Mechanics of Fiber Reinforced Polymer Composite Structures
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Module-04
Macromechanics of Lamina-II
Lecture-07
Strength Failure Criteria-Part I

Hello, welcome to the 4th module of this course mechanics of fiber reinforced polymer composite structures. In the last module we have been discussing the macromechanics of lamina and we understood the stress-strain relationship of a lamina with reference to the principal material axis as well as with reference to the global axis and we also understood the influence of the direction dependent stiffness of lamina on the stress-strain behavior and specially we understood how this behaviour is different from that of isotropic materials, like the existence of coupling terms like shear normal response coupling terms, even the normal-normal response term in orthotropic lamina is different from that in isotropic material. So, we understood also the influence of the fiber angle on the stress-strain behaviour of such lamina.

The importance of understanding the behaviour of lamina in the design of a laminate is also discussed. For example, if we want to design a laminate and the requirement is such that the laminate should have a good stiffness along x, we must put some of the laminae whose fibers are directed along x, ie. 0° fiber orientation. But having known that those laminae are actually weak in the transverse direction, therefore if the requirement is also to have a good stiffness in the transverse direction, we must also place some lamina with 90° fiber orientation. The number of 0° and 90° lamina depends upon what is the stiffness requirement. Suppose the stiffness in the x and y directions required are equal then we must put equal number of 0° and 90° lamina. Suppose in addition to that we also need the laminate to have a good shear resistance, then we must put some lamina with 45° fiber orientation because we have understood that the shear modulus is maximum when the fibers are oriented at an angle of 45°.

Number of such lamina in the laminate that is again decided by what is the kind of stiffness required. So, having understood that, another important aspect of laminate design is that we also need to know how a laminate performs under load whether subjected to a load whether it is safe, therefore we must also know the strength of a laminate.

Now because the number of laminae actually stacked together to form a laminate, therefore the strength of a laminate is definitely decided by the strength of the individual lamina comprising that laminate. Therefore, it is important to understand the strength of a lamina. So, in this module we will basically try to understand the strength of a lamina.

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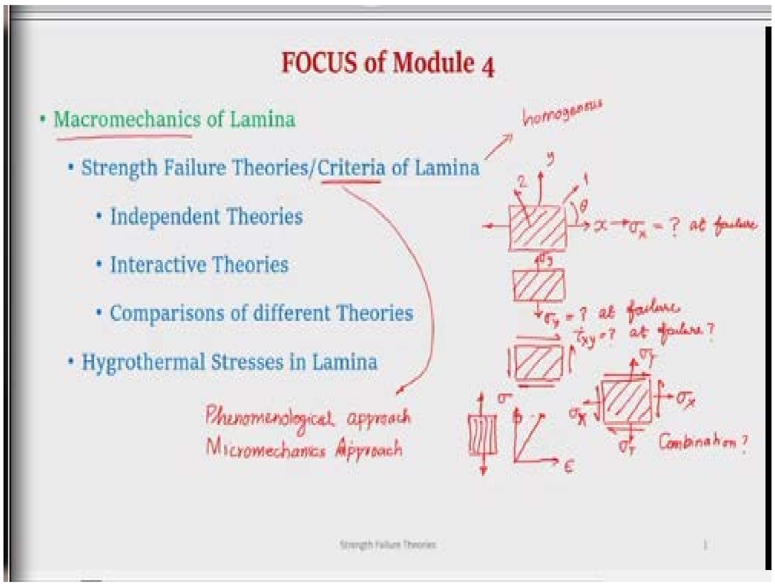

Note that this is again under macromechanics of lamina. Therefore, the strength failure theories, we will be discussing is also under macromechanics, meaning that we will not consider the individual constituents of the lamina. Like in the stress-strain relationship we have considered the lamina to be homogeneous, here also we will consider the lamina to be homogeneous, represented by the average properties and we will consider macromechanics failure theory.

Now, what is strength of a lamina? Suppose we consider a lamina subjected to a stress σ_x along x, (x-y is the global axes and 1-2 is the materials axes). So, we would like to know what is the value of σ_x at failure or in other words what is the maximum value of σ_x it can withstand before it fails. It could be even σ_y, and we would be interested to know what is the σ_y at failure. Similarly, it could also be subjected to say shear τ_{xy} and we will be interested to know what is the maximum in-plane shear stress it can withstand before it fails. So, this is what exactly is the strength of a lamina. Now it may not be always a single stress, it could be that a lamina is experiencing all these stresses acting together as combined stresses and we need to know what combination of these stresses will lead to the failure? So, this is what exactly we mean by the strength of a lamina, we need to know that what maximum stress, a single stress or a combination of stresses a lamina can actually withstand before it fails. Again, it is under macromechanics, therefore the failure theories or failure criteria discussed will be basically considering the lamina to be homogeneous and off course orthotropic.

Now, there are two types of approaches in the failure theories viz. phenomenological approach where we are concerned with the phenomenon and what is the consequence. In this we do not go into much of it is cause. For example, subjected to load, a lamina fails and we want to know what the strength of a lamina is. So, we take a lamina and suppose load it in UTM and we look at the stress-strain curve. It fails at a particular stress value and this is the failure point and the corresponding stress is the failure stress or the strength of that particular lamina. Now how this failure has taken place, the mechanisms, etc are not discussed.

