Thermal Engineering Basic and Applied
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Lecture – 11
Limitation of Carnot Cycle, Simple Rankine Cycle and Analysis

I welcome you all to this session of thermal engineering and today we shall discuss about the limitations of Carnot cycle and then we shall go to discuss about the actual cycle starting with the simple Rankine cycle you know that in the last class we have discussed about the ideal vapour power cycle that is the Carnot cycle. We have also tried to estimate the heat which is added to the cycle and the work which is extracted from the cycle during different processes.

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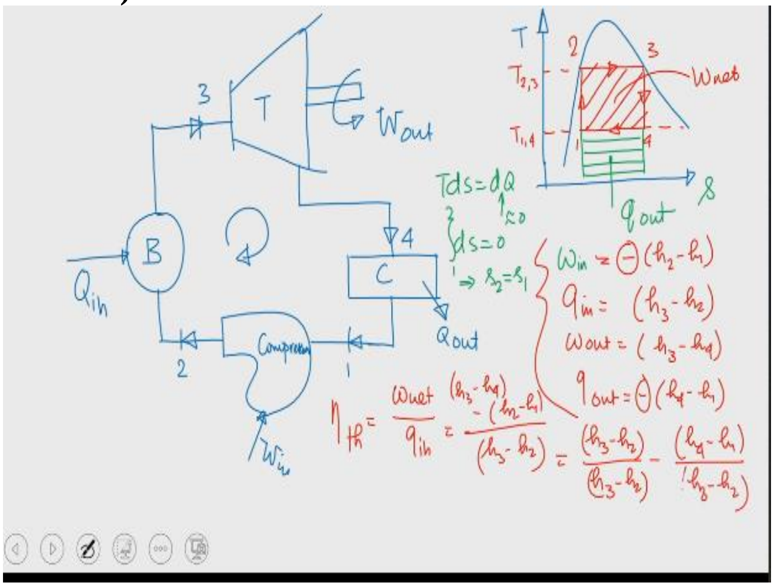

So, let’s try to just draw the schematic of the simple power plant as per the diagram in the slide where W_{out}, Q_{out}, Q_{in} & W_{in} has been also shown. We have discussed about the T-s diagram and if we use another colour to represent so this is 1, this is 2, this is 3, this is 4. So, this is $T_{2,3}$ this is$T_{1,4}$. we can represent the state points in the T-s diagram as shown in the diagram, where temperature $T_{2,3}$ & $T_{1,4}$ are marked. You know that is what we have discussed in the last class and the hatched portion (in red) in the T-s diagram is W_{net} and the area (in green) represents the heat rejection q_{out}.

So, basically 1-2 is the compression process and we have discussed that this process can be represented by a reversible adiabatic process. So this is not a device in which heat exchange takes place between system and surroundings rather it is a work interacting device, in particular this is a work absorbing device. So, this process is reversible adiabatic process. We know for reversible adiabatic process

$$Tds = dQ = 0$$

$$\Rightarrow \int_1^2 ds = 0 \quad \Rightarrow s_2 - s_1 = 0 \quad \Rightarrow s_2 = s_1$$

As entropy is remaining constant, so basically 1-2 is isentropic process.

$$\text{For } 1-2 \text{ Process; } \ s_2 = s_1$$

Now 2-3 is constant temperature heat addition or reversible isothermal process and 3-4 is reversible adiabatic expression that is the process that takes place while steam is passing through the turbine and finally 4-1 is constant temperature heat rejection.

We can write the equations in specific form by using first law of thermodynamics applied to steady state steady flow processes

$$\text{Work done compressor,} \qquad W_{in} = -(h_2 - h_1)$$

$$\text{Heat addition to Boiler,} \qquad q_{in} = h_3 - h_2$$

$$\text{Work done Turbine,} \qquad W_{out} = h_3 - h_4$$

$$\text{Heat rejected from the condenser,} \qquad q_{out} = -(h_4 - h_1)$$

Here in the first equation, negative sign indicates that work is added to the system and in the last expression, the negative sign indicates that heat is rejected from the system. So, this is the sign convention we have talked about in one of the previous classes that in thermodynamics, you can follow a particular sign convention. Normally it is followed that heat added to the system is taken as positive and work taken away from the system is taken as positive.

