

Advanced Thermodynamics and Combustion
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Module - VIII
Combustion and Flames
Lecture - 30
Laminar Diffusion Flame

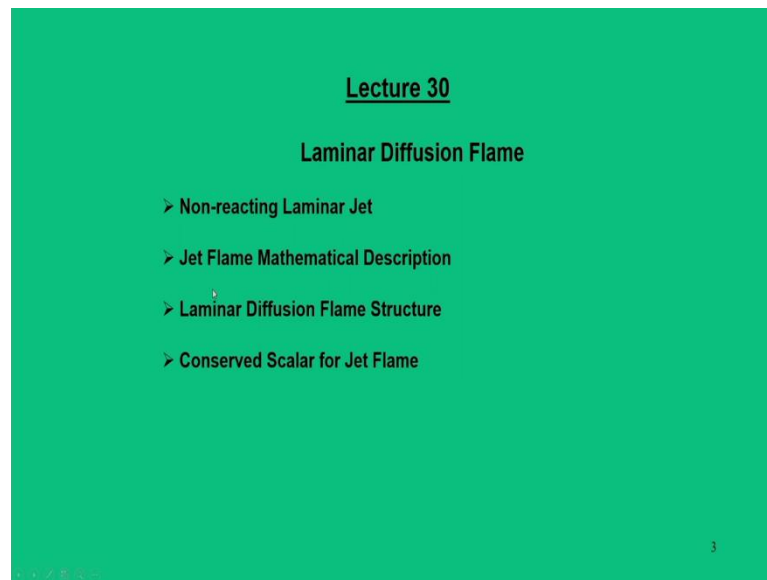
Dear learners, we are in the course Advanced Thermodynamics and Combustion module 8 Combustion and Flames.

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In the last two lectures of the module, we have deliberately discussed laminar premixed flame. Today we are going to discuss about Laminar Diffusion Flame.

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Now, in this laminar diffusion flame we will touch upon the following topics, first one is non-reacting laminar jet, then jet flame mathematical descriptions. Here, we will not go deep into the governing equations and solutions rather we will focus on the end results of those equations and subsequently we will discuss about the physical aspects.

The next important point to be discussed is laminar diffusion flame structure and one of the parameter which is called as a conserved scalar which was introduced in earlier lectures, we will try to see what is its significance with respect to jet flame. So, let us start the first thing to give some brief introduction about non-reacting laminar jet, we will see that the laminar diffusion flames importance will be realized when we are burning jets of fuels.

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Non-reacting Laminar Jet

- The importance of laminar diffusion flames are realized while burning jets of fuel. The study of laminar jet flames emphasizes formation of soot in diffusion burning.
- Many residential gas appliances such as cooking ovens employ laminar jet flames in which the fuel stream is partially premixed with air to provide non-sooting operation.
- A familiar example of non-premixed jet flame is the Bunsen-burner outer-cone flame.
- The prime concerns of any system utilizing laminar jet flames is the flame geometry with short flames and fuel types.
- Before entering into proper jet flames, the discussions of laminar constant-density jets are emphasized because they have analogous behavior of laminar jet flames.
- It is possible to describe the general characteristics of velocity and nozzle-fluid concentrations fields of laminar jets and understand Reynolds number dependence on spreading behavior of the jet.
- The nozzle-fluid concentration distribution is identical for equal flow rates which translates to flame lengths based on flow rates for a given fuel-oxidizer combination.



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So, while burning jets of fuel we mean? This is something like when the fuel is injected through nozzle, for example in a fuel injector in a diesel engines, in such cases what happens diffusion flame is generated. So, basically diffusion flame, when it is generated, it means that fuel and air mixture is pre mixed to some extents and again when the jet comes out it tries to get rest quantity of oxidizers from the air. So, typical example we can see in a Bunsen burner flame, the outer cone of this diffusion flame is nothing but the diffusion flame.

Whereas the inner cone is mainly the premixed flame. Now, when you deal premixed laminar flame, the treatment was different. Now, when you deal with a diffusion flame then there are some other issues it comes as a jet and the fuel when it comes out from this nozzle or fuel injector, it tries to spreads and it tries to diffuse with respect to air.

So, all sorts of phenomena that comes into picture and in a actual scenario what happens when the fuel is injected from a nozzle, it tries to diffuse within the air and side by side the combustion happens, combustion happens means that the density of the mixture keeps on changing within this flame structure or within the flame domain. As a result, equivalence ratio also changes.

But what we are trying to analyze here we will try to see that in a very hypothetical way we view that as if a fuel jet is being injected from a nozzle and it tries to diffuse in the air and during this diffusion process one thing that we are assuming that they are not reacting;

that is first thing, second thing is that during this process they are not reacting by non reactive mean that density do not change during the reaction process.

So, such a thing can be modeled if you assume a constant density reacting mixtures. So, you can choose some fuel and or air, we can choose a fuel whose density is also similar to that of air. So, without combustion being happened, it will tell us that how this jet behavior when it comes out and how the jet spreads?

How the jet diffuses? What is its spreading rate? And how the mole fraction, mass fraction or fuel fraction changes along the axial length of the jet? All sorts of things are the discussion pattern or discussion in these lectures.

So, first thing that we say we need to emphasize that laminar diffusion flames are very important and it finds many applications, one application I said in the diesel engines, in some cases in the gas appliances in cooking stoves also we use partially premixed air to provide non-sooting operations.

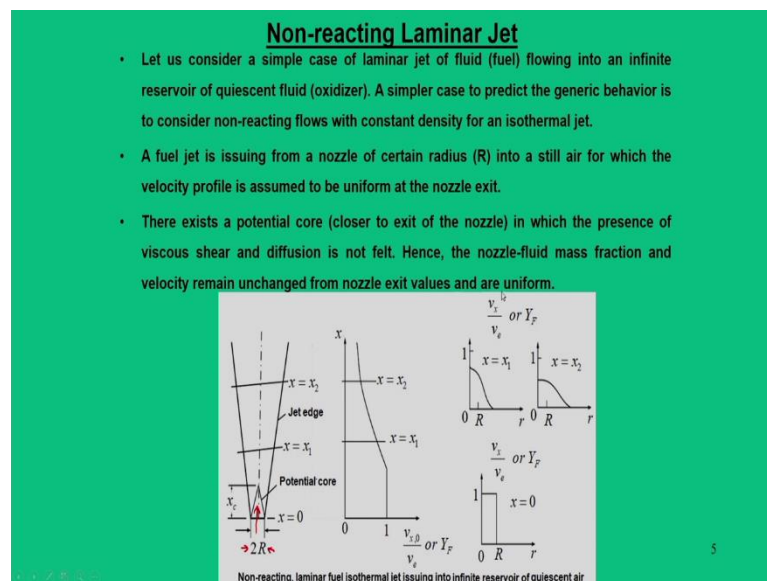
Now, here one more important point is that we intentionally provide a partially premixed mixture to the air just to provide a non-sooting; that means, soot does not form; that means, the mixing has taken place earlier and when it sees the oxidizer combustion needs to happen without having any unburnt products.

