Thermal Engineering: Basic and Applied
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Lecture – 30
Mass Flow Rate of Steam in Nozzle, Critical Pressure Ratio

I welcome you all to the session of thermal engineering and today we shall discuss about the mass flow rate, in fact in the last class, we have started deriving the expression of mass flow of steam through a nozzle and then we will discuss about the critical pressure ratio.

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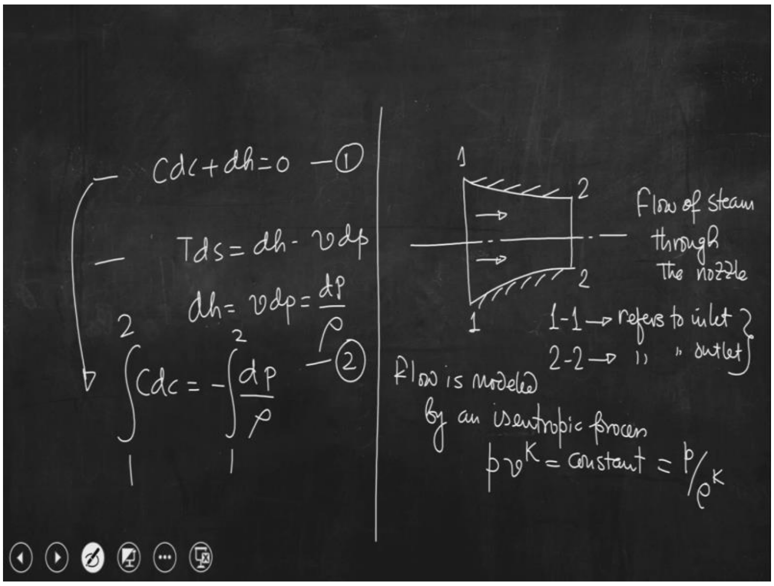

So, in the last class, we started our discussion on the mass flow rate steam through a nozzle. So there is the diagram of nozzle in the slide and there is a flow of steam through the nozzle and the section 1 1 refers to inlet and section 2 2 refers to outlet. So, let us again write the expression that we have discussed in the last class. So by applying the steady flow energy equation we could write

$$cdc + dh = 0 ----(1)$$

This equation is obtained from the steady flow energy equation applied between the section 1 1 and section 2 2. And we have assumed that the length of the nozzle is short, so that the change in elevation between these 2 sections can be neglected. And also we could write the property relation which is

$$Tds = dh - vdp$$

This is for any process at any state point. So, the flow of steam through the nozzle is modeled by an isentropic process. And that is represented mathematically by this equation that you have studied many times

$$pv^k = \text{constant} = \frac{p}{\rho^k}$$

Here k is index of expansion for the isentropic process that we have considered. Now if the process is isentropic we can write

$$dh = vdp = \frac{dp}{\rho} - - - - (2)$$

So, if we write the equation 2 in equation 1 then we can write

$$cdc = -\frac{dp}{\rho}$$

Now we can integrate between these 2 sections 1 and 2.

$$\int_1^2 cdc = -\int_1^2 \frac{dp}{\rho}$$

That means when the steam is flowing from section 1 1 to section 2 2, then the change in velocity that is from c_1 to c_2 and the change in pressure can be relatable. Since it is not a flow of an incompressible fluid rather the flow is of a compressible fluid, so, you know this pressure and density are related and that is what is modeled here like this, using this equation.

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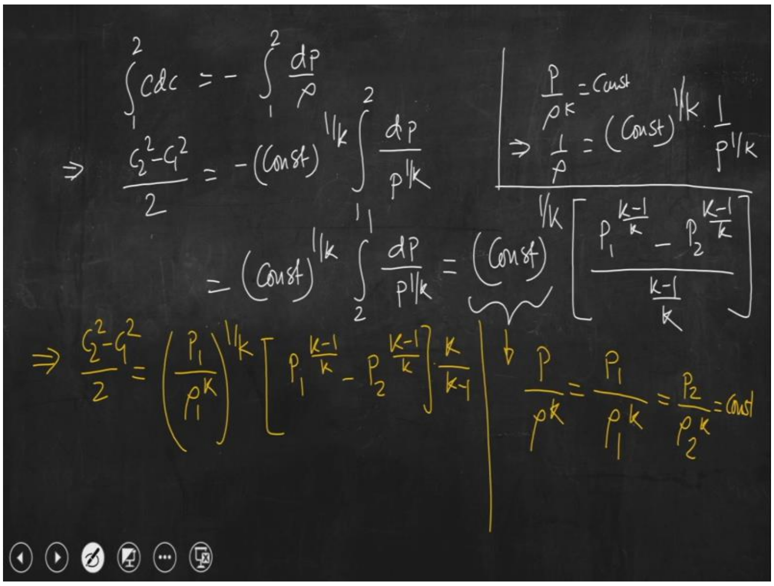