So, work which is coming out from the system should be taken as positive considering the sign convention, W_{in} is the work which is added to the system, so that is why it is negative. Heat q_{in} which is added to the system is taken as positive. Considering the sign convention q_{out} is the amount of heat which is being rejected from the cycle that is why it is known as negative. So, what about the thermal efficiency?

$$\eta_{th} = \frac{W_{net}}{q_{in}}; \ \ W_{net} = W_{out} - W_{in}$$

$$\Rightarrow \eta_{th} = \frac{W_{out} - W_{in}}{q_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

$$\Rightarrow \eta_{th} = \frac{(h_3 - h_2)}{(h_3 - h_2)} - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

$$\Rightarrow \eta_{th} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

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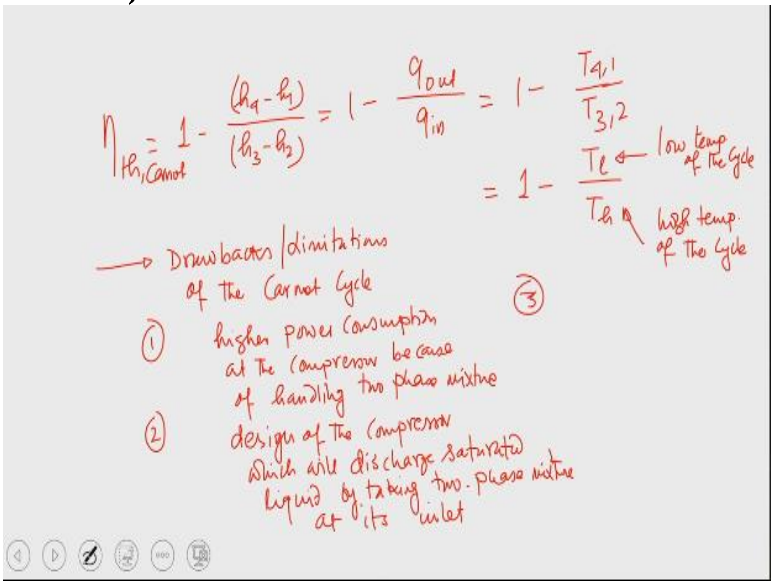

$$\eta_{th,Carnot} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_{4,1}}{T_{3,2}}$$

$$= 1 - \frac{T_l}{T_h}$$

T_l ← low temp of the cycle

T_h ← high temp. of the cycle

→ Drawbacks / limitations of the Carnot cycle

① higher power consumption at the compressor because of handling two phase mixture

② design of the compressor which will discharge saturated liquid by taking two-phase mixture at its inlet

③

$$\text{See } (h_4 - h_1) = q_{out}$$

The negative sign is used previously to indicate that it is the amount of heat which is rejected from the system. So,

$$\Rightarrow \eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

In the last class we have discussed that the Carnot cycle is comprising of 2 reversible adiabatic processes and 2 reversible isothermal processes. So basically you know that the heat is rejected at a temperature which is $T_{4,1}$ and heat is added at a constant temperature that is $T_{3,2}$. So, we can write

$$\Rightarrow \eta_{th} = 1 - \frac{T_{4,1}}{T_{3,2}}$$

So, this we can write for the reversible processes that you have started in thermodynamics. So, you can write

$$\Rightarrow \eta_{th,carnot} = 1 - \frac{T_l}{T_h}$$

So T_l is the low temperature of the cycle and T_h is the high temperature of the cycle.

So, try to understand even though it is an ideal cycle, what we can understand from this expression that, if we reduce the low temperature part of the cycle, then efficiency can be increased contrary if we increase the high temperature part of the cycle, then the efficiency will increase. So, there are 2 ways by how thermal efficiency of the Carnot cycle can be increased. Either by reducing the temperature of the low temperature part of the cycle or by increasing the temperature of the high temperature part of the cycle, we can increase the efficiency of the cycle.

So, you know that if you would like to reduce the low temperature part of the cycle that means, you need to reduce $T_{4,1}$. So, if you would like to reduce the temperature of $T_{4,1}$ that can be done, but by doing this, we are also going to invite another problem that we shall discuss in one of our subsequent classes.

So, basically try to understand that this condenser part is the low temperature thermal reservoir. So, if we reduce the temperature of this low temperature thermal reservoir we can increase the efficiency of the cycle.