As I mentioned, the familiar example of a non premixed jet flame is the Bunsen burner outer cone flame. In our study the prime concern of these laminar jet flames is to discuss about the flame geometry with short flames and fuel types. So, before entering all such things, the important discussion that we need to emphasize here we are going to consider a laminar constant density jet; constant density jets I mean that in a combustion system we have a fuel and oxidizers.

Normally oxidizer is air and air have certain density at given conditions of pressure and temperatures. So, we are choosing a fuel whose density is of similar number with respect to air. So, in that way we can say it is a constant density mixture and by doing so, we are also saying that it is non-reacting so that means, if you simulate the actual behavior density keeps on changing during this combustion process, because the products are formed. But when you say constant density, we assume that it is a non-reacting, but we are choosing a fuel such a way that the behavior is analogous to the actual jet flame.

So, through this analysis it will be possible to establish the general characteristics of velocity, nozzle fluid concentrations fields of laminar jets and understand the Reynolds number dependence of spreading behavior of the jet. So, we will look into a nozzle fluid concentration diffusion which is identical for equal flow rate which translates to frame length based on the flow rates for a given fuel oxidizer combinations. Let us see how things we are going to discuss.

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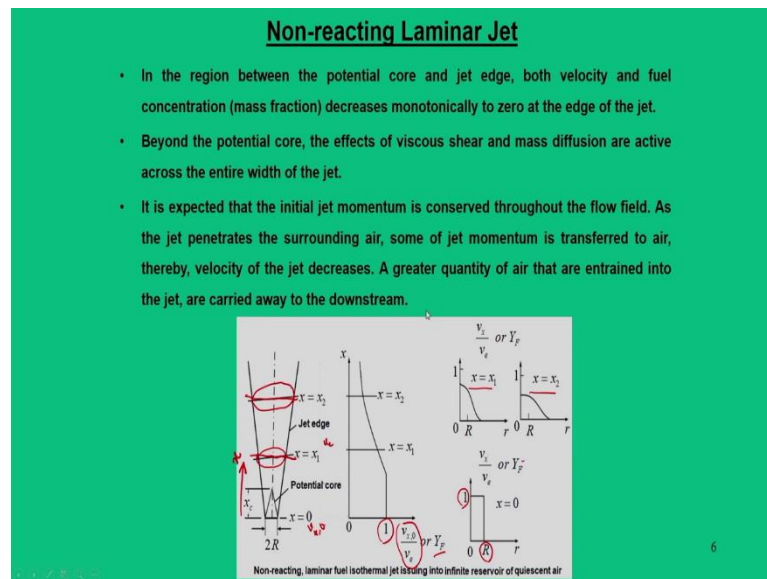


So, what we are considering is a simple case in which a laminar jet of fluid that means, fuel which flows into an infinite reservoir of quiescent fluid oxidizer. So, if you look at this particular figure, we are seeing that a fuel which is getting injected through a nozzle of diameter $2r$ and it comes as a jet and when it comes as a jet, it tries to spread and the outer medium is nothing but your stagnant fluid and in this case it is oxidizer and nothing but air.

So, if you say the nozzle of radius r and fuel jet that comes from a nozzle of radius r and it enters into a still air, the velocity profile is assumed to be uniform at the nozzle exit. So, this one assumption we are making seeing that at the nozzle exit; that means, at x is equal to 0 that is nozzle exit, at that point velocity profile will have uniform and now we will have a potential core and that potential core lasts for a distance x_c and this is nothing but a initial cone and the tip of this cone is nothing but the axial distance x_c from the exit of the nozzle.

So, this is something to be similar to a fully developed flow in a pipe. So, what we say is that there exists a potential core closer to nozzle exist in which the presence of viscous shear and diffusion is not felt; that means, within this potential core, the fuel moves freely without any viscous dissipation or diffusion. And in fact, it has sufficient momentum. The nozzle fluid mass fractions velocity remains unchanged from this nozzle exit value and they are uniform.

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Now, what happens outside? So, what we are looking at we are saying that if $2r$ is the diameter of the nozzle and this is the axial distance x which we are looking at and if you take two arbitrary locations x is equal to x_1 and x is equal to x_2 and if you try to plot, what is the value of the ratio, one is fuel mass fraction and other is the velocity at any x with respect to velocity at x is equal to 0.

So, x is equal to 0, this represent v_{x0} , velocity and at any x we say it is exit velocity at any x and if you try to plot what we can see it starts with 1 and if you keep on going along the axial distance its value drops. And then another way we can represent that what happens to fuel and what happens to fuel mass fractions at the two different locations.

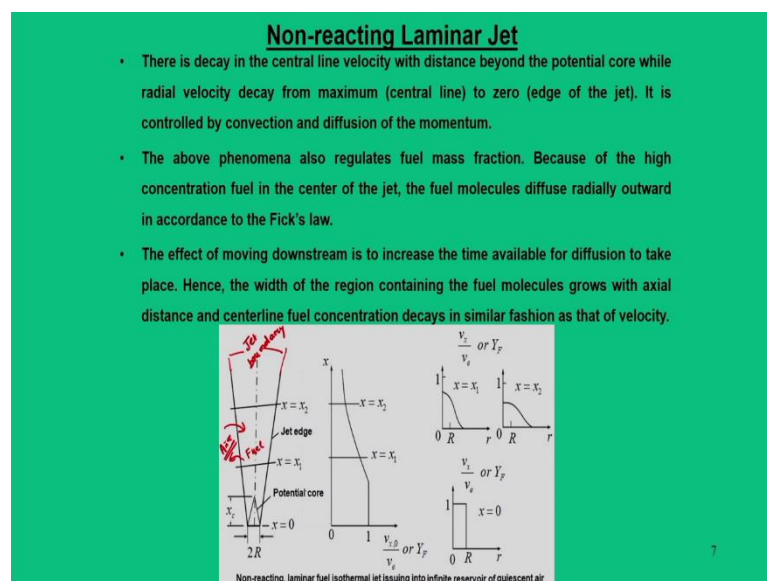
So, first thing let us say at x is equal to 0 we have radius r and whether you take the velocity ratios $\frac{v_x}{v_e}$ or fuel mass fractions they are 1. But when we go at any x . So, what happens this value keeps on changing. So, $\frac{v_x}{v_e}$ or Y_f this value monotonically comes down. So, maybe at

x is equal to 1, its value was initially 1, but at x is equal to x_1 it has come down to this value. And even at further distance at x is equal to x_2 this value again further drops.

