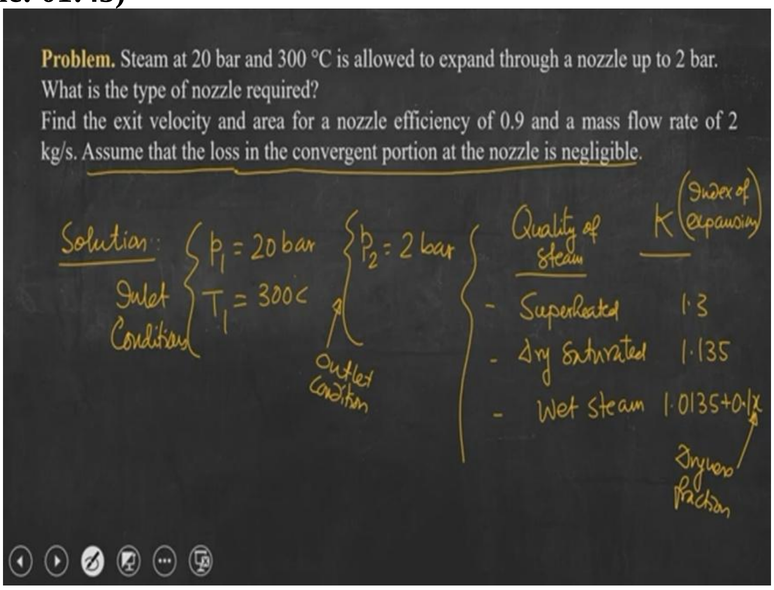
Thermal Engineering: Basic and Applied

Prof. Dr. Pranab K Mondal
Department of Mechanical Engineering
Indian Institute of Technology - Guwahati

Lecture - 34
Problem on Flow Nozzle

I welcome you all to the session of thermal engineering basic and applied and in today's class we shall solve our numerical problem on flow nozzle. We have discussed about the efficiency of the nozzle, knowing the flow physics when a steam is flowing through a channel having convergent divergence shape. So we have discussed that nozzle efficiency is influenced by a few factors, those are friction, velocity of steam and finally the length of the nozzle. Importantly friction plays an important role on the efficiency of the flow nozzle, but we have also seen that velocity of steam as well as the length of the nozzle indirectly affects the efficiency.

(Refer Slide Time: 01:45)



Problem: Steam at 20 bar and 300°C is allowed to expand through a nozzle up to 2 bar. What is the type of nozzle required?

Find the exit velocity and area for a nozzle efficiency of 0.9 and a mass flow rate of 2 kg/s. Assume that the loss in the convergent position at the nozzle is negligible.

So this is the problem statement. Let us first read the problem statement then we shall move to solve this. It is written that steam at 20 bar and 300°C is allowed to expand through a nozzle up to 2 bar. What is the type of nozzle required? So this is the first part of this problem that means inlet pressure is given, inlet temperature is given and also exit pressure is given. You may recall in the last class when you have talked about identifying several factors those affect the nozzle efficiency, we have discussed that inlet pressure is fixed, outlet pressure is also fixed and if the process is isentropic then exit velocity is also fixed. So inlet conditions are given that is pressure and temperature, outlet pressure is given that is 2 bar and from there we need to know which type of nozzle is needed for the steam power cycle.

The next part of the problem is find the exit velocity and area for a nozzle efficiency of 0.9. So basically it is given that the nozzle efficiency is 90% and for that we need to find the exit velocity. And mass flow rate of steam is 2 kg/s. It is given that assume that the loss in the convergent portion of the nozzle is negligible. If we underline this particular line then this line is giving us a clue about the type of the nozzle.

So it is given that the loss in the convergent portion of the nozzle is negligible. So if the nozzle is purely convergent type then perhaps it is an indication but since we really cannot trivially ignore the frictional losses, so perhaps it is not a purely convergent type. So this is giving us a clue but let us look at the actual case.

So let us start solving this problem. So first of all you need to understand which type of nozzle it is. So it is given

$$\text{Inlet Conditions: } p_1 = 20\ bar, T_1 = 300°C$$

$$\text{Outlet Condition: } p_{exit} = 2\ bar$$

Now when we have discussed about the critical pressure, we have also discussed that when a nozzle is said to be choked. That time we have discussed about the physical significance of the critical pressure ratio or critical pressure to be precise. Now if we try to find out the critical pressure for this then we need to know what would be the index of expansion of the steam following the isentropic expansion. You know the value of index of expansion for the ideal gas is constant but this is not a case for the steam. For steam, the index of expansion varies depending on the quality of steam at the inlet to the nozzle.

