Mechanics of Fiber Reinforced Polymer Composite Structures
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Module-04
Macromechanics of Lamina-II
Lecture-09
Hygrothermal Behaviour of Lamina

Hello and welcome to the third lecture of this fourth module.

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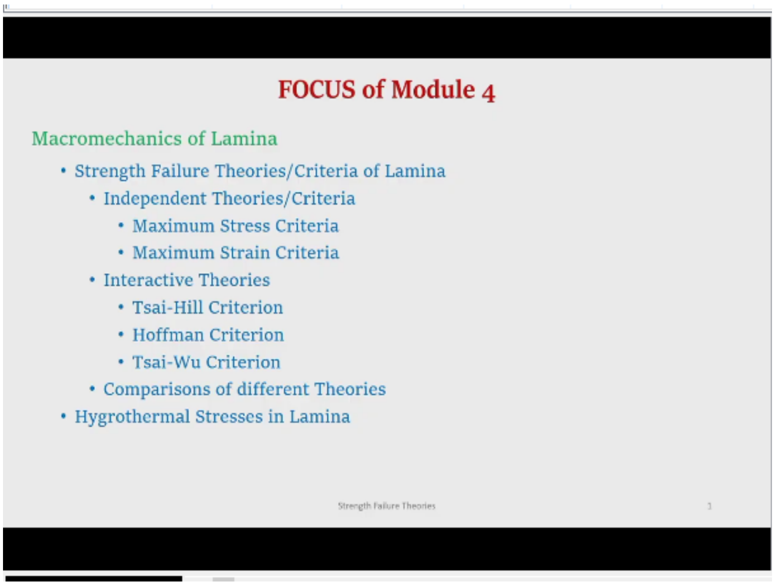

In our fourth module we have been discussing the macromechanics of lamina wherein first the stress strain relationships for both generally orthotropic lamina as well as for spatial orthotropic lamina have been developed in terms of the stiffness and the compliance matrices. Therefore, given the stresses, the corresponding strains could be determined and vice versa. Using those stress strain relationship in terms of compliance matrix, stiffness matrix or in terms of measurable engineering constants we can determine the stresses and strains both with reference to the global axis as well as the material axis. Then the transformations of compliance and stiffness matrix from material axis to global axis and vice versa have been discussed showing the influence of fiber orientation angle on the engineering constants in global axes. Finally, in the last two lectures applications of appropriate failure criteria (independent and interactive) to assess the safety or failure of a lamina have been discussed in details. In today's lecture, hygrothermal stresses in lamina will be discussed.

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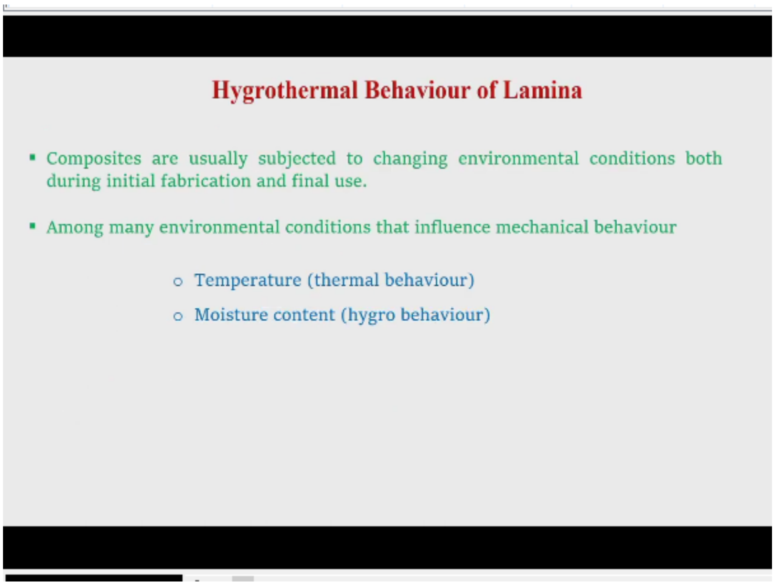

Till now in all the discussions on macromechanics of lamina, it was assumed that the lamina does not experience any environmental changes or there is no change in the environmental conditions. But in actual practice, the composites are actually subjected to changing environment conditions both during initial fabrication and during final use.

Among different environmental conditions that influence the mechanical behaviour the change in temperature and change in moisture content are very important especially for fiber reinforced polymer composites where these two factors have significant bearing on the behaviour of the polymer composites.

Now change in condition may be during fabrication like the laminate is actually cured at a particular temperature and pressure and then it is allowed to cool down to room temperature. Therefore there is a change in temperature. Similarly when it is exposed to the ambient conditions it may absorb moisture and therefore there may be moisture absorption as well as temperature change.

Also during service, depending upon the service conditions the operating temperature may be different from the ambient temperature or the service condition will be such that the environment is humid and it may actually absorb moisture. There are two principal effects of temperature and moisture content on the mechanical behaviour of polymer composites.