So, there is a monotonically drop in the velocity ratios, fractions and fuel mass fractions. So, we will try to quantify at what rate? Or what is the mathematical background behind this? Another important point needs to be emphasized that when the fuel comes out at the initial point that is x is equal to x_0 , it has a sufficient initial jet momentum and with this jet momentum it tries to penetrate into the air. And when it penetrates, with distance its momentum tries to drop.

So, these are the reasons, that is why the velocity ratio drops. And even the fuel concentration also drops, but one thing that we want to see that again, if you look at the radial distributions; that means, at any x is equal to x_1 there is a radial distribution of this fuel jet. So, this radial distribution and axial distribution of this fuel jet, we are going to understand how they behave.

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Another important point similar to the flow through pipe, we have a centerline velocity and we have velocity at the wall. In a same situations we can say that when fuel comes out it has a jet and there is a non physical boundary and that boundary call is a jet boundary.

So, of course, there is no existence of a physical boundary as it is there in the pipe flow, but here it is a jet, I mean it is just a non-physical boundary across which the property information of the jet as well as the air tries to match.

And while deciding this jet boundary we try to see that in this zone, we have mostly concentrated with fuel and we have other side of this jet we have mainly with air. And the boundaries is to be regulated with respect to the property value or matching value between this fuel and air based on these conditions that how fuel propagates into this medium.

So, what we can see is that air tries to penetrate towards the fuel and fuel tries to diffuse towards air. So, this phenomenon actually decides the boundary of the jet. Again, while talking about the fuel mass fractions, it is known that because of high concentration of the fuel in the center of the jet, the fuel molecule tries to diffuse outwards and with respect to Fick's law.

Now, effect of moving downstream is to increase the time available for the diffusion to take place and through this process, the width of the region of fuel molecule grows with the axial distance and the central line fuel concentration decays in a similar manner with respect to fuel velocity.

So, basically both velocity and fuel mass fraction or fuel concentration decays.

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Jet Flame Mathematical Description

- The momentum of flow of jet at any axial location is same as that of momentum of flow issuing from the nozzle at exit. Similarly, the mass of fluid issuing the nozzle is conserved with fuel mass fraction as unity at the nozzle exit.
- Momentum and species diffusivities are constant and equal. The non-dimensional parameter, Schmidt number expresses the ratio between these quantities.
- There are only two species: fuel and oxidizer. The basic governing equations are the boundary layer equations expressing conservation of mass, momentum and species.

Mass conservation: $\frac{\partial v_x}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r v_r) = 0$; Axial momentum: $v_x \frac{\partial v_x}{\partial x} + v_r \frac{\partial v_x}{\partial r} = \nu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_x}{\partial r} \right) \right)$

Species conservation: $v_x \frac{\partial Y_f}{\partial x} + v_r \frac{\partial Y_f}{\partial r} = D \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial Y_f}{\partial r} \right) \right)$ & $Y_{O_2} = 1 - Y_f$; $Sc = \frac{\nu}{D}$ (Schmidt number)

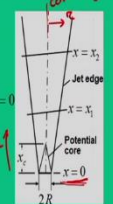
Along jet centreline ($r=0$): $v_r(0,x)=0$, $\frac{\partial v_x}{\partial r}(0,x)=0$, $\frac{\partial Y_f}{\partial r}(0,x)=0$

At large radii ($r \rightarrow \infty$), fluid is stagnant and no fuel is present: $v_x(x,\infty)=0$, $Y_f(x,\infty)=0$

At jet exit ($x=0$): $v_x(r \leq R, 0) = v_e$; $v_r(r > R, 0) = 0$; $Y_f(r \leq R, 0) = Y_{f,e} = 1$; $Y_f(r > R, 0) = 0$

Momentum flow of jet at any x , $J = 2\pi \int_0^\infty \rho(r,x) v_x^2(r,x) r dr = \rho_e v_e^2 (\pi R^2)$

Fuel mass at any x , $2\pi \int_0^\infty \rho(r,x) Y_f^2(r,x) Y_f(r,x) r dr = \rho_e v_e (\pi R^2) Y_{f,e}$



The diagram illustrates a jet flame. It shows a vertical axis x and a radial axis r . The jet originates from a nozzle at $x=0$ with radius R . The jet edge is defined by $x=x_1$ and $x=x_2$. The potential core is the region where the fuel concentration is high. The fuel mass is distributed within the jet, and the diagram shows the jet edge and potential core. Handwritten notes include 'conserved for $cr=0$ ' and '1'.

Now, let us see how we are going to model. What we are trying to say is that momentum of the flow set by any axial location is same as that of momentum issuing from the nozzle exit. So, it is nothing but the momentum balance or force balance at any exit it should match. Similarly, mass of the fluid issuing from the nozzle is conserved with respect to mass fractions as unity at the nozzle exit.

Another assumption we make that the momentum and species diffusivities are constant and equal. So, that is the momentum of the flow and at the same time there is a diffusion of species across the axial as well as the radial directions. So, these two parameters are governed through a non-dimensional number and we called as a Schmidt number and it expresses the ratio between these momentum and species diffusivity. So, basically, we have two species fuel and oxidizer for our analysis and based on that we need to frame the mass conservation, species conservation and axial momentum conservations.

And here since we are looking at the axial distance versus the radial distributions, appropriate coordinate system would be the cylindrical coordinates. So, based on that the three equations were derived. Here we have only axial velocity, $\frac{\partial v_x}{\partial x} + \left(\frac{1}{r}\right) \frac{\partial(rv_r)}{\partial r} = 0$. So, v_x and v_r is nothing but the axial velocity and radial velocity. Similarly, we have axial momentum equations and we have species conservation equations. Side by side we also define the mass fractions of fuel and oxidizer. And here we also have another number Schmidt number $Sc = \frac{\nu}{D}$ where ν is the viscosity and D is the diffusion coefficients.

Now, let us see how we understand this boundary conditions. So, basically there are two things, one is this is central line and this is at x is equal to 0 and central line means it is r is equal to 0. So, we are looking r in this directions and we are looking at x in this directions. So, this is x and this is r . ($r = 0$): $v_r(0, x) = 0$; $\frac{\partial v_x}{\partial r}(0, x) = 0$; $\frac{\partial Y_F}{\partial r}(0, x) = 0$

Because it is a fixed value. At r is equal to very high means very larger radius means if you are looking at sufficiently larger radius fluid is stagnant. So, as if there is no fuel present. So, under those conditions $v_x(\infty, x) = 0$; $Y_F(\infty, x) = 0$. And again, at the jet exit; at x is equal to 0 we have two possibilities either we can $r \leq R$ or $r > R$. So, accordingly we have $v_x(r > R, 0) = 0$.