$$\frac{p}{\rho^k} = \text{constant} \Rightarrow \frac{1}{\rho} = (\text{constant})^{\frac{1}{k}} . \frac{1}{p^{\frac{1}{k}}}$$

$$\int_1^2 cdc = -\int_1^2 \frac{dp}{\rho}$$

$$\Rightarrow \frac{c_2^2 - c_1^2}{2} = -(Const)^{\frac{1}{k}} \int_1^2 \frac{dp}{p^{\frac{1}{k}}}$$

$$= (Const)^{\frac{1}{k}} \int_2^1 \frac{dp}{p^{\frac{1}{k}}}$$

$$= (Const)^{\frac{1}{k}} \left[\frac{p_1^{\frac{k-1}{k}} - p_2^{\frac{k-1}{k}}}{\frac{k-1}{k}} \right]$$

So, this is very straight forward and trivial as well. Now we can write

$$\frac{p}{\rho^k} = \frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} = const$$

So, if we go one step further we can write

$$\Rightarrow \frac{c_2^2 - c_1^2}{2} = \left(\frac{p_1}{\rho_1^k}\right)^{\frac{1}{k}} \left[p_1^{\frac{k-1}{k}} - p_2^{\frac{k-1}{k}} \right] \frac{k}{k-1}$$

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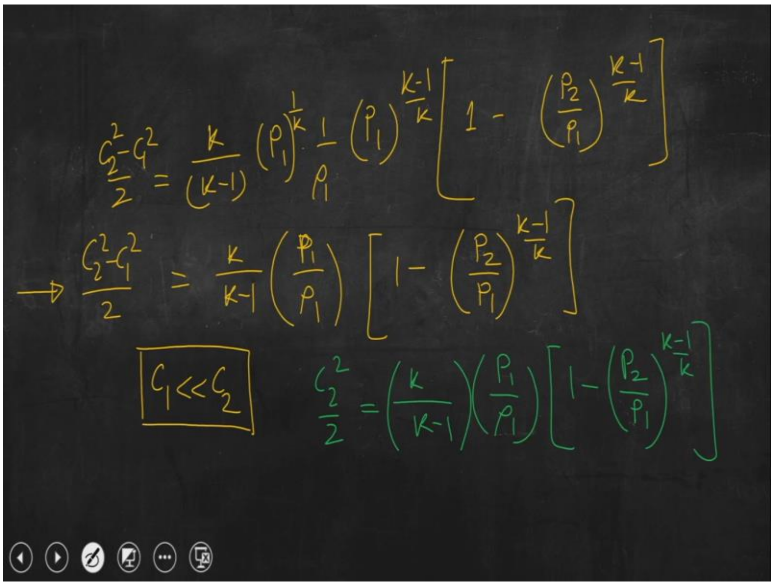

$$\Rightarrow \frac{c_2^2 - c_1^2}{2} = \frac{k}{k-1} (p_1)^{\frac{1}{k}} . \frac{1}{\rho_1} (p_1)^{\frac{k-1}{k}} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \right]$$

$$= \frac{k}{k-1} \left(\frac{p_1}{\rho_1}\right) \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \right]$$

So this is the final expression of change in velocity of steam as it passes through a convergent nozzle. So, if we go to the previous slide then we can see that the cross sectional area 2 2 is much less than the cross sectional area 1 1. So, if we calculate the velocity of steam at both sections, we will find that the velocity of steam at section 1 1 is much less than the velocity of steam which is there at section 2 2. So, from this order of magnitude analysis we can write

$$c_1 \ll c_2$$

$$\Rightarrow \frac{c_2^2}{2} = \left(\frac{k}{k-1}\right)\left(\frac{p_1}{\rho_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$$

So, this is the expression. So you know if we go one step further we can write

$$c_2^2 = \left(\frac{2k}{k-1}\right)\left(\frac{p_1}{\rho_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$$

