect. And hence it is called impulse reaction turbine or simply reaction turbine only to distinguish this special type of turbine from the impulse turbine.

So now we had also defined one term that is called degree of reaction. This is defined as the ratio of enthalpy drop of steam in the moving blade to the total enthalpy drop that is fixed blade plus enthalpy of steam in the moving blade.

$R = \text{Degree of Reaction}$

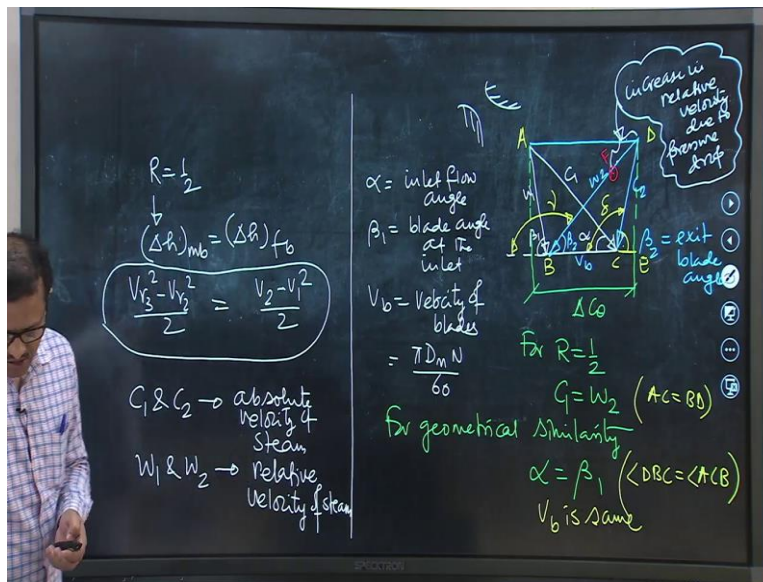
$$R = \frac{(\Delta h)_{mb}}{(\Delta h)_{fb} + (\Delta h)_{mb}}$$

So, that is the degree of reaction.

So, today we shall analyze the reaction turbine of course by drawing the velocity triangles, only to establish the mathematical form of the blading work or diagram work or the diagram efficiency or blading efficiency.

So, it is obvious that when  $R = 0$  then it is purely impulsive turbine. So, enthalpy drop of steam inside the moving blade is 0 and when  $R = 1$  then  $(\Delta h)_{fb} = 0$ , that means this is a purely reaction turbine. So, for  $R = \frac{1}{2}$  that is 50% reaction turbine; this is only one stage that we had drawn today to see the variation of pressure and velocity when steam is passing through the first row of nozzles or fixed blades and first row of blades or moving blades.

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So, if we try to draw the velocity triangle for this particular case that is  $R = \frac{1}{2}$  then what we can understand for this particular case is that the enthalpy drop of steam while passing through the moving blades is equal to enthalpy drop of steam passing through the nozzles or fixed blades.

$$(\Delta h)_{mb} = (\Delta h)_{fb}$$

$$\frac{V_{r3}^2 - V_{r2}^2}{2} = \frac{V_2^2 - V_1^2}{2}$$

So, basically  $V_2, V_1$  are the absolute velocity of steam. Here  $V_2$  is absolute velocity of steam entering the first row of moving blades and  $V_1$  is the absolute velocity of steam leaving the first row of nozzles. And  $V_{r3}, V_{r2}$  are the relative velocity of steam. Here  $V_{r3}$  is relative velocity of steam leaving the first row of moving blades and  $V_{r2}$  is the relative velocity of steam entering into the first row moving blades. But for the present analysis we will be using the

terminology  $C_1$  &  $C_2$  as the absolute velocity of steam and  $W_1$  &  $W_2$  are the relative velocity of steam. So, the velocity triangle for this particular case is drawn in the slide.