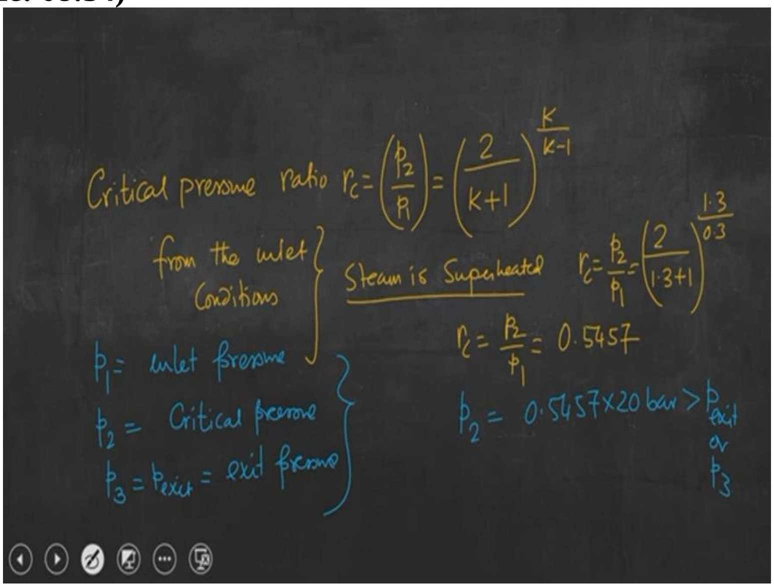
So let us now look at what would be the value of k for different cases of steam that is for different inlet qualities of steam from the following table.

Quality of steam	Index of expansion, k
Superheated	1.3
Dry saturated	1.135
Wet steam	$1.0135 + 0.1\,x;$ where $x =$ dryness fraction

So if the quality is superheated steam, the value of k is 1.3. So you can assume this value when you are solving any numerical problem. If the steam quality is dry saturated at the inlet to the nozzle, k is 1. 135. And finally if the steam is wet steam then it is$1.0135 + 0.1x$; where x is the dryness fraction. So these 3 are the different possibilities of steam that we may have at the inlet of the nozzle. Either the steam may be superheated steam or dry saturated or wet steam. So if the steam is wet at the inlet of the nozzle, then perhaps we can we know what is the dryness fraction and from there we can calculate k. So these are very important.

Now let us discuss something on the critical pressure ratio, because by knowing the critical pressure ratio only you can say what type of nozzle is needed for the operation of steam power plant, where this type of nozzles are there. So, next we can calculate critical pressure ratio.

(Refer Slide Time: 08:34)



$$\text{Critical pressure ratio}, r_c = \frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

So this is what we have derived. So when we have talked about mass flow rate per unit area at the exit of the nozzle, from there we could establish the expression of critical pressure ratio. Now, if we use the value of k = 1.3, because if we go back to the problem statement nothing is given.

See you can calculate the quality of steam at the inlet to the nozzle as pressure and temperature are given. So we have studied in thermodynamics that knowing these 2 properties, we can assess the quality of steam at the inlet of the nozzle from the steam table. So knowing the value of pressure and temperature which the stream is having at the inlet of the nozzle, we can calculate what is the state, whether the quality is 1 or it is less than 1. So from there we can assess about the quality of steam.

So now for this particular case, from steam table, the quality of steam is superheated. So you can assume at the inlet condition the steam is superheated. So $k = 1$

$$r_c = \frac{p_2}{p_1} = \left(\frac{2}{1.3+1}\right)^{\frac{1.3}{0.3}} = 0.5457$$

We can see from this particular expression that p_2 is the critical pressure. So p_1 is the inlet pressure and p_2 is the critical pressure and $p_3 = p_{exit}$ is the exit pressure. So we are assuming these 3. Now we can see that exit pressure is 2 bar that is given.

