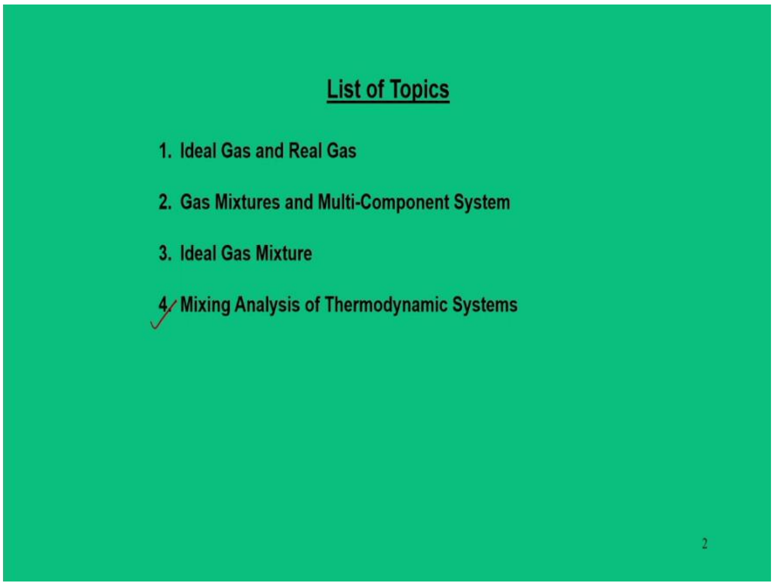
Advanced Thermodynamics and Combustion
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - IV
Properties of Gas Mixture
Lecture - 18
Mixing Analysis of Thermodynamic Systems

Dear learners, greetings from IIT, Guwahati. We are in the MOOCs course, Advanced Thermodynamics and Combustions; module 4, Properties of Gas Mixture.

(Refer Slide Time: 00:44)



List of Topics

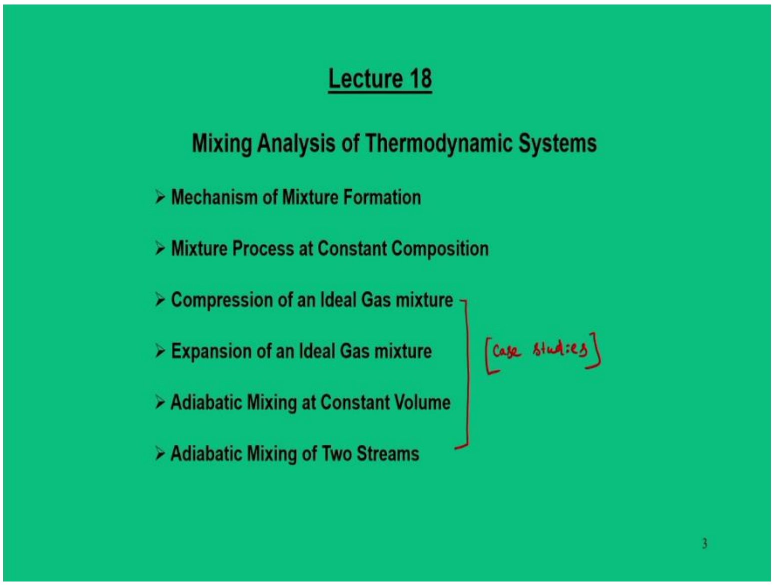
1. Ideal Gas and Real Gas
2. Gas Mixtures and Multi-Component System
3. Ideal Gas Mixture
4. ✓ Mixing Analysis of Thermodynamic Systems

So, in this module, we have covered three lectures. First one was Ideal Gas and Real Gas, its modeling; second one was intended for Gas Mixtures and Multi-Component Systems; in the third lectures, we have used the concept of Ideal Gas Mixtures and subsequently, we found out how thermodynamic properties for individual components as well as for the mixture are going to be calculated.

Now, in the last lecture of this module for today is the Mixing Analysis of Thermodynamic Systems; that means, system either it is mixed individually, the substances are in the pure component form. Now, they have mixed completely. Now, when they mix completely, how you are going to analyze. And in fact, here we will deliver some of the important applications of mixing process in various topics like in IC engines, in mixing of adiabatic streams or maybe in the psychrometrics you used to have the mixing process of air and water vapor, in the combustion study, it is the mixing process of air and the fuel.

So, all these things which are going to be used in our day to day life, we will talk about the modeling aspects of mixing process for various thermodynamic systems.

(Refer Slide Time: 02:20)



Lecture 18

Mixing Analysis of Thermodynamic Systems

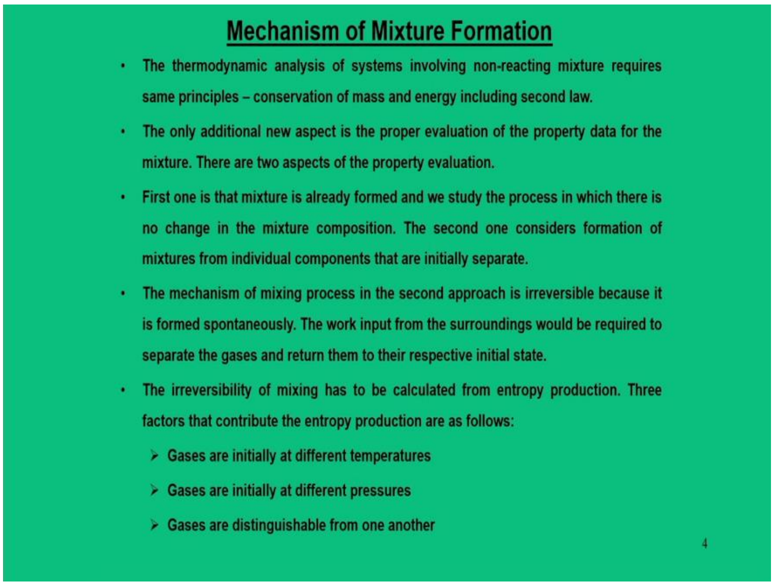
- Mechanism of Mixture Formation
- Mixture Process at Constant Composition
- Compression of an Ideal Gas mixture
- Expansion of an Ideal Gas mixture
- Adiabatic Mixing at Constant Volume
- Adiabatic Mixing of Two Streams

[Case studies]

So, theme of this topic for this lecture 18 is mixing analysis of thermodynamic systems. First, we will discuss about mechanism of mixing formations; then we will think about the mixing if it happens at constant composition, how the thermodynamic modeling is going to take place. Now, as a case studies, we will deal with the four important cases like compression of ideal gas mixture, expansion of an ideal gas mixture, adiabatic mixing at constant volume and adiabatic mixing of two streams. So, these things I will put them as some of the case studies.

So, these four case studies you are going to discuss the aspects and when you are going to discuss, you will try to find out what are the property evaluations in terms of enthalpy, entropy and internal energy.

(Refer Slide Time: 03:29)



Mechanism of Mixture Formation

- The thermodynamic analysis of systems involving non-reacting mixture requires same principles – conservation of mass and energy including second law.
- The only additional new aspect is the proper evaluation of the property data for the mixture. There are two aspects of the property evaluation.
- First one is that mixture is already formed and we study the process in which there is no change in the mixture composition. The second one considers formation of mixtures from individual components that are initially separate.
- The mechanism of mixing process in the second approach is irreversible because it is formed spontaneously. The work input from the surroundings would be required to separate the gases and return them to their respective initial state.
- The irreversibility of mixing has to be calculated from entropy production. Three factors that contribute the entropy production are as follows:
  - Gases are initially at different temperatures
  - Gases are initially at different pressures
  - Gases are distinguishable from one another

Now, let us start the formation of mixture or the concept of mixture formations. So, I call it as a mechanism of mixture formations. So, when you deal with the non-reacting mixtures, the thermodynamic analysis deals with the conservation of mass and energy principle as well as the second law. So, second law will give you the property entropy. So, this we normally used to do when we consider the working medium as a pure substance; but when you deal with the mixture, we require an additional data which is the composition of that component in the mixture.

So, there are two aspects of this; one is the component of that pure substance in the mixtures, second one is behavioral aspects of the mixture. So, this behavioral aspects of this mixture, we devoted in our last class that we will model it in as an ideal gas format. Now, coming back to the mixture, there are two categories. First one is that mixture is already formed; it means, already there is a mixture which is existed in a container or some medium. Now, we study the process in which there is no change in the mixture composition.

The second one consider the formation of mixtures from individual components that are initially separate. So, the second one is like a mixing of two streams or more number of streams, one can think that during combustions we have a fuel stream we have air stream. So, after combustion, we get a products, we call this as a combustion mixture. So, in fact, in psychrometrics, we have airstream and we have water vapour stream and they mix together.

