Advanced Thermodynamics and Combustion
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Module - 02
Entropy and Exergy
Lecture - 07
Entropy Analysis (Part IV)

Dear learners, greetings from IIT Guwahati. We are in this course Advanced Thermodynamics and Combustions that is module 2 Entropy and Exergy.

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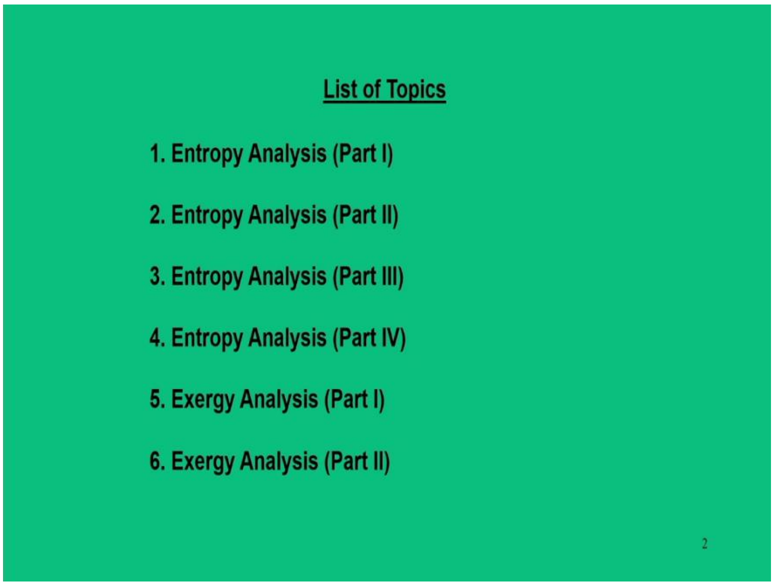

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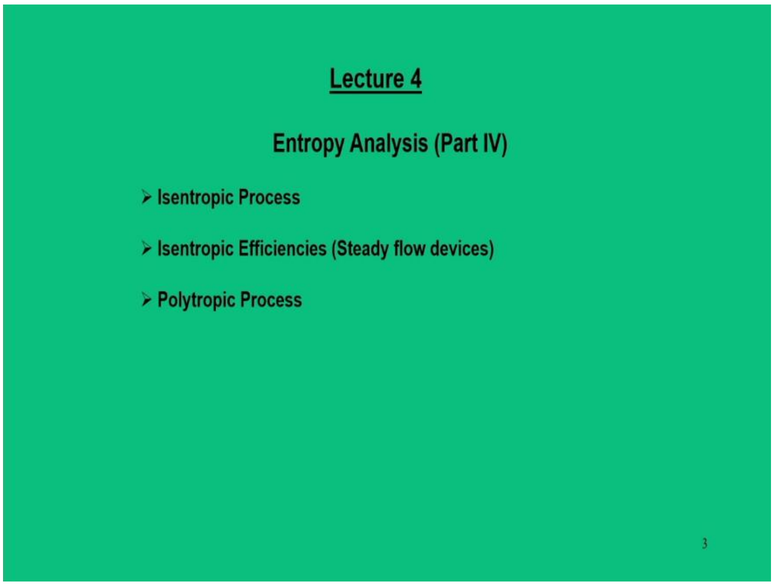

So, in this lecture we will cover the entropy analysis part 4. And in prior to this class we have exhaustively discussed about second law analysis and its consequence or statement in terms of entropy. This particular lecture we will focus mainly on isentropic process, isentropic efficiency and polytropic processes.

Just to emphasize the fact that at the UG level you might be aware of the term isentropic efficiencies which are normally applied for the steady flow devices. This steady flow devices includes compressors, turbines, nozzles, diffusers, pumps and they are the basic components for a steam or gas turbine power systems. Now with respect to this particular course although this is a kind of a repetitions.

But I will further emphasize the fact that entropy analysis plays a vital role and in particular, the term which we are going to discuss today that is isentropic efficiency is the benchmark for all practical devices. So in fact, this particular lecture is mainly devoted to the applied part of this course.