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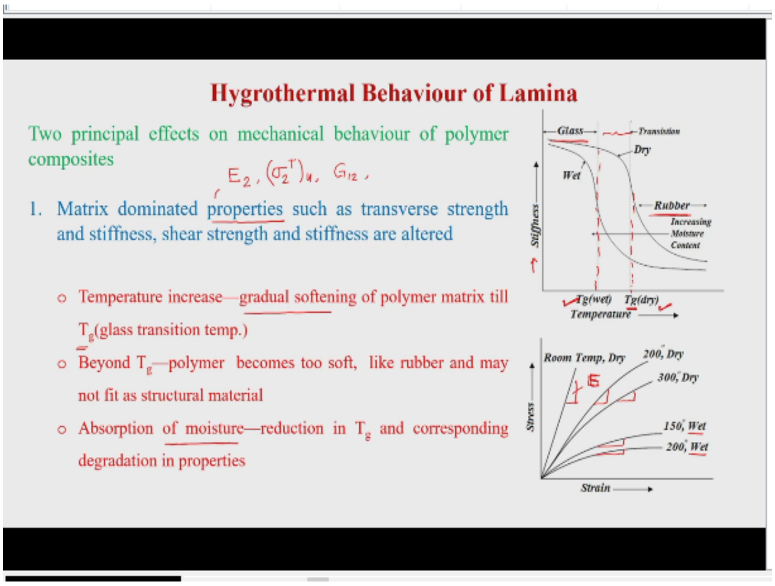

1. Matrix dominated properties such as transverse strength and stiffness, shear strength and stiffness are altered. With the increase in temperature there is a gradual softening (reduction in stiffness) of polymer matrix till the glass transition temperature (T_g) is reached. Beyond T_g, the polymer becomes too soft, like rubber and may not be suitable to be used as structural material. Again, absorption of moisture leads to reduction in T_g and corresponding degradation in matrix dominated properties as shown in the Fig. As could be seen in the figure that the dry glass transition temperature is higher than the wet glass transition temperature (when it absorbs moisture). That is on absorbing moisture the glass transition temperature actually decreases and it is seen that these polymers are highly sensitive to absorption of moisture. Sometimes 3 to 4% absorption of moisture leads to anything around 15 to 20% reduction in glass transition temperature. Thus increase in temperature as well as moisture absorption leads to the reduction in stiffness of polymer and therefore in a polymer matrix composites the change in temperature as well as moisture absorption has significant bearing on the matrix dominated properties like transverse stiffness, transverse strength.

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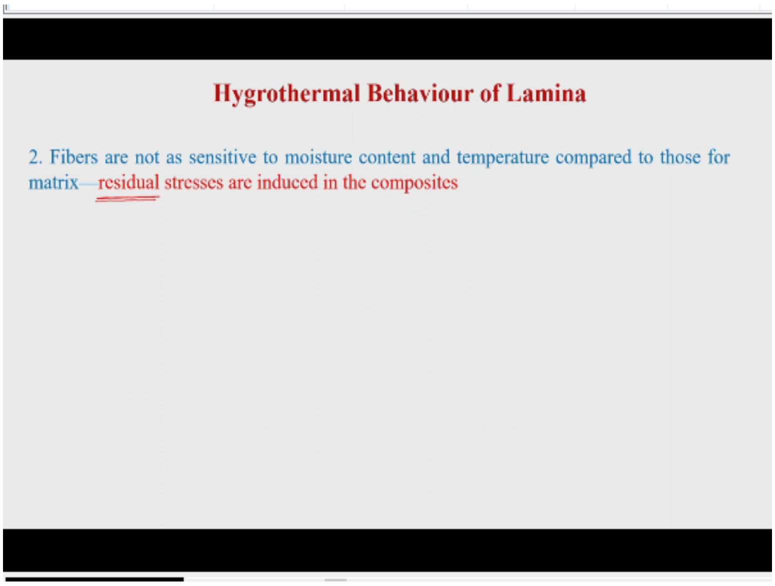

2. Second effect is due to the fact in fiber reinforced polymer matrix composites the fibers and matrix have different sensitivity towards the moisture absorption as well as temperature. The fibers are not really that sensitive to temperature and moisture absorption compared to that of the matrix and these results in a residual stress.

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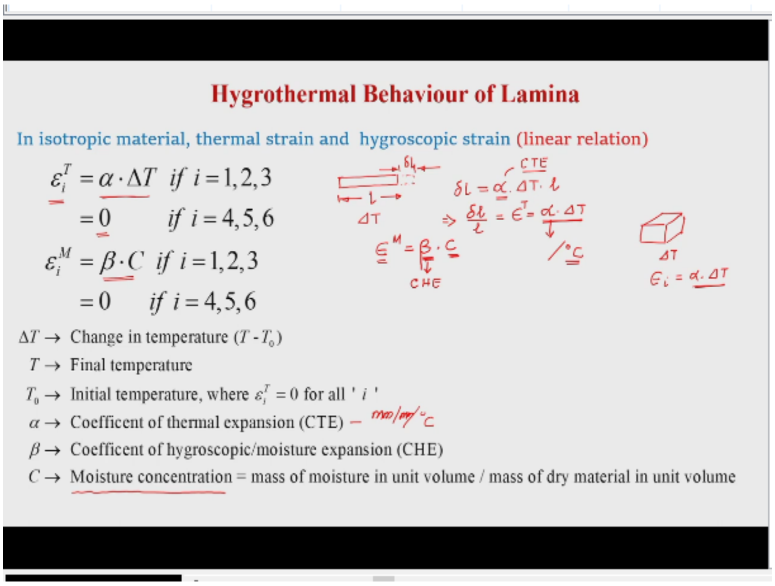

Now let us have a brief revisit of what exactly happens due to temperature rise. In an isotropic material like still, suppose there is an increase in temperature then what happens there is a change in dimension and the change in dimension is actually proportional to its initial dimension as well as increase in temperature.

As shown in the Fig. considering a steel rod of initial length l , and suppose it experiences a temperature change of ΔT then there is a change in length Δl and this change in length Δl is actually proportional to the temperature change ΔT and the initial length l and this is equal to $\Delta l = \alpha \Delta T l$ where α is the coefficient of thermal expansion.