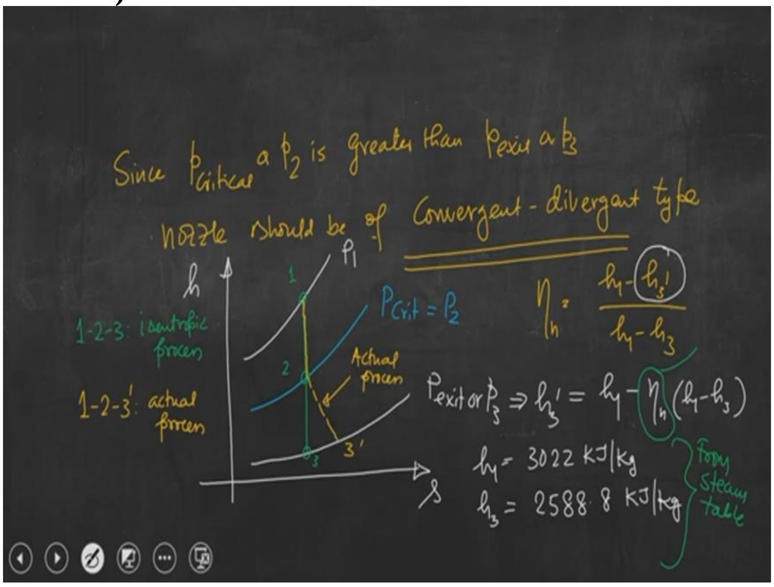
$$p_1 = 20\ bar, p_{exit} = 2\ bar$$

$$\text{Now } p_2 = 0.5457 \times 20 = 10.914$$

$$\text{That means } p_2 > p_{exit}$$

So the critical pressure is greater than the exit pressure. I had written in one of the previous classes that if the critical pressure is very important and if the magnitude of critical pressure is greater than the exit pressure then the nozzle should be of convergent divergent type.

(Refer Slide Time: 13:36)



So since $p_{critical}$ or p_2 is greater than p_{exit} or p_3, the nozzle should be of convergent divergent type. So this is very important to know. So this is the answer for the first part of the question. So this is convergent divergent type nozzle. So now let us look at the h-s diagram and try to map the expansion process therein. So if we go for the h-s plane, say we plot 3 pressure lines $p_{critical}$ or p_2, p_1 and p_3 or p_{exit}. Now we can plot the isentropic process 1-2-3 as shown in the slide.

We have discussed in the last class that this is the isentropic process but in real application, it is very difficult to have or achieve isentropic process. So accounting for the frictional losses the process will deviate from the isentropic process and the actual process would be different. For that we need to go back to the problem statement. The last line says that the loss in the convergent portion in the nozzle is negligible. So this statement is giving us a clue as I have already told that this is not a purely convergent type of nozzle. Now we need to assume that the loss due to frictional effect is negligible in the convergent part of the nozzle and if that is the case and if I try to draw the actual process, then I have to plot point 3'. So 1- 2- 3 is the isentropic process whereas 1- 2- 3' prime is the actual process. Then what we need to do? We need to calculate the exit velocity and area for a nozzle efficiency of 0.9 and mass flow rate 2 kg/s.

So let us now try to calculate.

$$\text{Nozzle efficiency}, \eta_n = \frac{h_1 - h_3'}{h_1 - h_3}$$

So this is what we have derived in the last class which is nothing but the ratio of actual heat drop to the isentropic heat drop. So this also can be related to the actual gain in kinetic energy and that due to isentropic process. Now from steam table we can calculate h_1 because we know the inlet conditions that is pressure and temperature is given. So we need to know h_3'. From the above expression we can write