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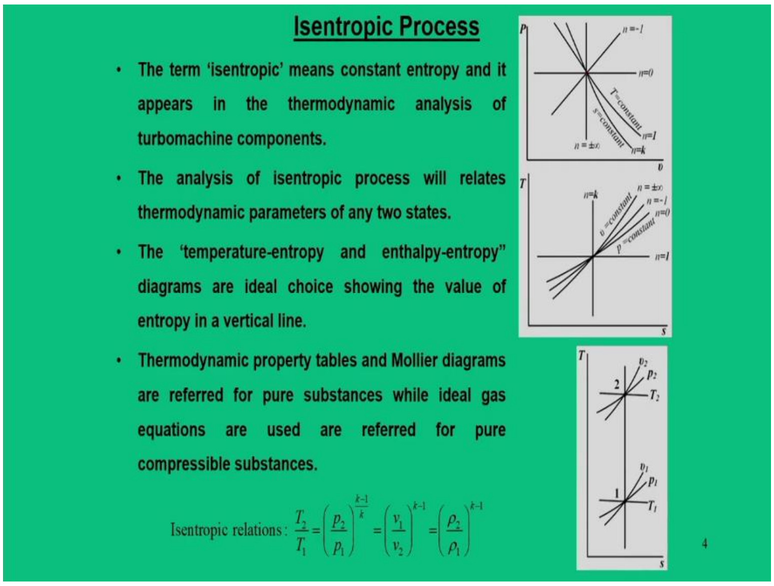

So, let me start first is isentropic process we have already discussed the term isentropic which means entropy is a constant parameter when the system undergoes a change of state. And this isentropic word is very vital in the thermodynamic analysis of turbomachine components. In fact, it relates the thermodynamic parameters between two states.

As you can see in the pressure volume diagram or temperature entropy diagrams one can draw various thermodynamic processes involving a constant pressure process, constant volume process, constant temperature process or isothermal process or it can be any processes that bears the name polytropic. And in fact, all these processes are quasistatic in nature which means we can use this thermodynamic equations as internal reversible process.

So, we have to analyze all these thermodynamic parameters for an isentropic process. The ideal choice that we should look is the temperature entropy or enthalpy entropy diagram that is because constant entropy line is represented as a vertical line. Now, when the process is analyzed for estimation of different parameters, we look for the thermodynamic property tables which are available in all fundamental thermodynamics books.

We also will explore Mollier diagrams which is nothing but the enthalpy entropy diagrams. And in fact, for a ideal gase we have derived the isentropic relations between two states that is pressure temperature, specific volume and density. So, this relation is very vital for analysis of isentropic process. $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{\rho_2}{\rho_1}\right)^{k-1}$

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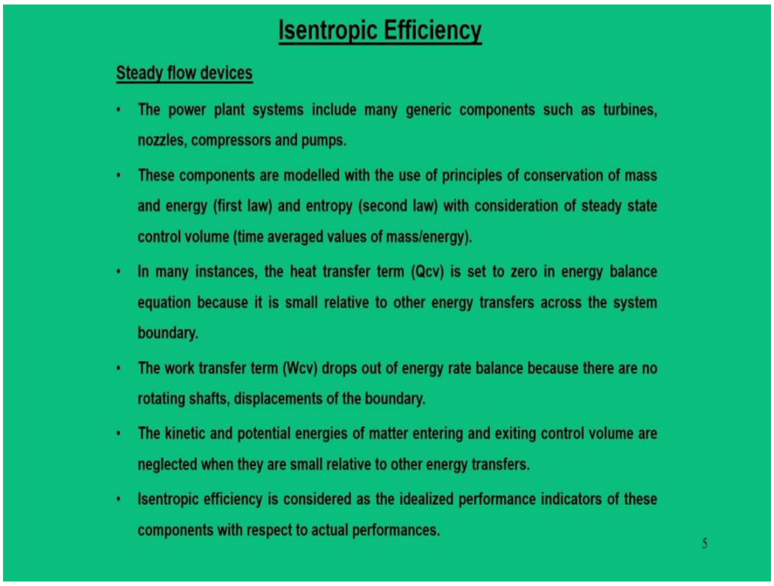

Now, let us move back to the importance of this isentropic process and when you deal with isentropic efficiency. In fact, all the steady flow devices that are used in the power plants, they undergo certain thermodynamic processes. And the analysis of these processes is dealt with one particular parameter that is if the process is internally reversible and for which the analysis can be done through isentropic process.

So, in a sense that the isentropic process is the benchmark for the idealized performance of the component. So, if I can summarize this fact, the steady flow devices includes components like turbines, nozzles, compressors and pumps. These components are modeled by two situations. One is the conservation of energy by using the first law of thermodynamics also we use mass balance equations. And most importantly it is also under consideration that is entropy which is used for steady state control volume.

Now basically analysis involve mass balance, energy balance and entropy balance for a control volume. Now when you deal with the entropy balance, we normally use the time averaged values of either entropy, mass or energy. Now because of the nature of the different components for example, turbine is meant for producing work, nozzle is meant for enhancing the velocity, compressors are used for raising the pressures. In fact, pumps are also used to rise in the pressure for liquids.