Therefore we can write this $\frac{\Delta l}{l}$ is nothing but the strain (change in length by initial length) and is equal to $\alpha\Delta T$. Thus thermal strain $\varepsilon = \alpha\Delta T$. So, the strain induced because of ΔT is decided by what is α, which is the property of that material known as coefficient of thermal expansion. Now this relation is linear only over a specified range of temperature and over a wide range of temperature, the variation it may not be linear.

Similarly suppose there is a like say polymer which is sensitive to moisture, these are porous and sensitive to moisture. Similarly, the thermal strain induced because of moisture is similarly given by βC, where β is the coefficient of hygroscopic expansion and C analogous to change in temperature. C is the moisture concentration which is the ratio of mass of the moisture in unit volume to the mass of the dry material in unit volume.

In the case of three dimension, these thermal and hygroscopic strains are is written as

$$\begin{aligned} \varepsilon_i^T &= \alpha \cdot \Delta T \qquad if\ i = 1,2,3 \\ &= 0 \qquad\qquad if\ i = 4,5,6 \\ \varepsilon_i^M &= \beta \cdot C \qquad if\ i = 1,2,3 \\ &= 0 \qquad\qquad if\ i = 4,5,6 \end{aligned}$$

$\Delta T \rightarrow$ Change in temperature $(T - T_0)$

$T \rightarrow$ Final temperature

$T_0 \rightarrow$ Initial temperature, where $\varepsilon_i^T = 0$ for all ' i '

$\alpha \rightarrow$ Coefficent of thermal expansion (CTE)

$\beta \rightarrow$ Coefficent of hygroscopic/moisture expansion (CHE)

$C \rightarrow$ Moisture concentration = mass of moisture in unit volume/mass of dry material in unit volume

In three dimensions there are six strains, three normal strains and three shear strains. For a three dimensional object (isotropic) subjected to a temperature change of ΔT will lead to ε_1, ε_2, ε_3 which are equal to α into ΔT. Similar expressions are there for hygroscopic strains also. Note that there is no shear strain due to ΔT and C.

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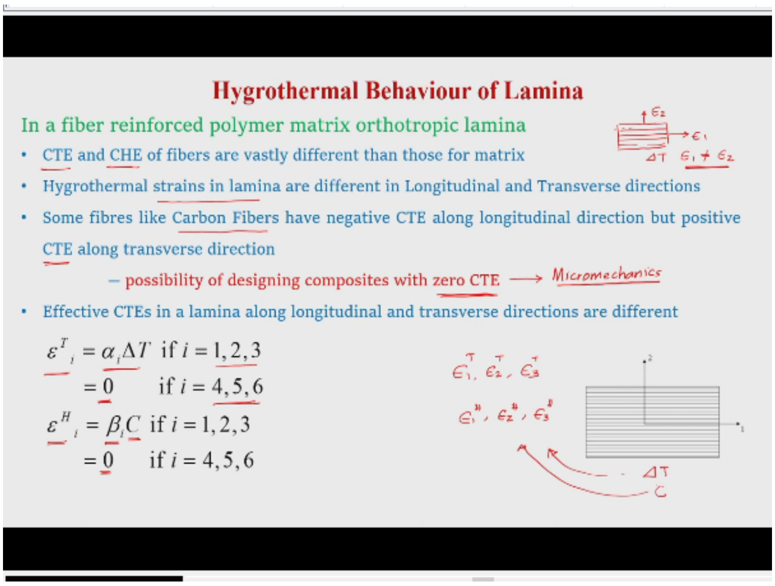

Having understood this let us see what residual stress is. Considering a fiber reinforced polymer matrix composites lamina as shown in the Fig., experiences ΔT, now the coefficient of thermal expansion of fiber is much less compared to the coefficient of thermal expansion of matrix. So, for ΔT, the fiber will have less expansion and the matrix will have more expansion if they are free to expand. This is because the coefficient of thermal expansion of matrix is more compared to that of the fiber. Now in the lamina, these fibers and the matrix perfectly bonded and because they are perfectly bonded, therefore they will have same expansion which is actually less than that of the free expansion of the matrix, but more than that of the free expansion of the fiber.

As a result the fibers experience an additional tensile strain ε_f and the matrix experiences an additional compression strain ε_m. Therefore the fiber is actually experiencing a tensile stress $\sigma_f=\varepsilon_f E_f$, where E_f is the Young's modulus of the fiber. The matrix experiences a compression stress $\sigma_m=\varepsilon_{fm}E_m$. This is the residual stress. Thus, there is a residual tensile stress in the fiber and there is a residual compression stress in the matrix because of the mismatch in coefficient of thermal expansion of the fiber and the matrix. Same is also true for moisture content. Fiber is almost insensitive to moisture, but the matrix is sensitive to moisture and hence the coefficient of moisture expansion in fiber and coefficient of moisture expansion in matrix are different and this results in residual stresses in the fiber and the matrix

There are major differences between the orthotropic and isotropic materials with reference to the hygrothermal behavior. Some fibers like carbon fibers have negative coefficient of thermal expansion along longitudinal direction, but in the transverse direction it is positive and that leads to the possibility of designing composites with zero coefficient of thermal expansion.