A lamina is actually made of a fiber and matrix and when the lamina is loaded, there are many localized failures, like maybe one of the fibers fails but failure of one fiber does not necessarily mean that the lamina has failed. Now that fiber breakage may lead to fiber matrix debonding, similarly there may be matrix crack, that matrix crack may lead to delamination. So, these are all local failures and a combined effect of all these actually leads to the final failure of a laminate. But in this phenomenological approach, in this macroscopic failure theory we do not discuss this. Here the main objective is to establish a mathematical expression which can correlate the actual failure points, so it is more of like fitting the data. Therefore, in that way it is actually kind of criterion, it is a failure criterion more than a theory. So, we will discuss these failure criteria, and we will also have a comparison and then finally in this module we will also see the hygrothermal stresses in a lamina.

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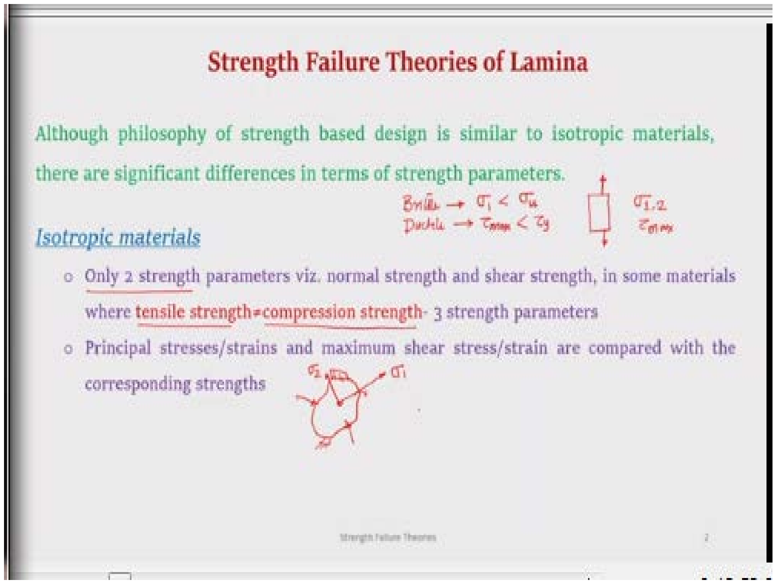

The philosophy of the strength-based failure theory is same as what we learned in the case of isotropic material where strength failure theories, like maximum normal stress theory, maximum shear stress theory, distortion energy theory etc. were discussed. The philosophy is same that means we find out the maximum stress and compare that with the corresponding strength to assess the failure or safety of a component subjected to load.

But there is a significant difference between the isotropic materials and the orthotropic materials. In isotropic materials we have only two strength parameters viz. , the ultimate tensile strength of that material (σ_u) and the shear stress at yield (τ_y). Say we take any component made of isotropic material subjected to load, we find out what are the principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$) and then we also find out what is the maximum shear stress ($\tau_{\max} = \dfrac{\sigma_1 - \sigma_2}{2}$). If the material is brittle, then we check whether σ_1, the maximum normal stress is less than the ultimate tensile strength of that material (σ_u) or if it is a ductile material, then we check that maximum shear stress ($\tau_{\max}$) is less than the shear stress at yield (τ_y). Therefore, basically there are two parameters in isotropic materials one is ultimate tensile strength or yield point strength and the shear strength.

However, in some cases there are some materials where the tensile strength is not equal to the compression strength, for example cast iron. In that case we need three strength parameters like $\sigma_u^T, \sigma_u^C, \tau_y$ (T and C stands for tension and compression) but in most of the cases materials like steel we go with two strength parameters, that is the normal strength and the shear strength. Also, the principal stress or strain and the maximum shear stress are compared to the corresponding strengths.

Now the strength of isotropic material is independent of direction, therefore it does not matter in which direction the maximum normal stress or the maximum principal stress is, we find out the maximum stress and then equate that to the corresponding strength because strength is same in all the directions. So, things are much simpler in isotropic material.

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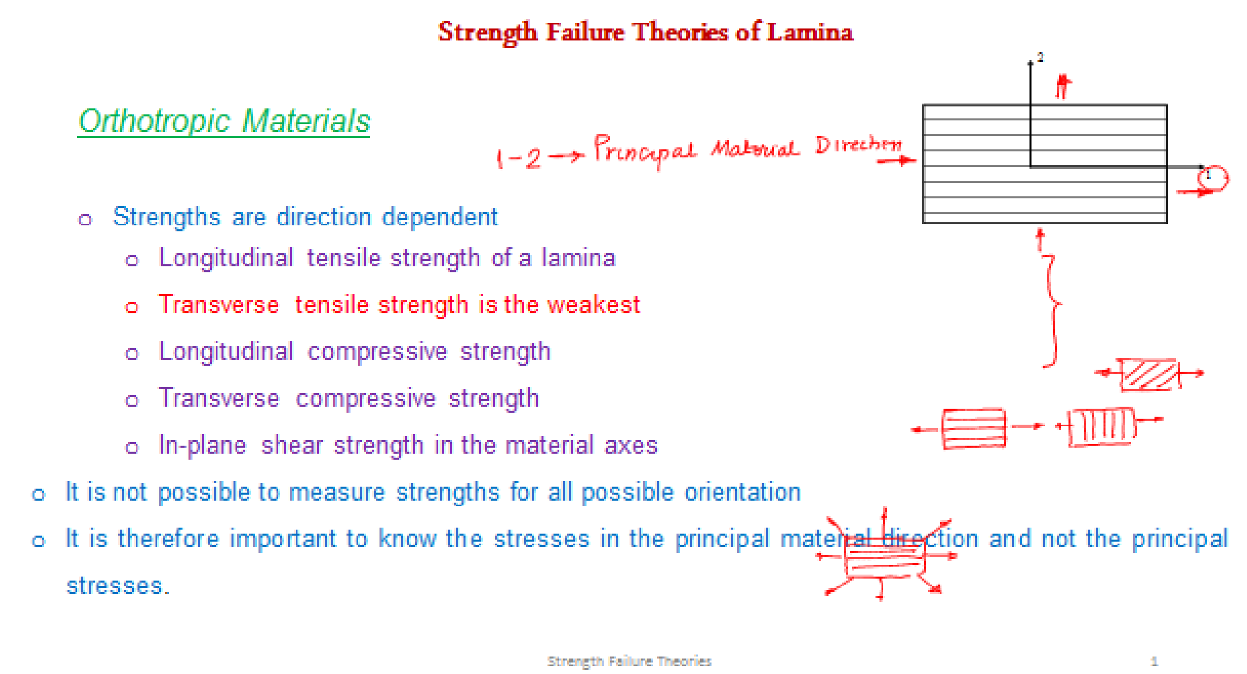