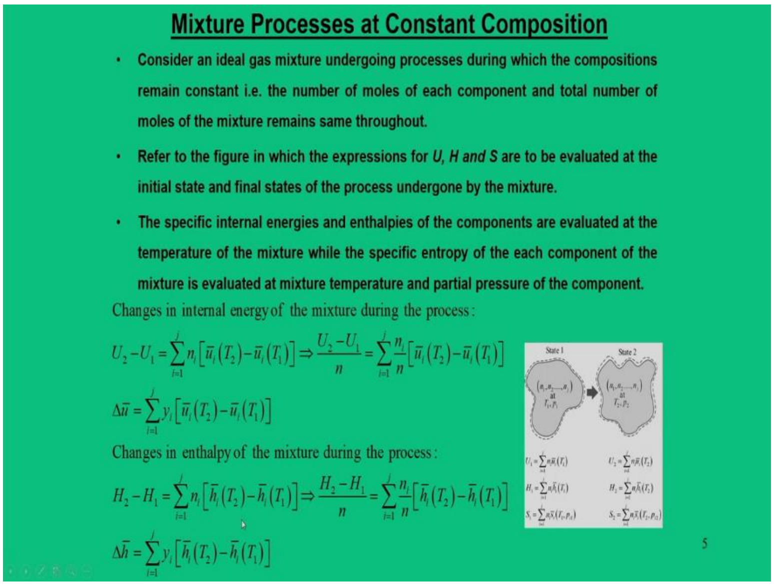
So, we call it as a mixture. But in the first one, the mixture is already formed like in an IC engine, the premixed air and fuel is in the combustion chamber. Then, once there is a change in the temperature and pressure that the composition also changes. So, these two situations, you are going to model.

Now, when you deal with the second aspects like when you talk about mechanizing of mixing process in the second approach, so this mixture process is always irreversible because this mixture is formed spontaneously and the work input from the surrounding and that means, if you want to separate the gases out of this mixture, then we would be requiring work input from the surroundings and this is never possible.

So, we call this process to be an irreversible process. So, we need to calculate; that means, when you deal with the thermodynamic properties, the parameter that needs to be going to evaluated for irreversibility calculation is the entropy. Why irreversibility term comes into picture? Because we said that the individual components that are initially separate.

So, when they are initially separate, they may be at different temperatures, they may have different pressures and they are also distinguishable from one another. That means, different gases. So, because of this reason, the process is irreversible in nature.

(Refer Slide Time: 07:00)



Mixture Processes at Constant Composition

- Consider an ideal gas mixture undergoing processes during which the compositions remain constant i.e. the number of moles of each component and total number of moles of the mixture remains same throughout.
- Refer to the figure in which the expressions for *U, H and S* are to be evaluated at the initial state and final states of the process undergone by the mixture.
- The specific internal energies and enthalpies of the components are evaluated at the temperature of the mixture while the specific entropy of the each component of the mixture is evaluated at mixture temperature and partial pressure of the component.

Changes in internal energy of the mixture during the process:

$$U_2-U_1=\sum_{i=1}^{j} n_i\left[\bar{u}_i(T_2)-\bar{u}_i(T_1)\right] \Rightarrow \frac{U_2-U_1}{n}=\sum_{i=1}^{j}\frac{n_i}{n}\left[\bar{u}_i(T_2)-\bar{u}_i(T_1)\right]$$

$$\Delta\bar{u}=\sum_{i=1}^{j} y_i\left[\bar{u}_i(T_2)-\bar{u}_i(T_1)\right]$$

Changes in enthalpy of the mixture during the process:

$$H_2-H_1=\sum_{i=1}^{j} n_i\left[\bar{h}_i(T_2)-\bar{h}_i(T_1)\right] \Rightarrow \frac{H_2-H_1}{n}=\sum_{i=1}^{j}\frac{n_i}{n}\left[\bar{h}_i(T_2)-\bar{h}_i(T_1)\right]$$

$$\Delta\bar{h}=\sum_{i=1}^{j} y_i\left[\bar{h}_i(T_2)-\bar{h}_i(T_1)\right]$$

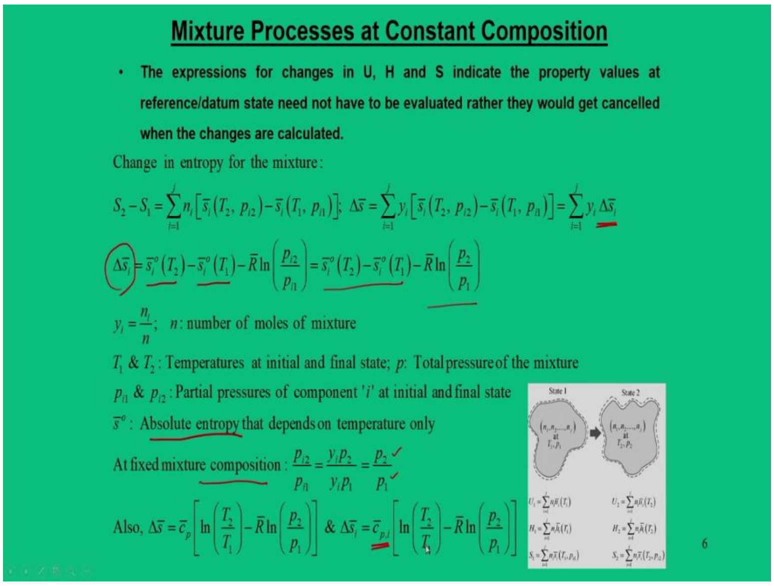
Now, let us target them one by one. First one is that we consider a mixing process at constant compositions. So, here if you look at this picture, we have a mixture at state 1 and state 2. So, what it means is that initially the mixture is formed with n number of gases with their mole fractions $n_1, n_2, \ldots, n_j$ and it is at initial pressure and temperature T_1 and p_1. Now, there is a change of state that happens, but at same composition. The composition do not change, but what changes is the pressure and temperatures.

So, because of this change of state, we will land off in the change of energies like we have initial internal energy U_1, we have well have final internal energy U_2. We have initial enthalpy H_1 and final enthalpy H_2, initial entropy S_1 and final enthalpy S_2. So, our previous studies or previous expressions enables us to calculate the changes in the internal energy of the mixture during the process.

So, if you can calculate that what is $U_2 - U_1$ due to change in the internal energy of the mixture between state 2 and 1. We can represent in its molar value $\sum_{i=1}^{j} n_i[\bar{u}_i(T_2) - \bar{u}_i(T_1)]$; that means, molar specific internal energy multiplied by its mole fractions. So, this molal value has to be considered at two different states that is temperature T_2, final state and temperature T_1, initial states.

By doing so, you can find out change in the molar specific internal energy which is the summation of the internal energy of any individual component i from state 2 and state 1 multiplied by its mole fraction. So, if you make summation of for all the components, we can calculate the total change in the molar specific internal energy $\Delta\bar{u} = \sum_{i=1}^{j} y_i[\bar{u}_i(T_2) - \bar{u}_i(T_1)]$. Similar philosophy can also be applied to calculate the change in the molar specific enthalpy. $\Delta\bar{h} = \sum_{i=1}^{j} y_i[\bar{h}_i(T_2) - \bar{h}_i(T_1)]$

(Refer Slide Time: 09:26)



Mixture Processes at Constant Composition

- The expressions for changes in U, H and S indicate the property values at reference/datum state need not have to be evaluated rather they would get cancelled when the changes are calculated.

Change in entropy for the mixture:

$$S_2 - S_1 = \sum_{i=1}^{j} n_i\left[\bar{s}_i(T_2, p_{i2}) - \bar{s}_i(T_1, p_{i1})\right];\ \Delta\bar{s} = \sum_{i=1}^{j} y_i\left[\bar{s}_i(T_2, p_{i2}) - \bar{s}_i(T_1, p_{i1})\right] = \sum_{i=1}^{j} y_i \Delta\bar{s}_i$$

$$\Delta\bar{s}_i = \bar{s}_i^o(T_2) - \bar{s}_i^o(T_1) - \bar{R}\ln\left(\frac{p_{i2}}{p_{i1}}\right) = \bar{s}_i^o(T_2) - \bar{s}_i^o(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)$$

$y_i = \frac{n_i}{n}$; n: number of moles of mixture

T_1 & T_2: Temperatures at initial and final state; p: Total pressure of the mixture

p_{i1} & p_{i2}: Partial pressures of component 'i' at initial and final state

$\bar{s}^o$: Absolute entropy that depends on temperature only

At fixed mixture composition: $\frac{p_{i2}}{p_{i1}} = \frac{y_i p_2}{y_i p_1} = \frac{p_2}{p_1}$ ✓

Also, $\Delta\bar{s} = \bar{c}_p\left[\ln\left(\frac{T_2}{T_1}\right) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right]$ & $\Delta\bar{s}_i = \bar{c}_{p,i}\left[\ln\left(\frac{T_2}{T_1}\right) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right]$

Then, moving further, we can also calculate the change in the entropy. But when you calculate the change in the entropy, entropy change is to be calculated at pressures for the each component i which will have its own partial pressure and this partial pressure is going to be different for the initial state and final state. So, at the initial state, the partial pressure for the component i is p_{i1} and then, the final state 2 the partial pressure for the component i is p_{i2}.