Now, considering an orthotropic lamina, the coefficients of thermal expansion and the coefficient of moisture expansion are direction dependent and as shown in Fig., if 1-2-3 are the principal materials directions, then

$$
\begin{aligned}
\varepsilon^{T}{}_{i} &= \alpha_i \Delta T && \text{if } i = 1,2,3 \\
&= 0 && \text{if } i = 4,5,6 \\
\varepsilon^{H}{}_{i} &= \beta_i C && \text{if } i = 1,2,3 \\
&= 0 && \text{if } i = 4,5,6
\end{aligned}
$$

Note that $\alpha_i(\alpha_1, \alpha_3, \alpha_3)$ and $\beta_i(\beta_1, \beta_3, \beta_3)$ like other properties of a lamina are also different in different directions. Again like isotropic material here also these are $= 0 \qquad \text{if } i = 4,5,6$ meaning that ΔT and C do not lead to any shear strain in the material axes.

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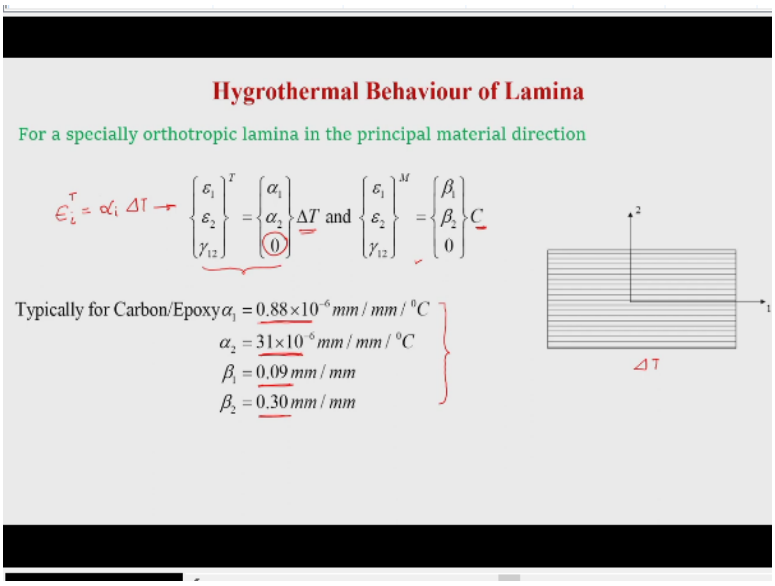

Considering a lamina subjected to ΔT and C, the hygrothermal strains in the material axes is

$$
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}^T = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \Delta T \text{ and } \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}^M = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} C
$$

$$
\begin{aligned}
\varepsilon_i^H &= \varepsilon_i^T + \varepsilon_i^M && if \ i = 1,2 \\
&= 0 && if \ i = 6
\end{aligned}
$$

Like the stress strain relationship in case of a specially orthotropic lamina, application of normal stress leads to normal strain and no shear strain. To have a understanding the typical value of coefficient of thermal expansion and coefficient of moisture expansion are

$$\alpha_1 = 0.88 \times 10^{-6} mm/mm/^0C$$
$$\alpha_2 = 31 \times 10^{-6} mm/mm/^0C$$
$$\beta_1 = 0.09 \, mm/mm$$
$$\beta_2 = 0.30 \, mm/mm$$

Note that this α_1, α_2, β_1, β_2 are actually functions of the coefficients of thermal expansion of the fiber and the matrix and their relative proportions. Similarly β_1, β_2 are the functions of coefficient of moisture expansion of the fiber and the matrix and their relative proportions. But in macromechanics of lamina, here we are considering the overall property of the lamina. In the study of micromechanics a detailed understanding of the influence of the individual properties on the lamina will be discussed.

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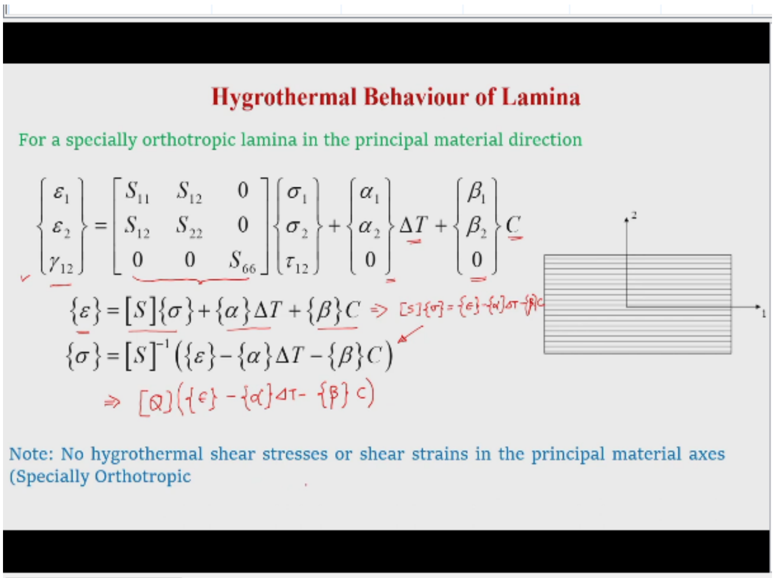

Now considering a specially orthotic lamina subjected to stresses σ_1, σ_2 and τ_{12} (in material axes), the total strain considering the hydrothermal and mechanical is given by

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} + \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \Delta T + \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} C$$

In short

$$\{\varepsilon\} = [S]\{\sigma\} + \{\alpha\}\Delta T + \{\beta\}C$$

$$\{\sigma\} = [S]^{-1}\left(\{\varepsilon\} - \{\alpha\}\Delta T - \{\beta\}C\right)$$

Again note that there are no hygrothermal shear stresses or strains in the principal material axes, because it is specially orthotropic.