So, this also can be interpreted for fuel fractions as well. So, $Y_F(r \leq R, 0) = Y_{F,e} = 1$; $Y_F(r > R, 0) = 0$. Another point that needs to be emphasized that momentum of the jet

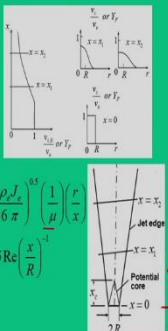
at any x . So, at any x is defined by $J = 2\pi \int_0^\infty \rho(r, x) v_x^2(r, x) r dr = \rho_e v_e^2 (\pi R^2)$ and that has to be equalized with respect to momentum jet momentum at x is equal to 0 that is initial jet momentum.

Similarly, the fuel mass has to be calculated with respect to its value at x equal to 0. So, this is the very basic bottom line through which these equations needs to be solved.

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Jet Flame Mathematical Description

- The solution is not valid near the nozzle but far away from the nozzle. The velocity decays inversely with axial distance and directly proportional to the jet Reynolds number. Fuel mass fraction also decays in similar manner.
- The velocity decay is more rapid with lower Reynolds number jets. This behavior occurs because the relative importance of initial momentum becomes smaller in comparison with viscous shearing action.



Nondimensional axial velocity distribution: $\frac{v_x}{v_e} = 0.375 \text{ Re} \left(\frac{x}{R} \right)^{-1/2} \left(1 + \frac{z^2}{4} \right)^{-3/2}$

Fuel mass fraction, $Y_f = 0.375 \text{ Re} \left(\frac{x}{R} \right)^{-1/2} \left(1 + \frac{z^2}{4} \right)^{-3/2}$; Similarity parameter, $z = \left(\frac{3\rho_e J_e}{16\pi} \right)^{1/2} \left(\frac{1}{\mu} \right) \left(\frac{r}{x} \right)$

Centreline velocity: $\frac{v_{x,c}}{v_e} = 0.375 \text{ Re} \left(\frac{x}{R} \right)^{-1/2}$; Centreline mass fraction: $Y_{f,c} = 0.375 \text{ Re} \left(\frac{x}{R} \right)^{-1/2}$

Reynolds number, $\text{Re} = \frac{\rho_e v_e R}{\mu}$; Initial jet momentum, $J_e = \rho_e v_e^2 (\pi R^2)$

Above solutions are valid far from nozzle when, $\left(\frac{x}{R} \right) \geq 0.375 \text{ Re}$ & $\text{Sc} = \frac{\nu}{D} = 1$ (Schmidt number)

And ultimately, we are not going into derivations for this, but what we are going to find is the solutions for this. So, while talking about the solutions we are talking in terms of ratios first ratio is the non non-dimensional axial velocity distribution that is $\frac{v_x}{v_e}$. So, I mentioned that v is the exit velocity, v_x is the velocity at any x .

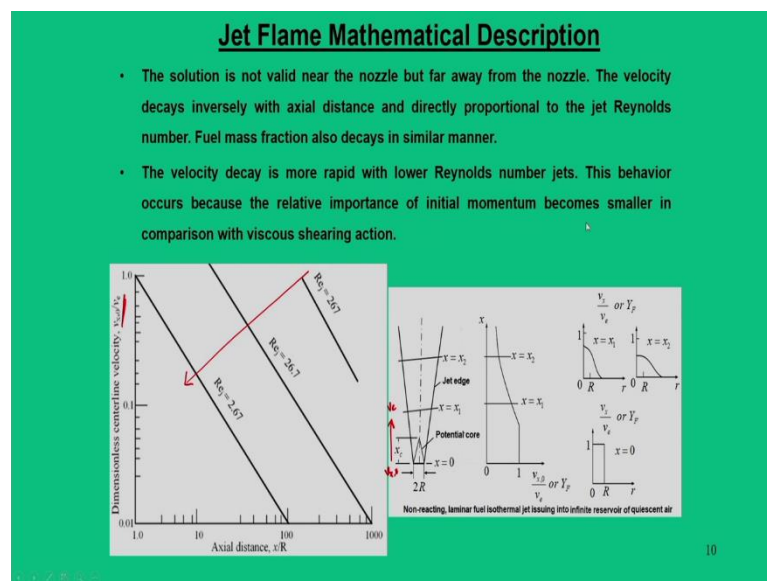
So, if you say this is x is equal to 0, we are talking about v_x at any x and that is a function of two parameters that is non-dimensional axial location that is x by r and other parameter that is known as similarity parameters; what is the importance of similarity parameter? Because, we are looking at the velocity distributions as well as the fuel mass fraction simultaneously.

So, the similarity parameter that controls both the parameters simultaneously is defined by a z and this z is correlated with respect to viscosity r/x and jet momentum at the initial momentum ρ_e and with this similarity parameters, we try to derive the expressions for the non-dimensional axial distributions for velocity and fuel mass fractions.

So, if you see these expressions here, if you see both the expressions one similar solution is that their expressions are almost same. So, that is the reason if you can see the decay for velocity distribution and fuel mass fractions are exactly same in nature. And from these equations we can find out the central line velocity as a function of Reynolds number and axial locations, we can also find out the centerline mass fractions which is typically needs to be the highest and with x as well as r they are going to decay.

Another term Reynolds number that is calculated based on the radius of the jet $Re = \frac{\rho_e v_e R}{\mu}$; and we also know the initial momentum of the jet; because you know the exit conditions of density and velocity. And we also know the radius of the nozzle. So, from this you can find the non-dimensional form of velocity ratio and fuel mass fractions. But however, when deriving the solutions, it is always taken care that it is valid far from the nozzle for these conditions. But however, it gives a more or less similar physical nature of the actual situations of jet flame.