So, because of this nature of the requirement, the terms that is associated in the energy balance one is heat transfer term through the control volume or the work transfer term for the control volume that can be set to zero or a negligible number as compared to other parameters.

So, when the Q_{cv} that is heat transfer term can be set to zero if it is small relative to other energy transfers across the system boundary. Now, work transfer terms drops to zero when there is no rotating shaft or physical displacement of boundary. So, in such cases we can neglect this term in some instances like turbines and compressors the kinetic and potential energies are negligible.

So, for all this component analysis, if we neglect this term and then we can define a term which is called isentropic efficiency and which is considered as the idealized performance indicators with respect to actual performance of the component.

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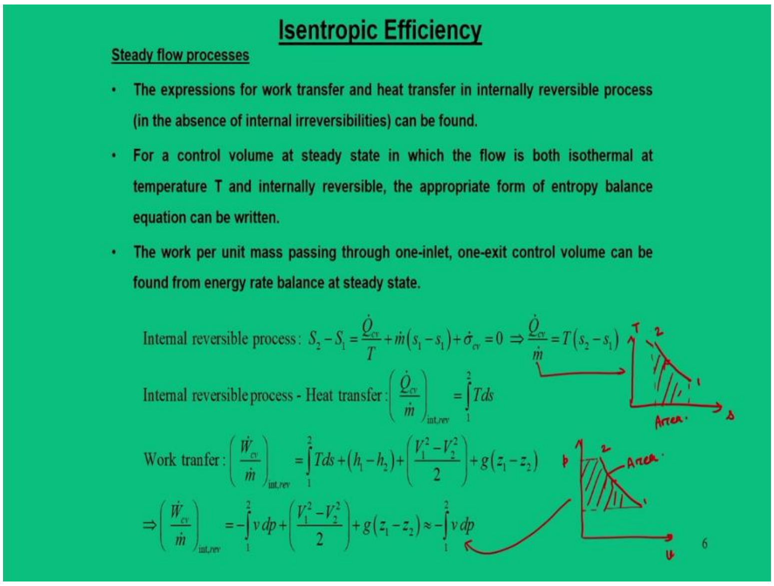

All these components use steady flow equations and the analysis is done through a control volume. And when I say control volume there is a possibilities of mass transfer as well as the energy transfer. So, for a control volume at steady state in which the flow is both isothermal at constant temperature as well as internal reversible, appropriate entropy balance equations can be written. Obviously, we also know the work or heat transfer per unit mass through one inlet, one exit control volume; that means, energy balance equations can be written.

So, in our last class we derived the relations for the internal reversible process that is entropy change. We also dealt with the heat transfer term for an internal reversible process work transfer term. And ultimately we end up in having the expressions for heat transfer and work transfer in a control volume analysis in a simplified term like heat transfer is nothing but $\int_1^2 Tds$, work transfer through the control volume per unit mass is $\int_1^2 vdp$.

Basically it means that on a T-s diagram, when the process undergoes a change of state the area under this curve represents the heat transfer. Similarly in a p-v diagram for an internal reversible process, when the system undergoes a change of state and this area represents the term vdp. So, this is the practical significance of work transfer term and heat transfer term for internal reversible processes. In fact, these two equations will be heavily used in the analysis of steady flow components.

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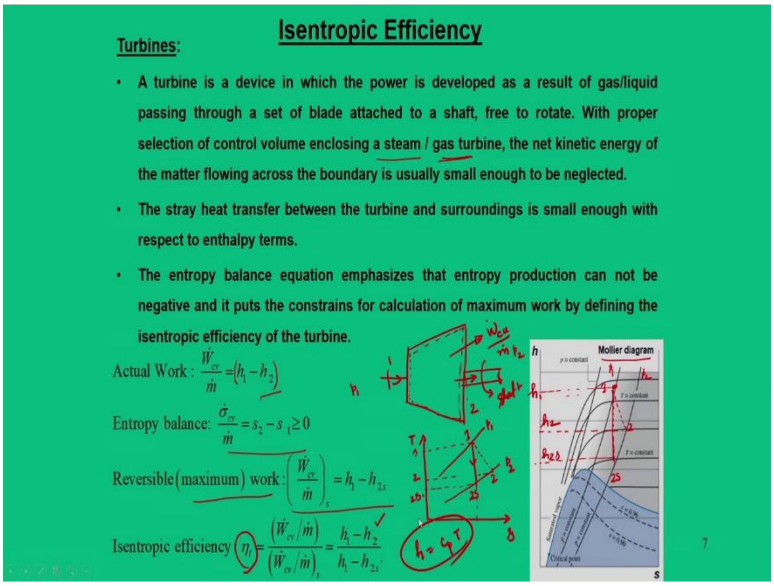

So, let me start the first important component for which we are going to calculate the isentropic efficiency and the component is turbines. So, we all know the turbine produces power. So, it is a device which is designed to produce power as a result of gas or liquid passing through a set of blades which is attached to shaft and the shaft is free to rotate.

