Thermal Engineering Basic and Applied
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Lecture – 21
Problems of Steam Power Cycle - Cont.

I welcome you all to the session of thermal engineering and today in continuation of my last lecture, we shall solve another problem from the steam power cycle. So, the problem that we are going to solve today is based on the ideal reheat Rankine cycle. So, let us first read out the problem. Then we will discuss the solution procedure.

A steam power plant that operates on an ideal reheat Rankine cycle between the specified pressure limits is considered. The pressure at which reheating takes place, the total rate of heat input to the input in the boiler and the thermal efficiency of the cycle are to be determined. Given that boiler is working at 15 MPa, condenser pressure is 10 kPa, intermediate reheating pressure is 2.15 MPa, and steam temperature at the exit of the boiler is 500°C.

So, with this given data set, we need to calculate the heat input to the boiler. So, we need to calculate 1) heat input to the boiler, 2) thermal efficiency of the cycle. So, it is an ideal reheat Rankine cycle. So, if you try to revisit what we have discussed in the reheat Rankine cycle is that it is the modification of the Rankine cycle. We have seen many modifications like increasing the boiler pressure, reducing the condenser pressure and super heating steam beyond the saturated vapor state. I mean all these modifications are considered essentially for the increase in the thermal efficiency of the system, but we have also discussed about several demerits associated with these modifications. And out of these three important modifications, we could discuss superheating steam beyond point 3 that is the state point at the inlet to the turbine.

If we can superheat the steam, efficiency can be increased and efficiency will not increase at the cost of the input energy to the boiler, but at times we also can increase the

quality of steam at the exit of the turbine. But the issue with this is the metallurgical consideration of the blade of the turbine. So, if such a turbine blade material is available, which can withstand such high temperature without inviting any operational issue like generation of thermal crack, then superheating steam could be an ideal way of increasing the thermal efficiency of steam power plant. But it is because of this restriction the concept of reheat Rankine cycle came into the picture and I am not going to discuss the detail of this, but let us first draw the schematic.

So q_{in} amount of energy is added to the boiler. Here we are considering per unit mass flow rate of the working substance. And then we have high pressure stage turbine and after it does work in the high pressure stage turbine, steam is taken to the low pressure stage turbine and eventually we are getting w_{out}. So, then there is low pressure stage turbine and finally, from the low pressure stage turbine, steam is taken to the condenser, wherein heat is rejected to the media which is circulated and then the collected condensate is pumped back to the boiler. So, in the pump, we need to give energy input in the form of work. So, basically this is the schematic depiction of the reheat Rankine cycle. Now, let us draw the T-s plane because we have seen that if we can represent all the processes in T-s plane then we can easily calculate the energy interaction either in terms of heat or work between system and the surroundings.

So, basically you know that q_{in} amount of heat is added to the boiler and at the cost of that input energy we are getting work output w_{out}. This is not the net work output because this is the work output we are getting from the turbine, but we also need to give some input work w_{in} to the pump for its operation. So, now, so we have to understand that $w_{net} = w_{out} - w_{in}$. And to obtain that net work, we are supplying q_{in} amount of heat. So, from there we can calculate the efficiency.

But only thing is that if we try to recall, then we have discussed in the context of this course that we have applied steady flow energy equation. So, if the steady state steady flow equation is applied to all the devices which are there like boiler, high pressure stage turbine, low pressure stage turbine, condenser and pump, then we can figure out the energy interaction between system and surroundings. So, basically if it is a work interacting device, we can easily calculate the work either added or work extracted from

the system in terms of the enthalpies. Similarly, if it is a heat interacting device, whether heat is added to the system or heat is rejected from the system, it can be calculated in terms of the enthalpies.

So, let us quickly draw the T-s diagram that will help us to proceed further. So, the condenser pressure that is $P_{condenser}$ and P_{boiler} line is drawn on the T-s plane. Now we can plot state point 1-2 then 2-3. So 3 is the state at the exit of the boiler. Now, from 3, we have learnt from previous discussion that steam is allowed to expand in the high pressure stage turbine up to an intermediate pressure and that pressure is given 2.15 MPa. And then it is again reheated. After doing some work, steam is taken back to the boiler and it is allowed to pass through this heating coil wherein again upon receiving heat, it is heated up to the temperature of the initial superheat. So this process is shown in T-s plane as 3- 4, 4 -5 and 5- 6. So, this is the T-s diagram.

