Thermal Engineering Basic and Applied

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Lecture – 20

Problems of Steam Power Cycle

I welcome you all to this session. In this session today, we shall solve one or two numerical problems related to stream power cycle. So, if you try to recall, in this module of this course we have discussed about different steam power cycles, starting from the Carnot cycle to the binary fluid cycles. Now, we have seen that efficiency of the Carnot cycle is the maximum that is what you have read even in your thermodynamics course. But identifying a few issues from the perspective of operation as well as design, there are a few practical cycles and we have also discussed about them. So, we have discussed about that efficiency of the Carnot cycle is maximum, but if we talk about simple ideal Rankine cycle, reheat Rankine cycle, and regenerative cycles, then starting from the simple ideal Rankine cycle to several other modifications that we have discussed, are essentially for the improvement of the thermal efficiency of the cycle.

So, till now we have discussed about this particular aspect, thermal efficiency from a qualitative perspective. That means you know that Carnot efficiency is maximum 100%, while in case of the ideal Rankine cycle or Rankine cycle with reheating, the efficiency reduces. So, to have a quantification of the thermal efficiency in a steam power plant, we need solve a few problems. So, accounting for this aspect today we shall solve two different problems.

I have taken the first problem from the simple ideal Rankine cycle. We shall illustrate this problem here, only to have fair idea about the solution of a problem. So, let us first read out the problem statement.

A simple ideal Rankine cycle with water as the working fluid operates between the specified pressure limits. The rate of heat addition and rejection and the thermal efficiency of the cycle are to be determined. Given that boiler pressure is 6 MPa, condenser pressure is 20 kPa and steam temperature at the exit of the boiler is given 500°C.

So, from these input data we need to calculate the heat addition, heat rejection and thermal efficiency. In fact, if we can calculate amount of heat which is added to the cycle or rejected from the cycle, then knowing these two parameters, we can quantify the efficiency.

So, if we try to solve this problem and you know that it is a very simple problem. So, whenever you are solving any numerical problem on this particular topic, the first objective should be to draw the schematic depiction of the steam power cycle. So, let us draw it as per the slide. We have boiler then turbine and after doing work, steam is taken to the condenser. Then from there, the collected steam is pumped back to the boiler. So, the processes are named as 1-2-3-4. Basically to the pump work (W_{in}) is added, at the condenser Q_{out} amount of heat is rejected. Then Q_{in} is the amount of heat added to the boiler and we get W_{out} as the work output from Turbine. So, this is the schematic depiction we have discussed so many times.

So, now it is given that the cycle is operating between two specified pressure limits. So, then what we need to do? We need to write down the quantities which are given in the problem statement. But before going to write them, let us draw the T-s diagram. So, if we try to draw the T-s diagram, then there is constant pressure line that is $P_{condenser}$ & P_{boiler}. So, this is the simplest cycle we know. The modification of the Carnot cycle is done because in Carnot cycle we have seen that there are issues like partial condensation, two phase mixture in pump cannot be used. So, compressor is used. And while we are using compressor and that too with two phase mixture, so because of the higher specific volume of stream, power consumption by the compressor is a substantial part of the work output. So, that is why it is not a real cycle. Now in the problem statement, it is mentioned whether steam is reheated or not, steam is superheated or not. We know that steam temperature at exit of the boiler is given 500°C. Perhaps this is the line which is giving us a clue about the quality of steam at the exit of the boiler or at the inlet of the turbine.

So, let us complete the T-s diagram first. So, we can plot point 1 and point 2. But we do not know whether point 3 will be on the saturated vapour line or it will be in the superheated regime, it will lie on this pressure line. So, as of now we know given the quantities.

$$P_2 = P_3 = 6 \text{ MPa}$$

$$P_1 = P_4 = 20 \text{ kPa}$$

$$T_3 = 500 \text{ °C}$$

Since we could not locate point 3 on the T-s plane, we also could not locate point 4, because 3-4 is the isentropic process that is occurring inside the turbine. So, now it is most important to understand that at this point, no matter whether point 3 is on the saturated vapour line or superheated regime, it will lie on the boiler pressure line.

So, to understand that let us quickly look at the steam table. You have learned to read data from steam table in your thermodynamics course, so I am not going to discuss all those part. We know the exit temperature of the steam at the boiler, which is also the condition of steam at the inlet to the turbine. And the point 3 will be on this special line, so, we know also pressure that is $P_3 = 6$ MPa. We know $T_3 = 500$ °C. So, now we can read this point, I mean whether the point will be in the superheated regime or the saturated vapor line by looking at the steam table.