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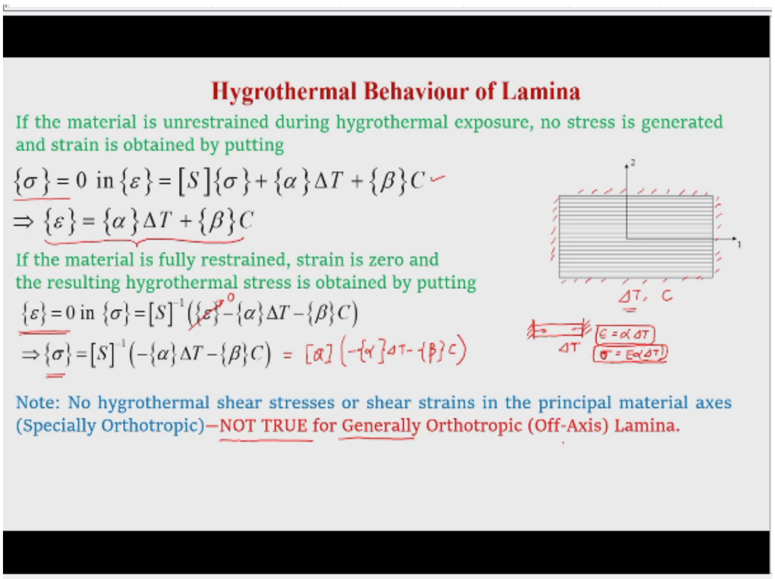

Considering a lamina and which is unrestrained that means it is free, and it is subjected to say ΔT and C. Now if it is free, there is no stress then

$$\{\sigma\}=0 \text{ in} \{\varepsilon\}=[S]\{\sigma\}+\{\alpha\}\Delta T+\{\beta\}C$$
$$\Rightarrow \{\varepsilon\}=\{\alpha\}\Delta T+\{\beta\}C$$

is the total strain as it is free to expand.

Now considering a fully restrained lamina, subjected to say ΔT and C but it is not allowed to change its dimension. Then there is no strain and

$$\{\varepsilon\}=0 \text{ in } \{\sigma\}=[S]^{-1}\left(\{\varepsilon\}-\{\alpha\}\Delta T-\{\beta\}C\right)$$
$$\Rightarrow \{\sigma\}=[S]^{-1}\left(-\{\alpha\}\Delta T-\{\beta\}C\right)$$

is the stress when the lamina is fully restrained.

Note that if ΔT is positive then the stress is negative (compressive) and if ΔT is negative the stress will be positive (tensile). Also note that in the material axis there is no associated shear strain or stresses. But this is not true if we try to establish the same hygrothermal stresses for a generally orthotropic off axis lamina.

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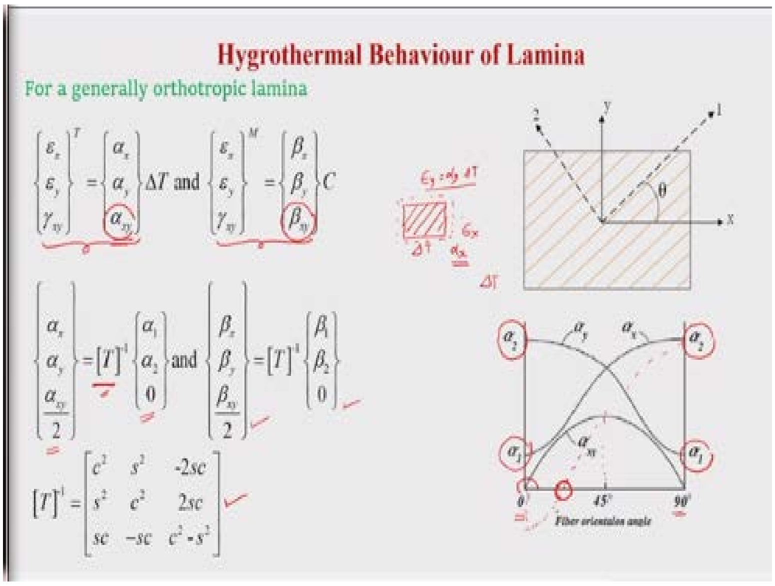

Considering a generally orthotropic lamina where the material axes 1-2-3 do not coincide with the global axes x-y-z, subjected to ΔT and C, the thermal and hygroscopic strains are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^T = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T \quad \text{and} \quad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^M = \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} C$$

Where $\alpha_x, \alpha_y, \alpha_{xy}$ and $\beta_x, \beta_y, \beta_{xy}$ are the coefficients of thermal expansion and coefficients of hygroscopic expansion in the x-y axes. Note that there is α_{xy} and β_{xy} which were not present in the material axes. The relationship between $\alpha_x, \alpha_y, \alpha_{xy}$ and $\alpha_1, \alpha_2, \alpha_{12}$ and between $\beta_x, \beta_y, \beta_{xy}$ and $\beta_1, \beta_2, \beta_{12}$ are

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \dfrac{\alpha_{xy}}{2} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \beta_x \\ \beta_y \\ \dfrac{\beta_{xy}}{2} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix}$$

$$where \quad [T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}$$

This is similar to the strain transformation matrix. Because α_x is nothing but the indication of strain along x- and α_y is the indication of strain along y- due to ΔT. Same is true for β_x and β_y. Therefore we can relate the material axis coefficient of thermal expansion to the global axis by the same strain transformation matrix. Figure shows the variation of $\alpha_x, \alpha_y, \alpha_{xy}$ and $\beta_x, \beta_y, \beta_{xy}$ with fiber orientation angle as could be seen that for θ=0° and 90° they do coincide with the corresponding values along 1 and 2. α_{xy} is 0 for θ^o=0 and 90^o and is maximum at for θ=45^o.