Now, with the selection of control volume enclosing a steam or gas turbines net kinetic energy of the matter flowing across the boundary is small which can be neglected. So, I told the turbine is designed for producing the power. So, the change in the velocity associated with the system is normally neglected because these are very small numbers as compared to other parameters.

So, typically the turbine processes is represented by simplified diagram like this in which we get work $\frac{W_{cv}}{\dot{m}}$. So, the control volume is chosen in such a way that the gas that enters the to the turbine from the state 1 expands to its full capacity and when the gas leaves that is at state 2, it is fully expanded and whatever work is developed through this process, it rotates the shaft.

So, the system undergoes a change of state from one to two and this turbine process, we can represent in a temperature entropy diagram. So, this is an expansion process. So, initial state your p_1 is higher and final state p_2 is lower. So, in a T-s diagram we can draw two constant pressure lines the first term first point is p_1 and second pressure line is p_2. So, the state is located at point 1 when it expands isentropically it goes to state 2s.

So, you can remember that this is an isentropic process and is a vertical line, but if there are internal irreversibility then the actual process will land at point 2, but at a higher temperature. So, in other words we can say that gas in a non isentropic process has not expanded fully. So, there arises the term isentropic efficiency of the turbine. So, from this expressions we can find out the actual work is $h_1 - h_2$.

So in fact, this $h_1 - h_2$ term comes from the steady flow equations and when you do the entropy balance for the control volume we can write $s_2 - s_1 \geq 0$. Now, if it is greater than 0; that means, there is irreversibilities associated in the expansion process. If it is equal to 0; that means, there is no irreversibilities in this process. So, from this when $s_2 - s_1 = 0$ we get the reversible work and which is considered as the maximum work.

So, $h_1 - h_{2s}$; so thereby we represent the isentropic efficiency as $(h_1 - h_2)/\ (h_1 - h_{2s})$. So, h_{2s} is your isentropic enthalpy when the gas at is at state 2. So, here let us talk something else that the isentropic efficiency is expressed in terms of enthalpy and when you use the steam. So, normally the expression is the general term when you talk about steam turbine we refer the diagram what we call as a Mollier diagrams.

So, this is the snapshot for this Mollier diagram which you can represent. So, these are the constant pressure lines we can write one is at p_1, other is at p_2. So, depending on the initial state conditions I can represent this state as 1. Now, has process been isentropic then I would have come to 2s, if since this process is not isentropic I land up in h_2.

So, here we can directly get the values of enthalpies from this Mollier diagram. So, we can say this value is h_1 and this value will be h_{2s} and this number will be h_2. So, efficiency is $(h_1 - h_2)/\,(h_1 - h_{2s})$. Now, if we use a gas turbine; that means, $h = C_pT$. Now if we use that so instead of Mollier diagrams, we can directly use the temperature entropy diagrams as I have explained.

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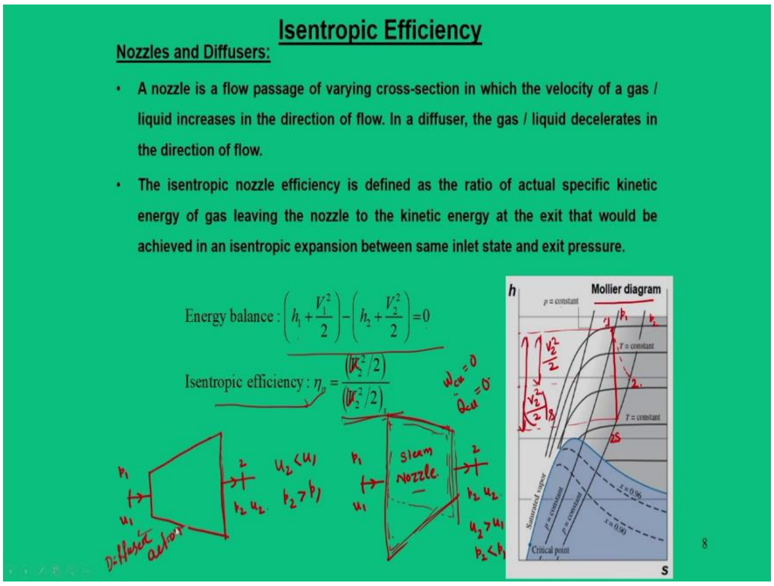

Now, another analogous term to expansion is nozzles. So, what does this mean is that, so normally the nozzles are used to generate higher velocity. So; that means, schematically they are represented as that of turbine. But their main intention is initial state is defined in terms of pressure p_1 and I can say velocity u_1 and final state is decided by pressure p_2 and velocity u_2. So, when the nozzle action is performed $u_2 > u_1; p_2 < p_1$.