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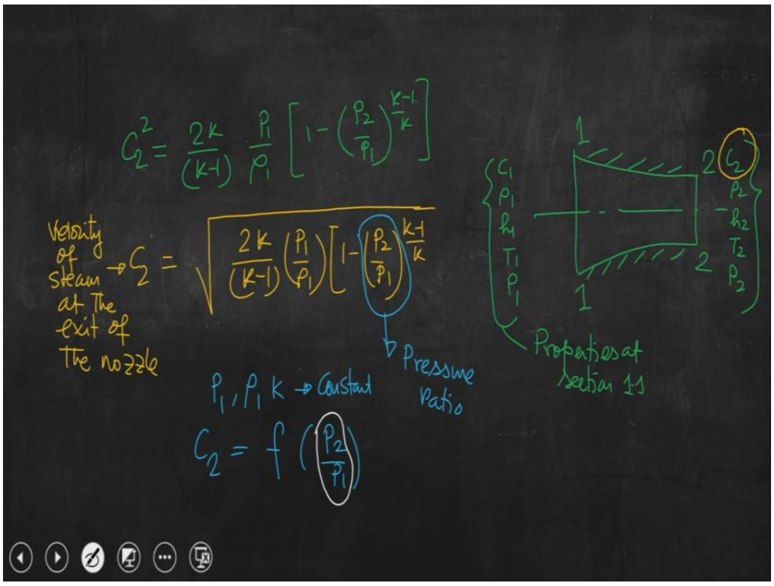

So from the diagram of a nozzle, there is the section 1 1 and section 2 2. The properties at section 1 1 are $c_1, \rho_1, h_1, T_1, p_1$. Similarly properties at section 2 2 are $c_2, \rho_2, h_2, T_2, p_2$. And we are interested in the velocity of steam at section 2 2 that is c_2.

$$c_2 = \sqrt{\left(\frac{2k}{k-1}\right)\left(\frac{p_1}{\rho_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]}$$

So this is the velocity of steam at the exit of the nozzle that is c_2. Once we have calculated the velocity of steam at the exit of the nozzle, we also can calculate the mass flow rate. Because we wanted to have the expression of mass flow rate of steam at the exit of the nozzle and that is very important. Now, from the expression that we have established today, we can see the velocity of steam at the exit of the nozzle.

So you can understand that $\frac{p_2}{p_1}$ is the pressure ratio. So this is the ratio of exit pressure to the inlet pressure. And we also can see k is the index of expansion coefficient. So from this we can understand that p_1, ρ_1, k are constant.

If this is the nozzle, then we know the steam pressure at the inlet to the nozzle because that is the pressure of steam at the exit of the boiler. If we can fix the temperature, pressure then density will be constant at that particular location. So if the pressure, density and the index of expansion; these 3 are constant at the inlet of the nozzle then we can see the exit velocity is function pressure ratio.

$$p_1, \rho_1, k \rightarrow constant$$

$$c_2 = f\left(\frac{p_2}{p_1}\right)$$

So under this condition, c_2 is only the function of pressure ratio, so why not to look at the value of $\left(\frac{p_2}{p_1}\right)$ for which the velocity of steam at the exit of the nozzle should be maximum.

Try to understand exactly what we have discussed in one of the previous classes that is the velocity of steam at the exit of the nozzle is very important and in fact the steam turbine is the assemblies of flow nozzle and turbine blades. When steam is entering into the flow nozzle, our main objective is to increase the kinetic energy of steam and that kinetic energy will essentially depend on the velocity of steam at the exit of the nozzle. From the expression that we have derived today, we can see that the velocity of steam at the exit of the nozzle is dependent on so many parameters like pressure, density at the inlet, index of expansion along with one particular ratio that is the pressure ratio.

p_1, ρ_1, k are constant at the inlet to the nozzle, because the pressure will be constant as that is the pressure of steam at the exit of the boiler. If we can maintain the temperature and pressure as constant, then probably density will remain constant. So now you can understand the velocity is the function of pressure ratio. Then why not to look at a particular pressure ratio for which the velocity will be maximum. And that should be the objective, when someone is designing the nozzle. So knowing the pressure of steam at the inlet of the nozzle, someone should try to calculate the pressure ratio means what would be the pressure range of operation for which the flow velocity at the exit of the steam should be maximum and in turn we will have the maximum kinetic energy of steam jet which will come out from the nozzle.