Boiler is the high temperature part of the cycle that is the high temperature thermal reservoir. So, if we increase the temperature of this reservoir, we also can increase the efficiency of the cycle. So, if you would like to increase the temperature of the high temperature part of the cycle that is the high temperature thermal reservoir, we need to have some more input energy that is the fuel used to supply heat to the boiler. So, again it is not good from economical point of view. On the contrary, if you would like to you know reduce the temperature of the lower temperature part of the cycle, we are also going to invite one problem related to the condenser pressure that we will discuss later.

Now, though we know that this is the ideal cycle but why do we need to study this particular cycle. So, this is essentially a cycle which is used to compare the performance of all the actual cycles and that is why this is important. Not only this, if we consider this cycle, then there are several issues, which are not very easy to achieve in practice, which are not easily attainable in practice.

So, let us now discuss about a few critical issues or or few limitations or few drawbacks, those are not easy to attain in practice. But knowing fully that those are not easily attainable in practice we study this particular cycle only to compare the performance of the actual cycles. So, if we try to discuss about the drawbacks or limitation of the Carnot cycle what are those?

1) If we start our discussion from this particular process that is process 1-2 which is this compression process, you know that compression process starts from state point 1 and the process ends at point 2. So, the process starts at point 1, which is the 2-phase mixture. So, basically the compression needs to handle 2-phase mixture corresponding to the state point 1 I mean the quality of this two phase mixture. So, since that compression needs to handle 2 phase mixture, we all know the specific volume of vapour is higher than the specific volume of liquid, so, handling 2-phase mixture will require higher power consumption. So, W_{in} that should be supplied to run the compressor will be very high accounting for the higher specific volume of the vapour corresponding to state point 1. So, first issue is higher power consumption at the compressor because of handling 2-phase mixture.

2) It is very difficult to design a compressor which will start from this 2-phase mixture and the process will end at state point 2. So basically starting with 2-phase mixture and terminating this process at the saturated liquid line is not so easy. So, designing a compressor, which will take 2-phase mixture as the intake fluid and eventually at the exit of the compressor we will be getting saturated liquid, is also another issue. So design of the compressor which will discharge saturated liquid by taking 2-phase mixture at its inlet.
So the 2 different drawbacks we have identified.

3) Now, you look at the process 3-4 and 4-1. If we look at process 4-1, the condensation process starts at point 4, so the exit steam quality is at 4. 21 50 The exit steam is also not just saturated steam at that particular temperature rather it is the 2-phase mixture. But the condensation process will terminate at state point 1 and it starts at point 4 where the working substance is 2-phase mixture and at point 1 it is also not the saturated liquid. So, this is also not very easy. So, the condensation up to state point 1 at which it is not even the saturated liquid but it is a mixture having liquid as well as vapour, so this is partial condensation.

So, designing a condenser which will terminate the process at state point 1, so it is not complete condensation rather partial condensation.

4) Here we have shown this state point 2 and state point 3 on the saturated liquid line and saturated vapour line. So, this saturated liquid line is SLL and this saturated vapour line is SVL. For the sake of generality, these 2 points could be shown to inside the vapour dome. However, we have shown these 2 points to be on the saturated liquid line and saturated vapour line. Now, if we start the expansion process from point 3 and the expansion process terminates at point 4. So, basically you know that the quality of steam at the exit of the turbine is far away from the saturated vapour line. So, the point 4 which is for away from the saturated vapour line and had it been a case that the state point 4 is closer to the saturated vapour line then quality of steam leaving the turbine could be closer to 1. But since the state point 4 is away from the saturated vapour line, so, the quality of steam at the exit of the turbine is not very good rather it is poor. So poor quality of steam at the exit of the turbine may leads to turbine pitting and erosion.

So, the poor quality of steam at the exit of the turbine following. If we allow the steam power plant to run using this Carnot vapour cycle then quality of the steam at the exit of the turbine is 4 and it will lead to the turbine blade and pitting problem. So, poor quality of steam at the turbine exit may lead to turbine blade pitting and erosion.

So, we have listed down several drawbacks associated with this Carnot vapour cycle. Not only that all process should be reversible process in practice, it is easy to attain all the processes to be reversible one. We have seen that there are of few issues that are the higher power consumption, design of the compressor which will take 2-phase mixture at its inlet and the process will end at the state point which is saturated liquid, partial condensation, the condensation process will end at state point 1 which is again at 2-phase mixture and the quality of the steam at the exit of the turbine is also very poor.