So this nozzle action is due to expansion of fluid from its initial state to final state and this nozzle can be a steam nozzle or it can be used for analysis of air. So; that means, working fluid could be a steam or air. Now when you say it is a steam nozzle we use again this Mollier diagrams.

So, now before you go further we can find out these energy balance equations which is $\left(h_1 + \frac{V_1^2}{2}\right) - \left(h_2 + \frac{V_2^2}{2}\right) = 0$. So, in this control volume we can simply write $W_{CV} = 0, Q_{CV} = 0$ there is no work transfer or no heat transfer.

So, using this equation we are getting this. Now the isentropic term is defined that if the gas has not expanded to its full capacity for generating the velocity. Then it would land off in having a efficiency $\frac{\left(\frac{V_2^2}{2}\right)}{\left(\frac{V_2^2}{2}\right)_s}$. So, how do you find out? So, initial state we can represent this in the Mollier diagrams we can say the gas expands from this pressure p_1 and to p_2.

So, from this initial state we can write if this process is isentropic then it will land off in reaching the point 2s, if it is a non isentropic it will land up in point 2. So, this particular term or scale will be $\frac{V_2^2}{2}$ and this particular term will represent $\left(\frac{V_2^2}{2}\right)_s$ if the process is isentropic. So, based on this we define the term isentropic efficiency.

So, the isentropic efficiency of the nozzle is defined as the ratio of actual specific kinetic energy of the gas leaving to the nozzle to the kinetic energy at the exit that would be achieved in an isentropic expansion. Now, I have also given another term diffuser. So, it is a just an opposite process that a nozzle does.

So, sorry I have made a mistake. So, it should be the other shape. So, this is the diagram for a steam nozzle where initial area is larger than the final area, for a diffuser the final area is larger than the initial area. So, we say let us say initial state is 1, velocity is u1, final state is 2, velocity is u_2. So, the other action would be so here $u_2 < u_1; p_2 > p_1$. So, this analysis also we can do through control volume and we call this as a diffuser action.

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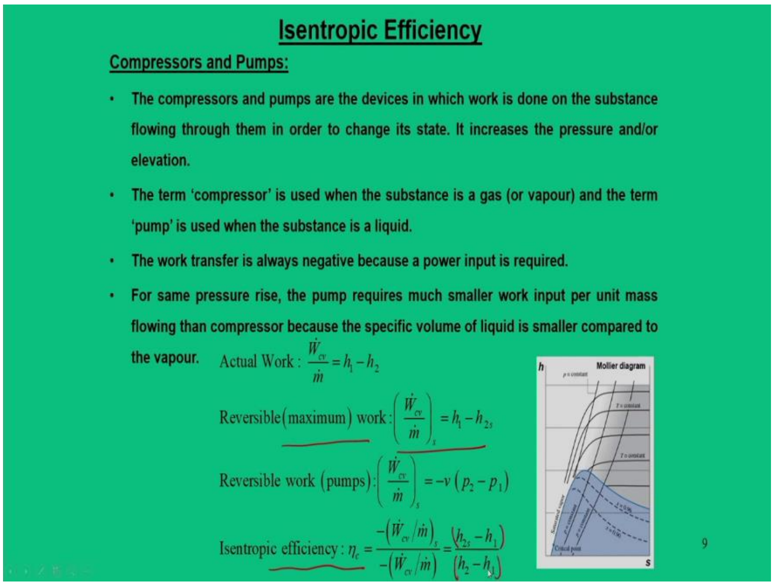

Now, we will move to compressors and pumps. The word compressor is used in the steady flow devices which takes work as a input; that means, work is done on the system and as a by virtue of the pressure of the fluid increases. So, in it and if we are using the air or gas as a working fluid the term compressor is most mostly preferred. If your working fluid is water or any liquid the term pump is fits as a most appropriate term.

So, in this in whether its a compressor or pump the work transfer is always negative because it requires the power input. We can see that for a same pressure rise, a pump requires smaller work per unit mass flowing through the compressors because specific volume of liquid is smaller compared to the vapour.

