Thermal Engineering Basic and Applied
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Lecture – 29
Flow Analysis of Steam in Nozzle – Mass Flow Rate

I welcome you all to the session of thermal engineering basic and applied. Today we shall discuss the flow through a steam nozzle, in fact we have started our discussion on this topic in the last class. And if we can recall in the last class we could derive the expression for area Mach number relationship. So, that particular relation, which we have established in the last class is known as the area Mach number relationship of flow nozzle. So, let us try to write what we had derived in the last class.

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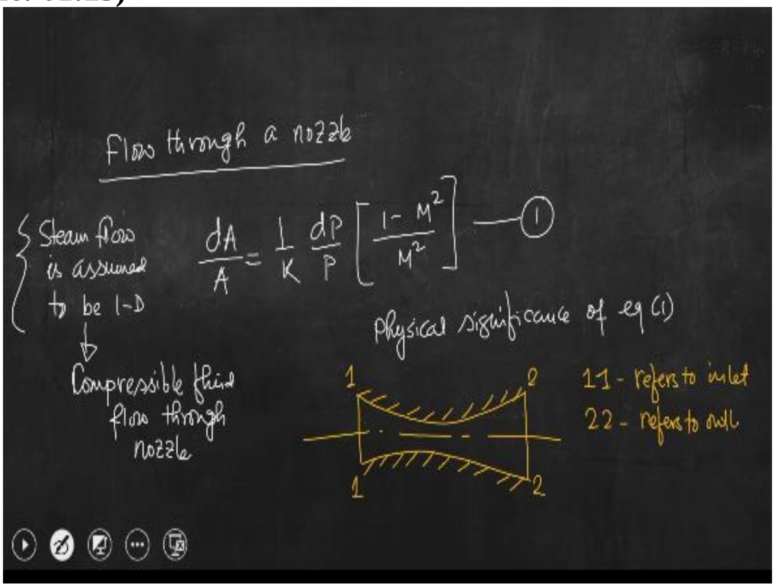

Flow through a nozzle

For the flow analysis which we had discussed in the last class, we had to take a few assumptions. What are those? The flow is frictionless, both internal and external. There is no heat interaction between the system and surroundings. If we assume that the flow is taking place through the nozzle, then walls of the nozzle are insulated. We had considered that the flow is 1 dimensional and we had also discussed that isentropic flow can be represented by an isentropic process. So in the last class, we could write the area Mach number relationship that is

$$\frac{dA}{A} = \frac{1}{k}\frac{dp}{p}\left[\frac{1-M^2}{M^2}\right] ------(1)$$

Here k is basically index of expansion for the isentropic process and the steam flow is assumed to be 1 dimensional and this is compressible fluid flow through nozzle.

Now let us discuss a few important points, in fact we can draw a few important conclusions from this particular equation. So first we shall discuss about the physical significance of equation 1. Today we shall try to have the expression of the mass flow rate through the nozzle or through the convergent divergent channel. But before that let us briefly discuss about the physical significance of equation 1, of course, concerning with the flow through a nozzle rather in the last class, we have assumed that it is the flow of a stream through a duct having varying area. So, the duct is drawn in the slide where, 1 1 refers to inlet and 2 2 refers to outlet.

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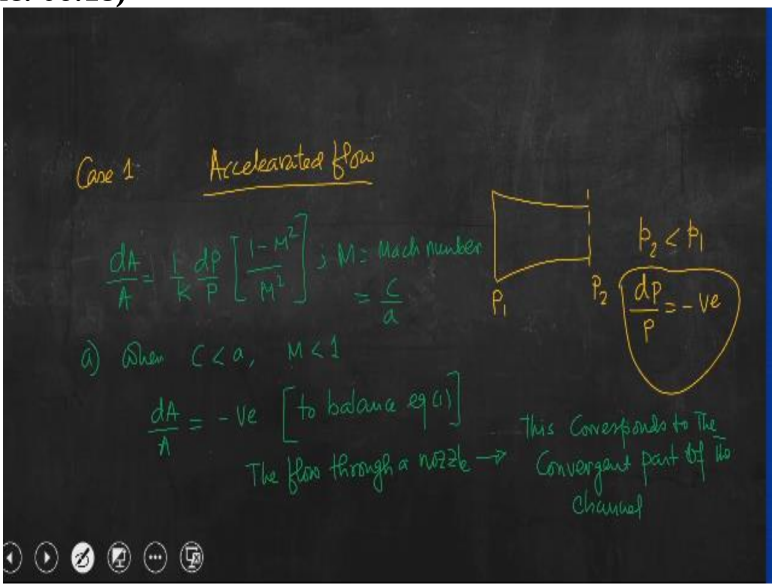

Case 1: Accelerated flow.

