Thermal Engineering Basic and Applied

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Lecture – 19

Analysis of Multi-fluid Cycle; Second Law Analysis of Steam Power Cycle

I welcome you all to the session of thermal engineering basic and applied and today we shall discuss about the multi fluid cycle. In particular, we shall do the analysis of this particular cycle to see the thermal efficiency. Then we shall discuss the second law analysis of the stream power cycle and while we shall be discussing this particular topic, we shall also discuss the necessity of studying this particular aspect in the context of this module.

So, if we try to recall in the last class, we have talked about the multiple fluid cycle which is also referred as the binary fluid cycle. We are going to see the mathematical analysis in terms of the heat added to the cycle and work extracted from the cycle. Essentially these components are needed to calculate the thermal efficiency. Now, just for the recapitulation we have seen that the concept of binary fluid cycle or multiple fluid cycle is coming because of a few shortcomings of the water which is used as the working substance in the previous cycles.

So, for brief recapitulation let us try to recall the important qualities that the working substance should have. Those are high critical temperature, low triple point temperature, high heat transfer characteristics, and high enthalpy of vaporization and finally, low condenser pressure is not allowed. In addition to this the working substance should not be corrosive, should not be toxic, and should be chemically stable and readily available.

So, if you now consider water, it is very difficult to have all these properties in water during the higher temperature part of the cycle. We cannot change the behavior of water at the high temperature part of the cycle right and this aspect allows us to think of different working substances. That means we cannot reject water as the working substance, but we can replace water at the high temperature part of the cycle with more suitable fluids.

Binary vapor cycle

So, briefly we would like to discuss about this. We know that water is not used for the high temperature part of the cycle. So, it is replaced by more suitable fluids like mercury, sodium, potassium etcetera. That means we cannot reject water completely as the working substance in the steam power cycle rather we cannot use water for the high temperature part of the cycle. What we can do? We can replace this. So you can understand water will be used again in the low temperature part of the cycle, but for the high temperature part we are using either mercury or sodium or potassium.

Basically in the beginning of this class we have discussed the desired properties for working substance like high critical temperature, low triple point temperature, high enthalpy of vaporization, high heat transfer characteristics. So, all these properties is not in water at the high temperature part of the cycle. But the same working substance can be used at the low temperature part of the cycle. That means different fluids are having different attractive features, but not all of them. That means our objective should be to use both fluids. Though these two different fluids are having different attractive features, but not all of them. So, we shall discuss that in the high temperature part of the cycle mercury will be having a few attractive features, but water will not have. Similarly, at the low temperature part of the cycle water will be having a few attractive features, but mercury may not have. So, it is because of this reason, two different vapor cycles operating on two working fluids are combined together and the concept is binary vapor cycle.

Now we try to draw the schematic depiction. So you can see this steam water cycle drawn in the slide with pump, a heat exchanger, turbine and condenser. So, this is the low temperature part of the cycle because we are using water as the working substance as per our previous discussion. And similarly, the high temperature part of the cycle has a turbine and this is connected to electric motor or the mercury electric motor. The shaft of this steam turbine is also connected to the shaft of an alternator and that is called steam alternator. The pump in the high temperature cycle is Pump-2. Here the working substance is pumped back to the boiler upon receiving heat, and then to the turbine. So 1-2-3-4 is the high temperature part of the cycle or the Hg- cycle. Similarly, we can give the name 5-6-7-8 as the low temperature part or the steam-water cycle.

So, this is the binary vapour cycle and the idea here was to use both fluids. We are enjoying the attractive features of mercury which are usable at high temperature and also we are exploiting the attractive feature of water at the low temperature part of the cycle. Then let us try to draw the T-s diagram for the low temperature part that is 1-2-3-4 and for the high temperature part of the cycle that is 5-6-7-8.

So, basically this heat exchanger is having dual activities. Though it is a heat exchanger, but it is acting like a condenser for the Hg- cycle and it is also acting like a boiler for the steam water cycle. So, the heat lost by mercury is equal to the heat gain by the water. And that is how water is converted into steam and finally, it is allowed to expand isentropically in the turbine and we are getting work output.