So, because of this region when you deal with the steam power plant, pump work is assumed to be smaller or very small as compared to turbine work. Now when you deal with the gas turbine plants, the compressor work and the turbine work they are at par it cannot be neglected.

So, that sense we say that turbine always drives the compressor. So analysis is almost in a similar sense as we did it for the turbines, but very basic difference that we have here the isentropic work which is the maximum work will be less. If the process is isentropic, the pump will consume less work. If there are internal irreversibility is present, then pump work will be higher. So, when you define the isentropic efficiency for the compressors the numerator term represents the isentropic work and the denominator term represents the actual work. $\eta_C = (h_{2s} - h_1)/(h_2 - h_1)$

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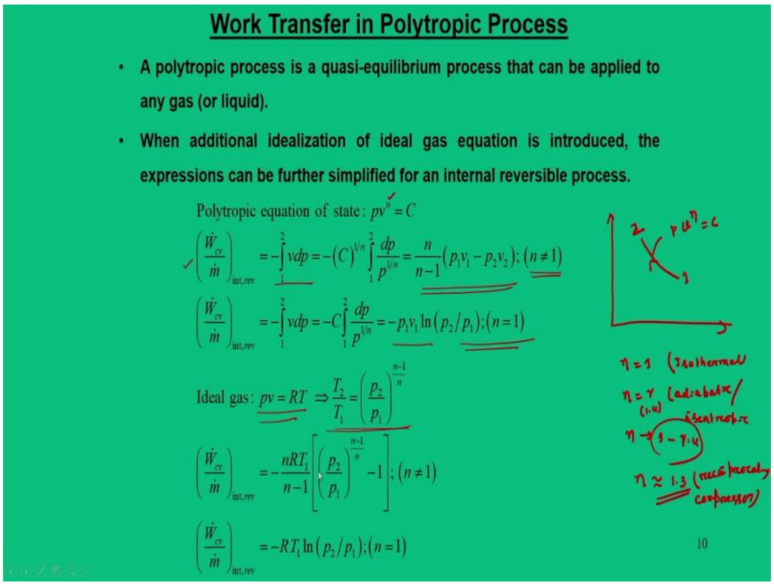

Now, in the last segment of this is a polytropic process. So, basically we have given a equation of state for a polytropic process as $pv^n = C$. Ideally the word polytrophic process is assigned when the system undergoes was quasi equilibrium change of states. And the equation of state can be written as $pv^n = C$ and these equations can be applied for any gas, but if it is an ideal gas this pv becomes RT.

So, in our analysis why we are dealing with this thing is that here the n quantifies the nature of the process. So, in a pv diagram the process that can happen is we can represent as as $pv^n = C$. Process goes from 1 to 2, this nature of the process depends on what value of n.

So, there are possibilities that n can be 1 which is isothermal; $n = \gamma$ which is adiabatic or sometimes if it is reversible we say isentropic. And if n is any value between 1 to γ; γ if you put as 1.4 for air. So, in between 1 to 1.4, so it becomes a polytropic equations. Typically, realistic number of n is about 1.3 which is normally used for reciprocating compressor.

So, why I am saying this is, reciprocating compressor is another area of application although it is not a steady flow device but its a cyclic device. But anyway our main intention is to quantify the work transfer for two situations. One is in fact, it is an internal reversible work which is represented as $\int_1^2 vdp$. Now when n is not equal to 1 it is represented in this form that is $\frac{n}{n-1}(p_1v_1 - p_2v_2)$.

Now, if n is equal to 1 the work transfer equation becomes $-p_1v_1 \ln\left(\frac{p_2}{p_1}\right)$. Now here you impose the ideal gas state equations which is $pv = RT$. So, when you use this takes the term of writing the equation of states what happens to the temperatures. So, we can have a equation like $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$.

Now based on these things, the work transfer equations further modified in this form. So, basically we are replacing the specific volume in terms of temperature.

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\text{int},rev} = -\frac{nRT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]; (n \neq 1),$$

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\text{int},rev} = -RT_1 \ln\left(\frac{p_2}{p_1}\right); (n = 1)$$