So, accelerated flow of steam through a duct having varying cross sectional area means it decreases initially then again increases. So, for flow is accelerated flow, if we have just drawn the diagram in slide. For the accelerated flow, we have learned from fluid mechanics course that in the direction of flow velocity will increase. If that is the case, then let us look into this particular configuration. So if we are considering this is the case so you know that this is P_1 and this is P_2. When the flow is accelerating that means velocity will increase in the direction of flow. So P_2 is much less than P_1 and $\frac{dp}{p}$ is negative.

So in the direction of flow, velocity will increase, that means pressure should fall and it implies that, $\frac{dp}{p}$ should be negative. Now if we recall the expression that is

$$\frac{dA}{A} = \frac{1}{k}\frac{dp}{p}\left[\frac{1 - M^2}{M^2}\right]$$

$$M = \text{Mach number} = \frac{c}{a}$$

So, this Mach number is the ratio of the flow velocity at any particular section at any given condition to the local sonic velocity at that section at that pressure and temperature. We have understood that if the flow is accelerated flow, then $\frac{dp}{p}$ is negative.

So $\frac{dp}{p}$ is negative and k is index of expansion for the isentropic process, now the role will be played by the Mach number. So now I am discussing the sub cases.

a) When $c < a$, that means $M < 1$.

If Mach number is less than 1 and $\frac{dp}{p}$ is negative, so to balance this equation $\frac{dA}{A}$ should be negative.

What does it indicate? If the flow is accelerated flow and if the local velocity at any particular section is less than the sonic velocity at given conditions, then we can see for this particular case, $\frac{dA}{A}$ must be negative. It indicates that it is a flow through a nozzle. In other words, this part this corresponds to the convergent part of the channel. So if we go back to this previous slide we can see that there is a channel having varying cross sectional area. Now if we look at this particular case, then this case corresponds to flow through the convergent part of the channel.

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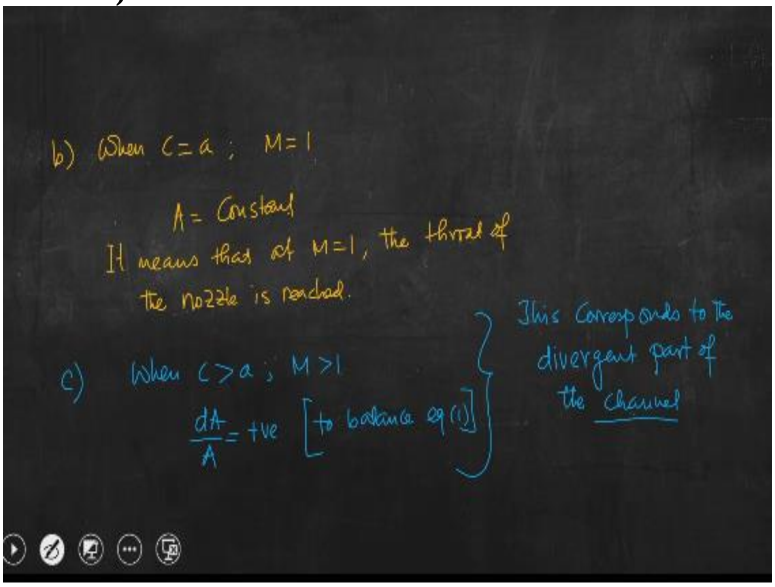

Now we go to the next sub case.

b) When $c = a$ that is $M = 1$.

When c = a, that means local fluid velocity at any particular section is equal to the sonic velocity at that section at that particular condition that is pressure and temperature. Then we can write that this is the accelerated flow, so $\frac{dp}{p}$ is negative. So to balance this equation

$$M = 1 \Rightarrow \frac{dA}{A} = 0$$

So we can see that $A =$ constant. So when $M = 1$ then $A =$ constant. So it means that when $M = 1$ the throat of the nozzle is reached. Perhaps you have studied this aspects when you have studied the compressible fluid flow part in fluid mechanics course. So you have seen that when $M = 1$ that is Mach number = 1, you can say that the flow has reached at the throat of the channel. Then let us move on to the last case of this particular part is that is accelerated flow.

c) When $c > a$ that is $M > 1$.