So, let us quickly look at the steam table. As we have seen that pressure is 6 MPa and temperature is 500°C, so, we shall first go to saturated water pressure table, not the temperature table because pressure is mentioned. So, when we look at the saturated water pressure table, corresponding to that pressure that is 6 MPa, if we find out that the saturation temperature is less than 500°C then definitely point 3 will be in the superheated regime or if it is equal to the saturation temperature at that pressure, then it would be on the saturated vapor line.

So, let us look at the saturated water pressure table; the given temperature is 500°C. So, it is well above than the saturation temperature. So, definitely point 3 will be in the superheated region. Now we can locate point 3 in the superheated regime of the T-s diagram. Now we can also locate point 4. So, state point 3 is in the superheated regime.

Identifying the location of point 3 in this T S plane, next what we need to do? We need to apply the steady flow energy equation that we have discussed across all the components. So, basically if we apply steady flow energy equation to the boiler, to the turbine, for the process which is there in the condenser as well as the process which is there in the pump. Then we can find out what would be the work and heat interaction.

Essentially if we need to quantify the efficiency as well as the amount of heat which is added or rejected to the cycle, then we need to apply the steady flow energy equation to different components. And knowing fully that boiler is a heat interacting device, we can calculate the amount of heat being added to the boiler. Turbine is a work interacting device, so by applying the steady flow energy equation, we can quantify the work output from the turbine. Similarly, condenser is again a heat interacting device and as there is no work output from the condenser, so we can calculate Q_{out}. And finally, the amount of work which is added to the pump also can be calculated by applying steady flow energy equation to the process which is there in the pump. I am not going to discuss this because already you have also established the mathematical form. So, straight away we can start solving this problem.

Process 1-2

Since a pump cannot handle two phase mixture, essentially the thermodynamic state of the working substance at point 1 should be saturated liquid. We know the pressure.

$$h_1 = h_{f@20kPa}$$

$$P_1 = P_4 = 20\ kPa$$

$$\text{Specific volume}, v_1 = v_{f@20kPa}$$

Again we need to look at the steam table. If we go to the steam table, then again we have to go to saturated table because of saturated water pressure and temperature given.

$$\text{So at Pressure} = 20\ \text{kPa};\quad v_f = 0.001017\frac{m^3}{kg}$$

Similarly you also can get h_f from the enthalpy table.

$$\text{So at Pressure} = 20\text{ kPa}; \quad h_f = 251.42\frac{kJ}{kg}$$

$$h_1 = 251.42\frac{kJ}{kg} \ \& \ v_1 = 0.001017\frac{m^3}{kg}$$

$$\text{Now, Work added to the Pump,} \qquad w_p = v_{f@20kPa}(P_2 - P_1)$$

I had taken enough time to discuss this particular aspect only to make you understand more carefully. So, now, we had discussed that work added to the pump is not *pdv* rather it is *vdp*. So, whether the process is a reversible isothermal process or reversible adiabatic process, the work done is always *vdp* that is what we had established in one of the previous classes. So, what is *vdp*? So, *v* is the volume of liquid corresponding to condenser pressure and *dp* is the change in pressure. In fact, the pump is responsible to build up this amount of pressure.

$$w_p = v_{f@20kPa}(P_2 - P_1) = 0.001017(6000 - 20) = 6.08\ kJ/kg$$

So, this is basically the amount of work added to the pump. We had written small w_p p because it is specific work as we are trying to express quantities per unit mass or per kg of the working substance.

Process 2-3.

Now, we need to go to the next stage that is the boiling. So, the amount of heat required to be added to the boiler for conversion of water into steam is nothing but change in enthalpy and that is what we could write from the application of steady flow energy equation to the process in the boiler. So, I can straight away write that small q_{in} because again I am trying to express the quantity in the specific form that means per unit kg of the working substance.

$$q_{in} = h_3 - h_2$$

You can calculate h_3 because you know pressure and temperature. So, again I am just trying to recall whatever we have learnt from our thermodynamics course that is two independent intensive properties are required to specify the state. You have studied this in state postulate. So, you try to understand that once we know temperature and pressure, then perhaps we can calculate other properties. Had it been an ideal gas, we could have applied the ideal gas equation to calculate other properties, but since it is working substance, we need to consult steam table. So, by looking at the steam table or superheated water table to be precise, we can calculate other properties like enthalpy, entropy, internal energy etcetera.