On the other hand, for orthotropic materials, like stiffness, strengths are also direction dependent. For example the longitudinal tensile strength of a lamina is far higher compared to the transverse tensile strength. For a lamina the stiffness in the longitudinal direction E_1 is greater than E_2. The strength here, the longitudinal tensile strength also is greater than the transverse tensile strength.

The reason is what is longitudinal tensile strength is the tensile strength along the longitudinal direction (direction 1) of the fiber and the transverse tensile strength is the tensile strength transverse (along direction 2) to the fiber direction. Now because the fibers are very strong and stiff in the longitudinal direction, therefore the strength of the lamina in direction 1 in the longitudinal tensile strength is actually decided by the strength of the fiber. On the other hand, in the transverse direction it is not the fiber but the fibers are bonded by impregnating into the matrix, therefore it is actually dominated by the matrix. Similarly, we have different strengths in the longitudinal compression and transverse compression and then the in-plane shear strength. Hence the strengths are also direction dependent and the transverse tensile strength is the weakest link, it is the minimum strength of a lamina.

So, depending on the directions, the strengths are different, and it is not possible to measure strength for all possible direction. Considering a lamina, now if it is loaded along 1, it has strength. Now if it is rotated by 90° and loaded, it is different, suppose it is rotated by 45°, strength is different. Therefore, the strengths are different and it is not possible to measure strength for all orientations.

So, how to then specify the strength of a lamina? It is therefore important to know the stresses in the principal material direction 1-2 (1 parallel to fiber, 2 perpendicular to the fiber, 1 is known as longitudinal direction, 2 is known as transverse direction) because, the strengths are specified in the principal material direction. Thus, while addressing the strength failure theories for orthotropic material, this is the difference compared to that in the isotropic material. In the case of isotropic materials we have 2 or maximum 3 strength parameters but in the case of an orthotropic materials, we have 5 strength parameters.

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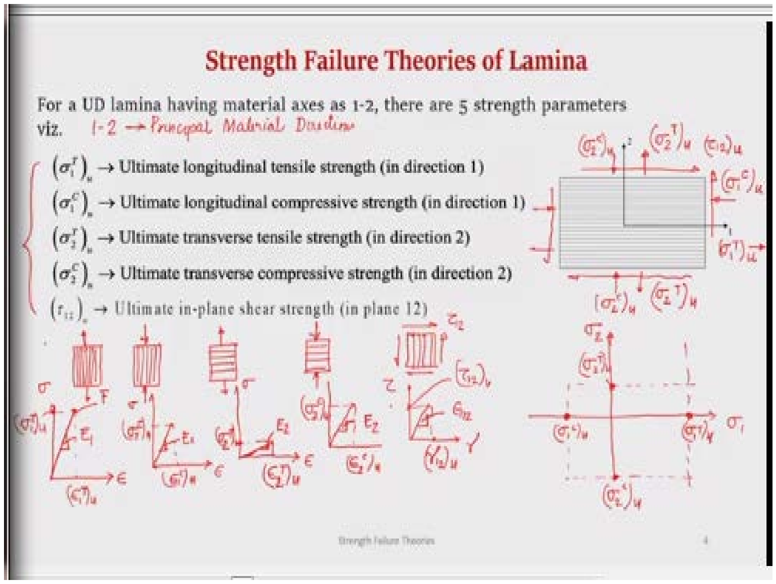

As shown in the Fig., with reference to the principal material directions, the five strength parameters are

1. $\left(\sigma_1^T\right)_u$, the ultimate longitudinal tensile strength.
2. $\left(\sigma_1^C\right)_u$, the ultimate longitudinal compression strength
3. $\left(\sigma_2^T\right)_u$, the transverse tensile strength
4. $\left(\sigma_2^C\right)_u$, the transverse compression strength and
5. $\left(\tau_{12}\right)_u$, is the ultimate in-plane shear strength.

So, these are the five strength parameters which are required to specify the strength in an orthotropic lamina. As shown in the Fig., these strengths could be obtained experimentally by applying corresponding loading in the corresponding directions. From the stress strain curves, the strengths and the corresponding modulus could also be obtained.

Again, we could only find out the failure stress or strengths and not the mechanisms by which a a lamina fails. The failure may be due to fiber breakage, matrix cracking, fiber microbuckling, fiber matrix debonding etc. but are not addressed and hence – macroscopic failure theories. However, in microscopic failure approaches all these things are actually considered to understand what the mechanism of different type of failure is.