If we increase the kinetic energy of jet and if we can deflect that particular jet when the jet is striking the turbine blade, we can have higher momentum change and that momentum will be absorbed by the rotor of the turbine wheel and from there we can get work output. So objective should be now to look at the magnitude of this particular pressure ratio for which the velocity of the steam at the exit of the nozzle will be maximum.

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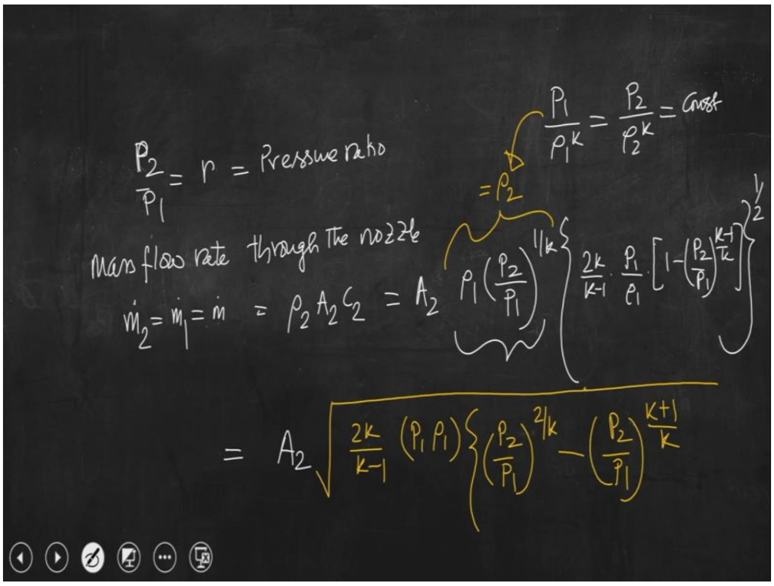

So now let us consider that

$$\frac{p_2}{p_1} = r = \text{Pressure ratio}$$

Our objective should be to find out the value of r rather the critical value of r, for which the velocity would be maximum. Let us first write the mass flow rate through the nozzle.

$$\dot{m}_2 = \dot{m}_1 = \dot{m}$$

So this is what we can write from the continuity equation.

$$\dot{m}_2 = \dot{m}_1 = \dot{m} = \rho_2 A_2 c_2$$

Now we know that

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} = const$$

$$\Rightarrow \rho_2 = \rho_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{k}}$$

So

$$\dot{m}_2 = \dot{m}_1 = \dot{m} = A_2 \rho_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{k}} c_2$$

Now putting the value of c_2

$$\dot{m}_2 = \dot{m}_1 = \dot{m} = A_2\rho_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{k}}\left[\left(\frac{2k}{k-1}\right)\left(\frac{p_1}{\rho_1}\right)\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right\}\right]^{\frac{1}{2}}$$

$$= A_2\sqrt{\frac{2k}{k-1}(p_1\rho_1)\left\{\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}}-\left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right\}}$$

So that is the expression of mass flow rate of steam at the exit of the nozzle. Again we can see that for the constant value of p_1, ρ_1, k mass flow rate of steam is also dependent on the pressure ratio p_2/p_1. And now we should try to find out what is the critical value of p_2/p_1, for which the velocity should be maximum, which in turn will give the maximum mass flow rate of steam at the exit of the nozzle. So now let us try to write the mass flow rate of steam per unit area.

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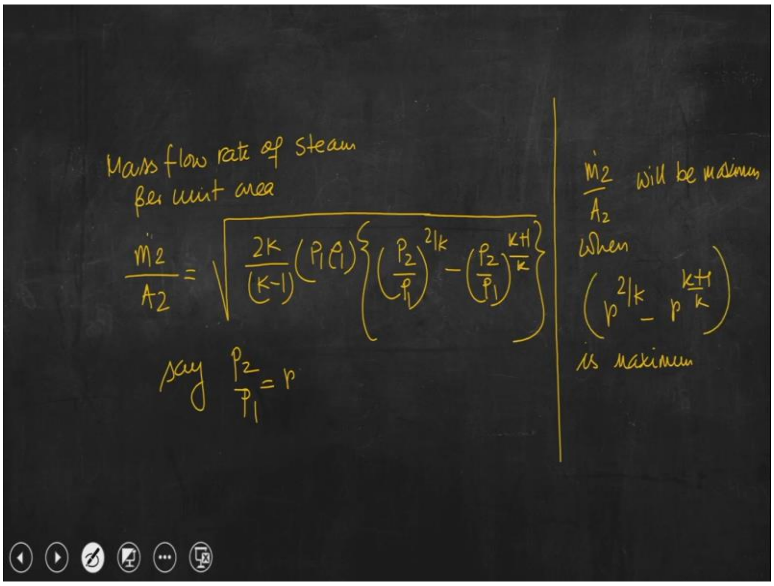