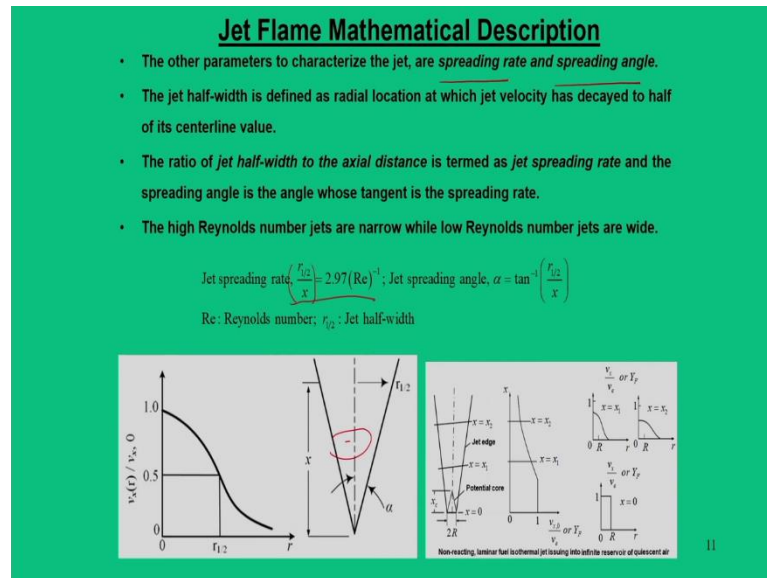
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Now, this particular plot shows the non-dimensional centerline velocity with respect to axial distance. So, we are looking at the axial distance x and its dependence with v_{x0}/v_e means at this location this is v_{x0} and v_e is any location that is non dimensional ratio. So, this ratio is plotted against axial distance. We see that in other words the decay is more rapid when the Reynolds number is small.

That means instead of going off if you come back from decreasing order of Reynolds number the decay is faster whereas, decay is slower, you can see the decay is very faster rate may be 10 times 100 times, 1000 times. So, if you say that lower Reynolds number of jet has higher rapid velocity decay. And this is occurring because the relative importance of initial momentum becomes smaller in comparison with the viscous shearing action.

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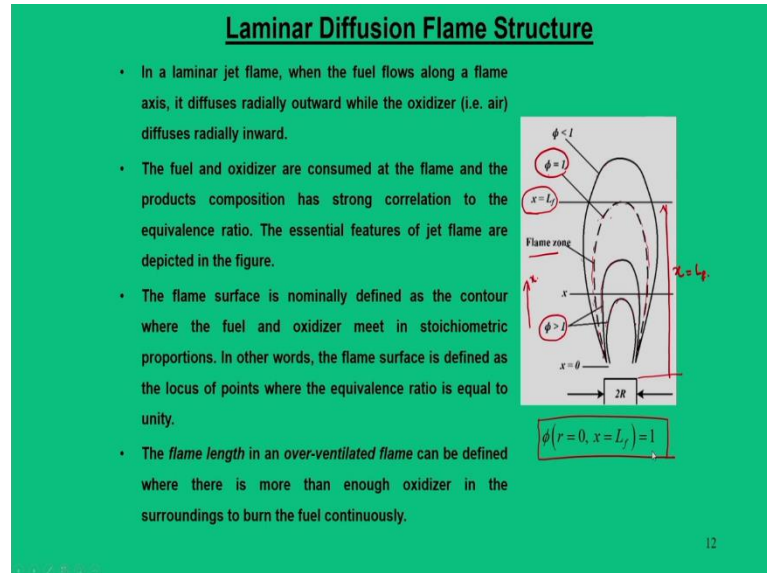


The next parameter that we are looking at is the jet spreading rate and the spreading angle. So, we define this jet spreading rate with respect to jet half width. So, what does this mean that if you plot the non-dimensional velocity with respect to radius; that means, along this radial profile we find that at one particular location that is r_1 or jet half width, we will find this ratio value drops from 1 to 0.5 means the ratio of jet half width to the jet axial distance is referred as jet spreading rate. And this jet half width is defined as the radial locations at which the jet velocity has decayed to half of its central line value.

So, once we know this jet spreading rate, we can find the jet spreading angle is the angle whose tangent is the jet spreading rate; that things we can represent in mathematical form as jet spreading rate is equal to $\frac{r_{1/2}}{x} = 2.97((\text{Re})^{-1})$. So, this says that if your Reynolds number is high, then jets are narrow; while the low Reynolds number jets are normally wide.

So, this information gives how a jet spreads in the oxidizer or how fuel spreads in the oxidizer.

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The other important aspects of laminar diffusion flame is to find the length of the flame. So, if you look at the actual scenario, we will see that we have different zones and that different zones normally represent the concentration of fuel and air ratio. Some location; that means, in the vicinity of the potential core or towards the nozzle side I can say we can find equivalence ratio is more.

And as and when we go along x, we will see that equivalence ratio keeps on coming down. So, when it keeps on coming down, it means that we are reaching more towards the air zone.

And one particular and in fact, the flame surface or what we say the zone is normally decided by a surface that normally tries to see the regions in which the air and fuel properties match. And one particular point or one particular flame zone where we can find there is a phi equal to 1. So, which means that at the entire contour of this flame zone or entire locations on this flame zone, if you calculate the equivalence ratio, they are they are equal which means that complete oxidation or complete combustion has taken place on this particular flame surface.

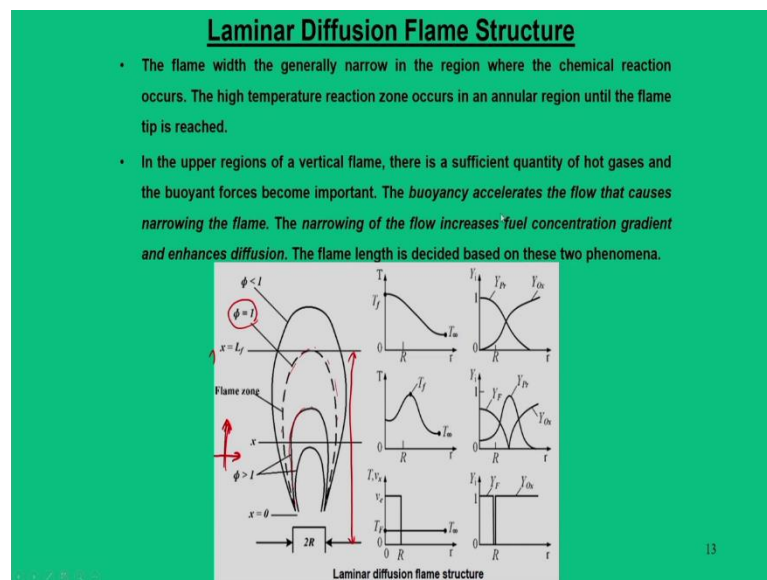
Now, if you look at this particular surface it is not the case of complete combustions, in this zone will find there are some fuel particles still they are not burnt. And when you move beyond this flame zone where ϕ is equal to 1, we have sufficient oxidizer, but there is no fuel to be burnt. So, this mismatch tells us that there is a particular length what we call as x equal to L_f , that is the length of the flame and that length of the flame is calculated as axial distance from this exit of this nozzle.

And that is nothing but the flame zone which is defined that beyond this length there are enough oxidizer in the surroundings to burn the fuel completely.