Note that in the case of an orthotropic lamina, σ_1, σ_2 and τ_{12} represent the material axes stresses and do not confuse σ_1, σ_2 as the principal stresses which are commonly used notations in isotropic materials. Suppose for a lamina, only σ_1 (along 1) is applied, the maximum stress it can withstand is $\left(\sigma_1^T\right)_u$ if σ_1 is +ve and $\left(\sigma_1^C\right)_u$ if σ_1 is –ve. Similarly for +ve and –ve applied σ_2, the maximum stress it can withstand $\left(\sigma_2^T\right)_u$ and $\left(\sigma_2^C\right)_u$ respectively. For applied in-plane shear stress, τ_{12}, the maximum stress it can withstand is $\left(\tau_{12}\right)_u$.

But actual loading may not be like this, it is actually may be a combination of σ_1, σ_2, τ_{12}. Therefore, we need to know subjected to combined loading how the failure takes place. So, this is the objective of developing the macroscopic failure theories.

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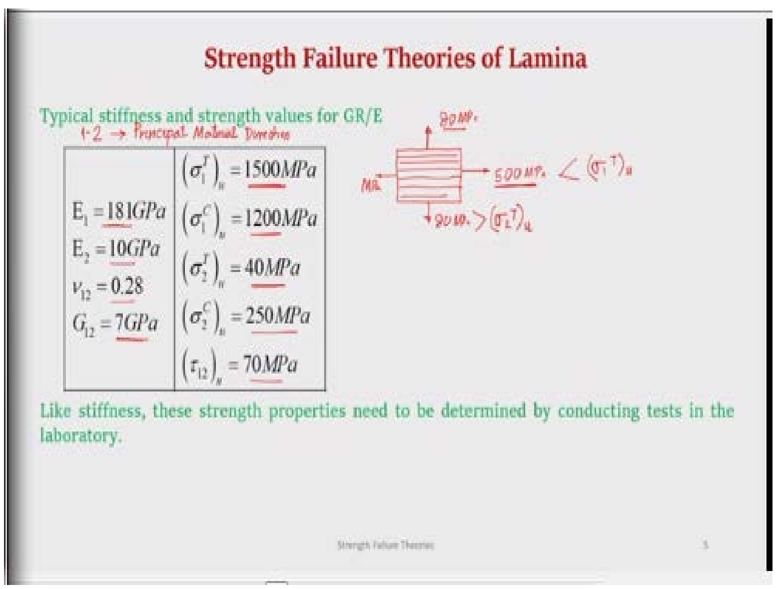

Therefore unlike isotropic materials, in the case of orthotropic materials subjected to load, we actually find out the materials axes stresses σ_1, σ_2, τ_{12} and not the principal stresses. The reason is this, because the strengths are specified with respect to the material axes directions.

For a typical graphite epoxy lamina the following are the strength and stiffness properties.

$E_1 = 181GPa$ $E_2 = 10GPa$ $\nu_{12} = 0.28$ $G_{12} = 7GPa$	$\left(\sigma_1^T\right)_u = 1500MPa$ $\left(\sigma_1^C\right)_u = 1200MPa$ $\left(\sigma_2^T\right)_u = 40MPa$ $\left(\sigma_2^C\right)_u = 250MPa$ $\left(\tau_{12}\right)_u = 70MPa$

Suppose a 45^o graphite epoxy lamina subjected to load and the stresses developed along 1 and 2 happened to be the principal stresses say 500 MPa and 80 MPa and there is no shear stress. So these are the principal stresses. So, the major principal stress 500 MPa is actually less than the strength 1500 MPa but that does not mean that it will not fail, it will fail because in the other direction 80 MPa is greater than the strength ie 40 MPa.

Therefore, since the strengths are direction dependent, the principal stresses do not have much bearing in the analysis of the strength of a lamina. It is important that we actually find out the stresses in the material direction and then compare those stresses with the corresponding strengths in the material direction. Now the strength properties in the material directions are actually determined by conducting laboratory tests. Laboratory tests could be conducted independently following certain standards and the five strengths parameters $\left(\sigma_1^T\right)_u$, $\left(\sigma_1^C\right)_u$, $\left(\sigma_2^C\right)_u$, $\left(\sigma_2^T\right)_u$ and $\left(\tau_{12}\right)_u$ could be known with respect to the principal material direction.

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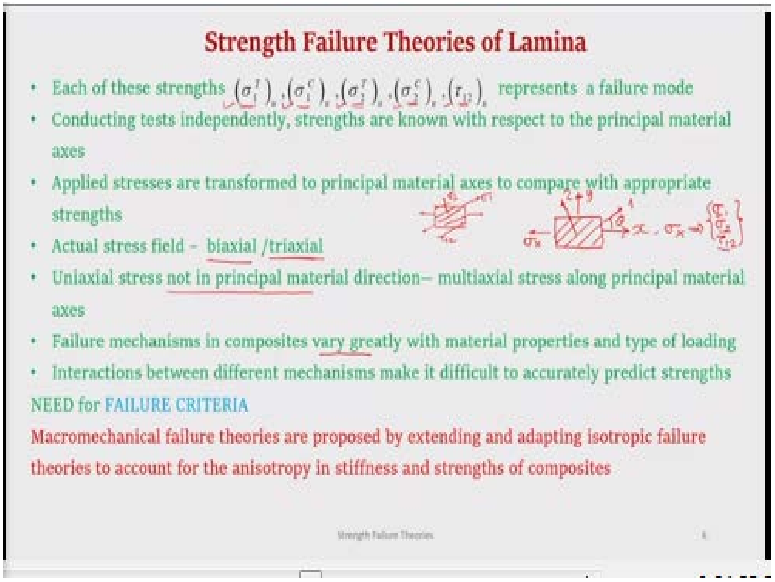

So, the applied stresses are actually transformed to principal material axis and then compared to the appropriate strengths to assess whether it is safe or whether it fails.