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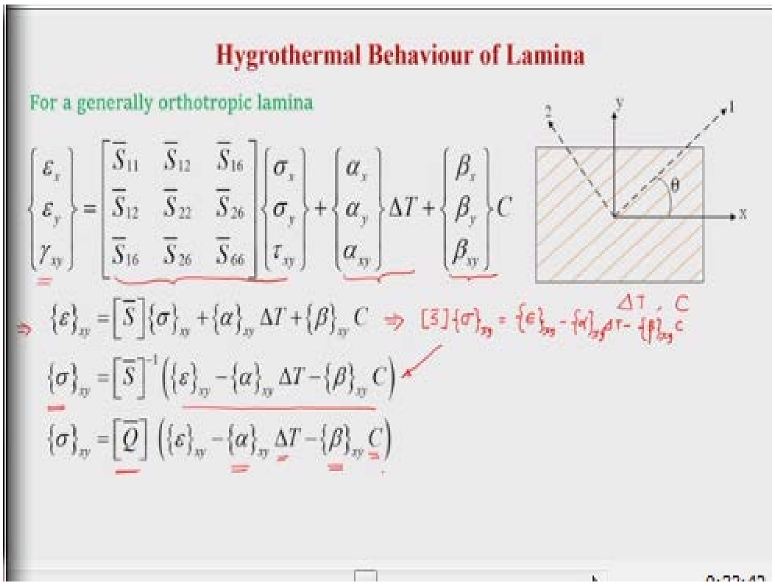

Similarly for a generally orthotropic lamina subjected to ΔT and C, total strain is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T + \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} C$$

and in short

$$\rightarrow \{\varepsilon\}_{xy} = \left[\bar{S}\right]\{\sigma\}_{xy} + \{\alpha\}_{xy}\,\Delta T + \{\beta\}_{xy}\,C$$

$$\rightarrow \{\sigma\}_{xy} = \left[\bar{S}\right]^{-1}\left(\{\varepsilon\}_{xy} - \{\alpha\}_{xy}\,\Delta T - \{\beta\}_{xy}\,C\right)$$

$$\rightarrow \{\sigma\}_{xy} = \left[\bar{Q}\right]\left(\{\varepsilon\}_{xy} - \{\alpha\}_{xy}\,\Delta T - \{\beta\}_{xy}\,C\right)$$

So, knowing [Q], reduced transform stiffness matrix, $\{\alpha\}_{xy}$, and $\{\beta\}_{xy}$ and change in temperature and the change in moisture content, we can actually find out what is the stress induced in the global axis.

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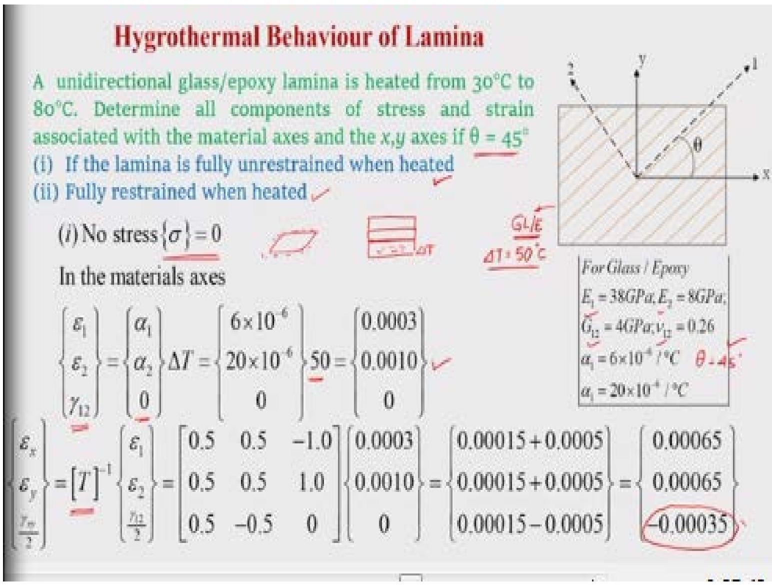

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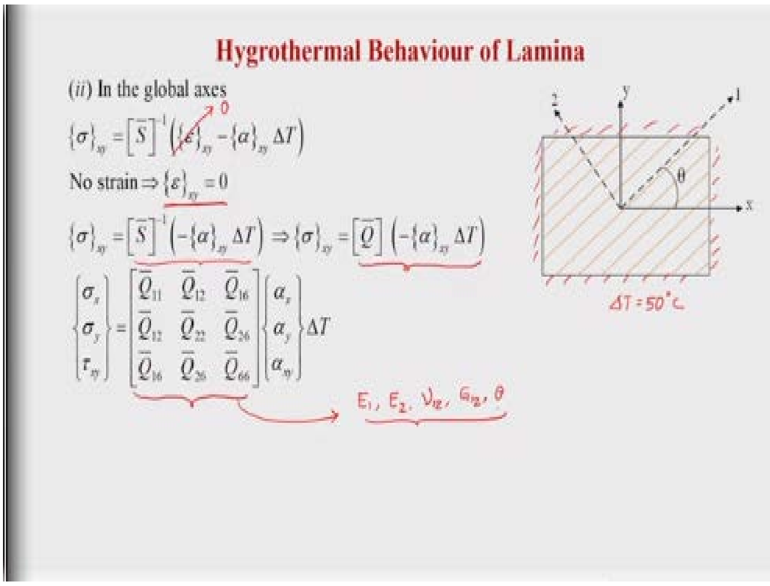