But below this length there is not enough oxidizer and normally when you talk about this flame surface, it is nominally defined as the contour where fuel and oxidizer meet in the stoichiometric proportions or in other words, we say that flame surface is the locus of points where the equivalence ratio is equal to unity. So, this is the catch mark point which needs to be known and for every flame situations or every laminar diffusion flame we need to calculate this flame length.

So, this contour is normally defined by this expressions in terms of equivalence ratio that is $\phi(r = 0, x = L_f) = 1$

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Now, while talking about this flame length we will also have flame width. There are two phenomena that takes place here we talked about flame length let us talk about what is

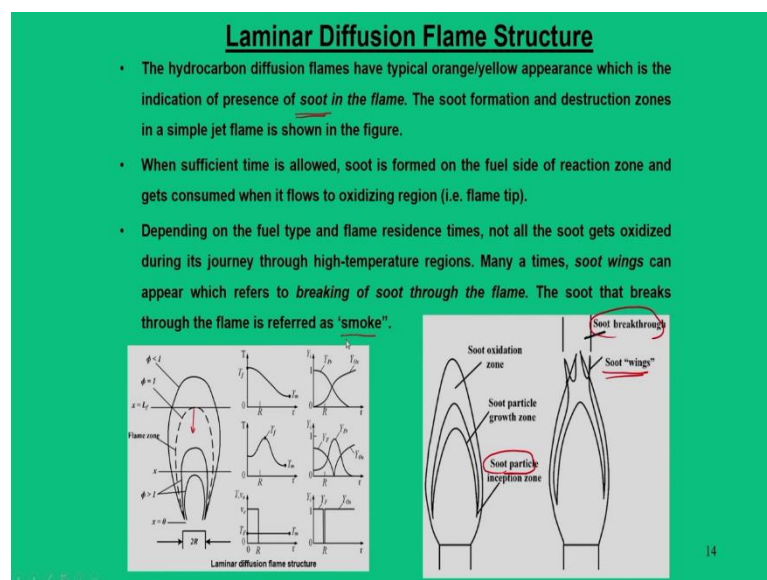
flame width. Flame width means if you go along the length of the flame, we will find the effect of buoyancy.

So, basically the buoyancy accelerates the flow that causes the flow to narrow. So, when the fuel gets pushed off towards the air so the flame tries to be narrow. At the same time when the flow becomes narrow flame it increases the fuel concentration gradient and enhances the diffusions.

So, basically fuel tries to diffuse outwards at the same time buoyancy also pushes off. So, the buoyancy pushes off and fuel also tries to diffuse outwards. So, this combined two effects gives such kind of surface and that surface we call this as a flame surface. And one particular surface = we can find for ϕ is equal to 1 and for that ϕ is equal to 1, we can find a particular x , that at what x it appears.

So, that is nothing but x is equal to L_f . So, the length of the flame is decided based on these two parameters one is buoyancy other is the diffusion.

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Another significant part of this flame is what we can say soot formations. So, formation of soot in a flame. We already told that when you see the flame surface below this ϕ is equal to 1; this is basically all the surface have equivalence ratio more than 1. So that means, they are rich in fuel.

But they do not find sufficient oxygen or oxidizers. So, this causes that there are some unburned fuel particles that remains. Now, when you look at this particular figure these unburned particles leads to the formation of soot and we call this as soot formation and the soot formation normally initiated at the exit point of this nozzle. So, maybe it starts with a potential core and because of momentum of this flow and the soot particles tries to move up.

So, basically through this process what we see is that we have soot particle inception zone then there is a zone what we call as soot particle growth zone and the soot particles strives to oxidize and if they are sufficient to oxidize, there is no further possible, this is a normal convention that even if during its propagation the fuel particles is has oxidized.

Now if at all for variety of reason, if it does not then what happens, we will have a formation of soot wings; in this soot wings we will find some zone at some flame surface, we will find there is a soot break through.

That means this soot particles try to break and when they tried to break and we normally refer this as a smoke. So, depending on the fuel residence type, not all the fuel gets oxidized during its journey from high temperature regions. So, many a times soot wings are formed and which refers as breaking of suits through the flame. The soot that breaks through the flame is referred as the smoke.

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Laminar Diffusion Flame Structure

- The other important feature of laminar jet diffusion flame is the relationship between flame length and initial condition. In a circular port flames, the flame length depends only on the flow rate which is the combined effect of initial velocity and port diameter.
- The flame length is indeed directly proportional to the initial volumetric flow rate and inversely related to the stoichiometric fuel mass fraction.
- Thus, fuels that require less air for complete combustion produce shorter flame.

Flame length, $L_f \approx \left(\frac{3}{8\pi} \right) \left(\frac{Q_f}{DY_{F,sto}} \right)$ (3)

Initial volume flow rate, $Q_f = v_f (\pi R^2)$

R : Radius of flame port; D : Species diffusivity

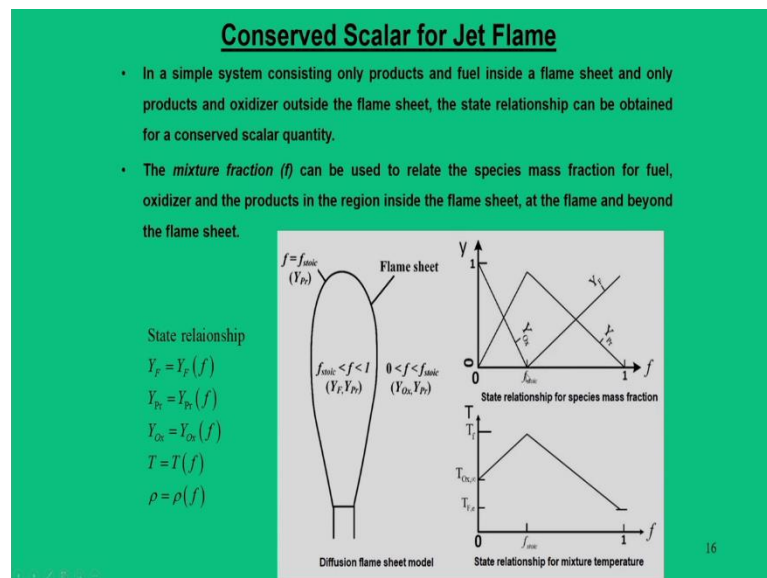
v_f : Velocity of the fuel at port exit

$Y_{F,sto}$: Stoichiometric fuel mass fraction

And the last point that I need to emphasize that from our understanding of our analysis or mathematical expressions, one can find out the length of the flame for which ϕ is equal to 1 as approximately equal to the $\left(\frac{3}{8\pi}\right)\left(\frac{Q_F}{DY_{F,stoi}}\right)$ where Q_F is nothing but the initial volume flow rate and that is decided by the initial jet velocity and area of the nozzle jet or nozzle port, then D stands for species diffusivity v_e is the velocity of the fuel at the port exit and Y_F is nothing but the stoichiometric fuel mass fractions.