But there also the temperature of the component i with its final state at T_2 and initial state at T_1. So, this particular expressions can be represented in summation for $S_2 - S_1$ in terms of the molar specific change in the entropy multiplied by its mole fraction. Now, here the important aspects that we need to calculate that what is this $\Delta\bar{s}_i$ that is for component i, we want to find out molar specific internal energy.

$$S_2 - S_1 = \sum_{i=1}^{j} n_i[\bar{s}_i(T_2, p_{i2}) - \bar{s}_i(T_1, p_{i1})]\,;\Delta\bar{s} = \sum_{i=1}^{j} y_i[\bar{s}_i(T_2, p_{i2}) - \bar{s}_i(T_1, p_{i1})]$$

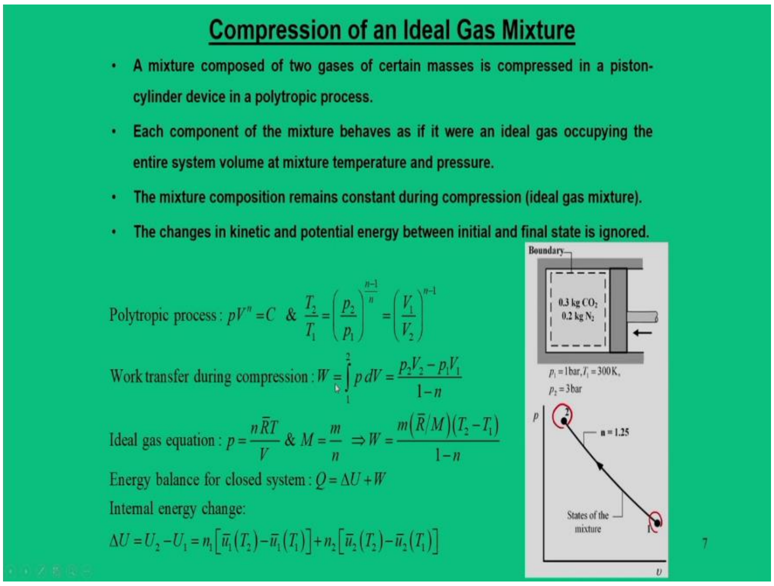
$$= \sum_{i=1}^{j} y_i \Delta\bar{s}_i$$

And by definition or the entropy calculation earlier, this has to be calculated with the concept of absolute entropy. Now, when you deal with the absolute entropy, the choice of pressure is meaningless. So, we have to calculate the absolute entropy for the component i at temperature T$_2$; absolute entropy for the component i at temperature T$_1$ and then, $\Delta \bar{s}_i = \bar{s}_i^o(T_2) - \bar{s}_i^o(T_1) - \bar{R}\ln\left(\frac{p_{i2}}{p_{i1}}\right)$.

Now, when I say this is a fixed mixture composition, $\frac{p_{i2}}{p_{i1}} = \frac{p_2}{p_1}$. So, this is how we have to calculate. This $\bar{s}_i^o(T_2) - \bar{s}_i^o(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)$ gives the change in the molar specific entropy as a function of absolute entropy change and $\bar{R}\ln\left(\frac{p_2}{p_1}\right)$. This is one way of evaluation; other way of evaluation is that we can also calculate the same things through c$_p$ calculations.

So, by definition entropy change is $\Delta \bar{s} = \bar{c}_p\left[\ln\left(\frac{T_2}{T_1}\right) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right]$. Then, this c p for the component i is represented by $\bar{c}_{p,i}$and knowing this value, we can use this expressions because we all know that what is the final state p$_2$ and T$_2$ and what is the initial state p$_1$ and T$_1$.

(Refer Slide Time: 12:28)



Compression of an Ideal Gas Mixture

- A mixture composed of two gases of certain masses is compressed in a piston-cylinder device in a polytropic process.
- Each component of the mixture behaves as if it were an ideal gas occupying the entire system volume at mixture temperature and pressure.
- The mixture composition remains constant during compression (ideal gas mixture).
- The changes in kinetic and potential energy between initial and final state is ignored.

Polytropic process : $pV^n = C$ & $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$

Work transfer during compression : $W = \int_1^2 p\,dV = \frac{p_2V_2 - p_1V_1}{1-n}$

Ideal gas equation : $p = \frac{n\bar{R}T}{V}$ & $M = \frac{m}{n}$ $\Rightarrow W = \frac{m(\bar{R}/M)(T_2 - T_1)}{1-n}$

Energy balance for closed system : $Q = \Delta U + W$

Internal energy change:

$\Delta U = U_2 - U_1 = n_1\left[\bar{u}_1(T_2) - \bar{u}_1(T_1)\right] + n_2\left[\bar{u}_2(T_2) - \bar{u}_2(T_1)\right]$

Then, we will move to different case studies. First case is the Compression of an Ideal Gas Mixture. So, here, the compression of an ideal gas mixture means that for the sake of an example you can say that we have a piston cylinder device which contains certain gas. Here, you have written as some mass of carbon dioxide and some mass of nitrogen and this mixture is being compressed by this piston.

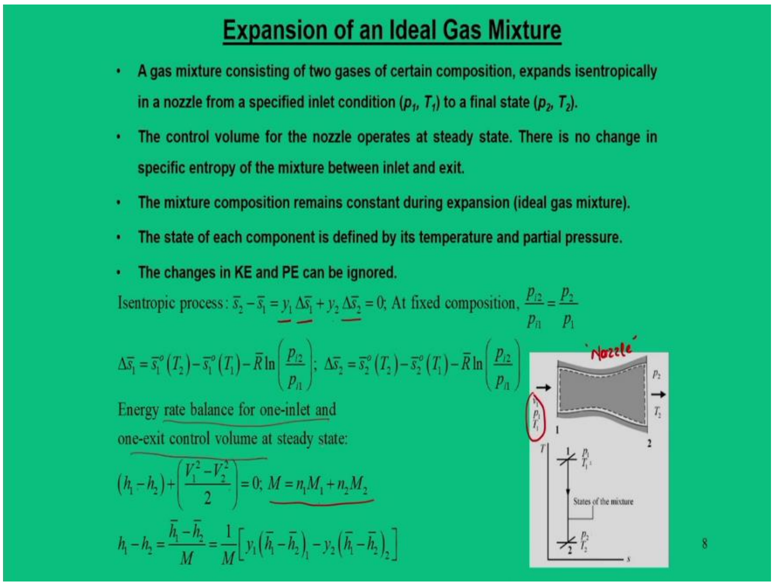
Now, when it is compressed, the initial state of the mixture which is state 1 moves to state 2 in a polytropic process with some polytropic index, some value of n. Now, how you are going to calculate the overall properties of the mixture? So, there are certain assumptions that we are going to say is that the changes in the kinetic energy and potential energy between initial and final state is ignored.

Each component of the mixtures behaves as if it were an ideal gas occupied in the entire system volume at mixture temperature and pressure. Then, mixture composition remains constant during the compressions. Having such this assumptions, we can write the polytropic equation $pV^n = C$; side by side, we can find the pressure, temperature and volume relations between initial state and the final states.

Then, work transfer during compression process; work transfer can be calculated as $W = \int_1^2 pdV$. Then, from this equation, you can calculate work transfer. Now, since it is a closed system analysis, so we can find out the energy balance equation for the closed system; $Q = \varDelta U + W$ and of course, we know from Q and W, we can also calculate the internal energy change $U_2 - U_1$.

This U 2 and U 1 has to be calculated with respect to state 2 and state 1 by knowing the composition of each individual gases.

(Refer Slide Time: 14:42)



Expansion of an Ideal Gas Mixture

- A gas mixture consisting of two gases of certain composition, expands isentropically in a nozzle from a specified inlet condition (**p_1, T_1**) to a final state (**p_2, T_2**).
- The control volume for the nozzle operates at steady state. There is no change in specific entropy of the mixture between inlet and exit.
- The mixture composition remains constant during expansion (ideal gas mixture).
- The state of each component is defined by its temperature and partial pressure.
- The changes in KE and PE can be ignored.

Isentropic process: $\bar{s}_2 - \bar{s}_1 = \underline{y_1}\,\underline{\Delta\bar{s}_1} + y_2\,\underline{\Delta\bar{s}_2} = 0$; At fixed composition, $\dfrac{p_{i2}}{p_{i1}} = \dfrac{p_2}{p_1}$

$$\Delta\bar{s}_1 = \bar{s}_1^o(T_2) - \bar{s}_1^o(T_1) - \bar{R}\ln\left(\frac{p_{i2}}{p_{i1}}\right);\ \Delta\bar{s}_2 = \bar{s}_2^o(T_2) - \bar{s}_2^o(T_1) - \bar{R}\ln\left(\frac{p_{i2}}{p_{i1}}\right)$$

Energy rate balance for one-inlet and one-exit control volume at steady state:

$$(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) = 0;\ \underline{M = n_1M_1 + n_2M_2}$$

$$h_1 - h_2 = \frac{\bar{h}_1 - \bar{h}_2}{M} = \frac{1}{M}\left[y_1\left(\bar{h}_1 - \bar{h}_2\right)_1 - y_2\left(\bar{h}_1 - \bar{h}_2\right)_2\right]$$

The next example that we are going to discuss is the Expansion of an Ideal Gas. So, when I say expansion, one typical expansion process as you can see that a gas from its initial state p_1, v_1, T_1 expands in a nozzle. So, this is a schematic diagram of nozzle. So, the characteristics feature of the nozzle, it increases; the flow expands in the nozzle and the process in this nozzle is normally treated to be isentropic process.