Now even though the strengths are actually determined conducting a single test applying one particular stress at a time but actual stress fields are mostly biaxial or triaxial. Even with applied uniaxial stress, but not along the material axis might lead to biaxial stresses. For example, suppose a lamina, (x-y global axes and 1-2 material axes) with fiber orientation θ, is loaded with uniaxial stress σ_x. Now this will lead to σ_1, σ_2 and τ_{12}, along the material axes which could be obtained by the stress transformation.

So, uniaxial stress not in the principal material direction may lead to multi axial stress along the principal material axis. Now failure mechanism in composites vary greatly with material properties and type of loading and a failure may be because of the combination of all these different modes and interactions between different mechanisms make it really difficult to accurately predict the strengths and therefore a failure criterion is needed. Now in macromechanical failure criteria, so the macromechanical failure theories are proposed by extending and adopting the failure theories which are applied to isotropic material but it takes into account the anisotropy in stiffness and strength of the lamina though the philosophy remains same.

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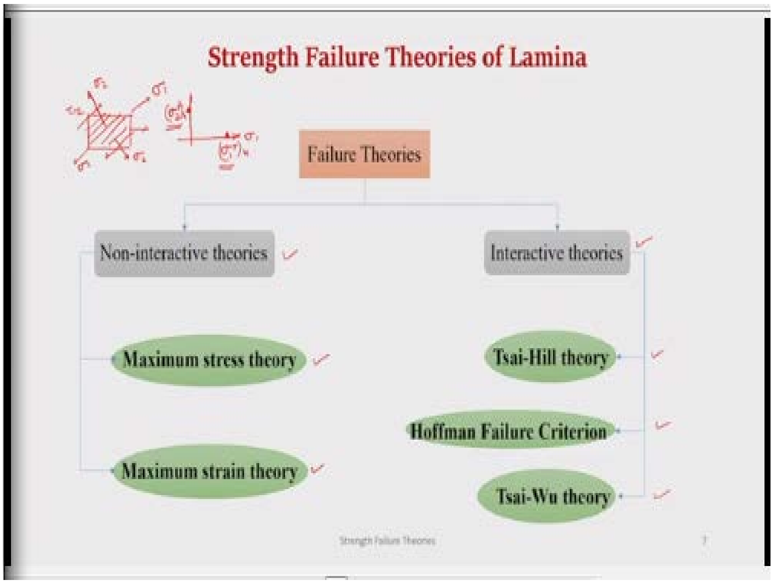

Macroscopic failure theories / criteria are actually classified into two types viz. non-interactive or independent theories / criteria and interactive theories / criteria.

In non-interactive theories it is assumed that there is no interaction among the stresses and each stress is independently compared to the corresponding strength. As shown in Fig., if we apply +ve σ_1 then the failure point is $\left(\sigma_1^T\right)_u$, if you apply +ve σ_2 the failure point is $\left(\sigma_2^T\right)_u$. But what happens if we apply σ_1 and σ_2 together because the failure is actually influenced by both the σ_1 and σ_2 and there is interactions. So, the non-interactive failure theories do not take into account this interaction and considers the individual failure independently.

On the other hand, the interactive theories take into account the interactions between the different stresses which lead to the failure. So, under non-interactive theories there are two theories viz. Maximum stress theory and Maximum strain theory. Similarly, there are number of interactive theories which are actually put forward at different point of time. Like Tsai-Hill theory, then Hoffman failure criterion, then Tsai-Wu theory etc. We will discuss all this one by one.

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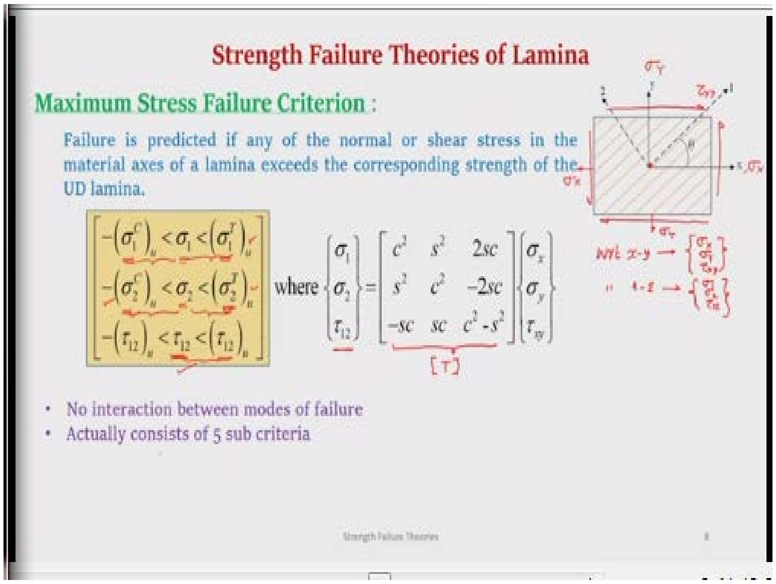

Maximum Stress Failure Criterion

As shown in Fig., given the state of stress at appoint in a lamina as subjected to σ_x, σ_y and τ_{xy}, we find out the stresses with reference to the principal material directions (1-2) using the stress transformation. So, with respect to the global x-y, the stresses are σ_x, σ_y, τ_{xy} and with respect to the principal material direction 1-2 the stresses are σ_1, σ_2 and τ_{12}.