Considering all these drawbacks, the concept of the actual power cycles came into the picture. Again I am telling we have discussed about the Carnot cycle that is the ideal vapour cycle, we have seen that to complete the cycle, we have considered all processes are to be reversible processes. Even if we considered that processes will be reversible process, we have identified 4 different drawbacks from the practical point of view.

Now, considering all these issues the actual power cycle came into the picture and the first actual cycle is Rankine cycle. So this cycle was conceptually developed by a Scottish mechanical engineer, William John McCole Rankine. So, in the name of this Scottish engineer, this cycle is known as Rankine cycle.

If we now look at the TS diagram of ideal cycle we can see that quality of seeing at point 4 is very poor. Not only that, if we try to circumvent the drawbacks associated with the Carnot cycle that is if we can extend the condensation process up to the saturated liquid line, then the complicated design of the condenser which will be allowed to have partial condensation can be eliminated.

If we can allow condensation process to be terminated up to the saturated liquid line, then, instead of using a compressor, we can use a pump because pump cannot handle 2 phase mixture. So, if we can have this condensation process up to the saturated liquid line, then instead of having two-phase mixture at state point 1, we can have only saturated liquid at state point 1 if we can extend the condensation process up the saturated liquid line. So, it is not a partial condensation. So, the problem associated with partial condensation can be eliminated.

At the end of the condensation if the state is on the saturated liquid line, we also can use pump. If we can use pump then we can eliminate these 2 drawbacks that is power consumption will be less because pump will handle only liquid and again the problem will not be there with the design of the compressor which will take saturated liquid as the inlet and the process will terminate at the saturated liquid. So basically we can remove all these problems, remaining problem is 4.

So, if we can remove the problem associated with partial condensation, we also can remove the problems which we have listed down at point 1 and point 2 as these are interrelated. Now the problem we have is poor quality of steam. Can we allow the boiling process to be further extended beyond the saturated vapour line? So, if steam at the exit of the boiler can be superheated beyond this point 3, then perhaps if we allow steam to expand inside the turbine following this reversible adiabatic process, then the point 4. So, idea is if we allow so the possibility of superheating the steam beyond point 3 if we do like this then we can have this is 4'. So, superheating the steam beyond point 3 would be another way out to increase the quality of steam at the exit of the turbine which we can see from the T-s diagram, that if we extend rather if we superheat the steam from point 3 to 3', we can see that at the exit of the turbine, the state will be 4' and this is very close to the saturated vapour line. So, in our way we are increasing the quality of the steam. So, all these remedies that we have discussed now can be accumulated in the actual cycle. In fact, the Scottish engineer William John McCole Rankine, who considered all these drawbacks and introduced the actual cycle that is Rankine cycle in which all these drawbacks can be eliminated.

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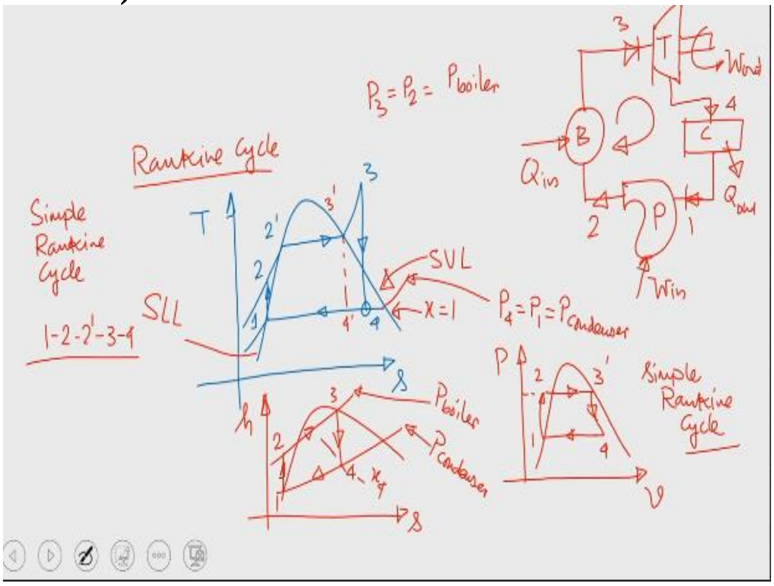

Rankine cycle

So, let us first discuss about the Rankine cycle, here I will be discussing about the TS diagram. So this is T-s diagram shown in slide. So, let us now discuss one by one, you can see that point 1 is on the saturated liquid line. So, the condensation process now is not the partial condensation. So, condensation process is getting terminated up to the saturated liquid state. So, now, instead of using compressor, we can use pump. So, we can try to draw the schematic again as shown in slide.