Now, here in our previous analysis, we used to say that only one gas expands in the nozzle. So, analysis of the treatment was little bit different and simple when you dealt with the control volume analysis for the nozzle. But here, a mixture expands in the nozzle. When a mixture expands; one thing we are assuming that they have constant composition.

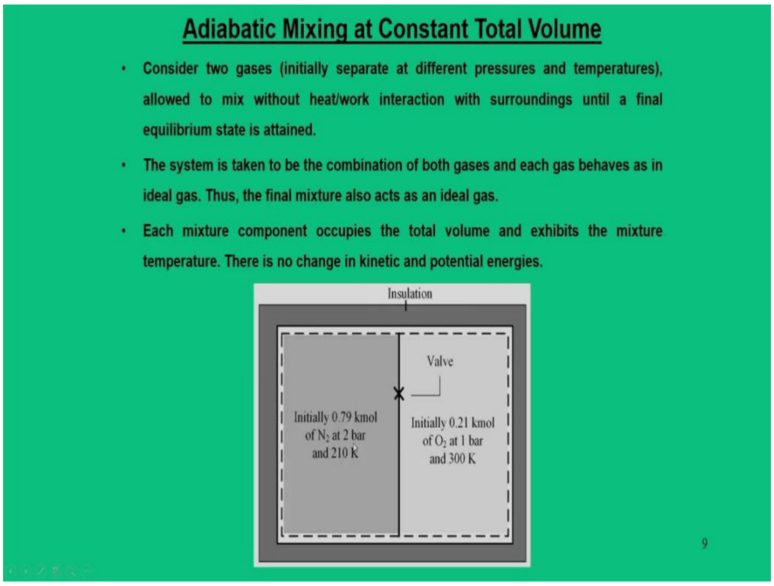
Mixture composition remains constant and the state of each component is defined by its temperature and the partial pressures. Same way, we also can say that kinetic energy and potential energy effects can be ignored and the process in a nozzle is typically considered to be an isentropic process. So, isentropic process means the change in the entropy between state 2 and 1 is 0.

So, we can calculate $\bar{s}_2 - \bar{s}_1 = y_1 \Delta\bar{s}_1 + y_2 \Delta\bar{s}_2$ and again, if you can recall our previous relations, $\Delta\bar{s}_1$ and $\Delta\bar{s}_2$ can be calculated with the concept of absolute entropies and partial pressures of each gas. And normally, when you use the word nozzle, we used to say that it has to increase the flow velocity.

$$\Delta\bar{s}_1 = \bar{s}_1^o(T_2) - \bar{s}_1^o(T_1) - \bar{R}\ln\left(\frac{p_{i2}}{p_{i1}}\right); \Delta\bar{s}_2 = \bar{s}_2^o(T_2) - \bar{s}_2^o(T_1) - \bar{R}\ln\left(\frac{p_{i2}}{p_{i1}}\right)$$

So, the flow velocity is associated with the enthalpy calculations. And of course, here we are going to use another relations in terms of the molecular weight and this molecular weight is required for calculation of enthalpy and enthalpy and flow velocity relations can be related as $(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) = 0$. And this model was done for an energy rate balance for an one-inlet and one-exit control volume at steady state.

(Refer Slide Time: 17:34)



Adiabatic Mixing at Constant Total Volume

- Consider two gases (initially separate at different pressures and temperatures), allowed to mix without heat/work interaction with surroundings until a final equilibrium state is attained.
- The system is taken to be the combination of both gases and each gas behaves as in ideal gas. Thus, the final mixture also acts as an ideal gas.
- Each mixture component occupies the total volume and exhibits the mixture temperature. There is no change in kinetic and potential energies.

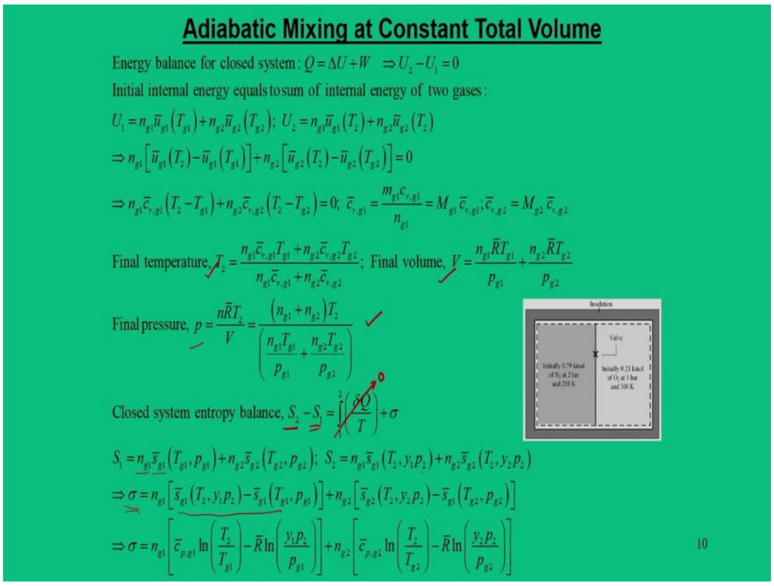
Next type of mixing, we call this as Adiabatic Mixing at Constant Volume. Now in our previous study, previous two cases, we have seen that the mixture was at constant compositions and during this process of change of state, the composition do not change. But and the mixture expands or compress during the change of state and now, we will consider a situations, where the components are initially separate and after certain situations, they form a mixtures.

One such case is adiabatic mixing at constant volume. So, if you look at this particular figure, it says that there are two gases; nitrogen and oxygen and they are separated by a partition. And entire system is insulated means there is no heat transfer into or out of the systems. Now, you want to mix. So, when we are trying to mix, we allow this partition to come to open through this valve.

Now, when this opens, the initial state of nitrogen and the initial state of oxygen is going to change. So, what the consequence could be? The initial mole fraction of oxygen could change and the number of moles is going to change. And since they are at different pressures at steady state, there will be pressure equalization and will have an effective temperature of the mixture.

So, to analyze such a things, we assume certain things like system is taken to be the combination of both the gases and each gas behaves as an ideal gas. The final mixture also acts as ideal gas and another important aspect is that each mixture component occupies total volume; that means, after the valve or this partition is over, each mixture components occupies the total volume and exhibits the mixture temperature and they have the common uniform temperatures. There is no change in the kinetic and potential energy.

(Refer Slide Time: 20:02)



Adiabatic Mixing at Constant Total Volume

Energy balance for closed system: $Q = \Delta U + W \Rightarrow U_2 - U_1 = 0$

Initial internal energy equals to sum of internal energy of two gases:

$$U_1 = n_{g1}\bar{u}_{g1}(T_{g1}) + n_{g2}\bar{u}_{g2}(T_{g2});\; U_2 = n_{g1}\bar{u}_{g1}(T_2) + n_{g2}\bar{u}_{g2}(T_2)$$

$$\Rightarrow n_{g1}\left[\bar{u}_{g1}(T_2) - \bar{u}_{g1}(T_{g1})\right] + n_{g2}\left[\bar{u}_{g2}(T_2) - \bar{u}_{g2}(T_{g2})\right] = 0$$

$$\Rightarrow n_{g1}\bar{c}_{v,g1}(T_2 - T_{g1}) + n_{g2}\bar{c}_{v,g2}(T_2 - T_{g2}) = 0;\; \bar{c}_{v,g1} = \frac{m_{g1}c_{v,g1}}{n_{g1}} = M_{g1}\bar{c}_{v,g1}; \bar{c}_{v,g2} = M_{g2}\bar{c}_{v,g2}$$

Final temperature, $T_2 = \frac{n_{g1}\bar{c}_{v,g1}T_{g1} + n_{g2}\bar{c}_{v,g2}T_{g2}}{n_{g1}\bar{c}_{v,g1} + n_{g2}\bar{c}_{v,g2}}$; Final volume, $V = \frac{n_{g1}\bar{R}T_{g1}}{p_{g1}} + \frac{n_{g2}\bar{R}T_{g2}}{p_{g2}}$

Final pressure, $p = \frac{n\bar{R}T_2}{V} = \frac{(n_{g1} + n_{g2})T_2}{\left(\frac{n_{g1}T_{g1}}{p_{g1}} + \frac{n_{g2}T_{g2}}{p_{g2}}\right)}$ ✓