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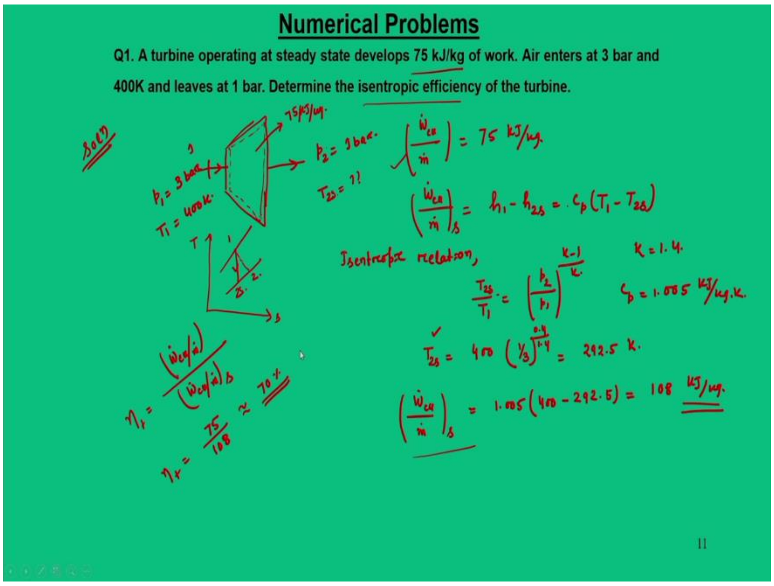

Now, we are going to discuss some of the numerical problems. So, the first problem is based on a turbine which operates at steady state and it develops 75 kJ of work. And this turbine is used in a maybe you can say it is a gas turbine power plant system in which air enters at 3 bar and 400 K and leaves at 1 bar.

So, basically your working fluid is air to start with first thing what you have to do you have to do the schematic diagram. So, for schematic diagram we can represent the nature of a turbine process, the gas expands. So, the state 1, pressure p_1 is 3 bar and T_1 is 400 K.

And final state your p_2 is 1 bar, T_2 is not known, but if we can assume this is a control volume; the work output it produces 75 kJ/kg. So, we can assume that this is your actual work. So, we can say $\frac{\dot{W}_{cv}}{\dot{m}} = 75\ kJ/kg$ and what we do not know is $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s$. Because the turbine process is isentropic if you can assume.

So, in your T-s diagram the process can be written as expansion of gas from state 1 to 2; 2s is the isentropic process.

$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_1 - h_{2s} = C_p(T_1 - T_{2s})$; Isentropic Relation: $\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$; $T_{2s} = 400\left(\frac{1}{3}\right)^{\frac{0.4}{1.4}} = 292.5\ K$; $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = 1.005\ (400 - 292.5) = 108\ kJ/kg$; $\eta_T = \frac{\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)}{\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s} = \frac{75}{108} \approx 70\%$. So, this is how you can find isentropic efficiency for the turbine.

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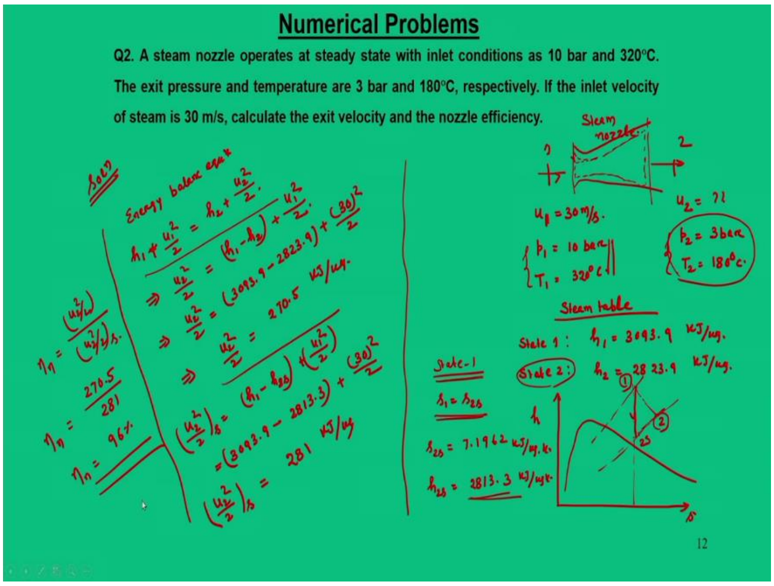

The next problem will is talk about steam nozzles. So, normally steam nozzles is used to generate thrust. So, schematically the steam nozzles shape is defined in this manner it is a basically convergent divergent nozzle and the control volume can be represented in this manner. So, your initial state is 1, final state is 2. So, your main intention is u_1 is your initial velocity which needs to be increased to u_2.

So, in this problem the initial velocity is given as 30 m/s, final velocity is not known, pressure condition is 10 bar, temperature is 320 C and exit pressure p_2 is given as 3 bar; T2 is 180 C. So, its a steam nozzle, which means we need to use steam table to evaluate enthalpy.