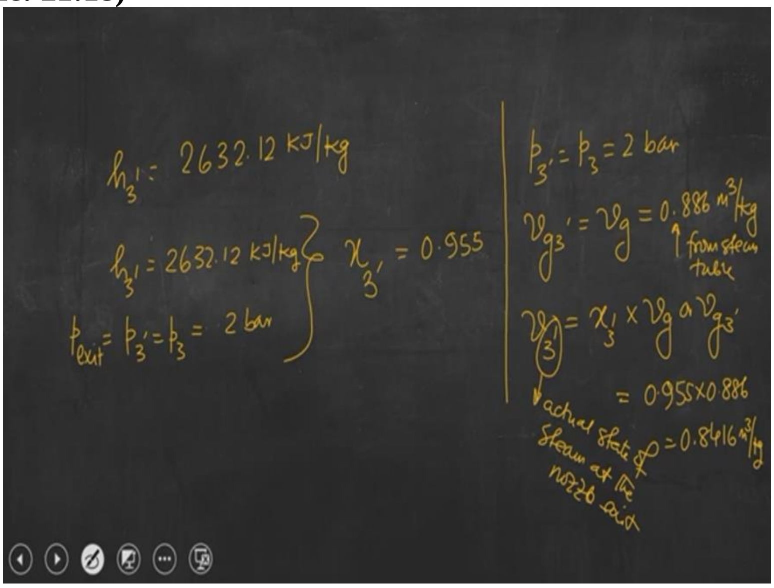
$$h_3' = h_1 - \eta_n(h_1 - h_3)$$

Here we can calculate h_1 from steam table and h_3 also. Because if we can calculate entropy at point 1 then entropy at point 3 is also the same and we know the exit pressure. So knowing these 2 properties, you can calculate enthalpy from the steam table.

$$h_1 = 3022\,\frac{\text{kJ}}{\text{kg}}; \;\; h_3 = 2588.8\,\frac{\text{kJ}}{\text{kg}}$$

I have already mentioned by how we can calculate h_1 and h_3. Pressure is given 20 bar and temperature is given 300°C. With these inlet conditions from steam table, we can get h_1. Pressure at exit is given that is 2 bar and $s_3 = s_1$because process 1-2-3 is isentropic process. So entropy at point 3 is equal to entropy at point 1. From inlet condition, we can calculate entropy again from stream table. So knowing entropy at point 3 and pressure at point 3 that is p_3, we can calculate h_3 and that is what we have calculated. Now if we plug in the value of h_1 and h_3 in the above expression then we can get $h_3{}'$.

(Refer Slide Time: 21:18)



$$h_3' = 3022 - 0.9(3022 - 2588.8) = 2632.12\,\frac{\text{kJ}}{\text{kg}}$$

Now next issue is what would be $x_3{}'$?

$$h_3' = 2632.12\,\frac{\text{kJ}}{\text{kg}};\ p_3{}' = p_3 = p_{exit} = 2\ \text{bar}$$

So by knowing this 2 properties and by looking at the Mollier diagram h-s plane, we can get the dryness fraction. So that is the beauty of the Mollier diagram that we can directly get the quality of steam. So corresponding to this state, by looking at the Mollier diagram, we can find out the dryness fraction at point 3' is 0.955. So if we go back to the previous slide and try to draw the dryness fraction line. So the line corresponding to $x = 1$ is drawn in the h-s diagram. So you can understand that point 3' prime is close to $x = 1$ that is not the saturated steam but close to the saturated steam.

So now what next we need to do? See exit pressure is 2 bar.

$$p_3{}' = p_3 = 2\ \text{bar}$$

And we can calculate what would be v_{g3}'. So that means at that particular condition what would be the specific volume of vapor?

$$v_{g3}' = v_g = 0.886 \frac{\text{m}^3}{\text{kg}}$$

So at that pressure the specific volume of vapor is given $0.886 \frac{\text{m}^3}{\text{kg}}$. So that means if we know the pressure $p_3 = 2$ bar and $p_1 = 20$ then we can calculate v_g for that pressure. Now the specific volume at 3' is v_3' and

$$v_3' = x_3' \times v_g \text{ or } v_{g3}'$$

So try to understand we have already calculated dryness fraction at point 3'. We know that point 3' is also on the p_3 line that is exit pressure line. We have already calculated $v_{g3}' = v_g = 0.886 \frac{\text{m}^3}{\text{kg}}$. So