So, the design should be in such a way that eventually the steam which would be coming at the exit of this heat exchanger is superheated, otherwise the problem associated with turbine blade erosion will be there. That we have discussed extensively when we have discussed about several other cycles. So, now our objective should be to analyze or to calculate the thermal efficiency. So, we are assuming that m kg of mercury is circulated per kg of stream generation. You know that mercury which is pumped by this pump P2 to the boiler, where we supply heat of Q_{in} amount. And Q_{out} amount of heat is rejected in the condenser of steam-water cycle. So virtually there is no heat rejection in the Hg cycle. But there is heat rejection, and that rejected heat is gained by the water in the steam-water cycle.

So, effectively you know there is the high temperature part of the reservoir, wherein heat is added and the low temperature part of the reservoir wherein heat is rejected. So, we have assumed that the mass of mercury which is circulated in this cycle is m kg and that mercury would be converted to mercury vapor in the boiler and then it will go into the turbine for its expansion. So we consider that per kg of steam, we need to supply or circulate m kg of mercury.

Analysis of the binary fluid cycle

So, you know that heat supplied in Hg boiler. We are using the prefix Hg to indicate that boiler is associated with the Hg cycle.

$$\text{So amount of heat added to the Hg} - \text{cycle,} \qquad Q_{in} = mh_{hg,i}$$

This quantity is basically amount of heat supplied per kg of Hg vapour formed in the boiler. So, if we assume that m kg of mercury vapour is produced in the boiler then this is the total amount of heat that is Q_{in}. And then what is work done?

$$\text{Work done in the Hg} - \text{cycle,} \qquad W_{hg} = mw_{hg}; \;\; w_{hg} = \text{specific work done}$$

So, this w_{hg} is again work done per kg of mercury in the cycle. Because we are assuming m kg of h g, so above expression is for the total work done. Next is what is the work done in the steam cycle I am not going to write steam water cycle.

$$\text{Work done in the steam} - \text{cycle,} \qquad W_{st} = 1 \times w_{st}$$

This is because we have assumed that for 1 kg of steam generation, we need to supply m kg of mercury. So w_{st} is the work done per kg of steam in the steam cycle.

So, now what is the total work done? So, if we go back to the previous slide, we are getting work output from the mercury turbine and we are also getting work output from the steam turbine. So, we need to calculate the total work output first and we are getting this total work output at the cost of this heat input Q_{in}. So, from there we can calculate the overall efficiency or total efficiency of the cycle.

$$\text{Total Work output} = W_{total} = mw_{hg} + w_{st}$$

$$\text{Overall efficiency of the binary cycle,} \qquad \eta_{ov} = \frac{W_{total}}{Q_{in}} = \frac{mw_{hg} + w_{st}}{mh_{hg,i}} \quad ----(1)$$

So, this is the work done for the binary cycle and overall efficiency of the binary cycle. We can also calculate the efficiency of mercury cycle.

$$\text{The efficiency of mercury cycle,} \qquad \eta_{hg} = \frac{mw_{hg}}{mh_{hg,i}} \quad ----(2)$$

So, we are supplying $mh_{hg,i}$ amount of energy and at the cost of that energy we are getting only mw_{hg} amount of work. So, this is the expression. So, we have calculate efficiency of the mercury cycle.

Till now we have calculated the overall efficiency as well as the efficiency of the Hg cycle. So this Hg-cycle is also known as topping cycle and this steam-water cycle is known as bottoming cycle. So, our next objective should be to look at the processes of the bottoming cycle and from there we should try to calculate the thermal efficiency of this cycle.

Try to understand if you like to calculate efficiency of the stream cycle, then we need to know the work output and energy input. We know that work output is w_{st} as 1 kg of stream should be generated. Now what is the input energy for that work output? There is no heat source which is supplied externally rather the temperature of the Hg vapor provides the input energy. After doing certain amount of work in the Hg turbine, it is taken through the heat exchanger. And you have studied in heat transfer course that this is just like counter flow heat exchanger. So, two streams are flowing opposite to each other and during the course of their flow, this heat exchange takes place. The water which is pumped to the heat exchanger will be heated up and it is designed in such a way that at the exit of the heat exchanger, it should be superheated steam. So this heat exchanger is also known as Hg condenser for the Hg cycle and steam boiler for the steam cycle.