So, for the initial state 1, so for the based on the condition p_1 and T_1; we can find out h_1 as 3093.9 kJ/kg and based on the condition p_2 and T_2; your h_2 is 2823.9.

So, let us talk about the Mollier diagram representation where the gas from its initial state expands to the final state; that means, initial state if the process is isentropic reaches at 2s. If the process is non isentropic it process reaches at 2. But what the data was given condition 1 and 2 and; obviously, they must represent the actual state. Now to find if this gas or a steam would have expanded as in an isentropic manner it would have raised to 2s. Now on this line on the on this line 1 - 2s process your entropy remains constant. So, I can say $s_1 = s_{2s}$.

Now, for this p_1 and T_1 I can find $s_{2s} = 7.1962\ kJ/kg-K$ and for this value we can find $h_{2s} = 2813.3\ kJ/kg-K$. So, this number we get from the data of state 1, this number we get from the data state 2. So, now, we are in the situation that we can write the energy balance equation which says $\left(h_1 + \frac{u_1^2}{2}\right) = \left(h_2 + \frac{u_2^2}{2}\right)$; $\frac{u_2^2}{2} = (3093.9 - 2823.9) + \frac{30^2}{2} = 270.5\ kJ/kg$.

So, then we need to find out $\frac{u_{2s}^2}{2}$. So, again we can rewrite this equation as $\left(\frac{u_2^2}{2}\right)_s = (h_1 - h_{2s}) + \frac{u_1^2}{2} = (3093.9 - 2813.3) + \frac{30^2}{2} = 281\ kJ/kg$. So, now, we can find the isentropic efficiency for the nozzle that is $\eta_n = \frac{\left(\frac{u_2^2}{2}\right)}{\left(\frac{u_2^2}{2}\right)_s} = 96\%$.

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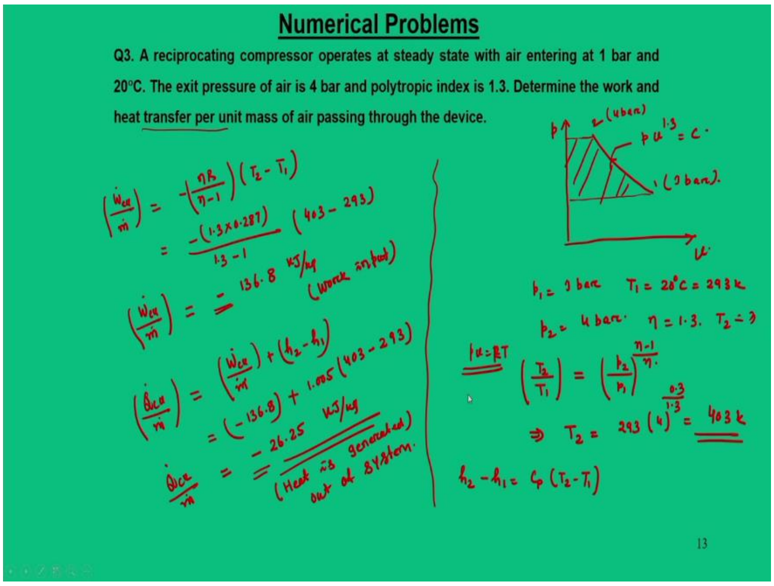

Now, we will move to third problem which is based on a reciprocating compressor and for this reciprocating compressors we are going to find out work and heat transfer per unit mass of air and this is nothing but for an internal reversible process. So, first thing we have to draw since it is a compressors.

So, a p-v diagram will be most appropriate that is we can say pressure volume diagram, the system goes from 1 to 2 and this equation is written as $pv^{1.3} = C$. So, basically this is nothing but your area under this diagram, pressure at this point 2 is 4 bar, initial pressure is 1 bar. So, we have T_1 20 C or 293 K. So, T_2 is not known.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}; T_2 = 293(4)^{\frac{0.3}{1.3}} = 403\ K; \frac{\dot{W}_{cv}}{\dot{m}} = -\left(\frac{nR}{n-1}\right)(T_2 - T_1)$$
$$= -\frac{1.3 \times 0.287}{1.3 - 1}(403 - 293) = -136.8\ kJ/kg$$

So, the negative sign indicates work input to the compressor.

Now, in similar way we need to find out the heat transfer. So, we can recall our steady flow energy balance equations $\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) = \frac{\dot{W}_{cv}}{\dot{m}} + C_p(T_2 - T_1) = -136.8 + 1.005(403 - 293) = -26.25 \frac{kJ}{kg}$.

So, this term negative means heat is generated and it is coming out. So, by virtue of this the air temperature increases. So, with this I conclude this lecture for today.

Thank you for your attention.