So, instead of using compressor, now we can use pump. Since pump cannot handle 2-phase mixture, consumption of power will be less because it is used to handle only liquid. So, you can see it will be pumped to state point 2 and then 2-3 is the heating inside the boiler at constant pressure. So, instead of having constant temperature heating, which we have seen from the T-s diagram for the Carnot vapour cycle, now we are having heating that is not at the constant temperature but at the constant pressure.

And finally, we are having expansion of the steam inside the turbine 3-4. Now, see what I was discussing that if the heating process ends at 3', then if we allow steam to expand from 3', you can see from this T-s diagram that the quality of steam at the exit of the turbine will correspond to the state point 4. Since the point 4' is away from the saturated vapour line, because quality of this particular state is x = 1. So, the point 4' is away from x = 1 line so, the quality will be very poor. But by superheating the steam beyond state point 3', up to point 3 we can see that the quality of steam at the exit of this turbine is improving, so quality is better.

Now, you can understand that the problems or drawbacks associated with the Carnot cycle can be eliminated by using this Rankine cycle. This cycle is known as simple Rankine cycle. Still we can call it simple ideal Rankine cycle, because we are assuming that the processes are reversible process. So, 1-2 is reversible adiabatic process that is pumping then this 2-3 is constant pressure heat addition & we are again considering the process is reversible process.

And you can understand from the process 3-4 that is in the turbine, is again reversible adiabatic expansion because entropy at 3 is equal to entropy at point 4. In actual practice process will not be like this, some degree of irreversibility will be there. And finally, the heat rejection process 4-1 is again not a constant temperature process rather it is constant pressure heat rejection.

$$P_3 = P_2 = P_{boiler}$$

$$P_4 = P_1 = P_{condenser}$$

Now, there are 2 things I would like to discuss. First of all, if we try to draw the P-V diagram which is very important, so, we can plot state point 4, 3', 1 & 2. Now, the thing is that if we even do not go for superheating the steam beyond 3', this is also the simple Rankine cycle. So, the simple Rankine cycle is 1-2-2'-3-4 and at least we have eliminated the problem of designing a compressor which will take 2-phase mixture at the inlet and at the outlet it will be saturated liquid. And if we are not using compressor because the quality at state point 1 is only saturated liquid, so high power consumption is not there because pump is only handling saturated liquid.

I have told you that handling 2-phase mixture needs higher power consumption, because specific volume of vapour is higher than the specific volume of the liquid at any given temperature or pressure. So you know that here we have drawn the P-V diagram of the simple Rankine cycle that is 1-2-2'-3-4. So, now provision is there using the cycle that we can superheating the steam even beyond 3' up to 3 essentially to increase the quality of steam at the exit of the turbine.

So, in simple Rankine cycle we can see that we can heat up to the saturated vapour line then we can allow steam to expand isentropically. So, this is the P-V diagram for the simple Rankine cycle and also we can draw the h-s diagram, which is also very important diagram. We can plot state points 1, 2, 3, 4. So, in the diagram $P_{condensor}$ & P_{boiler} has been marked.

So, you know the beauty of this diagram is you also can get the quality from the same diagram, I am not going to discuss this particular issue. So, the idea of simple Rankine cycle is we can eliminate the problem associated with the ideal Carnot cycle and if we can eliminate those, then the cycle becomes more practical. So, the processes which are there in real practice can be compared by using the processes which we have mapped in T-S plane, but all these processes correspond to Rankin cycle. So, all these processes constitute together to form a cycle which is known as the Rankine cycle. So, again what we need to do? We need to go for the mathematical quantification of the efficiency.