So, it clearly tells that length of the flame is directly proportional to the volume flow rate and inversely related to the fuel mass fractions.

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The other important aspect that I need to emphasize is the conserved scalar. So, the conserved scalar is a quantity which is normally used in all types of combustion problems which says that any scalar quantity that remains conserved throughout the combustion process. One such parameter is nothing but the mixture fractions and the fraction of mass or mole fraction of the component which originates from the fuel.

And which remains conserved throughout its journey in the combustion process. So, based on that what we see is that if you look at a flame sheet we can have three distinct locations, one is on the flame surface where f is equal to stoichiometric value, at the point your almost exactly entire combustion has happened and $Y_{products}$ are is equal to 1. And within this

flame we will find there is some mass fraction of the fuel as some products, outside the flame we will not find fuel we will find oxidizer and products.

So, there is a mass balance between the flame surface or flame sheet inside the flame and outside the flame. And distribution of temperature and mass fractions can be represented in this manner as it is a very clear understanding that temperature keeps on increasing till stoichiometric value again further drops with respect to fuel mass fraction. And if you see the mixture fraction value with respect to the mass fraction Y then for oxidizer, for fuel and products, then it can be plotted at that fuel stoichiometric value, its value for mixture fraction is 0.

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Conserved Scalar for Jet Flame

- In a simple system consisting only products and fuel inside a flame sheet and only products and oxidizer outside the flame sheet, the state relationship can be obtained for a conserved scalar quantity.
- The mixture fraction can be used to relate the species mass fraction for fuel, oxidizer and the products in the region inside the flame sheet, at the flame and beyond the flame sheet.

Mixture fraction, $f = \frac{\text{Mass of material originating in fuel stream}}{\text{Mass of mixture}} = \frac{m_f + m_{pr}}{m_{mix}}$

1 kg fuel + ν kg oxidizer $\rightarrow (\nu+1)$ kg products

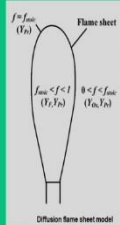
$\Rightarrow f = Y_f + \left(\frac{1}{\nu+1}\right) Y_{pr}$ & $f_{mix} = \left(\frac{1}{\nu+1}\right)$

Inside the flame ($f_{mix} < f \leq 1$): $Y_f = \frac{f - f_{mix}}{1 - f_{mix}}$; $Y_{O_2} = 0$; $Y_{pr} = \frac{1 - f}{1 - f_{mix}}$

Outside the flame ($0 < f \leq f_{mix}$): $Y_f = 0$; $Y_{O_2} = 1 - \frac{f}{f_{mix}}$; $Y_{pr} = \frac{f}{f_{mix}}$

At the flame ($f = f_{mix}$): $Y_f = 0$; $Y_{O_2} = 0$; $Y_{pr} = 1$

Y_f : kg fuel per kg mixture; Y_{pr} : kg products per kg mixture; Y_{O_2} : kg oxidizer per kg mixture



So, a complete understanding we can make what happens. So, by definitions we can say a mixture fraction is the fractions that of the mass of the material that originates in the fuel stream divided by the mass of the mixtures. So, based on that we have fuel fractions, we have the fuel stoichiometric value; when 1 kg of fuel mixes with ν kg of oxidizer to form $\nu + 1$ kg of products.

So, we have three distinct zones one is inside the flame and under that conditions based on this relations, we can find within the flame what is the mass fraction of the fuel, what is the mass fraction of the oxidizer and what is the mass fraction of products.

Similarly, outside the flame we can find what is the mass fraction of fuel, oxidizer and products? And that relations we can represent in terms of stoichiometric values.

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Numerical Problems

Q1. A jet of ethylene exits from a 8 mm diameter nozzle into still air at 300 K and 1 atm. Calculate the spreading angles and axial locations where the jet centerline mass fraction drops to stoichiometric values for the following jet exit velocities: (a) 0.08 m/s; (b) 0.008 m/s. The viscosity of ethylene is 102.3×10^{-7} N.s/m².

Constant density C_2H_4 , $(Mw)_{C_2H_4} = 28$, $(Mw)_{air} = 29$.

(a) Spreading angle (α)

$$\alpha = \tan^{-1} \left(\frac{r_{c,0.5}}{x} \right) = \frac{r_{c,0.5}}{R_{0.5}}$$

$$(R_{0.5})_1 = \frac{\rho U_{0.1} R}{\mu} = \frac{1.18 \times 0.08 \times 0.004}{102.3 \times 10^{-7}} = 35.65$$

$$(R_{0.5})_2 = \frac{\rho U_{0.2} R}{\mu} = 3.565$$

$$\alpha_1 = \tan^{-1} \left(\frac{3.97}{35.65} \right) = 4.75^\circ$$

$$\alpha_2 = \tan^{-1} \left(\frac{3.97}{3.565} \right) = 39.8^\circ$$

Low velocity jets $\Rightarrow$ higher spreading angle

So, this is all discussions with respect to laminar jet flames. So, based on our understanding let us try to solve some numerical problems with respect to jet flame. So, the first problem that talks about we have a jet of ethylene; that means, ethylene jet that comes out of a nozzle and that nozzle diameter that $2R$ is 8 mm and the ambient conditions that comes out is air and air condition is 300 kelvin and pressure is one atmosphere.

So, under these conditions we need to find out the spreading angle and other is we have to find the axial location at which the jet central line mass fraction drops to the stoichiometric value. And these things we need to find out for two velocities one is 0.08 meter per second other is 0.008 meter per second.

So, basically we know $v_{e1} = 0.08 \text{ m/s}$, $v_{e2} = 0.008 \text{ m/s}$. For these two conditions we need to find out jet spreading angle and axial locations. So, let us start the solution the first thing that we need to check whether it is a constant density or not. So, constant density I mean if you look at ethylene. So, ethylene is C_2H_4 and its molecular weight is 28 and molecular weight for air is approximately 29.

So, more or less we can say a constant density situation may be assumed and again it is a non-reacting when you say constant density, non-reacting also will have similar

resemblance. So, considering this that whatever mathematical treatment we have done for this study we can make use of that.

$$\alpha = \tan^{-1} \left(\frac{r_1}{x} \right); \frac{r_1}{x} = \frac{2.97}{R_{ej}}$$

$$R_{e1} = \frac{\rho v_{e1} R}{\mu} = \frac{1.14 \times 0.08 \times 0.004}{102.3 \times 10^{-7}} = 35.65; R_{e1} = \frac{\rho v_{e2} R}{\mu} = 3.565$$

$$\alpha_1 = \tan^{-1} \left(\frac{2.97}{35.65} \right) = 4.75^\circ; \alpha_2 = \tan^{-1} \left(\frac{2.97}{3.565} \right) = 39.8^\circ$$

So, we see that the low velocity jets means low Reynolds number jets implies higher spreading angle. So, this information we get that is also naturally true.