So, we can calculate h_3 from the superheated table, but we also need to calculate h_2. But how we can calculate h_2? Look at the process. So, we know enthalpy at state point 1 and we also have calculated work which is added to the pump. So, again if you apply steady flow energy equation you can see

$$h_2 = h_1 + w_p$$

$$\Rightarrow h_2 = 251.42 + 6.08 = 257.50 \frac{kJ}{kg}$$

It is very important to write unit otherwise you may come up with wrong answer. So, this is h_2. So, we have calculated h_2 just by applying the steady flow energy equation to a process which is there in the pump.

Now, we need to know h_3. So, already we have discussed that point 3 is lying in the superheated regime. So, we have to go to the superheated table to calculate the enthalpy knowing the pressure and temperature at that point.

$$P_3 = 6 \text{ MPa}; T_3 = 500°C$$

So, from the superheated table we can calculate h_3. So, for this problem trying to consult steam table repeatedly only to make you understand, how we can read data from the steam table. So, again if we go to the steam table where we should go? We should go to the superheated table. So, we know pressure and temperature. Temperature is given at the extreme left column. But either we can look at the temperature and from there we can calculate other properties, or we know pressure is given that is $P_3 = 6$ MPa, so we can directly check pressure 6 MPa table. At the top left corner we can see that temperature is given 500°C. And now all these corresponding several properties are available in the table like first one is specific volume then internal energy, third column is the enthalpy and fourth column is the entropy. So, we can calculate both enthalpy and entropy.

$$h_3 = 3423.1\frac{kJ}{kg};\;\; s_3 = 6.8826\frac{kJ}{kg.K}$$

So, entropy is also required and I shall explain the reason immediately next to this task.

$$q_{in} = h_3 - h_2 = 3423.1 - 257.50 = 3165.6\frac{kJ}{kg}$$

So, this is the amount of heat added to the boiler. So, this is the first part of this problem. If we go back to the problem statement, then we are asked the rate of heat addition. This is the total amount of heat added to this boiler, and if we know the mass flow rate of the working substance that is $\dot{m}$. So, if we multiply this quantity with $\dot{m}$, then we can calculate the rate at which heat is added to the cycle.

Now, we also need to calculate heat rejection that would be in the condenser and finally, if we can calculate the amount of heat rejected from there, then we can quantify the thermal efficiency. So, we can go to the next process.

Process 3-4

So, this is isentropic expansion. Few minutes back I told that we need to calculate entropy as well. So, we had calculated entropy corresponding to that particular state point 3. If we know the entropy at state point 3 then that would be entropy of state point 4. Why? Let us look at the T-s diagram. So, you know 3-4 is isentropic process. So, if we know the entropy at state point 3 then entropy at 4 will be equal to will be equal to entropy at 3.

$$s_3 = s_4$$

From state postulate you need to know two intensive independent properties to calculate other properties. So, you understand very well that till now, we have calculated the work added to the pump, heat added to the boiler. And finally, we need to calculate the turbine work output. So, if we apply the steady flow energy equation to the process that is occurring inside the turbine then the work interaction between system and surrounding can be written in terms of the enthalpy drop. So, basically you know that

$$\text{Work output}, w_{out} = s_3 - h_4$$

Somehow we have calculated s_3, but we have to calculate h_4. For that if we focus at this particular point on the T-s plane, we know only pressure, because point 4 is lying on this condenser pressure line. And by knowing only one property, we cannot calculate other properties from the stream table. What we need to do? We need to know at least another intensive property. So, now knowing the process is isentropic and we have measured the entropy at state point 3. So, we can write

$$s_4 = s_3 = 6.8826 \frac{kJ}{kg.K}; \quad P_4 = 20 \text{ kPa}$$

So, these two properties are good enough to find out the enthalpy at state point 4. How? For that again there is something we need to understand. So, if we go to the T-s diagram the state point 4 is inside the vapor dome. So, that is I mean quality of point 4 is neither pure liquid nor pure vapor. So, it is a two phase mixture. So, point 4 is not lying on the saturated vapor line nor it is lying on the saturated liquid line, rather it is inside the vapor dome. So, it is a two phase mixture. Again you have studied in thermodynamics that if we know pressure and entropy at state point 4 then we need to calculate what would be enthalpy.

$$h_4 = h_{f@20kPa} + x_4 h_{fg@20kPa}$$

So, we will get $h_{f@20kPa}$ & $h_{fg@20kPa}$ from stream table. We know x_4 is the quality of the mixture. Had it been pure steam, x_4 would have been 1, but it is not the pure steam as it the point is not lying on the saturated vapour line.