One thing which is not given in the problem statement. We are taking steam and what is done here is that steam is allowed to go into the high pressure stage turbine, after doing some work it is taken back to the boiler for further reheating and it is reheated at an intermediate pressure. So, basically when steam is doing work in the high pressure stage turbine, pressure will fall. So, this expansion is allowed up to an intermediate pressure that is 2.15 MPa. So, that means $P_4 = P_5$. These two points are on the same pressure line. And you know that steam is expanded up to state point 4 and then again upon receiving heat in the boiler, it is reheated, but it is not given up to what temperature it is reheated. So, this is very important, if nothing is given in the problem statement then we have learnt that usually steam is reheated up to the initial super heat temperature. So, you can understand 3 is the superheat temperature before it enters into the turbine. So, the reheating is continued until the temperature becomes equal to the initial superheat temperature that is T_3. So, it is very important that $T_3 = T_5$. And that is what we have learnt from the reheat Rankine cycle.

If this reheating temperature is given, that is well and good, but if it is not given then we have to assume that this reheating is done up to the initial superheat temperature that is T_3 and when it enters into the low pressure stage turbine, it again expands isentropically up to the state point 6 and it does work. The turbine blades in both the high pressure stage

and low pressure stage turbines are connected to a common shaft. So, from there we are getting work output. So, this is what is very important. Now, let us move to see the calculation steps.

You can understand this easily from our previous discussion as well as the problem, we have solved in the last class. Point 1 is on the saturated liquid line and that is 10 kPa.

$$P_1 = P_6 = 10 \text{ kPa}$$

$$h_1 = h_{f@10kPa}$$

And that you can easily get from steam table. I am not going to show this steam table today. You just need to go to the saturated water pressure table or temperature table. Let me tell you one thing here pressure is given 10 kPa. So, we can easily calculate the enthalpy at state point 1 that is nothing, but enthalpy of saturated liquid corresponding to 10 kPa. Instead of pressure, if temperature is given, then also you can calculate enthalpy at state point 1 which is saturated enthalpy corresponding to that temperature. This is very important that we can understand point 1 is on the saturated liquid line.

$$h_1 = h_{f@10kPa} = 191.81 \frac{\text{kJ}}{\text{kg}}$$

$$v_1 = v_{f@10kPa} = 0.00101 \frac{m^3}{kg}$$

Though all the processes, which I have tried to show in this T-s plane are not drawn as per scale, but still if you look at the T-s diagram you can see that temperature at point 2 is slightly higher than temperature at point 1. So, let us draw again. So, basically there is the constant pressure line and there is another compression line. Point 1 & 2 are plotted on those line, so that the temperature at point 2 is greater than temperature at point 1.

The state at 1 is saturated liquid and that is again incompressible. So, we are pumping. We are pressurizing that liquid to develop pressure, but this is incompressible. So, you know rate of volumetric dilution is 0, we cannot compress it. Even then temperature is increasing, because of the viscous dissipation. So, this pump is not a heat interacting device, this is a work interacting device and that too the process which we have seen is

isentropic process and you can see from this T-s plane that it is reversible adiabatic process.

So, long as the process is reversible, entropy generation is equal to 0 and the process is not interacting with the surrounding in terms of heat. So, when water is taken to the pump and by receiving energy in terms of work, its pressure is increased, but it is not interacting with the surroundings in terms of heat. Even then we can see a slight increase in temperature that is because of this viscosity. So, basically we can calculate that work pump

$$w_p = v_1(P_2 - P_1)$$

So, this is vdp it is not pdv. So, again let me discuss something that whether the process is reversible adiabatic or the process is reversible isothermal, as long as the process is steady state steady flow process, irrespective of the process, the work done expression is –vdp..

$$w_p = v_1(P_2 - P_1) = 15.14\frac{kJ}{kg}$$

This is because we are trying to calculate quantities per unit mass flow rate of the working substance.

$$h_2 = h_1 + w_p = 191.81 + 15.14 = 206.95\frac{kJ}{kg}$$

As I told you that if we apply steady state steady flow equation to several processes, which are getting executed in different devices, we will be getting energy interaction whether it is work interaction or heat interaction in terms of the enthalpies. So, we have calculated enthalpy at state point 2, and at state point 1. So, essentially if we can calculate enthalpy at several state points, then everything else we can calculate. So, next target should be to calculate enthalpy at state point 3, 4, 5 and 6.

State point 3

$$T_3 = 500°C; \; P_3 = 15 \text{ MPa}$$

Again from steam table, we can find the value of T_g.

If $T_{g@15MPa} < T_3$; Then point 3 will be in the superheated region

If $T_{g@15MPa} = T_3$; Then point 3 will be on the saturated vapour line

So, now it is in the superheated regime because this is very intuitive. So if we know that the point 3 is in the superheated regime, then by looking at the superheated steam table at that pressure, we can calculate enthalpy.