Now we need to calculate the amount of heat that is added to the water and we have discussed that heat lost by mercury is equal to the heat gain by the steam. So, from there we can calculate the amount of heat added to the steam water cycle.

$$\text{Heat lost by mercury vapour} = \text{Heat gained by water}$$

$$mh_{hg,o} = 1 \times h_{st} \;\; \Rightarrow m = \frac{h_{st,i}}{h_{hg,o}} - - - - (3a)$$

So, the loss of energy by the mercury during the cycle is the energy that should be gained by the water. So, can't we write the expression of efficiency from the perspective of energy balance? So, we know that $mh_{hg,i}$ is the amount of energy added to the hg cycle, but out of this energy we are going to reject $mh_{hg,o}$ amount of energy to the water. So, the net energy that is used to get the work output from the mercury turbine.

$$\text{The efficiency of mercury cycle,} \qquad \eta_{hg} = \frac{mh_{hg,i} - mh_{hg,o}}{mh_{hg,i}}$$

$$\Rightarrow \eta_{hg} = \frac{mh_{hg,i} - mh_{st,i}}{mh_{hg,i}}$$

Why we are doing this? Because we are going to calculate the thermal efficiency of the steam cycle soon. For that we need to relate this h_{st} because that is the amount of energy added to the steam water cycle per kg of steam generation.

$$\Rightarrow \eta_{hg} = 1 - \frac{1}{m}\frac{h_{st,i}}{h_{hg,i}} ------ (3b)$$

So, if we use the expression of m from equation 3 a and if we plug in the equation 3 b, then we can write

$$\eta_{hg} = 1 - \frac{h_{st,i}}{m\ h_{hg,i}} = 1 - \frac{h_{st,i}}{\frac{h_{st,i}}{h_{hg,o}}\ h_{hg,i}}$$

So $h_{st,i}$ is the enthalpy of steam at the inlet to the cycle. Basically enthalpy of mercury that is $h_{hg,o}$ is equal to the enthalpy of steam at the inlet to the cycle that is $h_{st,i}$. So, that is why I am using this nomenclature s t i.

$$\eta_{hg} = 1 - \frac{h_{hg,o}}{h_{hg,i}}$$

So, this is the efficiency of the mercury cycle in terms of the enthalpy. So, from here, we can write the thermal efficiency of the steam cycle.

$$\text{Efficiency of the steam cycle,} \qquad \eta_{st} = \frac{1 \times w_{st}}{1 \times h_{st,i}} = \frac{w_{st}}{h_{st,i}}$$

So, this $h_{st,i}$ is the input energy per kg of steam to the cycle and at the cost of that we are getting work output that is w_{st}. So, this is the efficiency that you can understand very easily, which is work output by the heat input. So, $1 \times h_{st,i}$ is the amount of energy added to the steam cycle per kg of steam and we are getting total work output that is $1 \times w_{st}$.

$$\eta_{st} = \frac{w_{st}}{h_{st,i}} = \frac{(h_{st,i} - h_{st,o})}{m\ h_{hg,o}}$$

If we look at the stream cycle then $h_{st,i}$ is the enthalpy per kg of steam which is entering into the cycle or given to the cycle, but there is heat rejection because we need to run it in a cyclic manner. So, per kg of steam, the amount of heat that must be rejected from the cycle is Q_{out} which is nothing, but $h_{st,o}$ per kg of steam. So, and that is so enthalpy in minus enthalpy out is w_{st}. So, the above expression is for the efficiency of the steam cycle.