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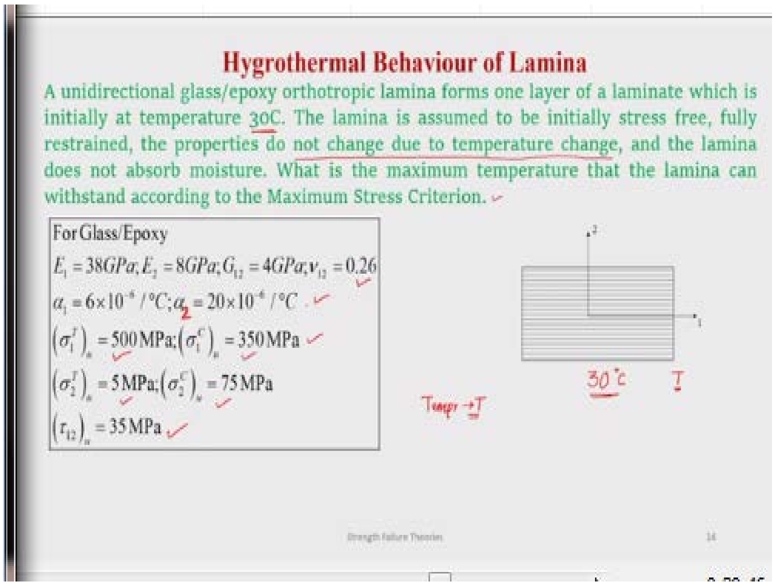

We shall solve a few example problems to understand what we have discussed.

Example 1: A unidirectional glass/epoxy lamina is heated from 30°C to 80°C. Determine all components of stress and strain associated with the material axes and the *x*,*y* axes if θ = 45°

(i) If the lamina is fully unrestrained when heated

(ii) Fully restrained when heated

$$\text{Given} \quad \boxed{\begin{array}{l} For\,Glass\,/\,Epoxy \\ E_1 = 38GPa; E_2 = 8GPa; \\ G_{12} = 4GPa; \nu_{12} = 0.26 \\ \alpha_1 = 6\times10^{-6}\ /\ °C \\ \alpha_1 = 20\times10^{-6}\ /\ °C \end{array}}$$

Solution:

$$(i)\ \text{No stress}\ \{\sigma\} = 0$$

$$\text{In the materials axes} \quad \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \Delta T = \begin{Bmatrix} 6\times10^{-6} \\ 20\times10^{-6} \\ 0 \end{Bmatrix} 50 = \begin{Bmatrix} 0.0003 \\ 0.0010 \\ 0 \end{Bmatrix}$$

Strains in the global axes

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 & -1.0 \\ 0.5 & 0.5 & 1.0 \\ 0.5 & -0.5 & 0 \end{bmatrix} \begin{Bmatrix} 0.0003 \\ 0.0010 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.00015 + 0.0005 \\ 0.00015 + 0.0005 \\ 0.00015 - 0.0005 \end{Bmatrix} = \begin{Bmatrix} 0.00065 \\ 0.00065 \\ -0.00035 \end{Bmatrix}$$

(ii) In the global axes $\{\sigma\}_{xy} = \left[\overline{S}\right]^{-1}\left(\{\varepsilon\}_{xy} - \{\alpha\}_{xy}\,\Delta T\right)$

No strain $\Rightarrow \{\varepsilon\}_{xy} = 0$

$\{\sigma\}_{xy} = \left[\overline{S}\right]^{-1}\left(-\{\alpha\}_{xy}\,\Delta T\right) \Rightarrow \{\sigma\}_{xy} = \left[\overline{Q}\right]\left(-\{\alpha\}_{xy}\,\Delta T\right)$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}\Delta T$$

Example 2: A unidirectional glass/epoxy orthotropic lamina forms one layer of a laminate which is initially at temperature 30C. The lamina is assumed to be initially stress free, fully restrained, the properties do not change due to temperature change, and the lamina does not absorb moisture. What is the maximum temperature that the lamina can withstand according to the Maximum Stress Criterion? Given

For Glass/Epoxy

$E_1 = 38GPa; \quad E_2 = 8GPa;$

$G_{12} = 4GPa; \quad \nu_{12} = 0.26$

$\alpha_1 = 6\times10^{-6}\ /\,^\circ C; \alpha_1 = 20\times10^{-6}\ /\,^\circ C$

$\left(\sigma_1^T\right)_u = 500\,\text{MPa}; \left(\sigma_1^C\right)_u = 350\,\text{MPa}$

$\left(\sigma_2^T\right)_u = 5\,\text{MPa}; \quad \left(\sigma_2^C\right)_u = 75\,\text{MPa}$

$\left(\tau_{12}\right)_u = 35\,\text{MPa}$

Solution:

$$\text{Using}\quad \begin{aligned}\{\varepsilon\} &= [S]\{\sigma\} + \{\alpha\}\Delta T + \{\beta\}C\\ \{\sigma\} &= [S]^{-1}\left(\{\varepsilon\} - \{\alpha\}\Delta T - \{\beta\}C\right)\end{aligned}$$