So, at first the Inlet velocity triangle is drawn. Steam is coming out from the nozzle and  $\alpha$  is the flow angle and  $\beta_1$  is the blade angle. Then the exit velocity triangle has been superimposed on the same plane. So, let me again identify the velocity components, basically  $C_1$  the absolute velocity of steam. So, instead of  $V_1$  we are assuming this as  $C_1$  as said that I am we are going to consider  $C_1$  &  $C_2$  as the absolute velocity of steam and it is not  $V_1$  &  $V_2$ . This is up to you to use  $V_1$  &  $V_2$ . So, now in the diagram velocity  $C_1$ , flow angle  $\alpha$  and blade angle  $\beta_1$  has been marked. And likewise  $C_2, W_2$  has also been marked in the velocity triangle. So  $C_2$  &  $W_2$  are the absolute velocity of steam leaving the nozzle and relative velocity of steam leaving the nozzle and entering into the first row of moving blades. And  $V_b$  is the blade velocity.

$$V_b = \frac{\pi D_m N}{60}$$

So, this is calculated based on the mean diameter  $D_m$ . Now the exit blade angle is  $\beta_2$  and  $\Delta C_\theta$  is drawn. So, now we will write it because we have also done similar type of analysis for the impulse type of turbine. Since we are going to consider a special case for which degree of reaction is half for such a case we can write that

$$\text{For } R = \frac{1}{2}; C_1 = W_2$$

This we will be getting from the expression of enthalpy drop across the moving blade and enthalpy drop across the nozzles or fixed blades. There is similarity in geometry of blades because essentially blades are extruded from the same die. So, for geometrical similarity we can write  $\alpha = \beta_1$ . So, in fact this is a true case because blades of the turbine are extruded from a same die and geometrical similarity is maintained.

So, now let us give name of these 2 triangles that we have drawn here as say A B C & D. So, what we had seen that

$$C_1 = W_2 \Rightarrow AC = BD$$

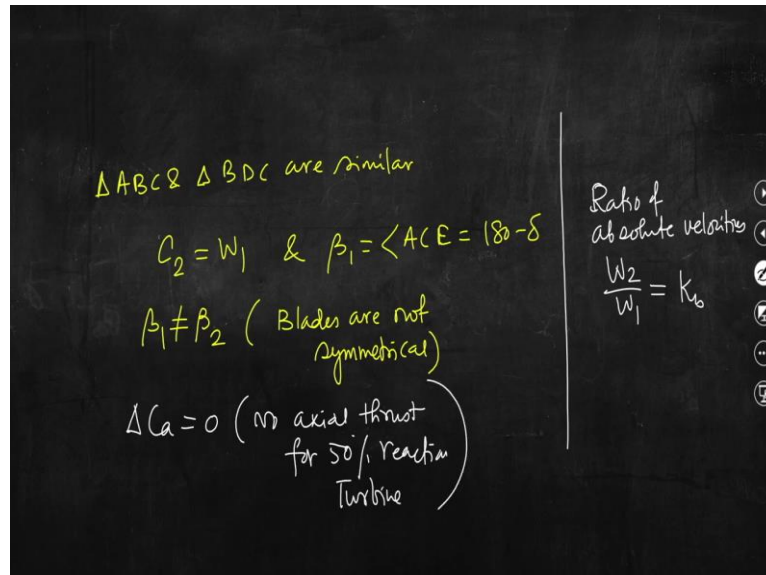
$$\alpha = \beta_2 \Rightarrow \angle DBC = \angle ACB$$

And  $V_b$  is common. So, blade velocity  $V_b = \frac{\pi D_m N}{60}$ .

So, we have studied similarity in our class 10. So now, we can compare these 2 triangles  $\Delta DBC$  &  $\Delta ACB$  from the velocity diagram. Now we can see that there is a common

side  $BC$  and other 2 sides are equal that is  $AC = BD$  and 2 angles are same that is  $\angle DBC = \angle ACB$ . So, from the understanding of Side-Angle-Side similarity of 2 triangles we can now compare. So, basically we can write  $\Delta DBC$  &  $\Delta ACB$  are similar.

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If these 2 triangles are similar then we can write that

$$C_2 = W_1$$

All angles are measured in the clockwise direction and angle  $\gamma$  &  $\delta$  has been marked in the velocity triangle. So, you can see that as these 2 triangles are similar, hence

$$\beta_1 = \angle ACE = 180^\circ - \delta$$

Basically it is very uncommon that the blades will be symmetrical. So, you can assume that this is the geometrical similarity of the rotor disc but if blades are not symmetrical then  $\beta_1 \neq \beta_2$ . You know that we had discussed for the impulse turbines that  $\beta_1 = \beta_2$ . It is true case for the impulse turbine because most of the impulse turbines are having symmetrical blades. So, this is a true case for the impulse turbine but this is not a case for the reaction turbine.