If you look at the equation, when Mach number greater than 1, then $\left[\frac{1-M^2}{M^2}\right]$ is negative. And $\frac{dp}{p}$ is already negative because the flow is accelerating. So the consequence of these would be that $\frac{dA}{A}$ is positive. If $\frac{dA}{A}$ is positive that means area increases. So that means, this corresponds to the divergent part of the channel. So that is the convergent divergent channel. So, we had considered flow through a channel having varying area. And we have considered 3 different cases. Now let us look into another important case that is case 2.

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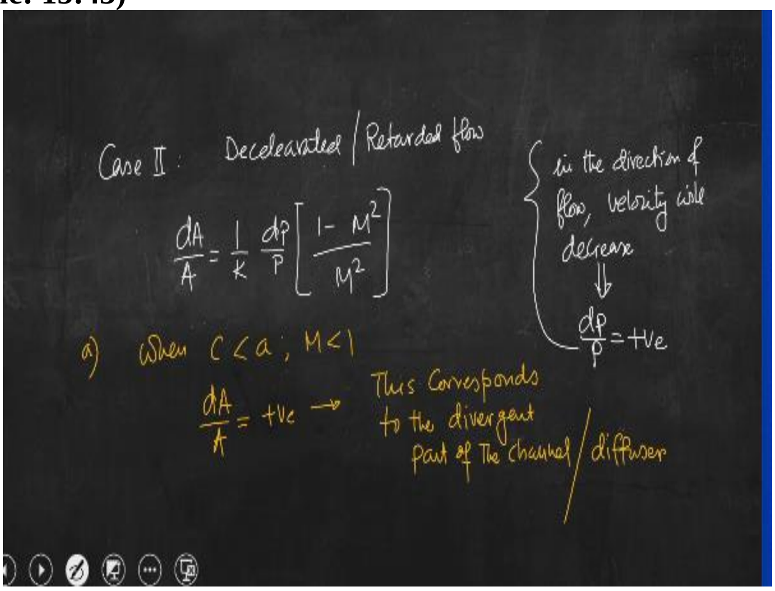

Case 2

So this is the decelerated flow or retarded flow. In this case you can understand flow is decelerating, that means in the direction of flow, velocity will decrease. This implies that $\frac{dp}{p}$ is positive. Now let us, again write the area Mach number relationship

$$\frac{dA}{A} = \frac{1}{k}\frac{dp}{p}\left[\frac{1 - M^2}{M^2}\right]$$

$$M = \text{Mach number} = \frac{c}{a}$$

So let us now discuss about several sub cases.

a) When $c < a$ that is $M < 1$

If Mach number is less than 1, then $\left[\frac{1-M^2}{M^2}\right]$ quantity is positive. Now $\frac{dp}{p}$ is positive. It implies that $\frac{dA}{A}$ is positive that means this corresponds to the divergent part of the channel. Let me tell you that it is diffuser. When velocity decreases in the direction of flow, it is not a nozzle rather it is the diffuser by definition. So diffuser is again a mechanical device and the flow configuration is such that in the direction of flow velocity will decrease and pressure will increase..

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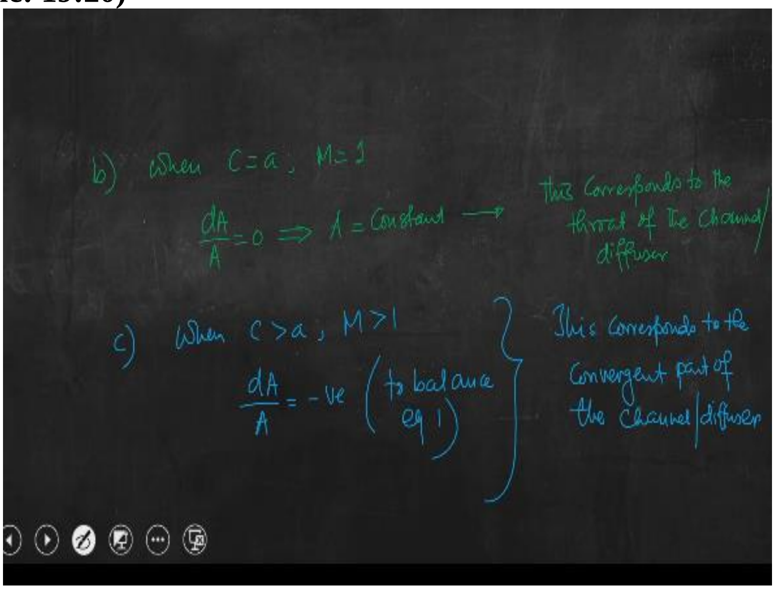