Now the five strength parameters $(\sigma_1^T)_u$, $(\sigma_1^C)_u$, $(\sigma_2^C)_u$, $(\sigma_2^T)_u$ and $(\tau_{12})_u$ with reference to the material axis are known, and the stresses σ_1, σ_2 and τ_{12} with reference to the materials axes are also known, the maximum stress criterion states that failure is predicted if any of the normal or shear stress in the material axes of a lamina exceeds the corresponding strength of the UD lamina. Mathematically,

$$\begin{bmatrix} -\left(\sigma_1^C\right)_u < \sigma_1 < \left(\sigma_1^T\right)_u \\ -\left(\sigma_2^C\right)_u < \sigma_2 < \left(\sigma_2^T\right)_u \\ -\left(\tau_{12}\right)_u < \tau_{12} < \left(\tau_{12}\right)_u \end{bmatrix} \text{ where } \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

Note that the stresses are compared with the corresponding strengths depending upon the sign. In case of τ_{12}, the sign of τ_{12} in the material axis does not have any bearing, Therefore, the sign of shear stress does not does not have any bearing in the material axis. Therefore, the shear stress in the material axis must be less than the corresponding shear strength. So, though it is a maximum stress failure criterion, it actually it consists of five different criteria corresponding to five different modes of failure viz. longitudinal tensile (LT) failure ($\sigma_1 < \left(\sigma_1^T\right)_u$), longitudinal compression (LC) failure ($-\left(\sigma_1^C\right)_u < \sigma_1$), transverse tensile (TT) failure ($\sigma_2 < \left(\sigma_2^T\right)_u$), transverse compression (TC) failure ($-\left(\sigma_2^C\right)_u < \sigma_2$) and in-plane shear (S) failure ($-\left(\tau_{12}\right)_u < \tau_{12} < \left(\tau_{12}\right)_u$). So, there is no interaction, each strength is compared independently and that is why it is called non-interactive theory or independent theory.

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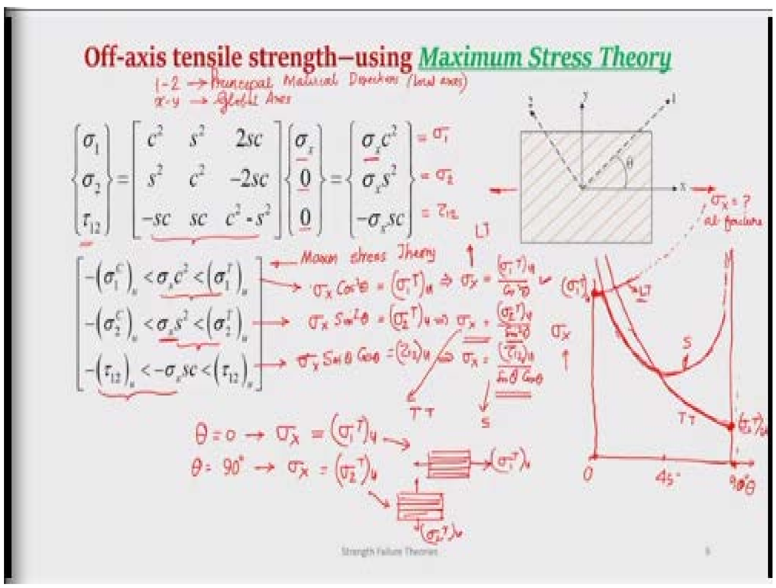

Now let us try to determine the off-axis tensile strength of a lamina using maximum stress theory. What is off-axis tensile strength? As shown in Fig., for a lamina (1-2 is the principal material direction and x-y is the global axis). If this lamina is subjected to uniaxial tensile stress σ_x, the maximum value of σ_x at failure is the off-axis tensile strength. It is called off-axis because it is along the x- axis, like the tensile strength along 1 that is $\left(\sigma_1^T\right)_u$?

Now the state of stress with reference to x-y is $\begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix}$. Therefore, with reference to the material axes the stresses are obtained using the transformation as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \sigma_x c^2 \\ \sigma_x s^2 \\ -\sigma_x sc \end{Bmatrix}$$

where c = cos θ and s = sin θ. Therefore, $\sigma_1 = \sigma_x \cos^2\theta$, $\sigma_2 = \sigma_x \sin^2\theta$ and $\tau_{12} = -\sigma_x \sin\theta\cos\theta$. Therefore, applying maximum stress theory, we have five conditions as

$$\begin{bmatrix} -\left(\sigma_1^C\right)_u < \sigma_x c^2 < \left(\sigma_1^T\right)_u \\ -\left(\sigma_2^C\right)_u < \sigma_x s^2 < \left(\sigma_2^T\right)_u \\ -\left(\tau_{12}\right)_u < -\sigma_x sc < \left(\tau_{12}\right)_u \end{bmatrix}$$