Closed system entropy balance, $S_2 - S_1 = \cancelto{0}{\int_1^2 \left(\frac{\delta Q}{T}\right)} + \sigma$

$$S_1 = n_{g1}\bar{s}_{g1}(T_{g1}, p_{g1}) + n_{g2}\bar{s}_{g2}(T_{g2}, p_{g2});\; S_2 = n_{g1}\bar{s}_{g1}(T_2, y_1 p_2) + n_{g2}\bar{s}_{g2}(T_2, y_2 p_2)$$

$$\Rightarrow \sigma = n_{g1}\left[\underline{\bar{s}_{g1}(T_2, y_1 p_2) - \bar{s}_{g1}(T_{g1}, p_{g1})}\right] + n_{g2}\left[\bar{s}_{g2}(T_2, y_2 p_2) - \bar{s}_{g1}(T_{g2}, p_{g2})\right]$$

$$\Rightarrow \sigma = n_{g1}\left[\bar{c}_{p,g1}\ln\left(\frac{T_2}{T_{g1}}\right) - \bar{R}\ln\left(\frac{y_1 p_2}{p_{g1}}\right)\right] + n_{g2}\left[\bar{c}_{p,g2}\ln\left(\frac{T_2}{T_{g2}}\right) - \bar{R}\ln\left(\frac{y_2 p_2}{p_{g2}}\right)\right]$$

So, having said this, we are going to analyze this particular case with energy balance and entropy balance equations. So, energy balance, we have used the first law which talks about the information since it is a closed systems, we have to deal with the internal energy $\Delta U + W$ and this equation there is no heat transfer and works transfer. So, $U_2 - U_1 = 0$.

Now, if you say U_1, this value is based on its molar composition. So that means, if it is gas 1, $U_1 = n_{g1}\bar{u}_{g1}(T_{g1}) + n_{g2}\bar{u}_{g2}(T_{g2})$ and similarly way, the internal energy for gas 2 can be calculated by its number of moles and its molar value of specific internal energy, $U_2 = n_{g1}\bar{u}_{g1}(T_2) + n_{g2}\bar{u}_{g2}(T_2)$.

So, this way, we calculate the total internal energy of the systems and then, similar way, we can calculate the total internal energy when it undergoes change of state. So, from this things, we can rearrange this equations and also, internal energy is a function of specific volume. So, you can introduce the specific volume of the gas, from mass and mole, we can get the molecular weight.

So, using these relations, ultimately what we are going to target that when the final state is reached, what is the final temperature T_2, what is the final volume V and final pressure p. So, this is how the energy balance equation will give you all these numbers.

$$\text{Final temperature}, T_2 = \frac{n_{g1}\bar{c}_{v,g1}T_{g1} + n_{g2}\bar{c}_{v,g2}T_{g2}}{n_{g1}\bar{c}_{v,g1} + n_{g2}\bar{c}_{v,g2}};$$

$$\text{Final volume}, V = \frac{n_{g1}\bar{R}T_{g1}}{p_{g1}} + \frac{n_{g2}\bar{R}T_{g2}}{p_{g2}}$$

$$\text{Final pressure}, p = \frac{n\bar{R}T_2}{V} = \frac{(n_{g1} + n_{g2})T_2}{\left(\frac{n_{g1}T_{g1}}{p_{g1}} + \frac{n_{g2}T_{g2}}{p_{g2}}\right)}$$

Then, we have to think about entropy balance equations. That means, here it is a closed system entropy balance; $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right) + \sigma$ and for this situations, dQ goes to 0 because there is no heat transfer. So, it will give you the entropy production through this change in the entropy.

Now, S_2 has to be calculated that means total entropy at state 2 has to be calculated with respect to final pressure, temperature. S_1 has to be calculated with respect to the initial state of this mixture. Now, if you look at here, now S_1 if you calculate, it is a function of its number of moles of this gas and molar specific internal energy which is a function of T_{g1}, p_{g1} for the gas 1 and it is function of T_{g2}, p_{g2} for the gas 2.

Similarly, when you go to the final state, final pressure of the gas 1 will be mole fraction y_1p_2 and mole fraction of for the gas 2 would be y_2p_2; p_2 is your final pressure which is this. Then by putting this, this will give you an expressions of entropy production during this mixing process.

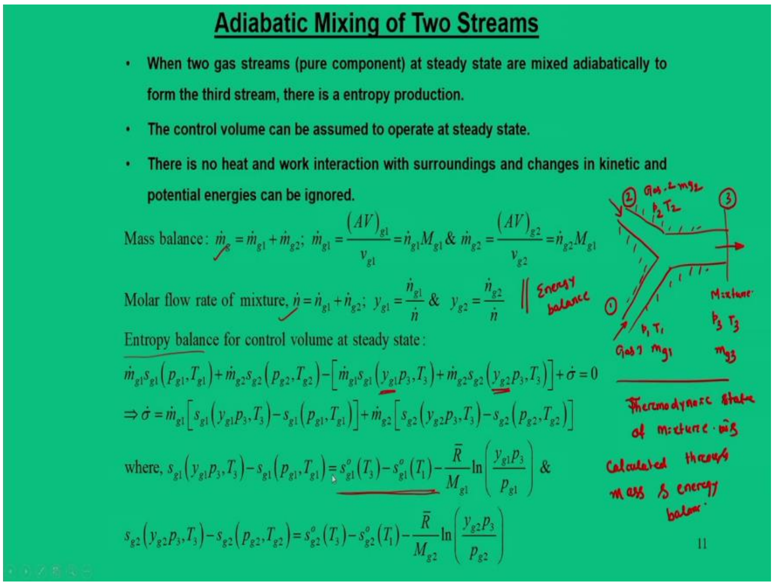
$$S_1 = n_{g1}\bar{s}_{g1}(T_{g1}, p_{g1}) + n_{g2}\bar{s}_{g2}(T_{g2}, p_{g2}); S_2 = n_{g1}\bar{s}_{g1}(T_2, y_1p_2) + n_{g2}\bar{s}_{g2}(T_2, y_2p_2)$$

$$\Rightarrow \sigma = n_{g1}[\bar{s}_{g1}(T_2, y_1p_2) - \bar{s}_{g1}(T_{g1}, p_{g1})] + n_{g2}[\bar{s}_{g2}(T_2, y_2p_2) - \bar{s}_{g1}(T_{g2}, p_{g2})]$$

Now while doing so, we can also bring out this change in the entropy which is a function of c_p and R. So, these relations has to be used. So, basically when you solve the problem and try to find out how I am going to introduce the property values, then the picture will be more clear. So, we will discuss them down the line.

$$\sigma = n_{g1}\left[\bar{c}_{p,g1}\ln\left(\frac{T_2}{T_{g1}}\right) - \bar{R}\ln\left(\frac{y_1 p_2}{p_{g1}}\right)\right] + n_{g2}\left[\bar{c}_{p,g2}\ln\left(\frac{T_2}{T_{g2}}\right) - \bar{R}\ln\left(\frac{y_2 p_2}{p_{g2}}\right)\right]$$

(Refer Slide Time: 24:04)



Adiabatic Mixing of Two Streams

- When two gas streams (pure component) at steady state are mixed adiabatically to form the third stream, there is a entropy production.
- The control volume can be assumed to operate at steady state.
- There is no heat and work interaction with surroundings and changes in kinetic and potential energies can be ignored.

Mass balance: $\dot{m}_g = \dot{m}_{g1} + \dot{m}_{g2};\ \dot{m}_{g1} = \frac{(AV)_{g1}}{v_{g1}} = \dot{n}_{g1}M_{g1}\ \&\ \dot{m}_{g2} = \frac{(AV)_{g2}}{v_{g2}} = \dot{n}_{g2}M_{g1}$

Molar flow rate of mixture, $\dot{n} = \dot{n}_{g1} + \dot{n}_{g2};\ y_{g1} = \frac{\dot{n}_{g1}}{\dot{n}}\ \&\ y_{g2} = \frac{\dot{n}_{g2}}{\dot{n}}$ || Energy balance

Entropy balance for control volume at steady state:

$$\dot{m}_{g1}s_{g1}(p_{g1},T_{g1}) + \dot{m}_{g2}s_{g2}(p_{g2},T_{g2}) - \left[\dot{m}_{g1}s_{g1}(y_{g1}p_3,T_3) + \dot{m}_{g2}s_{g2}(y_{g2}p_3,T_3)\right] + \dot{\sigma} = 0$$

$$\Rightarrow \dot{\sigma} = \dot{m}_{g1}\left[s_{g1}(y_{g1}p_3,T_3) - s_{g1}(p_{g1},T_{g1})\right] + \dot{m}_{g2}\left[s_{g2}(y_{g2}p_3,T_3) - s_{g2}(p_{g2},T_{g2})\right]$$

where, $s_{g1}(y_{g1}p_3,T_3) - s_{g1}(p_{g1},T_{g1}) = s^o_{g1}(T_3) - s^o_{g1}(T_1) - \frac{\bar{R}}{M_{g1}}\ln\left(\frac{y_{g1}p_3}{p_{g1}}\right)$ &

$$s_{g2}(y_{g2}p_3,T_3) - s_{g2}(p_{g2},T_{g2}) = s^o_{g2}(T_3) - s^o_{g2}(T_1) - \frac{\bar{R}}{M_{g2}}\ln\left(\frac{y_{g2}p_3}{p_{g2}}\right)$$

Thermodynamic state of mixture is calculated through mass & energy balance.