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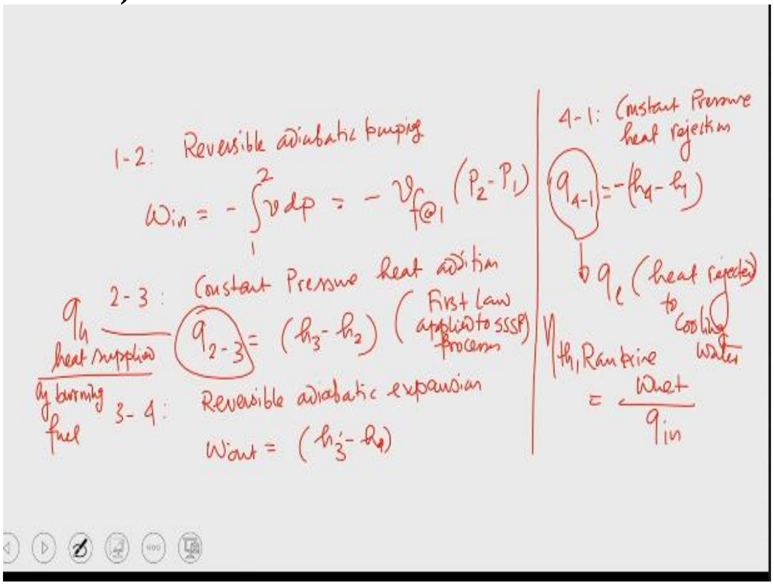

1-2: Reversible adiabatic pumping

$$W_{in} = -\int_1^2 v\,dp = -v_{f@1}(P_2 - P_1)$$

2-3: Constant Pressure heat addition

$$\boxed{q_{2-3}} = (h_3 - h_2)$$ (First Law applied to SSSF Process)

q_h heat supplied by burning fuel

3-4: Reversible adiabatic expansion

$$W_{out} = (h_{3'} - h_4)$$

4-1: Constant Pressure heat rejection

$$\boxed{q_{4-1}} = -(h_4 - h_1)$$

q_c (heat rejected to cooling water)

$$\eta_{th,Rankine} = \frac{W_{net}}{q_{in}}$$

So, you could understand that basically

$$\text{Process } 1-2: \text{ reversible adiabatic pumping}$$

$$W_{in} = -\int_1^2 vdp = -v_{f1}(p_2 - p_1)$$

So, this is boiler pressure minus condenser pressure.

$$\text{Process } 2-3: \text{ constant pressure heat addition}$$

$$q_{2-3} = (h_3 - h_2) = q_h \text{ (heat supplied)}$$

So it is not the heat addition at constant temperature rather it is constant pressure heat addition.

$$\text{Process } 3'-4: \text{reversible adiabatic expansion}$$

$$W_{out} = (h_{3'} - h_4)$$

$$\text{Process } 4-1\text{: constant pressure heat rejection}$$

$$q_{4-1} = -(h_4 - h_1) = q_l \text{ (heat rejected)}$$

So this is negative because it is the amount of heat which is rejected from the system.

So, now we can quickly calculate the efficiency. So, thermal efficiency of the Rankine cycle should be again

$$\eta_{th,Rankine} = \frac{W_{net}}{q_{in}}$$

So, let me tell you again W_{in} is basically work done

$$W_{in} = -\int_1^2 vdp = -v_{f1}(p_2 - p_1) = (h_2 - h_1)$$

$$v_{f1} = \text{specific volume of the liquid corresponding to state point 1}$$

So, if we know the condenser pressure, then at that pressure the volume of the saturated liquid and the pressure of the boiler pressure or the pressure at which boiler is operating, we can calculate what is W_{in}.

$$q_{2-3} = (h_3 - h_2) = q_h\text{(heat supplied by burning fuel)}$$

So, this is basically the constant pressure heat addition. This expression, we are getting from first law applied to steady state steady flow processes.

$$W_{out} = (h_{3'} - h_4)$$

So, this is the reversible adiabatic expansion & you can also get it from the first law applied to steady state steady flow processes.

$$q_{4-1} = -(h_4 - h_1) = q_l \text{ (heat rejected to cooling water)}$$

And finally this is the heat rejection. Now, this is the amount of heat which is rejected. If you have visited any thermal power plant, you will find that this is the amount of heat rejected in cooling tower.