Now because σ_x is positive, so both $\sigma_1 = \sigma_x \cos^2\theta$ and $\sigma_2 = \sigma_x \sin^2\theta$ are positive, therefore we will have to check the following

$$\sigma_x c^2 < \left(\sigma_1^T\right)_u \Rightarrow \sigma_x < \frac{\left(\sigma_1^T\right)_u}{\cos^2\theta}$$
$$\sigma_x s^2 < \left(\sigma_2^T\right)_u \Rightarrow \sigma_x < \frac{\left(\sigma_2^T\right)_u}{\sin^2\theta}$$
$$-\left(\tau_{12}\right)_u < -\sigma_x sc < \left(\tau_{12}\right)_u \Rightarrow \sigma_x < \frac{\left(\tau_{12}\right)_u}{\sin\theta\cos\theta}$$

So, these are the three conditions which need to be satisfied. So, violating any one of these conditions will be failure that means it should be less. If we plot σ_x vs. θ following these three conditions the curves look like as shown in Fig. where the first curve corresponding to the first condition represents LT and the second curve corresponding to the second condition represents the LT and the third curve represents the S condition of failure. Now naturally what will be the value of the off-axis tensile strength at failure, σ_x will be decide by the value of θ.

this is the first condition we will have to check. That means $\sigma_x \cos^2\theta$ in the limit could be equal to $\left(\sigma_1^T\right)_u$ as long as is less than $\left(\sigma_1^T\right)_u$, it is safe. Again, this is also positive, therefore this condition this leads to $\sigma_x \sin^2\theta = \left(\sigma_2^T\right)_u$ and this is $-\sigma_x$, so this is the condition. Because it is negative, but there is no bearing on of sign of shear stress in the material axis. Therefore, this leads to $\sigma_x \sin\theta\cos\theta = \left(\tau_{12}\right)_u$.

Therefore, which implies that $\sigma_x = \dfrac{\left(\sigma_1^T\right)_u}{\cos^2\theta}$ or $\sigma_x = \dfrac{\left(\sigma_2^T\right)_u}{\sin^2\theta}$ or $\sigma_x = \dfrac{\left(\tau_{12}\right)_u}{\sin\theta\cos\theta}$. So, these are the three conditions which need to be satisfied. So, violating any one of these conditions will be failure that means it should be less I have written equal to that means it is the limiting case. Now naturally what will be the value of σ_x that is decide by the value of θ. Suppose we try to see what are the strengths versus θ, say this is θ, σ_x.

For the first one (LT), as θ increases, cosθ decreases and we know that, cos 90° = 0. Therefore at θ = 0°, $\sigma_x = \left(\sigma_1^T\right)_u$ and the curve actually goes to infinity at 90°. Now suppose we plot the second one (TT) at θ = 0°, sin θ = 0 therefore it is infinite and at θ= 90°, $\sigma_x = \left(\sigma_2^T\right)_u$.

For the last condition (S), for both θ = 0° and θ = 90° it is infinity and at θ = 45° this is minimum. So, taking the minimum of these three curves, the variation of σ_x vs. θ is as hown. Here, for θ near 0°, the failure is due to LT and as θ increases the failure is due to S and near θ=90°, the failure is due to TT.

Now if you can clearly see for a small value of θ near 0° the failure is because of longitudinal tensile and after that up to certain angle it is actually dominated by shear and as we move away from 45° towards 90° it is actually the transverse tensile. So, you can clearly see that even the off-axis strength of a lamina we can see that there are three different modes of failure depending upon the value of fiber orientation angle θ and θ = 0°, $\sigma_x = \left(\sigma_1^T\right)_u$, θ = 90°, $\sigma_x = \left(\sigma_2^T\right)_u$.

Now let us take a very quick example to understand that how we actually decide the failure of a lamina using maximum stress theory.

$\underline{Example}$: A 60° GR/E lamina subjected to $\boxed{\sigma_x = 4p, \sigma_y = -6p, \tau_{xy} = 8p}$;

Using maximum stress theory, determine max. value of $p > 0$, so that the lamina is safe.

Given,

$$\left(\sigma_1^T\right)_u = 1725 MPa, \left(\sigma_1^C\right)_u = 1350 MPa$$

$$\left(\sigma_2^T\right)_u = 40 MPa, \left(\sigma_2^C\right)_u = 275 MPa$$

$$\left(\tau_{12}\right)_u = 95 MPa$$

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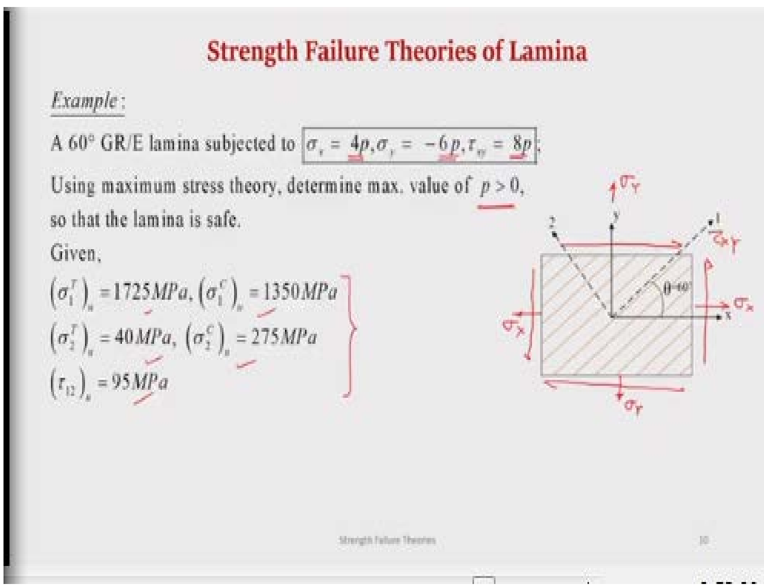