So, we could quantify the efficiency of the binary vapour cycle, we also could establish the efficiency of the mercury cycle, finally we have established the efficiency of the steam cycle. So, our next objective should be to see that what fraction of stream cycle efficiency is contributing the total efficiency of the cycle. For that we need to do further calculation. Because we need to know the contribution coming from both mercury cycle and steam cycle towards the overall efficiency. Is it 60% / 40% percent or is it 50% / 50% or 30% / 70%?

Overall Efficiency

$$\eta_{overall} = \frac{mw_{hg} + w_{st}}{mh_{hg,i}} = \frac{w_{hg}}{h_{hg,i}} + \frac{w_{st}}{mh_{hg,i}}$$

Now try to understand that $\frac{w_{hg}}{h_{hg,i}}$ is efficiency of the mercury cycle because $h_{hg,i}$ is the input energy and w_{hg} is the output work. So, this is basically mercury cycle efficiency η_{hg}.

$$\Rightarrow \eta_{overall} = \eta_{hg} + \frac{w_{st}}{h_{st,i}} . \frac{h_{st,i}}{mh_{hg,i}}$$

Why you are doing so? Because, you know that w_{st} is the work output for the input of energy $h_{st,i}$ to the steam cycle. So, the term $\frac{w_{st}}{h_{st,i}}$ is efficiency of the steam cycle.

$$\eta_{overall} = \eta_{hg} + \eta_{st} . \frac{h_{st,i}}{mh_{hg,i}}$$

Even it is not in the complete or in a closed form. So, from here also we cannot we cannot say what would be the contribution by this steam cycle towards the overall efficiency.

So, it is not the closed form. Rather we need to again try to write this expression in terms of the efficiencies of either steam cycle or mercury cycle. Now what about this term $\frac{h_{st,i}}{mh_{hg,i}}$? So, if we look at this particular part, we can replace the value of m from equation 3a.

$$\frac{h_{st,i}}{mh_{hg,i}} = \frac{h_{hg,o}}{h_{st,i}} \cdot \frac{h_{st,i}}{h_{hg,i}} = \frac{h_{hg,o}}{h_{hg,i}}$$

$$\text{Hence,} \qquad \eta_{overall} = \eta_{hg} + \eta_{st} \cdot \frac{h_{hg,o}}{h_{hg,i}}$$

Now we can write this expression in terms of the efficiencies. So, few minutes back, we have calculated this mathematical expression that is

$$\eta_{hg} = 1 - \frac{h_{hg,o}}{h_{hg,i}} \quad \Rightarrow \frac{h_{hg,o}}{h_{hg,i}} = 1 - \eta_{hg}$$

$$\text{Now we can write,} \qquad \eta_{overall} = \eta_{hg} + \eta_{st}(1 - \eta_{hg})$$

So this is the final expression of the overall efficiency of the binary cycle, which is now in the closed form. By looking at this expression, we can at least tell the contribution coming from both the mercury and steam cycle to towards the overall efficiency of the binary cycle. So this is basically the analysis part of this binary cycle and it is very easy, just you know things, we have discussed from our basic concept.

Now finally, I would like to discuss about the second law analysis. This is very important. See you have studied about second law of thermodynamics in the context of basic thermodynamics course, but now question is why we need to again discuss this particular aspect in the context of this module that is steam power cycle. So we have discussed about many cycles starting from the Carnot cycle to ideal Rankine cycle, then Rankine cycle with different modifications, reheat Rankine cycle, regenerative Rankine cycle and finally binary vapour cycle. So out of those cycles, only the Carnot cycle is the reversible cycle. If you try to recall then there are two reversible isothermal processes and two reversible adiabatic processes in Carnot cycle. Basically all the processes in Carnot cycle are both internally and externally reversible. There is no need of studying this particular topic. So, basically second law is vital to understand the degree of irreversibility present in any processes. So, this particular analysis again in the context of stream power cycle is important only to know the sources of irreversibility as well as degree of irreversibility present in any particular process.