And the other one is that Adiabatic Mixing of Two Streams. So, this is a situation, mainly we come across when you deal with the psychrometrics, when air and water vapors are mixed. So, when I say adiabatic mixing of two streams like I can assume that there is a passage two streams or two gases from state 1 and 2, they come and they mix and finally, the mixture goes toward state 3.

So, we say we can say gas 1, gas 2 and it is forms as a mixture of two gas. Conditions of gas 1 maybe gas 1 may be p_1, T_1, m_{g1} and condition for gas 2 may be p_2, T_2, m_{g2}; gas 3 could be final pressure p_3, T_3, m_{g3}. And mixing process is adiabatic, so there is no heat transfer into this medium. Now, for this situation, we have to write down the mass balance and energy balance simultaneously.

So, you have to consider this mass balance, molar mass of the mixtures. So, this will give you the mole fraction and mass fraction of this mixture. In addition to that we have to consider also energy balance equation. So, mass and energy balance equations need to be taken together to calculate the thermodynamic states of the mixture.

$$\text{Mass balance: } \dot{m}_g = \dot{m}_{g1} + \dot{m}_{g2}; \dot{m}_{g1} = \frac{(AV)_{g1}}{v_{g1}} = \dot{n}_{g1} M_{g1} \& \dot{m}_{g2} = \frac{(AV)_{g2}}{v_{g2}} = \dot{n}_{g2} M_{g1}$$

$$\text{Molar flow rate of mixture, } \dot{n} = \dot{n}_{g1} + \dot{n}_{g2}; y_{g1} = \frac{\dot{n}_{g1}}{\dot{n}} \& y_{g2} = \frac{\dot{n}_{g2}}{\dot{n}}$$

So, thermodynamic state of mixture is to be calculated through mass balance, and energy balance. Now, apart from this, we also need to find out the entropy balance and because it is an adiabatic mixing process and process is irreversible, so entropy balance will give you what is the entropy production rate.

$$\text{Entropy balance: } \dot{m}_{g1} s_{g1}(p_{g1}, T_{g1}) + \dot{m}_{g2} s_{g2}(p_{g2}, T_{g2}) - [\dot{m}_{g1} s_{g1}(y_{g1} p_3, T_3) + \dot{m}_{g2} s_{g2}(y_{g2} p_3, T_3)] + \dot{\sigma} = 0$$

$$\text{where, } s_{g1}(y_{g1} p_3, T_3) - s_{g1}(p_{g1}, T_{g1}) = s_{g1}^o(T_3) - s_{g1}^o(T_1) - \frac{\bar{R}}{M_{g1}} \ln\left(\frac{y_{g1} p_3}{p_{g1}}\right)$$

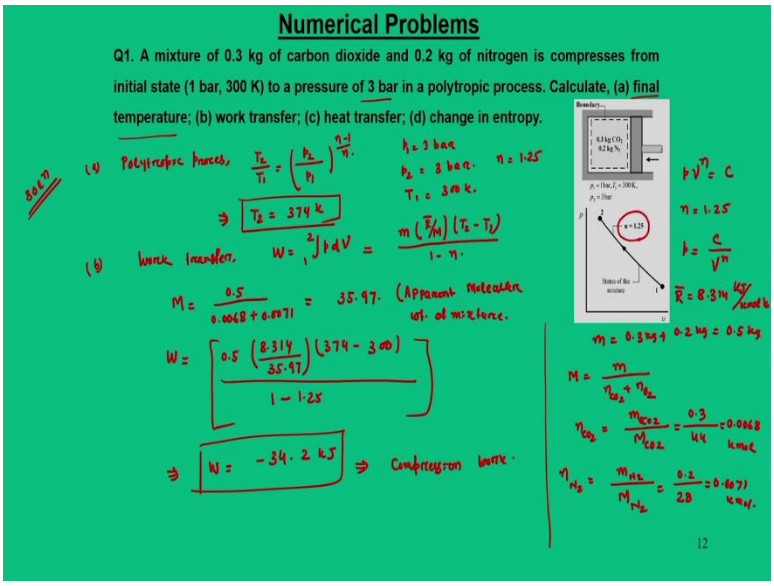
$$s_{g2}(y_{g2} p_3, T_3) - s_{g2}(p_{g2}, T_{g2}) = s_{g2}^o(T_3) - s_{g2}^o(T_1) - \frac{\bar{R}}{M_{g2}} \ln\left(\frac{y_{g2} p_3}{p_{g2}}\right)$$

So, we have to use in similar philosophy that what is the entropy at state 1 for the gas 1 and this is to be evaluated attemperature and pressure for the gas 1. For gas 2, it has to be evaluated at its corresponding pressure p2 and T2 and after the formation of the mixture, the composition of the components may change.

So, that is what we will land up having the mole fraction for the gas 1 and mole fraction for the gas 2 and once you know this mole fraction and final pressures, the entropy will be evaluated at that temperature and pressures. By inserting this value, we can calculate the entropy production term. Now, here important thing that need to be find out for entropy calculations, the change in the entropy for the gas 1 from its final state to initial state, we need to require the concept of absolute entropy and these absolute entropy values can be used to find the entropy change for respective gases.

So, here it is an example of two streams. So, there are situations, you can have multiple number of streams and for each streams, these terms needs to be evaluated.

(Refer Slide Time: 28:16)



Numerical Problems

Q1. A mixture of 0.3 kg of carbon dioxide and 0.2 kg of nitrogen is compresses from initial state (1 bar, 300 K) to a pressure of 3 bar in a polytropic process. Calculate, (a) final temperature; (b) work transfer; (c) heat transfer; (d) change in entropy.

Soln

(a) Polytropic proces, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$.

$p_1 = 1$ bar

$p_2 = 3$ bar.

$T_1 = 300$ K.

$n = 1.25$

$\Rightarrow \boxed{T_2 = 374\ K}$

(b) work transfer, $W = \int_1^2 p\,dV = \frac{m(\bar{R}/M)(T_2 - T_1)}{1-n}$.

$M = \frac{0.5}{0.0068 + 0.0071} = 35.97$. (Apparent Molecular wt. of mixture.

$W = \left[\frac{0.5\left(\frac{8.314}{35.97}\right)(374-300)}{1-1.25}\right]$

$\Rightarrow \boxed{W = -34.2\ kJ} \Rightarrow$ Compression work.

$pV^n = C$

$n = 1.25$

$p = \frac{C}{V^n}$

$\bar{R} = 8.314\ kJ/kmol\,K$

$m = 0.3\,kg + 0.2\,kg = 0.5\,kg$

$M = \frac{m}{n_{CO_2} + n_{O_2}}$

$n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{0.3}{44} = 0.0068$ kmol

$n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{0.2}{28} = 0.0071$ kmol.

12

This is all about the four different important cases that is going to be considered for the mixing analysis. Now, based on our analysis for the discussion for the lecture today, we are now going to solve some numerical problems which is nothing but the two different case studies, what we have studied.

First one is mixing process at constant compositions. So, when I say constant composition, I will use the same example or same problem figure which has been given in the earlier discussions. So, we will be using the values directly here. Now, let us see how you need to go about while solving the numerical problems.

So, here the problem statement is that we have a mixture of 0.3 kg of carbon dioxide and 0.2 kg of nitrogen and they are getting compressed from its initial state which is 1 bar, 300 K to a final pressure. So, final pressure of the mixture is 3 bar and this is a compression process; that means, we go from initial state to final state through a polytropic process and this polytropic index is given in this figure as $n = 1.25$.

Now, if this is the situation, let us see how you will go about. So, let us start the solutions. So, the first question that was asked we have to find out the final temperatures. We are given with the final pressure, but we are not given with the final temperatures. Now, to do this, final temperature calculations. Now, for a polytropic process, we can write we satisfy

$$pV^n = C \& \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = 374K.$$

Second part, we are going to evaluate what is the work transfer.

$$W = \int_1^2 pdV = \frac{m\left(\frac{\bar{R}}{M}\right)(T_2 - T_1)}{1-n}; m = 0.3 + 0.2 = 0.5kg;\ M = \frac{m}{n_{CO_2} + n_{N_2}};$$

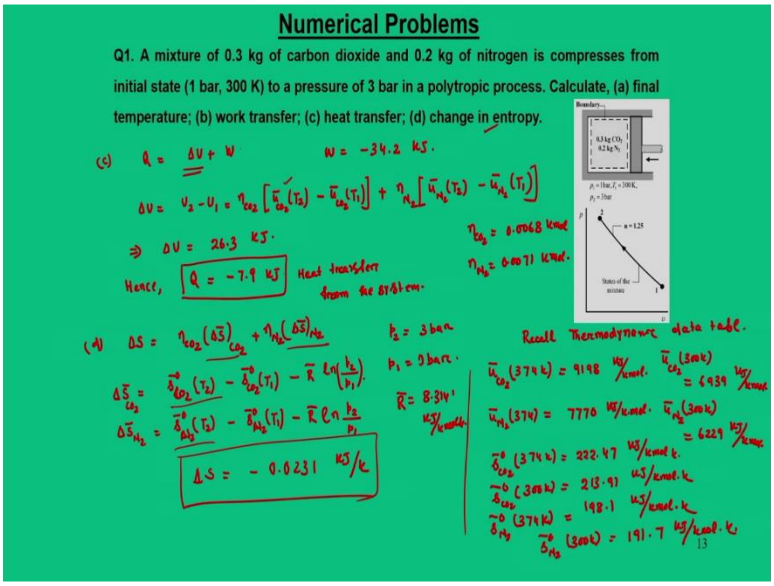
$$n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{0.3}{44} = 0.0068\ kmol;\ n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{0.2}{28} = 0.0071\ kmol$$

$$\Rightarrow M = \frac{0.5}{0.0068 + 0.0071} = 35.97\ (Apparent\ molecular\ weight)$$

$$\Rightarrow W = \frac{m\left(\frac{\bar{R}}{M}\right)(T_2 - T_1)}{1-n} = -34.2\ kJ$$

Negative sign implies that it is a compression work. So, now next part is that we need to calculate the heat transfer. So, for heat transfer, we need to go for this heat transfer by first law.