So, now we go back to the velocity triangles and I am marking a particular point say F. So, let me write here that for steam turbine when steam is passing through the moving blades or blades the ratio of absolute velocity is

$$\frac{W_2}{W_1} = K_b = \text{Blade friction factor}$$

So, because of the losses due to friction  $W_2 < W_1$ . So, that means the relative velocity of steam when it is entering into the first row of blades or moving blades is  $W_1$  but the relative velocity

of steam coming out from the first row of blades or moving blades is  $W_2$  and  $W_2 < W_1$  and it is because of the frictional losses. So, this  $K_b$  is always less than 1.

And now if you try to recall the velocity triangles for the impulse turbine, we had seen that there is basically drop in axial velocity but we can understand that the relative velocity is now extended up to point D. So, basically this increase in relative velocity of steam when it is coming out from the first row of blades or moving blades is due to the increase in velocity of steam when it is passing through the blades in case of a reaction turbine. And that is solely due to drop in pressure.

So, let me tell you this once again. So, for the impulse turbine  $W_2 < W_1$  and we can see for this particular case is that  $W_2$  is extended up to point D. So, this increase in relative velocity of steam when it is passing through the first row of blades or moving blades is solely due to pressure drop and that is what we had discussed here.

So, basically it is because of this pressure drop, there will be an increase in kinetic energy of the steam which in turn is also responsible for the increase in relative velocity that we can see from here. So, this is increase in relative velocity is due to pressure drop. So, if we now see this diagram we can see that there is no axial thrust.

So, there is no axial thrust because  $\Delta C_a = 0$  that means for 50% reaction turbine for which  $C_2 = W_1$  and this is the blade angle at the inlet, there is no axial thrust. But we cannot say like this as there will be a small amount of axial thrust that is solely due to change in pressure when stream is passing through the first row or second row or third row of blades or moving blades. So, basically a change in pressure of stream when it passes through the first row of blades or second row blades, a small amount of axial thrust would be there and but there is no axial thrust that we can see particularly for this type of turbine that is 50% reaction turbine from this diagram. So, there is no change in axial velocity.

So, basically  $C_{a1} = C_{a2}$  that we can see from this diagram hence  $\Delta C_a = C_{a1} - C_{a2} = 0$ . So, there is no axial thrust but as I said a small amount of axial thrust would be there and it is because of this pressure drop when steam is passing through the blades.

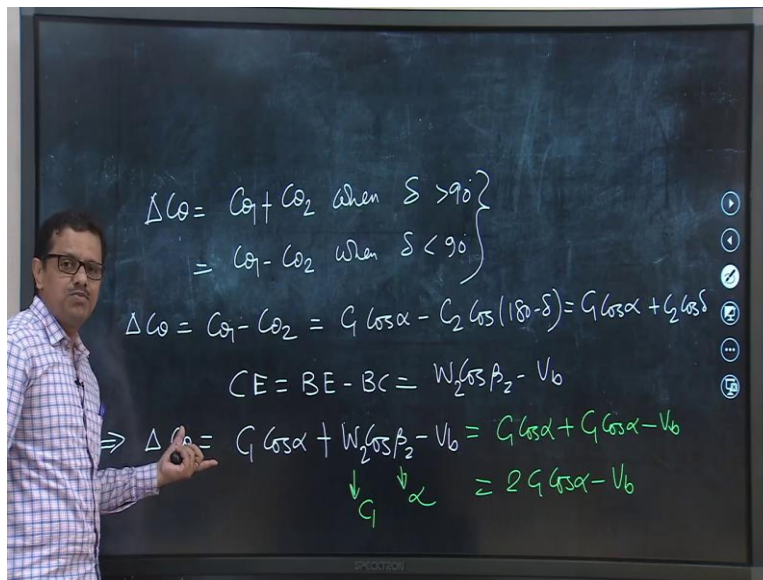


So, now our next objective should be to calculate  $\Delta C_\theta$  that is the change in swirl component of velocity or wall component of velocity which is responsible for the tangential thrust. So, basically there is no axial thrust that we have discussed. So,

$$\Delta C_a = C_{a1} - C_{a2} = 0$$

So there is no axial thrust for 50% reaction turbine. But as I said small amount of axial thrust would be there due to change in pressure of the steam as it passes through the blades.