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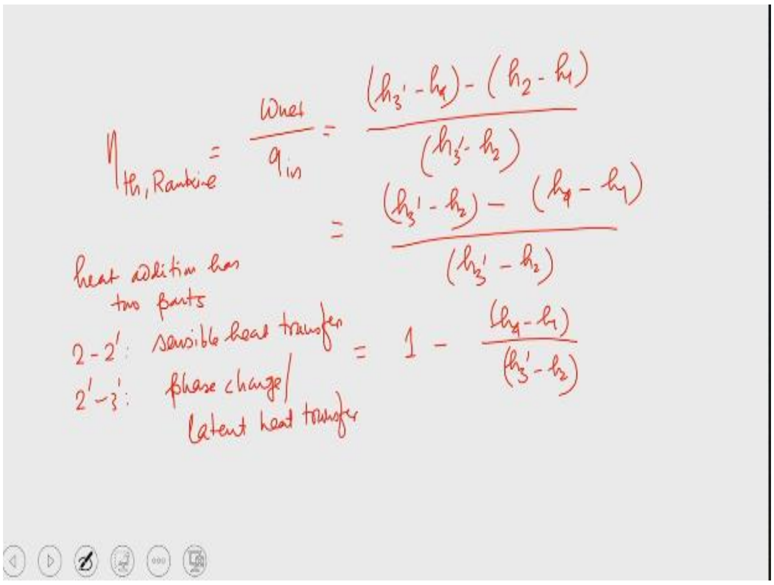

$$\eta_{th,Rankine} = \frac{W_{net}}{q_{in}} = \frac{(h_{3'} - h_4) - (h_2 - h_1)}{(h_{3'} - h_2)}$$

$$= \frac{(h_{3'} - h_2) - (h_4 - h_1)}{(h_{3'} - h_2)}$$

$$= 1 - \frac{(h_4 - h_1)}{(h_{3'} - h_2)}$$

heat addition has two parts

2-2': sensible heat transfer

2'-3': phase change/ latent heat transfer

$$\eta_{th,Rankine} = \frac{W_{net}}{q_{in}} = \frac{(h_{3'} - h_4) - (h_2 - h_1)}{(h_{3'} - h_2)}$$

Here I am assuming that the 1-2-2'-3'-4' is the simple Rankine cycle without superheating the steam. So, there is no modification just simple ideal Rankine cycle. So, you know, this is the amount of heat added following this line 2 to 3.

$$\eta_{th,Rankine} = \frac{(h_{3'} - h_2) - (h_4 - h_1)}{(h_{3'} - h_2)} = 1 - \frac{(h_4 - h_1)}{(h_{3'} - h_2)}$$

So, this is the expression of the thermal efficiency of the Rankine cycle.

$$W_{in} = -\int_1^2 vdp = (h_2 - h_1)$$

This $\int_1^2 vdp$ is nothing but change in enthalpy of the liquid because of this pumping process. So, you know this efficiency of Rankine cycle quantity again can be compared with the efficiency that we have following the Carnot vapour cycle and if we compare, we will find that the efficiency of the ideal simple Rankine cycle is less than the efficiency of the ideal Carnot cycle. That means if the thermal efficiency of the Rankine cycle is compared with the thermal efficiency of the Carnot cycle, we will find that the efficiency of the simple Rankine cycle is less than the efficiency of the ideal Carnot cycle. The reason is quite obvious, because if we look at this particular T-s diagram, the heat is added not at constant temperature which was the case for the Carnot cycle, but in this case the heat is added at constant pressure.

So, this heat addition has 2 sub parts; one is 2-2' and next is 2'-3'. This 2-2' is sensible heat transfer and 2'-3' is phase change or the latent heat transfer. So, in the section 2-2' that is the sensible heat transfer, you can see that the temperature increases. So, liquid temperature increases from T_2 to T_2, and then again heat is continuously added but there is a phase change. So, it is because of this particular section average temperature of heat addition reduces. So, the average temperature at which heat is added to the boiler for this particular case that is following this Rankine cycle is less than the temperature at which heat is added in the Carnot cycle.

So, if we consider 1'-2'-3'-4' is the Rankine cycle but then the problem associated with the partial condensation has not been there. And we are having the liquid at point 1 that is saturated liquid we are pumping. Earlier it was compressed from 1' to 2' and then constant temperature heat addition but it is now pumped from 1' to 2 up to the boiler pressure and then 2- 2' that small segment is where the sensible heat transfer takes place. And this 2-2' part lowers the main temperature at which heat is added to the boiler and it is because of this reason the efficiency of the simple ideal Rankine cycle is less than the efficiency of the Carnot cycle.

We will see mathematically that the efficiency of the simple ideal Rankine cycle is less than the efficiency of the Carnot cycle and this part will take up in the next class. So with this I stop here today and we shall continue our discussion in the next class. Thank you.