As I have mentioned that Carnot cycle is fully reversible cycle, but in addition to this Carnot cycle, we have also studied about simple ideal Rankine cycle, re-heat cycle and we have seen that these cycles are not reversible. These cycles are internally reversible, but are not externally reversible. To understand why this is so, let us briefly draw the schematic. In the schematic we have shown the basic components like boiler, turbine, pump and condenser. For the Regenerative cycles and even for the Reheat cycle, we need additional components. So for now, if we consider this simple ideal Rankine cycle, we can say that this is the schematic which is used to describe the processes in simple ideal Rankine cycle. So we can see that all the processes are internally reversible.

In the boiler, heat is added from high temperature thermal reservoir to the boiling liquids and finally, if we look at this component, condenser then heat is rejected from condensing steam to the low temperature reservoir. So, although processes are internally reversible, but these two processes, that are addition of heat from high temperature thermal reservoir to boiling liquid and rejection of heat from condensing steam to the low temperature part of the reservoir, are not reversible externally. These are highly irreversible process. So, we cannot call this cycle as the completely reversible cycle because though the processes are internally reversible, but these two processes that is heat addition to the boiler and heat rejection from the condenser, are externally irreversible. So, if these two processes are externally reversible we cannot call it as a reversible cycle. It is because of this irreversibility whether during heat addition to the boiler or during heat rejection from the condenser, some degree of irreversibility will be there. And as long as the irreversibility is there in any process, it will try to destroy the available energy. So, this cycle has the potential to do some work, but that potential to do some work will be destroyed because of this irreversibility present during in these two processes.

So, it is because of this reason we also need to study the second law analysis. Second law analysis will give us a picture about the presence of the irreversibility, the sources of irreversibilities as well as it will also help us to measure the degree of irreversibility present in any processes. So, let us quickly do it. You have studied the term exergy destruction in thermodynamics. What do you mean by that? Exergy destruction is nothing, but the resource degradation. That means these two processes like, heat transfer from high temperature thermal reservoir to the boiling liquid and heat rejection from condensing steam to the low temperature thermal reservoir, will destruct the energy at which this cycle will work.

So if we somehow can remove these two processes, or if we can somehow ensure that heat should be added to the boiler without inviting any irreversibility and heat must be rejected from the system to the surroundings without inviting any thermodynamic irreversibility, then perhaps the efficiency of the cycle would have been better as compared to the case where these two processes will invite irreversibility and that is the real process. That means these two processes are not allowing the system to operate at the best exergetic efficiency. So exergetic efficiency will be destroyed only because of the irreversibility is present.

So this exergy destruction is the measure of the deviation of the exergetic efficiency of any particular system.

$$\text{Exergy destruction for steady flow process,} \qquad \dot{x}_{des} = T_0 \dot{s}_{gen}$$

$$\dot{s}_{gen} = \text{Entropy generation}$$

So, basically you know that this two process will lead to the generation of entropy and because of this entropy generation, there will be resource degradation that means the available energy would be degraded and as a result of which exergetic efficiency will be reduced. So, probably you have studied this equation in thermodynamics and I am not going to derive it. So, this is the exergy destruction for a steady flow system.

Let me discuss about one particular case which is drawn in the slide. Say if we consider one system and its system boundary, so $\dot{x}_{in}$ is exergy in and $\dot{x}_{out}$ is exergy out. Basically the system will be having exergy of the system (x_{sys}) plus exergy destruction (x_d). So now we can write that

$$\dot{x}_{in} = \frac{dx_{sys}}{dt} + \dot{x}_{out} + x_d$$

So, you can understand that this exergy in, which is added to the system will change the exergy of the system, there will be some amount of exergy destruction and some amount of exergy will come out from the system. So, if we have multiple inlet to the system and multiple exit from the system then

$$\sum \dot{x}_{in} = \frac{dx_{sys}}{dt} + \sum \dot{x}_{out} + x_d$$

So, some amount of exergy destruction will be there.

$$x_d = 0; \text{ if the process is reversible}$$

$$x_d > 0; \text{ if the process is irreversible}$$

That means for all the irreversible processes this exergy destruction is greater than equal to 0. Now, we will try to write this expression in the context of stream power cycle, which is our focus today.