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Now let us try to quantify  $\Delta C_\theta = C_{\theta 1} - C_{\theta 2}$ . So, now say if we measure the angles in the clockwise direction then  $\delta > 90^\circ$ . So, when  $\delta > 90^\circ$  then to get this change in swirl velocity  $\Delta C_\theta$  these 2 components are added together.

$$\Delta C_\theta = C_{\theta 1} + C_{\theta 2}; \text{ when } \delta > 90^\circ$$

$$\Delta C_\theta = C_{\theta 1} - C_{\theta 2}; \text{ when } \delta < 90^\circ$$

Let me tell you once again, we have measured the angle in the clockwise direction. Now  $\delta > 90^\circ$ , for this case if you would like to quantify or estimate the change in swirl component of velocity  $\Delta C_\theta$  then we have to add  $C_{\theta 1}$  &  $C_{\theta 2}$  together. That means we need we have to add the swirl component of velocity together to obtain the change in swirl component of velocity. But when  $\delta < 90^\circ$  then  $C_{\theta 2}$  should be subtracted from  $C_{\theta 1}$ .

$$\Delta C_\theta = C_{\theta 1} - C_{\theta 2} = C_1 \cos \alpha - C_2 \cos(180 - \delta) = C_1 \cos \alpha + C_2 \cos \delta$$

$$\text{Now; } CE = BE - BC = W_2 \cos \beta_2 - V_b$$

So, we can write again

$$\Delta C_\theta = C_{\theta 1} - C_{\theta 2} = C_1 \cos \alpha + W_2 \cos \beta_2 - V_b$$

Now if we go back to the previous slide we can see that for  $R = \frac{1}{2}$

$$C_1 = W_2$$

You will be getting this if you equate the enthalpy drop across the moving blade and enthalpy drop across the fixed blades and  $\alpha = \beta_2$  because of the geometric similarity.

$$\begin{aligned}\Delta C_\theta &= C_{\theta 1} - C_{\theta 2} = C_1 \cos \alpha + W_2 \cos \beta_2 - V_b \\ \Rightarrow \Delta C_\theta &= C_{\theta 1} - C_{\theta 2} = C_1 \cos \alpha + C_1 \cos \alpha - V_b \\ &\Rightarrow \Delta C_\theta = 2C_1 \cos \alpha - V_b\end{aligned}$$

So, basically you are trying to calculate the change in swirl component of velocity which should be responsible for the tangential thrust. It is also possible that you can calculate this because

$$\Delta C_\theta = W_1 \cos \beta_1 + W_2 \cos \beta_2$$

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The image shows handwritten notes on a blackboard. On the left side, the following equations are written in green:

$$\begin{aligned}\Delta C_\theta &= W_1 \cos \beta_1 + W_2 \cos \beta_2 \\ &= C_1 \cos \alpha - V_b + C_1 \cos \alpha \\ &= 2C_1 \cos \alpha - V_b\end{aligned}$$

An upward-pointing arrow is drawn next to the first equation. Below the equations, it says "Change in Swirl Component of velocities".

On the right side, the notes are written in yellow and green:

Tangential thrust  
 Work transfer diagram (Per unit mass flow of steam) work =  $\Delta C_\theta V_b$   
 Input energy per unit mass flow of steam =  $\frac{C_1^2}{2} + \frac{W_2^2 - W_1^2}{2}$

$$\Rightarrow \Delta C_\theta = C_1 \cos \alpha - V_b + C_1 \cos \alpha = 2C_1 \cos \alpha - V_b$$

So, we can understand that this change in swirl component of velocity is  $2C_1 \cos \alpha - V_b$ . So, we can calculate  $\Delta C_\theta$  which should be responsible for the tangential thrust. Now we can calculate the tangential thrust. So, we can write that

$$\text{Tangential Thrust} = \Delta C_\theta \times m_s; m_s = \text{mass flow rate of steam}$$

So, from there we also can calculate the work transfer or diagram work.

$$\text{Work transfer or Diagram work} = \Delta C_\theta \times m_s \times V_b$$

$$\text{Work transfer or Diagram work per unit mass flow rate} = \Delta C_\theta \times V_b$$

Now we are getting this work transfer diagram work per unit mass of the steam flow rate. So, this is the amount of work that we can extract at the cost of some input energy. So, if we now try to quantify the blading efficiency or diagram efficiency, then we also need to know the input energy per unit mass flow of steam.

$$\text{Input energy per unit mass flow of steam} = \frac{C_1^2}{2} + \frac{W_2^2 - W_1^2}{2}$$

We had seen that for the reaction turbine there is an increase in kinetic energy of steam because of the decrease in pressure. And that effect also should be taken into account that what is the input energy given to get this amount of work output.