So mass flow rate of steam per unit area

$$\frac{\dot{m}_2}{A_2} = \sqrt{\frac{2k}{k-1}(p_1\rho_1)\left\{\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}}-\left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right\}}$$

So this is the expression of mass flow rate per unit area. And as I said that $\frac{p_2}{p_1} = r$, so we can try to find out the critical value of r for which is this mass flow rate per unit area should be maximum. Because area is constant so if this particular quantity becomes maximum, then mass flow rate also will be maximum for a constant exit area. So, if we try to find out the maximum value of $\frac{\dot{m}_2}{A_2}$ which in turn allow us to find out the maximum value of this quantity. So the pressure ratio for which the mass flow rate per unit area should be maximum. As I told you mass flow rate per unit area is maximum, because area is even a constant. The maximum value of this quantity for a particular value of $\frac{p_2}{p_1}$ is nothing but the maximum value of the quantity which is there in the bracket under root of the expression. So that means our objective is that $\frac{\dot{m}_2}{A_2}$ will be maximum when $(r)^{\frac{2}{k}} - (r)^{\frac{k+1}{k}}$ is maximum.

So to find out the maximum value

$$\frac{d}{dr}\left[(r)^{\frac{2}{k}} - (r)^{\frac{k+1}{k}}\right] = 0$$

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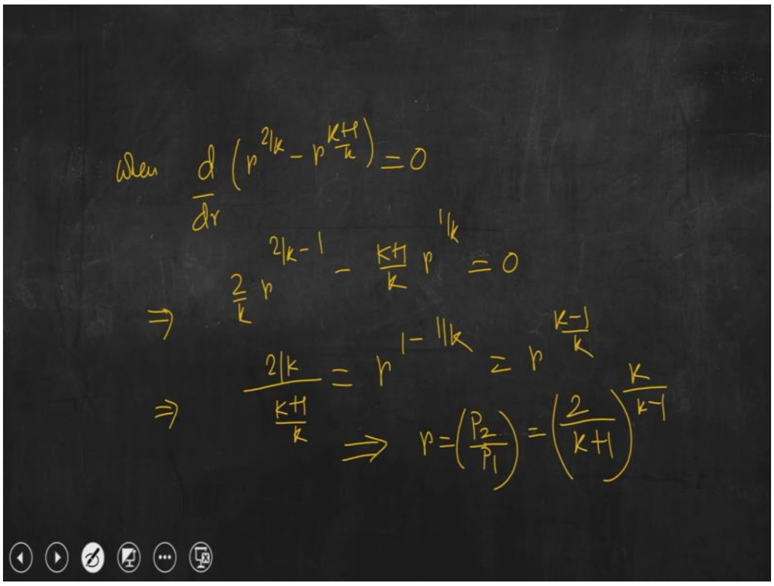

So again this is very trivial. If we do this then we can try to find out

$$\frac{2}{k}\, r^{\frac{2}{k}-1} - \frac{k+1}{k}\, r^{\frac{1}{k}} = 0$$

$$\Rightarrow \frac{2/k}{\frac{k+1}{k}} = r^{1-\frac{1}{k}} = r^{\frac{k-1}{k}}$$

$$\Rightarrow r = \frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

So, this is just the algebraic manipulation that you can do and you will find the value of r. So this is basically very important aspect that we have the value of r. So, this is the critical value of r which is nothing but the pressure ratio that is the ratio of exit pressure of steam to the inlet pressure of steam, for which the mass flow rate per unit area, which is again the mass flow rate of steam because area is constant, is maximum.

So our objective is whenever someone is designing the steam nozzle, the designer must be careful to find out the critical pressure ratio that is nothing but the ratio of exit pressure to the inlet pressure of steam when it is flowing through the nozzle, for which the mass flow rate of steam will be maximum. We shall discuss about the critical significance of the critical pressure ratio in the context of the nozzle operation together with the design aspects someone should consider while designing the flow nozzle from this particular expression. And that part we shall discuss in the next class. With this I stop here today, Thank you.