$$\dot{x}_{des} = T_0 \dot{s}_{gen}; \;\; T_0 = \text{reference temperature; (here, '0' refers to reference)}$$

Now for the steady flow system in the context of steam power cycle, if we consider simple ideal Rankine cycle, we can write that

$$\dot{x}_{des} = T_0 \dot{s}_{gen} = T_0 \left[\sum_{out} \dot{m}\, s + \frac{\dot{Q}_{out}}{T_{bound,out}} - \left(\sum_{in} \dot{m}\, s + \frac{\dot{Q}_{in}}{T_{bound,in}} \right) \right]$$

$$= T_0 (\dot{s}_{out} - \dot{s}_{in})$$

So, this equation is for the steady state steady flow system. Now, if we consider that we have single inlet, single exit and per unit mass of the working substance, then we can write

$$\dot{x}_{des} = T_0 \left[(s_e - s_i) + \frac{q_e}{T_{bound,e}} - \frac{q_{in}}{T_{bound,in}} \right]$$

$$q_{in} = \text{heat is added to the system}; \;\; q_e = \text{heat is rejected from the system}$$

Whether heat is added or rejected, it is done through the system boundary. So, these are the temperature of this boundary through which heat transfer takes place.

$T_{bound,e}$ = the boundary temperature at the exit through which heat is rejected

$T_{bound,in}$ = is the temperature of the boundary at which heat is added to the system

So, this is the expression for the single inlet, single exit and per unit mass of the working substance. Now we would like to write this for a cyclic process. So, this is the cyclic process, it is not that we are focusing only in the boiler, it is not a case that we are focusing only in the turbine, but we are focusing on the cycle. So we can write the above equation even for all the individual components, but if you would like to write this for the cycle then it can be written as below.

$$\dot{x}_{des} = T_0 \left[\frac{q_e}{T_{bound,e}} - \frac{q_{in}}{T_{bound,in}} \right]$$

T_0 = Sourrounding state

So, now basically you can apply the previous expression for all the components which are there in the cycle. And we can see from this final expression pertaining to this cycle, all the processes are getting executed in a cyclic manner.

Some amount of irreversibility is there and that irreversibility will destroy the exergetic efficiency. What are the sources of this irreversibility? You can say that one is heat rejection that isq_e and another is q_{in}. So, basically if we can figure out the sources of irreversibility then we also can calculate the degree of irreversibility that is present. So, pertaining to the cycle, as I told you in the beginning that although all the processes are internally reversible, but only problem is that addition of heat from high temperature thermal reservoir to the boiling liquid that is this term $\frac{q_{in}}{T_{bound,in}}$ and rejection of heat from condensing steam to the low temperature reservoir that is $\frac{q_e}{T_{bound,e}}$. So, these two factors are responsible to invite some degree of irreversibilities which in turn will reduce the system performance by destroying the exergetic efficiency.

$$\dot{x}_{des} = T_0 \left[(s_e - s_i) + \frac{q_e}{T_{bound,e}} - \frac{q_{in}}{T_{bound,in}} \right]$$

Considering this expression, if you supply certain amount of heat to be rejected from the turbine, even then some degree of irreversibility will be there. In real application that is there, but we are allowing the turbine surface to be insulated, so that the loss of energy from the turbine to the surroundings is neglected. So, you can understand that if we apply this equation even for the turbine and pump, then these two factors $\frac{q_e}{T_{bound,e}}$ & $\frac{q_{in}}{T_{bound,in}}$ are 0 because there is no heat interaction between system and surroundings. So, this is applied for any module which are there in the steam power cycle. But what about this $(s_e - s_i)$? So, basically when steam is entering into the turbine and steam is leaving, there is entropy transport associated with the flow of steam and entropy rejection associated with the steam rejection. So as long as the process is cyclic then perhaps the contribution from this term will not be there and that is how we could write the expression of exergy destruction for the cycle is like this. And having established this expression, we also could identify the sources of irreversibility in the context of stream power cycle that is the heat addition to the boiler and heat rejection from the condenser. So with this I stop here today and we shall continue our discussion in the next class. Thank you.