(Refer Slide Time: 35:47)



Numerical Problems

Q1. A mixture of 0.3 kg of carbon dioxide and 0.2 kg of nitrogen is compresses from initial state (1 bar, 300 K) to a pressure of 3 bar in a polytropic process. Calculate, (a) final temperature; (b) work transfer; (c) heat transfer; (d) change in entropy.

(c) $Q = \Delta U + W$ $\quad W = -34.2$ kJ.

$\Delta U = U_2 - U_1 = n_{CO_2}\left[\bar{u}_{CO_2}(T_2) - \bar{u}_{CO_2}(T_1)\right] + n_{N_2}\left[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)\right]$

$\Rightarrow \Delta U = 26.3$ kJ.

$n_{CO_2} = 0.0068$ kmol

$n_{N_2} = 0.0071$ kmol.

Hence, $\boxed{Q = -7.9 \text{ kJ}}$ Heat transfer from the system.

(d) $\Delta S = n_{CO_2}(\Delta\bar{s})_{CO_2} + n_{N_2}(\Delta\bar{s})_{N_2}$

$p_2 = 3$ bar

$p_1 = 1$ bar.

$\Delta\bar{s}_{CO_2} = \bar{s}^0_{CO_2}(T_2) - \bar{s}^0_{CO_2}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)$.

$\Delta\bar{s}_{N_2} = \bar{s}^0_{N_2}(T_2) - \bar{s}^0_{N_2}(T_1) - \bar{R}\ln\frac{p_2}{p_1}$

$\bar{R} = 8.314$ kJ/kmol.k

$\boxed{\Delta S = -0.0231 \text{ kJ/k}}$

Recall Thermodynamic data table.

$\bar{u}_{CO_2}(374k) = 9198$ kJ/kmol. $\quad \bar{u}_{CO_2}(300k) = 6939$ kJ/kmol

$\bar{u}_{N_2}(374) = 7770$ kJ/kmol. $\quad \bar{u}_{N_2}(300k) = 6229$ kJ/kmol

$\bar{s}^0_{CO_2}(374k) = 222.47$ kJ/kmol k.

$\bar{s}^0_{CO_2}(300k) = 213.91$ kJ/kmol.k

$\bar{s}^0_{N_2}(374k) = 198.1$ kJ/kmol.k

$\bar{s}^0_{N_2}(300k) = 191.7$ kJ/kmol.k

So, you can write $Q = \Delta U + W$. Now, to calculate ΔU, we need the information for mixture at its initial state and final states. $\Delta U = U_2 - U_1 = n_{CO_2}\left[\bar{u}_{CO_2}(T_2) - \bar{u}_{CO_2}(T_1)\right] + n_{N_2}\left[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)\right]$.

So, here we require the information about the molar specific internal energy at two different temperatures. So, we need to use the thermodynamic data table and this is available in any of the thermodynamic book towards the end.

$$\Delta U = 0.0068[9198 - 6939] + 0.0071[7770 - 6229] = 26.3\ kJ$$

Hence, Q would be $-7.9\ kJ$ which means that compression increases the temperatures, which means heat transfer from the system and last one is we need to calculate the change in the entropy.

$$\bar{s}_2 - \bar{s}_1 = n_{CO_2}\Delta\bar{s}_{CO_2} + n_{N_2}\Delta\bar{s}_{N_2}$$

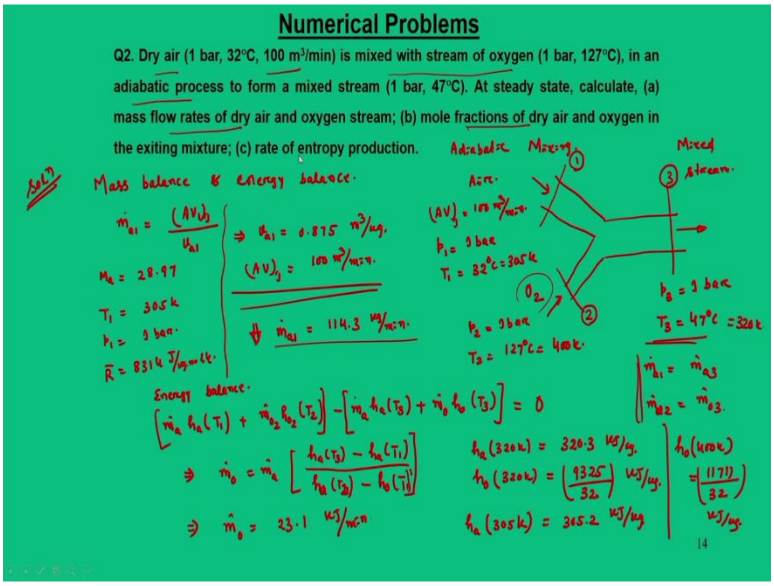
$$\Delta\bar{s}_{CO_2} = \bar{s}^o_{CO_2}(T_2) - \bar{s}^o_{CO_2}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right) = 222.47 - 213.91 - 8.314\ln\left(\frac{3}{1}\right)$$

$$\Delta\bar{s}_{N_2} = \bar{s}^o_{N_2}(T_2) - \bar{s}^o_{N_2}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right) = 198.1 - 191.7 - 8.314\ln\left(\frac{3}{1}\right)$$

$$\Delta s = -0.0231\ kJ/K$$

So, mixing process, it is a negative quantity. So, since heat transfer is negative, so you also get entropy as a negative value.

(Refer Slide Time: 43:39)



Numerical Problems

Q2. Dry air (1 bar, 32°C, 100 m³/min) is mixed with stream of oxygen (1 bar, 127°C), in an adiabatic process to form a mixed stream (1 bar, 47°C). At steady state, calculate, (a) mass flow rates of dry air and oxygen stream; (b) mole fractions of dry air and oxygen in the exiting mixture; (c) rate of entropy production.

Sol^n

Mass balance & energy balance.

$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}}$

$\Rightarrow v_{a1} = 0.875\ m^3/kg.$

$(AV)_1 = 100\ m^3/min.$

$\Downarrow\ \dot{m}_{a1} = 114.3\ kg/min.$

$M_a = 28.97$

$T_1 = 305\ K$

$p_1 = 1\ bar.$

$\bar{R} = 8314\ J/kmol\,K.$

Energy balance.

$[\dot{m}_a h_a(T_1) + \dot{m}_{O_2} h_{O_2}(T_2)] - [\dot{m}_a h_a(T_3) + \dot{m}_o h_o(T_3)] = 0$

$\Rightarrow \dot{m}_o = \dot{m}_a\left[\frac{h_a(T_3) - h_a(T_1)}{h_o(T_2) - h_o(T_3)}\right]$

$\Rightarrow \dot{m}_o = 23.1\ kg/min$

Adiabatic Mixing

Air.

$(AV)_1 = 100\ m^3/min.$

$p_1 = 1\ bar$

$T_1 = 32^\circ C = 305\ K$

O_2

$p_2 = 1\ bar$

$T_2 = 127^\circ C = 400\ K.$

$p_3 = 1\ bar$

$T_3 = 47^\circ C = 320\ K$

$\dot{m}_{a1} = \dot{m}_{a3}$

$\dot{m}_{o2} = \dot{m}_{o3}.$

$h_a(320K) = 320.3\ kJ/kg.$

$h_o(320K) = \left(\frac{9325}{32}\right)\ kJ/kg.$

$h_a(305K) = 305.2\ kJ/kg$

$h_o(400K) = \left(\frac{11711}{32}\right)\ kJ/kg.$

Now, in the next problem, we are going to talk about a situation in which a dry air is mixed with a stream of oxygen and this mixing process is an adiabatic process. So, it is a case of adiabatic mixing. So, adiabatic mixing means we have two streams they come in contact with each other. At state 1, we say it is air and it is coming at flow rate. So, flow rate means area times velocity that is $100\ m^3/min$; $p_1 = 1bar, T_1 = 32\ C = 305\ K$. State 2, it is oxygen, condition is $p_2 = 1bar, T_2 = 127\ C = 400\ K$. And the final mixing stream that is we say mixed stream, the mixed stream has a condition $p_3 = 1bar, T_3 = 47\ C = 320\ K$.