So from superheated steam table, $\quad h_3 = 3310.8 \dfrac{kJ}{kg}$ and $s_3 = 6.3480 \dfrac{kJ}{kg.\,K}$

If you look at steam table, you will find $T_{g@15MPa} < T_3$, it implies point 3 is in the superheated regime. So, you can calculate h_3 & s_3 from the superheated steam table. So, we have also calculated s_3. Now why we need to take the information about entropy at that point? If we go back to the previous slide, you can see that process 3-4 is isentropic process. Now our objective is we need to calculate enthalpy at all points. So we have calculated enthalpy at point 3 and we have to calculate enthalpy at state point 4. For that what we know? We know P_4 because this expansion process takes place up to an intermediate pressure that is 2.15 MPa. Since these two points 4 and 5 are on the same pressure line, so naturally $P_4 = P_5 = 2.15$ MPa. We have seen from the state postulate that if we try to place this particular state point in T-s S plane, we also need to know another important intensive independent properties. We have calculated entropy at state point 3 from the process that we have mapped in T-s plane. The process 3 -4 is isentropic process. So $s_4 = s_3$ and we know already P_4. So, we know P_4 from the process and we can see that $s_4 = s_3$, so we can easily get s_4 as well. So, knowing s_4 & P_4 we can calculate h_4. By knowing only P_4 we cannot calculate h_4.

State point 4

So the process 3-4 is an isentropic process and

$$s_4 = s_3 = 6.3480 \frac{kJ}{kg.K}$$

And given that, $P_4 = 2.15$ MPa

Now, let me tell you one important point. We know the pressure, so again we can go to the superheated table and initially we can go to the saturated water pressure table. By looking at the saturated water pressure table at that pressure, we need to find out s_g. So, I am telling you the procedure. So, basically at this pressure if we try to find out s_g that is entropy of this saturated vapor and if we find that the entropy $s_4 > s_g$ then there is no doubt that point 4 will be in the superheated region. Let me tell you once again. See we have drawn it, in fact, I have drawn it without knowing whether point 4 will be in the superheated region or lie on the saturated vapour line. We had no clue before to place this particular state point in T-s plane. But now time has come, because we know pressure. So, at that pressure if we try to find out the saturated entropy from the steam table; even by looking at the saturated water steam table, if we calculate s_g and if $s_4 > s_g$, then we can ascertain that point 4 should be in the superheated regime. So, step is to check $s_{g@2.15MPa}$, and if $s_4 > s_g$, then point 4 will be in the superheated regime. So I have drawn it correctly. As $s_4 > s_g$, so this is in the superheated regime.

Now, if we know s_4, then we also can calculate h_4.

$$h_4 = 2817.3 \frac{\text{kJ}}{\text{kg}}$$

Let me tell you another important point.

If the case is like this point 4 is on the saturated line, then $h_4 = h_g$ corresponding to that pressure.

Since, point 4 is in the superheated regime, we can directly calculate it from the superheated table.

A case may be when point 4 will be inside the vapor dome, because it all depends on the designer who is designing the turbine. depending on the requirement of the system. So, if point 4 lies inside the vapour dome, then how can we calculate enthalpy at that point? So, if we know that point 4 is inside the vapour dome, then

$$s_4 = s_f + x_4 s_{fg}$$

In that case you know the total entropy at point 4, because you got it from s_3. You need to calculate s_f & s_{fg} at the pressure at which the point 4 is defined, that is at P_4. So, basically I am discussing if point 4 is in vapour dome. If the case is like this, then we have to calculate first quality at state point 4 and once we calculate quality at state point 4 then we can go to calculate enthalpy h_4 by using following equation.

$$h_4 = h_f + x_4 h_{fg}$$

The common sense is that if you continue expansion in the high pressure stage turbine up to the point which is 4 and if point 4 is inside the vapor dome then again there will be some phase change heat transfer and again it would be heated in the superheated region. So, it is not advisable to design in such a way that point 4 would be inside the vapour dome. You may think why? This is of course from the consideration of the thermal efficiency of the plant. The best design would be not to allow expansion in the high pressure stage turbine to go up to 4 and 4 should be inside the vapor dome. So, we have calculated h_4 & h_3. So, we have calculated enthalpy up to state point 4.