Now let us discuss about another 2 sub cases.

b) When $c = a$ that is $M = 1$

If Mach number = 1 then $\frac{dA}{A}$ = 0. So it indicates that A= constant and this corresponds to the throat of the channel/ diffuser.

c) When $c > a$ that is $M > 1$

See If Mach number greater than 1, then $\left[\frac{1-M^2}{M^2}\right]$ quantity is negative. And $\frac{dp}{p}$ is positive pertaining to this case. So $\frac{dA}{A}$ will be negative; essentially this is to balance equation 1 and this corresponds to the convergent part of the channel/ diffuser. So that means simply we can say that if this is the case, then diffuser would be of convergent type.

So these are the cases and if we try to summarize then basically you know that we have discussed several cases by considering several values of Mach number or several regimes of Mach number that is whether Mach number is greater than 1 or less than 1 or equal to 1 for 2 different cases that are case-1: it is accelerating and case-2: it is decelerating. So now let us try to summarize whatever we have learned in the form of a table.

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Type of flow	Accelerated flow Falling pressure	Decelerated flow Rising pressure
Subsonic flow (M<1)	Convergent nozzle	Divergent diffuser
Supersonic flow (M>1)	Divergent nozzle	Convergent nozzle
Sonic flow (M=1)	A=constant at the throat of the nozzle	A=constant at the throat of the diffuser

So let us write like this type of flow then accelerated flow that is falling pressure and finally we have another category that is you know retarded flow rising pressure so we can see now type of the flow is a subsonic let me write here using in another color so subsonic flow that is M less than 1 if it is subsonic flow we have discussed if the flow is accelerating then this is the convergent part of the nozzle let us go into the first case accelerated flow subsonic flow that is M less than 1 that is the convergent part of the channel.

What do you mean by convergent part of the channel? We do understand if the channel is having convergent that is by common name known as nozzle so this is the nozzle. So let me write here so this is the convergent channel or I am writing nozzle so this is the flow direction and if the flow is subsonic for the retarded flow rising pressure, we have seen here that it is the divergent part of the diffuser so divergent part of the channel is that itself is the diffuser.

So I am, writing divergent diffuser, so this is the case, this is the flow direction, arrow indicates the direction of flow now if the flow is supersonic that is M greater than 1, if M greater than 1 we have seen from case 1, when M greater than 1 that is the divergent part of the channel divergent part of the nozzle so basically we can say that is divergent nozzle so this is divergent nozzle and so this is the flow direction and when the flow is supersonic pertaining to this case that is retarded flow we have seen that this is the convergent part of the channel or diffuser.

So, this would be convergent diffuser so this should be convergent diffuser and the shape would be like this, so this is the flow direction. Now last one is the sonic velocity sonic that is M = 1 so that is what we have seen A = constant at the throat of the nozzle so that is this part area = constant and this is also A = constant and at the throat of the diffuser so this is the flow direction

So here we have summarized in the tabular form of what we have discussed pertaining to the equation that we have derived in the last class.

So having this discussion, now we can see that if the flow is subsonic and the duct is convergent, then we can tell something more from this particular table. So if the flow is subsonic, duct is convergent type, then it is definitely nozzle. If the flow is subsonic, duct is of divergent type, then it is diffuser. So in the table I have written convergent nozzle and divergent nozzle. Because we have studied that we call a channel as nozzle, when its area decreases in the direction of flow. If the flow is supersonic and duct is convergent type, then it is diffuser. So just opposite, if the flow is supersonic and duct is of divergent type, then it is the nozzle. But for the sonic velocity, it is obvious that the throat is reached.

So with this I have completed the physical significance. Rather we have tried to discuss about the physical significance of the relation which is also known as the area Mach number relationship of the nozzle, when there is a flow of steam through the nozzle.

Now, let us look into the mass flow rate of the nozzle. Because our objective should be to get the area at the exit of the nozzle, so that the desired flow velocity of steam can be attained. Now to understand the velocity at the exit of the nozzle, let us now try to look at the expression of mass flow rate through the nozzle. Let me tell you that instead of the volume flow rate, here we are referring to the mass flow rate. Because we are considering flow of stream which is considered to be the compressible fluid.