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The image shows a chalkboard with the following handwritten work:

$$\begin{aligned} \text{Input energy} &= \frac{C_1^2}{2} + \frac{W_2^2 - W_1^2}{2} \\ &= \frac{C_1^2}{2} + \frac{C_1^2}{2} - \frac{W_1^2}{2} \quad [W_2 = C_1] \\ &= C_1^2 - \frac{W_1^2}{2} \end{aligned}$$

From  $\Delta ABC \Rightarrow \cos \alpha = \frac{C_1^2 + V_b^2 - W_1^2}{2V_b C_1}$

$$\Rightarrow W_1^2 = C_1^2 + V_b^2 - 2V_b C_1 \cos \alpha$$

$$\text{Input energy} = \frac{C_1^2}{2} + \frac{W_2^2 - W_1^2}{2}$$

$$\Rightarrow \text{Input energy} = \frac{C_1^2}{2} + \frac{C_1^2}{2} - \frac{W_1^2}{2} \quad [W_2 = C_1 \text{ for } 50\% \text{ reaction turbine}]$$

$$\Rightarrow \text{Input energy} = C_1^2 - \frac{W_1^2}{2}$$

Now we need to write  $W_1$  in terms of the known quantities that is the velocity of steam which will come out from the nozzle that we know and also we know the blade velocity  $V_b$ . So, if you look at the velocity triangle ABC then we can write

$$\cos \alpha = \frac{C_1^2 + V_b^2 - W_1^2}{2V_b C_1}$$

$$\Rightarrow W_1^2 = C_1^2 + V_b^2 - 2V_b C_1 \cos \alpha$$

So, if we write this expression of  $W_1$  in the input energy then we can write the diagram efficiency. So, let me tell you once again tangential thrust would be  $\Delta C_\theta \times m_s$ . Since we are trying to estimate the work transfer or diagram work per unit mass flow of steam so

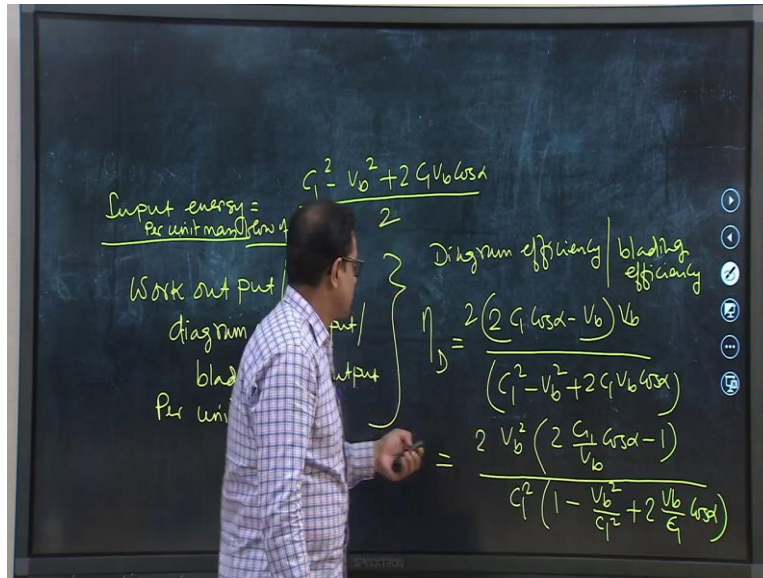
$$\text{Diagram work} = \text{Tangential thrust} \times V_b$$

$$\text{Diagram work per unit mass flow rate of steam} = \Delta C_\theta \times V_b$$

And this is the work that we are getting at the cost of this input energy and we could write the final expression of input energy as

$$\text{Input energy} = C_1^2 - \frac{W_1^2}{2}$$

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$$\Rightarrow \text{Input energy per unit mass flow rate of steam} = C_1^2 - \frac{C_1^2 + V_b^2 - 2V_b C_1 \cos \alpha}{2}$$

$$= \frac{C_1^2 - V_b^2 + 2V_b C_1 \cos \alpha}{2}$$

Now this is the input energy and at the cost of this input energy we are getting work output diagram work output or blading work output per unit mass flow of steam because while you are trying to estimate the efficiency, the unit will remain same.