Final temperature comes as 320 K which means that composition of air and oxygen are adjusted together so that final temperature is reached. Now, to do that we need to recall two things; one is the mass balance and energy balance. This mass balance and energy balance will give you the information about the mass flow rate and mole fraction, how do you do that?

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}}; M = 28.97, T_1 = 305\ K, p_1 = 1\ bar, \bar{R} = 8314 \Rightarrow v_{a1} = 0.875 \frac{m^3}{kg}$$

$$\Rightarrow \dot{m}_{a1} = \frac{100}{0.875} = 114.3\ kg/min$$

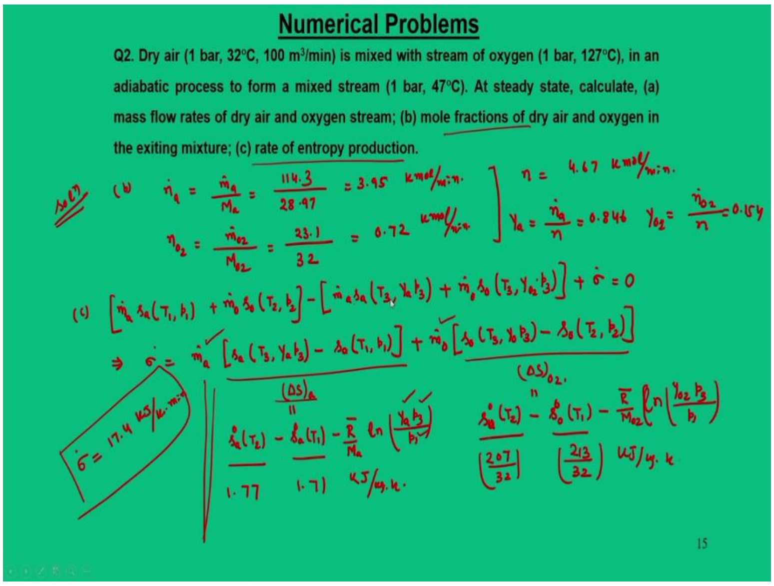
So, we have mass flow rate of air, we do not know mass flow rate of oxygen; we also do not know mass flow rate of mixed stream. Now, when I say mixed streams, I can assume that $\dot{m}_{a1} = \dot{m}_{O3}$; $\dot{m}_{O2} = \dot{m}_{O3}$, then we have to consider the energy balance.

$$(\dot{m}_{a1} h_a(T_1) + \dot{m}_{O_2} h_{O_2}(T_2)) - (\dot{m}_a h_a(T_3) + \dot{m}_O\ \ h_{O_2}(T_3)) = 0$$

$$\Rightarrow \dot{m}_O = m_a \left(\frac{h_a(T_3) - h_a(T_1)}{h_O\ \ (T_2) - h_O\ \ (T_1)} \right) = 114.3 \left(\frac{320.3 - 305.2}{\left(\frac{9325}{32}\right) - \left(\frac{11711}{32}\right)} \right) = 23.1\ kg/min$$

So, we require enthalpy information at different temperatures. So, this data, we get from the thermodynamics any thermodynamic books towards the end of this chapters or books. So, after inserting these numbers, we get $\dot{m}_O$ as 23.1 kilojoule per minute. So, this gives the information about mass flow rates.

(Refer Slide Time: 51:26)



Numerical Problems

Q2. Dry air (1 bar, 32°C, 100 m³/min) is mixed with stream of oxygen (1 bar, 127°C), in an adiabatic process to form a mixed stream (1 bar, 47°C). At steady state, calculate, (a) mass flow rates of dry air and oxygen stream; (b) mole fractions of dry air and oxygen in the exiting mixture; (c) rate of entropy production.

Sol^n

(b) $\dot{n}_a = \frac{\dot{m}_a}{M_a} = \frac{114.3}{28.97} = 3.95$ kmol/min.

$n_{O_2} = \frac{\dot{m}_{O_2}}{M_{O_2}} = \frac{23.1}{32} = 0.72$ kmol/min

$n = 4.67$ kmol/min.

$Y_a = \frac{\dot{n}_a}{n} = 0.846$ $Y_{O_2} = \frac{\dot{n}_{O_2}}{n} = 0.154$

(c) $\left[\dot{m}_a s_a(T_1, p_1) + \dot{m}_o s_o(T_2, p_2)\right] - \left[\dot{m}_a s_a(T_3, Y_a p_3) + \dot{m}_o s_o(T_3, Y_{O_2} p_3)\right] + \dot{\sigma} = 0$

$\Rightarrow \dot{\sigma} = \dot{m}_a \underbrace{\left[s_a(T_3, Y_a p_3) - s_a(T_1, p_1)\right]}_{(\Delta s)_a} + \dot{m}_o \underbrace{\left[s_o(T_3, Y_o p_3) - s_o(T_2, p_2)\right]}_{(\Delta s)_{O_2}}$

$(\Delta s)_a = \underbrace{s^o_a(T_2)}_{1.77} - \underbrace{s_a(T_1)}_{1.71} - \frac{\bar{R}}{M_a} \ln\left(\frac{Y_a p_3}{p_1}\right)$ kJ/kg.K.

$(\Delta s)_{O_2} = \underbrace{s^o_{O_2}(T_2)}_{\left(\frac{207}{32}\right)} - \underbrace{s^o_o(T_1)}_{\left(\frac{213}{32}\right)} - \frac{\bar{R}}{M_{O_2}} \ln\left(\frac{Y_{O_2} p_3}{p_1}\right)$ kJ/kg.K

$\boxed{\dot{\sigma} = 17.4 \text{ kJ/K.min}}$

15

Now, once you have mass flow rate, then we can get mole fraction, secondary information you require mole fractions.

$$\dot{n}_a = \frac{\dot{m}_a}{M_a} = \frac{114.3}{28.97} = 3.95;\ \dot{n}_{O_2} = \frac{\dot{m}_{O_2}}{M_{O_2}} = \frac{23.1}{32} = 0.72\ \Rightarrow n = 4.67\ kmol/min$$

$$Y_a = \frac{\dot{n}_a}{n} = 0.846; Y_{O_2} = \frac{\dot{n}_{O_2}}{n} = 0.154$$

Then, last part is entropy production. So, entropy production equation is written through the control volume equation and that is we have to write it clearly.

$$\dot{m}_a s_a(T_1, p_1) + \dot{m}_{O_2} s_{O_2}(p_2, T_2) - \left[\dot{m}_a s_a(y_a p_3, T_3) + \dot{m}_{O_2} s_{O_2}\left(y_{O_2} p_3, T_3\right)\right] + \dot{\sigma} = 0$$

$$\Rightarrow \dot{\sigma} = \dot{m}_a\left(s_a(y_a p_3, T_3) - s_a(T_1, p_1)\right) + \dot{m}_{O_2}\left(s_{O_2}\left(y_{O_2} p_3, T_3\right) - s_{O_2}(p_2, T_2)\right)$$

$$\Rightarrow \dot{\sigma} = \dot{m}_a(\Delta s)_a + \dot{m}_{O_2}(\Delta s)_{O_2}$$

$$= \dot{m}_a\left(\bar{s}^o_a(T_2) - \bar{s}^o_a(T_1) - \frac{\bar{R}}{M_a} ln\left(\frac{Y_a p_3}{p_1}\right)\right)$$

$$+ \dot{m}_{O_2}\left(\bar{s}^o_{O_2}(T_2) - \bar{s}^o_{O_2}(T_1) - \frac{\bar{R}}{M_{O_2}} ln\left(\frac{Y_{O_2} p_3}{p_1}\right)\right)$$

So, $\bar{s}_a^o(T_2)$ is 1.77; $\bar{s}_a^o(T_1)$ is 1.71. These values are kilo joule per kg kelvin. $\bar{s}_{O_2}^o(T_2)$ is $207/32$ and $\bar{s}_{O_2}^o(T_1)$ is $213/32$ kilo joule per kg kelvin. So, by inserting these values, we get $\dot{\sigma}$ is 17.4 kilojoule per kelvin minute. So, entropy production during this mixing process is 17 kilojoule per kelvin per minute.

So, just to summarize these things although you see these expressions are written, but you should understand the notation in which framework it is written. So, based on the which, it will be easy for you to calculate the property values directly from the book. So, only it is a matter of inserting the data in a given equations, even though notations are looks little bit complicated; but if you are used to it, then it will be very easy to solve this kind of problems.

Rather this solution is just an addition or subtractions. But the way of representing in terms of notations normally takes time.

With this viewpoint and understanding and I will close this lecture for today and also, we close this module 4.

Thank you for your attention.