Basically from the last part, we have seen the effect of same duct. So it is the convergent duct, which is behaving just like a nozzle, when the flow is subsonic. And the same convergent duct can be used as the diffuser, when the flow is changing from subsonic to the supersonic. So let me write here one important conclusion that the effect of same duct may be reversed depending on the behavior of the fluid and flow. So this is very important. So we can use the same duct, sometimes as the nozzle and sometimes as the diffuser.

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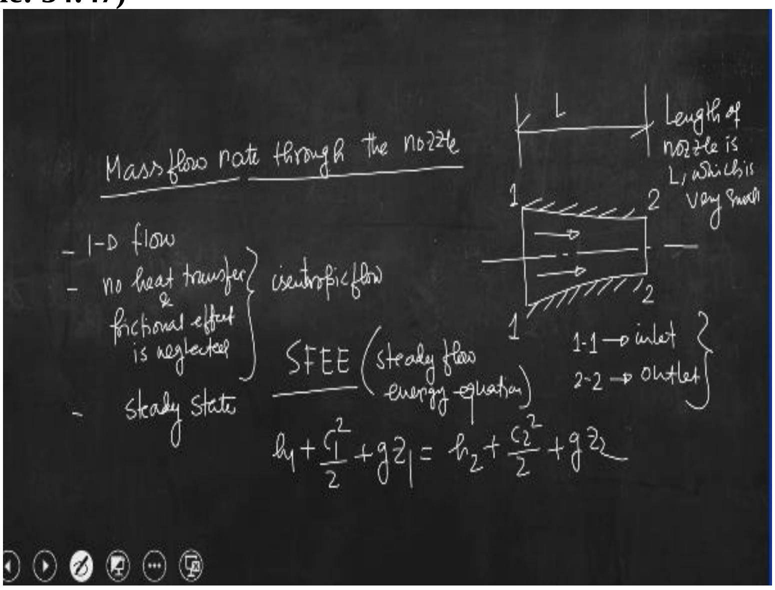

Mass flow rate through the nozzle

So now let us, discuss about the mass flow rate through the nozzle. We have considered the flow through a nozzle; 1 1 refers to inlet, 2 2 refers to outlet. And we are again considering the same set of assumptions that we had taken to establish the area Mach number relationship.

Assumptions

- The flow is 1 dimensional
- There is no heat interaction between the system and surroundings. So when there is a flow of steam through the nozzle, there is no heat loss from the flowing stream through the walls of the nozzle to the surroundings.
- Frictional effect is neglected

So we can assume we can model the flow by the isentropic flow process.

- Steady state analysis

Now we apply the steady flow energy equation between section 1 1 and section 2 2. We are assuming that there is no heat interaction and work interaction, so you can write

$$h_1 + \frac{c_1^2}{2} + gz_1 = h_2 + \frac{c_2^2}{2} + gz_2$$

You can understand that when we are writing this equation, we have not considered heat and work interaction. So because we have already assumed that there is no heat interaction and this is not a work interacting device. But you can argue, because we need to have some work to maintain the flow in the presence of pressure and that work done is already taken into account in this particular term that is enthalpy. Now assume that nozzle the length (L) is very small. There is little elevation difference. But if the length is very small, we can ignore the change in elevation that is $z_1 - z_2$.

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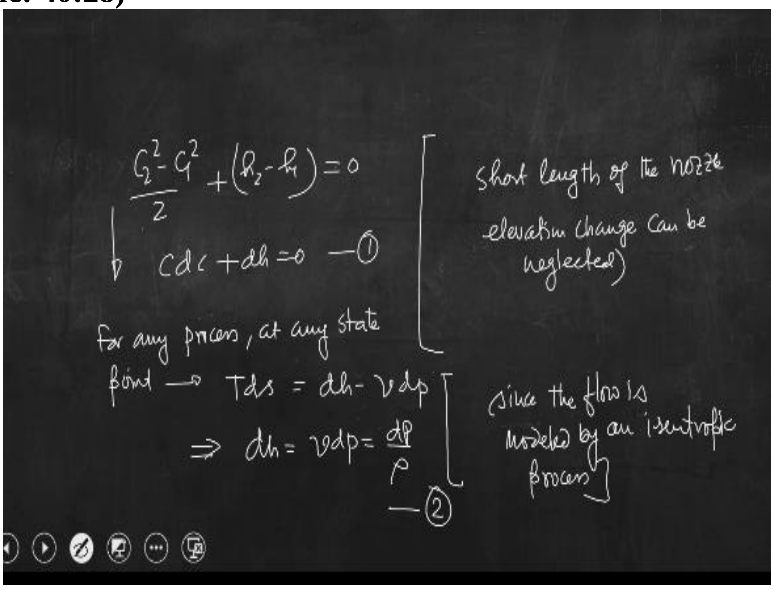