$$\text{Diagram efficiency, } \eta_D = \frac{2(2C_1 \cos \alpha - V_b)V_b}{C_1^2 - V_b^2 + 2V_b C_1 \cos \alpha}$$

$$\Rightarrow \eta_D = \frac{2V_b^2 \left( \frac{2C_1}{V_b} \cos \alpha - 1 \right)}{C_1^2 \left( 1 - \frac{V_b^2}{C_1^2} + 2 \frac{V_b}{C_1} \cos \alpha \right)}$$

So, this is the expression of blading work or diagram work. We can see that there is particular ratio that is  $V_b/C_1$ . So, that means we can try to cast this expression of blading efficiency or diagram efficiency in terms of the velocity ratio that is  $V_b/C_1$ .

$$\Rightarrow \eta_D = \frac{2 \left( \frac{V_b}{C_1} \right)^2 \left[ \frac{2 \cos \alpha}{\left( \frac{V_b}{C_1} \right)} - 1 \right]}{\left[ 1 - \left( \frac{V_b}{C_1} \right)^2 + 2 \left( \frac{V_b}{C_1} \right) \cos \alpha \right]}$$

That means we can see that the diagram efficiency of blading efficiency is a function of one ratio that is the velocity ratio.

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Handwritten notes on a blackboard showing the derivation of the optimum velocity ratio for maximum diagram efficiency. The notes include the following:

$$\eta_D = \frac{2 \left( \frac{V_b}{C_1} \right)^2 \left[ \frac{2 \cos \alpha}{\left( \frac{V_b}{C_1} \right)} - 1 \right]}{\left[ 1 - \left( \frac{V_b}{C_1} \right)^2 + 2 \left( \frac{V_b}{C_1} \right) \cos \alpha \right]}$$

$\frac{V_b}{C_1}$  = Velocity ratio

Particular value of  $\left( \frac{V_b}{C_1} \right)$  for which the diagram blading efficiency will be maximum is the optimum velocity ratio  $\left( \frac{V_b}{C_1} \right)_{opt}$

$$\left( \frac{V_b}{C_1} \right)_{opt} = C_1 \cos \alpha$$

So, we had discussed about this particular velocity ratio in the context of impulse turbine. So, again we can see that the diagram efficiency or blading efficiency of the reaction turbine can also be expressed in terms of velocity ratio. Now try to understand that there is a particular value of this ratio for which the diagram efficiency will be maximum and to obtain that optimum velocity ratio we need to differentiate  $\eta_D$  with respect to the velocity ratio and equate it to zero.

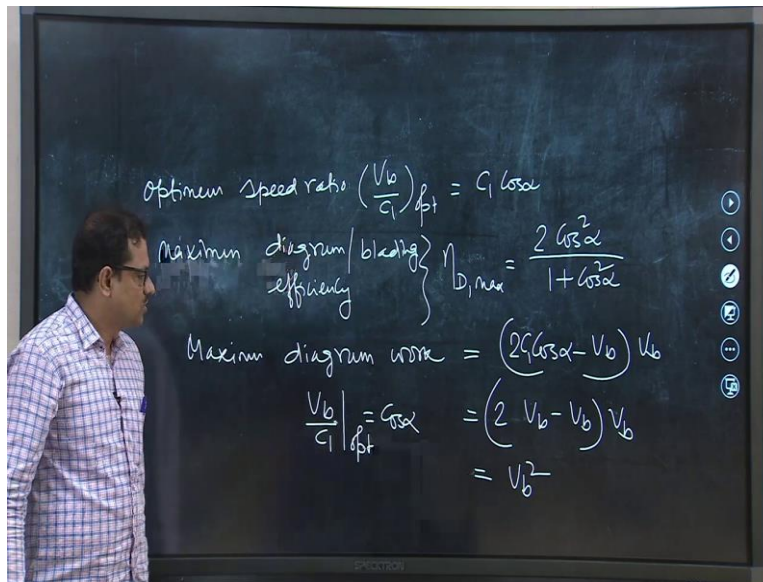
So, that means there is a particular value of  $\frac{V_b}{C_1}$  for which the diagram efficiency or blading efficiency will be maximum and for that we need to differentiate  $\eta_D$  with respect to  $\frac{V_b}{C_1}$  and then equating it to 0, we will be getting  $\left( \frac{V_b}{C_1} \right)_{opt}$ . This particular value of  $\frac{V_b}{C_1}$  for which the diagram efficiency will be maximum is the Optimum velocity ratio.