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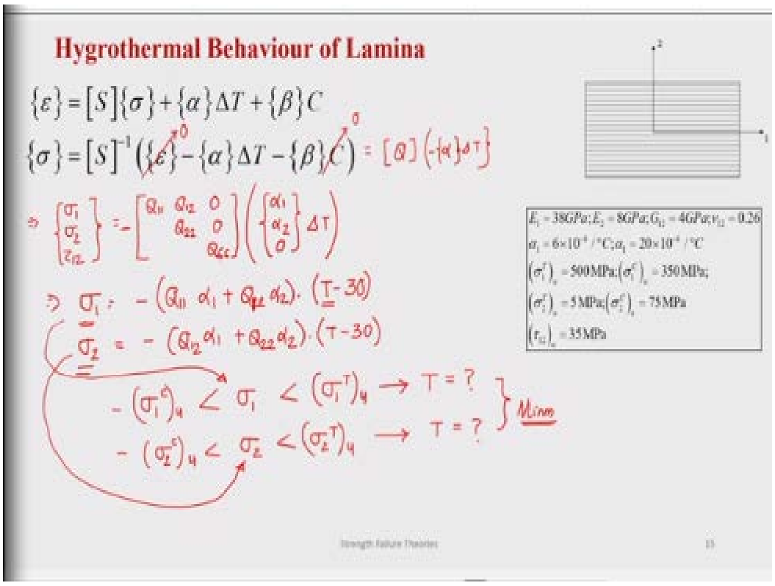


Now since it is fully restrained that means strain is 0

$$\{\varepsilon\} = [S]\{\sigma\} + \{\alpha\}\Delta T + \{\beta\}C$$
$$\{\sigma\} = [S]^{-1}\left(\{\cancel{\varepsilon}^{\to 0}\} - \{\alpha\}\Delta T - \{\beta\}\cancel{C}^{\to 0}\right)$$
$$\{\sigma\} = [S]^{-1}\left(\{0\} - \{\alpha\}\Delta T - \{\beta\}0\right) = [Q]\left(-\{\alpha\}\Delta T\right)$$

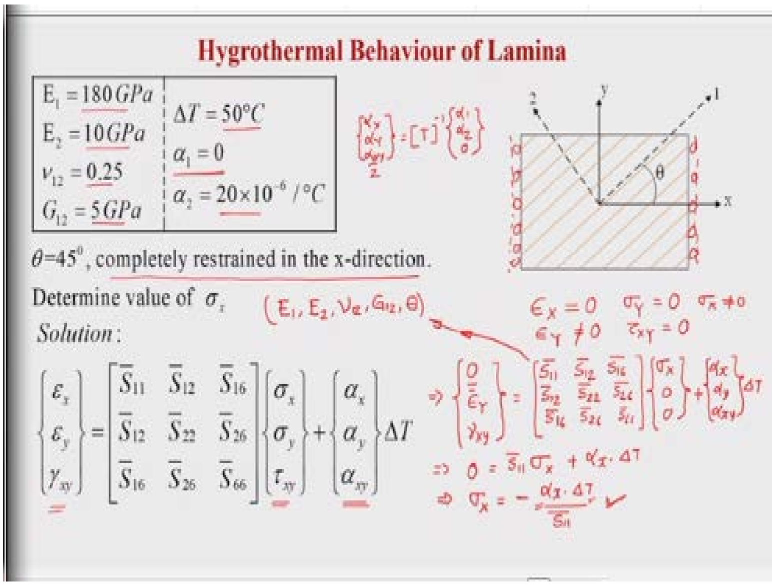
Therefore $\sigma_1 = -(Q_{11}\alpha_1 + Q_{22}\alpha_2)(T-30)$.

What is ΔT? Suppose T is the maximum temperature it can withstand, so

$\sigma_2 = -(Q_{12}\alpha_1 + Q_{22}\alpha_2)(T - 30)$ and there is no τ_{12}.

We can apply maximum stress criterion, to find out what is that maximum T it can withstand for safety. **(Refer Slide Time: 43:20)**

Example 3: A 45˚ graphite/epoxy lamina is completely restrained in the x direction as shown in the Fig and the properties of the lamina are given. It is experiencing a temperature change of 50°C, determine σ_x.



Solution:

Using the relation below where the strain along x is zero, we get

$$\begin{Bmatrix} \varepsilon_x \to 0 \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{S_{11}} & \overline{S_{12}} & \overline{S_{16}} \\ \overline{S_{12}} & \overline{S_{22}} & \overline{S_{26}} \\ \overline{S_{16}} & \overline{S_{26}} & \overline{S_{66}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T$$

$$0 = \overline{S}_{11}\sigma_x + \alpha_x \Delta T$$

$$\Rightarrow \sigma_x = -\frac{\alpha_x \Delta T}{\overline{S_{11}}}$$

Knowing $\bar{S}_{11}$ (from E_1, E_2, ν_{12}, G_{12} and θ), α_x and ΔT = 50, we can find out what is the stress induced along x and naturally it is as expected it is negative for positive ΔT.

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Hygrothermal Behaviour of Lamina

$$\begin{Bmatrix}\alpha_1\\ \alpha_2\\ 0\end{Bmatrix} \longrightarrow \begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix} \longrightarrow \begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}\Delta T$$

$$\begin{Bmatrix}\alpha_1\\ \alpha_2\\ 0\end{Bmatrix} \rightarrow \begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \gamma_{12}\end{Bmatrix} = \begin{Bmatrix}\alpha_1\\ \alpha_2\\ 0\end{Bmatrix}\Delta T \rightarrow \begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{Bmatrix} = [T]^{-1}\begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \gamma_{12}\end{Bmatrix}$$