State point 5

Next is h_5 and you know it is again important. So, it is given now that

$$P_4 = P_5 = 2.15 \text{ MPa}$$

I was talking about state postulates few minutes back that means only one property is not good enough to calculate other properties from steam table, until and unless you know at least another property. So, what we need is another property. So, we need to go back to the problem statement. It is given that $T_3 = T_5$. So, that means, steam is taken back to the boiler and it is reheated again up to the initial superheat temperature that is T_3. So, that means

$$T_5 = T_3 = 500°C$$

Now, again from the superheated table, we can find out the temperature at $P_5 = 2.15$ MPa pressure that is T_g. And if $T_g < T_5$, then point 5 will be in the superheated region. So, from the superheated table we can write

$$h_5 = 3466.61 \frac{kJ}{kg}$$

So, at that pressure we can find out what is T_g and then if $T_g < T_5$ then definitely point 5 will be in the superheated regime and we can go to the superheated table to calculate h_5. So, ideally we also have calculated h_5 and next is h_6.

State point 6

So you know that process 5-6 is an isentropic process.

$$s_5 = s_6 = 7.3988 \frac{kJ}{kg.K}$$

So, we can calculate s_5 from the steam table. By calculating s_5 we also can calculate s_6. Now, if we go to the T-s plane we can see that the point 6 is inside the vapour dome.

$$s_5 = s_6 = s_{f@10kPa} + x_6 s_{fg@10kPa}$$

$$\Rightarrow 7.3988 = 0.6492 + x_6 \times 7.4996$$

$$\Rightarrow x_6 = 0.9$$

Now, since we have calculated x_6, we also can calculate the value of h_6.

$$h_6 = h_{f@10kPa} + x_6 h_{fg@10kPa}$$

$$= (191.81 + 0.9 \times 2392.1) \frac{kJ}{kg} = 2344.7 \frac{kJ}{kg}$$

We have calculated $h_{f@10kPa}$ & $h_{fg@10kPa}$ in the beginning, when we have calculated pump work. It is very important to mention here that always you should write unit of all

these quantities. So, finally, we have also calculated h_6. So, now calculating enthalpy at every state point, next we can easily calculate the rate of heat supply.

Rate of heat supply

If we calculate per unit mass flow rate of the working substance, then rate of heat supply is as per the below expression.

$$\text{Rate of heat supply} = (h_3 - h_2) + (h_5 - h_4)$$

Why this expression? Because if we go back to the schematics, we can see that we are supplying $(h_3 - h_2)$ initially and again steam is entering at state point 4 and leaving at state point 5. So, the amount of heat that should be added to the boiler is equal to $(h_3 - h_2)$ plus the amount of heat required to increase the temperature of steam when it comes from high pressure stage turbine and goes to the low pressure stage turbine that is $(h_5 - h_4)$.

$$q_{in} = \text{Rate of heat supply} = \{(3310.8 - 206.95) + (3466.61 - 2817.2)\}$$
$$= 3753.26 \frac{kJ}{kg}$$

So, this is rate of heat supply per unit mass flow rate of the working substance. So, this is one part and another part is thermal efficiency.

$$\text{Thermal Efficiency} = 1 - \frac{q_{out}}{q_{in}}$$

$$q_{out} = h_6 - h_1 = 2344.7 - 191.81 = 2152.89 \frac{kJ}{kg}$$

$$\text{Thermal Efficiency} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2152.89}{3753.26} = 0.428 = 42.8\%$$

So that means, efficiency is 42.8%. So, we have seen that in this problem, we need to calculate the heat input and the thermal efficiency, but to calculate thermal efficiency you also had to calculate heat output. So, in addition to the heat input, we also had to calculate

heat rejection from the system and calculating these two quantities, we have quantified the thermal efficiency and it is coming as 42.8%.

So, to summarize, we have taken this example, we have discussed several issues while solving the problem. And finally, I would like to discuss one important point that the efficiency is 42.8% which is not even 50%. So, ideally efficiency should be higher. But what we can see from this calculation is that even though we have calculated efficiency as 42.8%, let me tell you, we have taken a few assumptions like the isentropic process, but it is very difficult to achieve these processes in practice. What what will happen? All these processes will invite thermodynamic irreversibility and thermodynamic irreversibility will disallow the system to run at the best exergetic efficiency. So that means, efficiency will be even less in real practice. So that means, to run the system, though we need to supply energy in the form of heat, but the work output that we are getting is less. And it is because of this reason, we have seen that heat is termed as low grade energy, while work is termed as the high grade energy in thermodynamics.

And second point is that from the amount of energy that we have added to the system in the form of heat, a part of that is converted to the useful work and remaining energy is getting lost from the system to the surroundings. We need to do it, otherwise we will not be able to run the system in a cyclic manner and that is what is the restriction imposed by the second law of thermodynamics. So, with this I stop here today and we shall continue our discussion in the next class. Thank you.