$$\text{Optimum velocity ratio} = \left( \frac{V_b}{C_1} \right)_{opt} = C_1 \cos \alpha$$

But I am not going to do this task here.

$$\text{Maximum diagram efficiency} = \eta_{D,max} = \frac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$$

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So, that means we have maximum diagram work and you can understand that

$$\text{Maximum diagram work} = \Delta C_{\theta} \times V_b = (2C_1 \cos \alpha - V_b)V_b$$

We have obtained that

$$\left(\frac{V_b}{C_1}\right)_{opt} = C_1 \cos \alpha$$

$$\Rightarrow C_1 \cos \alpha = V_b$$

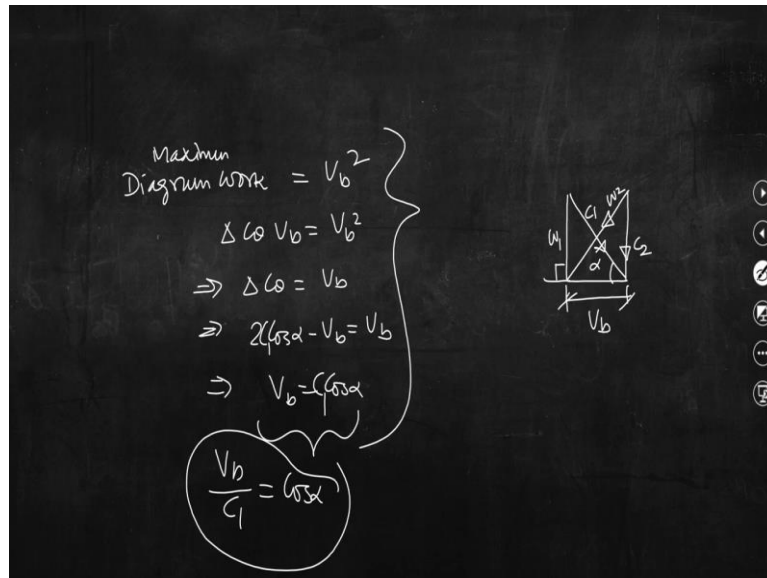
$$\text{Maximum diagram work} = (2C_1 \cos \alpha - V_b)V_b = (2V_b - V_b)V_b = V_b^2$$

Now very quickly I will tell what would be the velocity triangles particularly for this case when diagram efficiency should be the maximum efficiency and diagram work would be the maximum work for a 50% reaction turbine.

$$\text{Maximum diagram work} = V_b^2$$

$$\Rightarrow \Delta C_{\theta} V_b = V_b^2$$

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$$\begin{aligned} \Rightarrow \Delta C_\theta &= V_b \\ \Rightarrow 2C_1 \cos \alpha - V_b &= V_b \\ \Rightarrow V_b &= C_1 \cos \alpha \\ \frac{V_b}{C_1} &= \cos \alpha \end{aligned}$$

So, that is the optimum speed ratio. Now you can understand  $C_1 \cos \alpha = V_b$ . Now the velocity triangle for the 50% reaction turbine has been drawn in the slide with parameters like  $W_1, C_1, W_2, C_2, V_b, \beta_1$  and  $\beta_2$ . So,  $C_1 \cos \alpha = V_b$  that we can see from this velocity triangles for 50% reaction turbine. So, this is the velocity triangles for 50% reaction turbine producing maximum diagram work. So, let me tell you once again that this is the velocity triangle of the reaction turbine having degree of reaction equal to half when producing maximum work.

So to summarize, today we have discussed about the reaction turbine. From the velocity triangles we have tried to estimate the diagram work, tangential thrust and finally diagram efficiency. We have quantified the optimum ratio for which the diagram efficiency is maximum. From there we have also tried to predict the velocity triangles for a 50% reaction turbine producing maximum work. So, with this I stop here today and we shall continue our discussion in the next class, thank you.