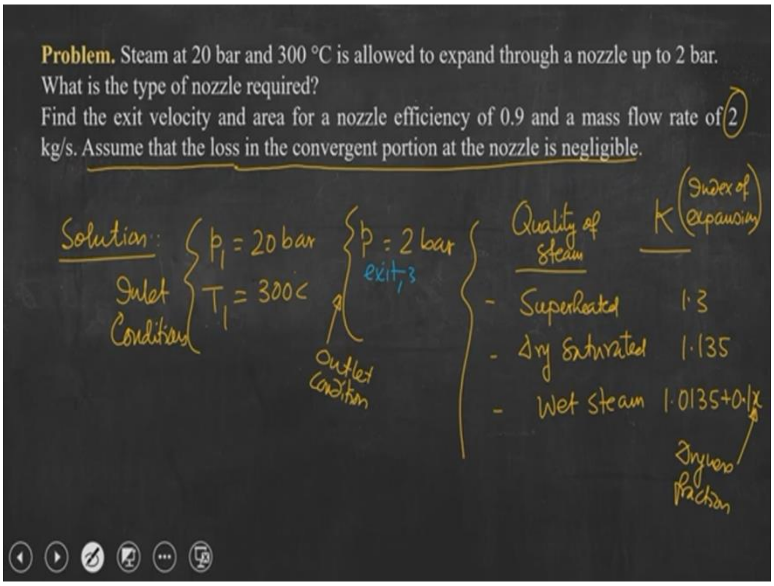
$$v_3' = x_3' \times v_g \text{ or } v_{g3}'$$

$$= 0.955 \times 0.886 = 0.8416 \frac{\text{m}^3}{\text{kg}}$$

So basically the specific volume of vapor at that pressure multiplied by the dryness fraction that will give the specific volume of steam at that particular point that is at the exit of the nozzle. So this 3' is the actual state of steam at the nozzle exit. So $0.8416 \frac{\text{m}^3}{\text{kg}}$ is the specific volume of steam at the exit of the nozzle.

So let me tell you once again that we can get specific volume of steam at that pressure from steam table. 3' is also located on the same pressure line. So at 3' we need to know this specific volume which is nothing but dryness fraction of that particular point multiplied by the specific volume of steam when the steam quality is saturated steam. So that is v_g and we have calculated it.

(Refer Slide Time: 27:31)



Now next we need to calculate what would be the exit velocity of steam and this would be again actual exit velocity of steam. In the last class we have already calculated what would be the exit velocity of steam c_3', if we ignore the velocity of steam at the inlet of the nozzle, assuming the fact that velocity of steam at the inlet of the nozzle is very, very small as compared to the velocity of steam at the exit of the nozzle. And our objective is to increase c_3' and if we recall that formula for it that is nothing but

$$c_3' = 44.2\sqrt{h_1 - h_3'}$$

So we have already calculated $h_1$and h_3' and if we plug in the values

$$c_3' = 44.2\sqrt{3022 - 2632.12}\,\frac{\text{m}}{\text{s}}$$

$$\Rightarrow c_3' = 883.04\,\frac{\text{m}}{\text{s}}$$

So the unit should be m/s and in fact I also have discussed the unit conversion. So after calculation we are getting exit velocity as 883.04 m/s. So can you imagine the velocity of steam at the exit of the nozzle in m/s? So the steam which will come out from the nozzle will be in the form of a jet. So that jet will now strike the turbine blade and it is because of this momentum change, some momentum will be absorbed by the shaft of the turbine wheel.

Now if we know the exit velocity, then we can write what would be area. So say exit area is A_3'.

$$A_3' = \frac{2 \times v_3'}{c_3'}$$

So if we go back to the problem statement then we are writing 2 because mass flow rate is given 2 kg/s. So we are writing this expression from the expression of mass flow rate of steam that is

$$\dot{m}_s = \frac{A_3' \times c_3'}{v_3'}$$

So this is the mass flow rate of steam. So basically it is area multiplied by velocity multiplied by density and instead of writing density, I have written the specific volume. So this is very straightforward formula. So this is mass flow rate of steam and that is given 2 kg/s. So I have written

$$A_3{}' = \frac{\dot{m}_s \times v_3'}{c_3'} = 19.1 \text{ cm}^2$$

So this is the answer. So let me tell you that 19.1 cm^2 is the area of the nozzle at its exit, $883.04 \frac{\text{m}}{\text{s}}$ is the velocity of steam at the exit and the type of the nozzle is convergent divergent type. So these 3 are the solutions for this particular problem.

So if we try to summarize, we have taken 1 example and we have seen that from the given data, how we can determine the type of a nozzle specific to that application then from the known data using the formula that we have derived till now, we could find out the velocity at the exit of the nozzle and area of the nozzle for this particular convergent divergent type of nozzle. So this is the last class of this module of this course. From next class onwards, we shall discuss about steam turbine. Thank you.