So, we need to first calculate h_4 for that $h_{f@20kPa}$ & $h_{fg@20kPa}$ can be known from steam table, but it is also essential to know x_4. So, it is very important that we know entropy.

$$s_4 = 6.8826 \frac{kJ}{kg.K} = s_{f@20kPa} + x_4 s_{fg@20kPa}$$

$$x_4 = \frac{6.8826 - s_{f@20kPa}}{s_{fg@20kPa}}$$

We can calculate entropy of saturated liquid and s_{fg} corresponding to 20 kPa from steam table. So, let us again go to saturated water pressure table and we can see corresponding to 20 kPa Pascal then we have

$$h_f = 251.42\frac{kJ}{kg};\ h_{fg} = 2357.5\frac{kJ}{kg}; s_f = 0.8320\frac{kJ}{kg.K}; s_{fg} = 7.0752\frac{kJ}{kg.K}$$

Check that $s_g = 7.90\ \frac{kJ}{kg.K}$, but the entropy at state point 4 is less than s_g. So, it indicates that point 4 is not on the saturated vapour line and it is in the two phase zone.

$$x_4 = \frac{6.8826 - 0.8320}{7.0752} = 0.8552$$

So, x_4 is 85%. So, 85% is stream and remaining almost 15% is water. So, this is two phase mixture. So, now, if we plug in the value of x_4 for h_4 calculation then

$$h_4 = h_{f@20kPa} + x_4 h_{fg@20kPa}$$

$$\Rightarrow h_4 = 251.42 + 0.8552 \times (2357.5) = 2267.5\frac{kJ}{kg}$$

So, we have calculated h_4. Now we can calculate the work done w_{out} from the turbine.

$$w_{out} = h_3 - h_4$$

You may ask me why it is not $h_4 - h_3$. Because of this enthalpy drop, we are getting work output. So, enthalpy will drop in the course of flow of steam through the turbine is the reason because of which we will be getting work output. Essentially thermal energy will be converted to the mechanical energy in terms of work. So, heat that is added to the steam, would be converted into work in the turbine.

$$w_{out} = h_3 - h_4$$

$$\text{From the superheated table, at } P_3 = 6\text{ MPa } \& \ T_3 = 500°\text{C};\quad h_3 = 3423.1\frac{kJ}{kg}$$

$$w_{out} = 3423.1 - 2267.5 = 1155.6\frac{kJ}{kg}$$

So, this is the amount of work output from the turbine or the work done by the steam on the rotating part of the wheel.

Process 4-2

This is the constant pressure heat rejection process. If we look at the T-s diagram, we can spot process 4-1. We have calculated h_4 by this linear interpolation and we already know h_1 is h_f corresponding to pressure 20 kPa.

$$q_{out} = h_4 - h_1 = 2267.5 - 251.42 = 2016.08\frac{kJ}{kg}$$

So, this is the second part of the problem that is amount of heat rejected from the cycle. So, this is another answer of this problem. Again I am telling if you would like to calculate rate at which heat is rejected, we need to know the mass flow rate of working substance. So, if you multiply q_{out} with $\dot{m}$ then the unit would be kJ/s. In fact, J/s is watt. So, basically the unit will be kW for the rate at which heat must be rejected.

Now finally, we need to calculate very important quantity that is thermal efficiency. So, if we assume that all the processes are internally reversible then

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 0.363 = 36.3\%$$

So, try to understand that efficiency is very very poor. By looking at this expression q_{out} is the amount of heat that must be rejected in the condenser. The efficiency is very poor because we could not consume the enthalpy in the turbine. That means you see, enthalpy at the inlet to the turbine is 3423.1 kJ/kg and enthalpy of steam at the exit of the turbine is 2267.5 kJ/kg. So, a substantial part of enthalpy is going out from the turbine without doing any work and it is because of this reason thermal efficiency of the cycle is only 36.3%. Before I go to complete for this session, I would like to tell that you also can check whether efficiency is coming as 36.3% but by using the other expression that is w_{out}/q_{in}. So, you know that while you are calculating w_{out}, basically you have to calculate w_{net}. As you know that we are also giving w_p amount of work input to the cycle. So, this much amount of work must be subtracted from the turbine work output and that is the net work output and that net work should be divided by the heat input and that is the thermal efficiency of the cycle.

So, to summarize, by illustrating this very simple example, we have seen the steps or the procedures rather we have discussed methodically all the steps which are involved to solve this type of problem. I would suggest that before going to solve this type of problem, you need to first draw the schematic diagram and then it would be convenient for you to solve the problem, if you can map all the processes in T-s plane. And afterward you have to just apply steady flow energy equation and you have to consult with the stream table. So, with this I stop here today. Thank you.