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Numerical Problems

Q1. A jet of ethylene exits from a 8 mm diameter nozzle into still air at 300 K and 1 atm. Calculate the spreading angles and axial locations where the jet centerline mass fraction drops to stoichiometric values for the following jet exit velocities: (a) 0.08 m/s; (b) 0.008 m/s. The viscosity of ethylene is 102.3×10^{-7} N.s/m².

Q1

$$Y_{F,sto} = Y_{F,0} = 0.375 Re \left(\frac{x}{R} \right)^{-1}$$

$$\Rightarrow x = \left[\frac{0.375 Re}{Y_{F,sto}} \right] R$$

Q2

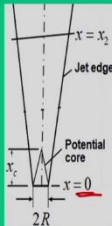
$$C_2 H_4 + a (O_2 + 3.76 N_2) \rightarrow x CO_2 + \frac{y}{2} H_2O + 3.76a N_2$$

$$(AF)_{sto} = \frac{m_{air}}{m_F} = \frac{4.76 \times 32 \times 2.9}{28} = 14.7$$

$$Y_{sto} = \frac{1}{14.7 + 1} = 0.6637$$

$$x_1 = \frac{0.375 \times 35.65 \times 0.004}{0.6637} = 0.84 \text{ m}$$

$$x_2 = \frac{0.375 \times 3.565 \times 0.004}{0.6637} = 0.684 \text{ m}$$



Jet edge
Potential core
x = x₂
x = 0
2R

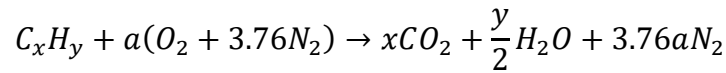
Handwritten notes:
R = 0.004 m
R₀ = R₀₀
R₀₂

Handwritten notes:
Fuel conc. decays
Same value for low velocity
id. w.r.t. high velocity
id.

And the second part of the discussion would be we need to find out the axial location of the jet central line where mass fraction drops to the stoichiometric value. So, for that we need to find out the expressions

$$Y_F = 0.375 Re \left(\frac{x}{R} \right)^{-1}; x = \left[\frac{0.375 Re}{Y_{F,sto}} \right] R$$

So, basically we need to find what is the $Y_{F,stoi}$, all the other parameters is known R is 0.004, we have R_{e1} and R_{e2} . So, to find the stoichiometric value we have to recall this fundamental equation.



$$\text{Fuel: } C_2H_4: x = 2, y = 4; a = x + \frac{y}{4} = 3$$

So, when you know all these things, we can write the stoichiometric value that is

$$AF_{stoi} = \frac{m_{air}}{m_f} = \frac{4.76a(MW)_{air}}{(MW)_f} = 14.7; Y_{stoi} = \frac{1}{14.7 + 1} = 0.0637$$

Once we will have the stoichiometric value then we can find the two parameters,

$$x_1 = \left[\frac{0.375 \times 35.65}{0.0637} \right] 0.004 = 0.84m, x_2 = \left[\frac{0.375 \times 3.565}{0.0637} \right] 0.004 = 0.084m$$

So, it says that fuel concentration decays to same value for low velocity jet with respect to high velocity jet.

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Numerical Problems

Q2. A jet of ethylene exits at a velocity of 0.008 m/s from a 8 mm diameter nozzle into still air at 300 K and 1 atm. What exit diameter of the nozzle is required to maintain the same flow rate if the exit velocity is increased by 10-times. Determine the axial location for which fuel mass fraction drops to stoichiometric value for this condition.

Handwritten solution:

Flow rate, $Q = U_1 A_1 = U_2 A_2$
 $\Rightarrow U_1 (\pi R_1^2) = U_2 (\pi R_2^2)$
 $\Rightarrow R_2^2 = \left(\frac{U_1}{U_2} \right) \times R_1^2$
 $\Rightarrow R_2 = 1.2 \text{ mm} \Rightarrow 2R_2 = 2.4 \text{ mm}$

$R_{eq} = \frac{\rho U_2 R_2}{\mu} = \frac{1.18 \times 0.08 \times 0.0012}{102.3 \times 10^{-7}} = 10.7$

$x = \left(\frac{0.375 R_{eq}}{Y_{stoi}} \right) R = \frac{0.375 \times 10.7}{0.0637} \times 0.0012$
 $\Rightarrow x \approx 0.875 \text{ m}$

Diagram: A diagram of a jet showing the potential core and jet edge. The jet edge is at $x = x_2$ and the potential core is at $x = 0$. The radius at the jet edge is $2R$. Handwritten notes indicate $U_1 = 0.008 \text{ m/s}$, $2R_1 = 8 \text{ mm}$, and $U_{eq} = 0.08 \text{ m/s}$.

And next is the same problem, but it is tuned in a different way saying that here we are looking at data that is given is $v_{e1} = 0.008 \text{ m/s}$, $2R = 8 \text{ mm}$.

Now, what is required is we have to find out the exit diameter of another the nozzle which is required to maintain the same flow rate, but exit velocity has to be increased 10 times 10 time means 0.08 meter per second. So, here the exit velocity is increased by changing the radius of the nozzle.

So, for that what remains same is the volume flow rate.

$$Q = v_{e1}A_1 = v_{e2}A_2; v_{e1}(\pi R_1^2) = v_{e2}(\pi R_2^2); R_2^2 = \left(\frac{v_{e1}}{v_{e2}}\right) R_1^2; R_2 = 1.2 \text{ mm}$$

So, this gives $2R_2$ or diameter of the nozzle exit diameter of the nozzle as 2.4 mm. So, to maintain this flow rate the exit diameter is has to be 2.4 mm. Now for this situation, we need to find out the axial location for which fuel mass fractions drops to stoichiometric value. So, again for this nozzle we need to find out what is Reynolds number of the jet.

$$Re_j = \frac{\rho v_{ej} R_2}{\mu} = \frac{1.14 \times 0.08 \times 0.0012}{102.3 \times 10^{-7}} = 10.7$$

$$x = \left[\frac{0.375 Re_j}{Y_{F,stoi}} \right] R = \left[\frac{0.375 \times 10.7}{0.0637} \right] 0.0012 = 0.075 \text{ m}$$

So, this is almost close to the previous cases of x value is 0.084 meter. So, this is all about the discussions for today.

Thank you for your attentions.