Now we can write

$$\frac{c_2^2 - c_1^2}{2} + (h_2 - h_1) = 0$$

So we are assuming the short length of the nozzle here, so elevation change can be neglected. In the last class, I have mentioned that nozzles are placed in a plane where the elevation is very small. Even if there is a small elevation change between these 2 sections that is inlet and outlet of the nozzle, since the length is very small, we can ignore the change in that. So that is why the change in elevation can be neglected. So we can write from this equation that

$$cdc + dh = 0 - - - - (1)$$

We have learned from thermodynamics that for any process, at any state point, we can write

$$Tds = dh - vdp$$

So this is the second Tds equation that you have learned from basic thermodynamics course. Now we can write this for any process, in fact we also can write $Tds = dh - vdp$ for any process. So for any process this equation can be written at any state point.

Now we have tried to model the flow by an isentropic process. Since the flow is modeled by an isentropic process so we can write

$$dh = vdp = \frac{dp}{\rho} - - - - (2)$$

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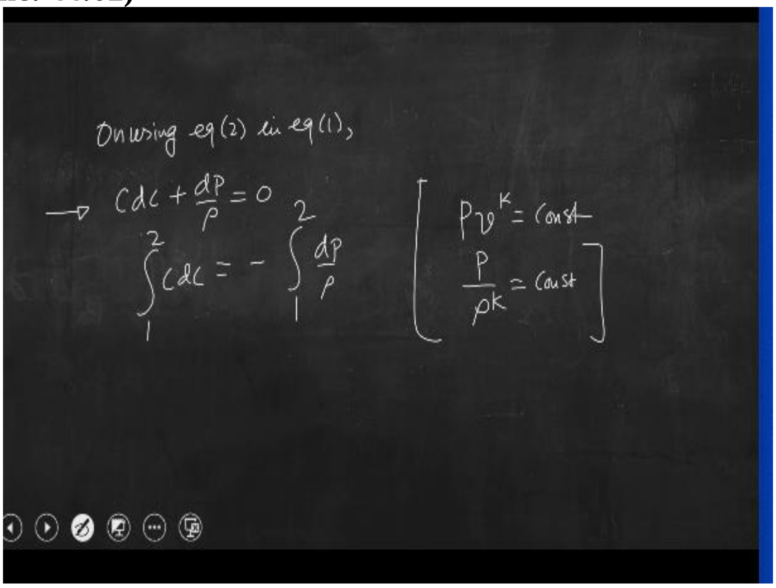

So on using equation 2 in equation 1 we can get

$$cdc + \frac{dp}{\rho} = 0$$

Now we can integrate this equation from section 1 to 2 then, we can relate the change in velocity of steam as it moves from the inlet section to the outlet section of the nozzle. So eventually we have this equation and if we try to integrate this equation, we can write the change in velocity of steam as it moves from section 1 to section 2. And we can relate it in terms of the pressure and density. So from there, we can at least get the expression of velocity at the exit of the nozzle.

Why we need to have the flow velocity at the exit of the nozzle? Because we are trying to figure out the expression of mass flow rate through the nozzle and the mass flow rate of nozzle is important at the exit of the nozzle. See mass flow rate at the exit of the nozzle should be function of density, velocity and the cross-sectional area.

Now the velocity at the exit of the nozzle is very important because that velocity will dictate the energy of steam that we are having before it strikes the blades. So that is why it is very important. So let us again process from this equation

$$\int_1^2 cdc + \int_1^2 \frac{dp}{\rho} = 0$$

$$\Rightarrow \int_1^2 cdc = -\int_1^2 \frac{dp}{\rho}$$

See we need to relate pressure and density. Basically we know that for isentropic process

$$pv^k = \text{constant} \Rightarrow \frac{p}{\rho^k} = \text{constant}$$

So if we plug in the value of ρ from this expression into this equation, then we can perform this integration and we can have the change in flow velocity of steam as it moves from the inlet section to the outlet section. And starting from this particular part, we shall try to calculate the mass flow rate of the stream at the exit of the nozzle and that part we shall take up in the next class. So with this I stop here today and we shall continue our discussion from this particular part in the next class. Thank you.