<u>**Solution:**</u>

Given the state of stress with reference to the x-y is $\begin{Bmatrix} 4p \\ -6p \\ 8p \end{Bmatrix}$, the material axes stresses are obtained using stress transformation for θ = 60° as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} 4p \\ -6p \\ 8p \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 3.4 \\ -5.4 \\ -8.3 \end{Bmatrix} p}$$

Now the conditions for safety are

$$\text{Condition for safety}: \begin{pmatrix} -1350\times10^6 < \sigma_1 < 1725\times10^6 \\ -275\times10^6 < \sigma_2 < 40\times10^6 \\ -95\times10^6 < \tau_{12} < 95\times10^6 \end{pmatrix}$$

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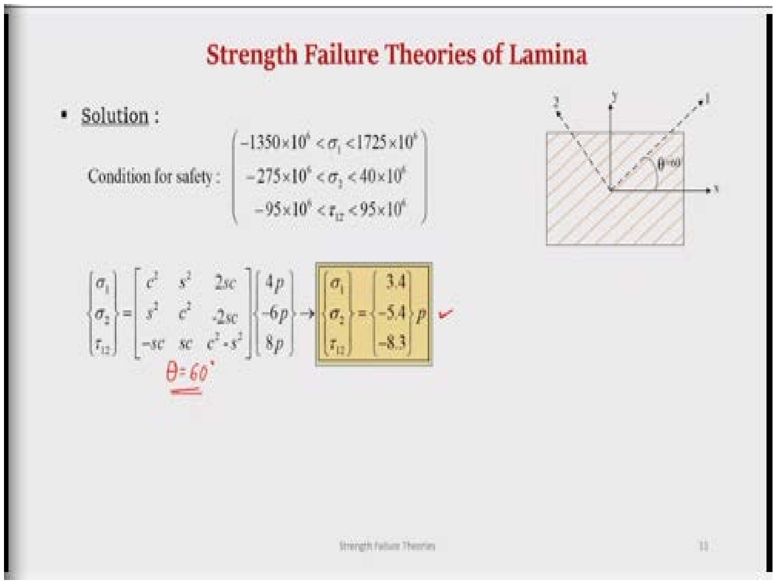

And putting the values of material axes stresses and taking care of the sign,

$$\begin{pmatrix} -1350\times10^6 < 3.4p < 1725\times10^6 \\ -275\times10^6 < -5.4p < 40\times10^6 \\ -95\times10^6 < -8.3p < 95\times10^6 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -397\times10^6 < p < 397\times10^6 \\ -7.8\times10^6 < p < 50.9\times10^6 \\ -11.4\times10^6 < p < 11.4\times10^6 \end{pmatrix}$$

And since $p > 0$, therefore the maximum value of p is

$$\mapsto \boxed{0 < p < 11.4\times10^6}$$

Therefore, as soon as $p > 11.4 \times 10^6$, the third condition is violated, then the lamina will fail. Using maximum stress theory, we could find out the maximum value of p, and we can also see the condition which is first violated the third condition which is actually shear. Therefore, we can say that the lamina will actually fail due to shear.

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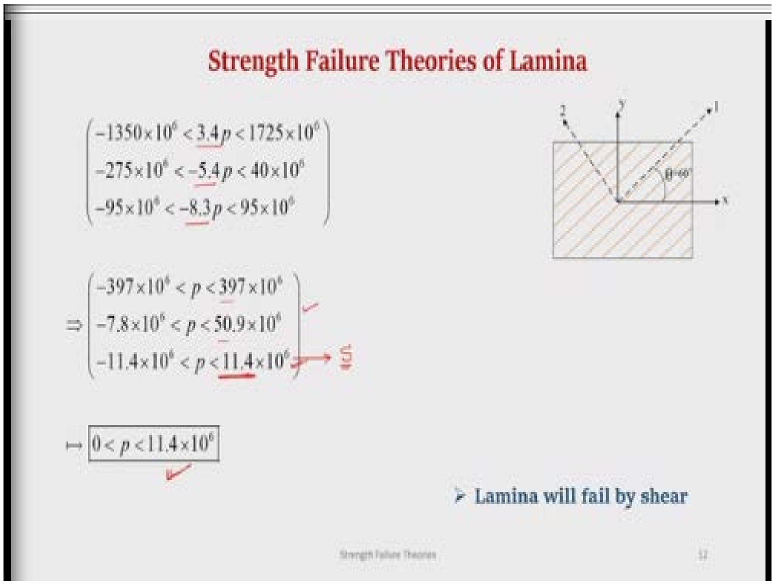

Next, let us try to determine the off-axis shear strength of a 60° glass epoxy lamina. The off-axis shear strength here means that when the 60° glass epoxy lamina subjected to pure shear τ_{xy}, what is the maximum shear stress τ_{xy} it can withstand before it fails.

The state of stress, only $\tau_{xy} \neq 0$, $\sigma_x = 0$, $\sigma_y = 0$. Therefore, from this state of stress with respect to x-y, the stresses in the material axis using this stress transformation matrix (θ = 60°) could be obtained as.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} 0.25 & 0.75 & 0.866 \\ 0.75 & 0.25 & -0.866 \\ -0.433 & 0.433 & -0.50 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \tau_{xy} \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 0.866 \\ -0.866 \\ -0.5 \end{Bmatrix} \tau_{xy}}$$

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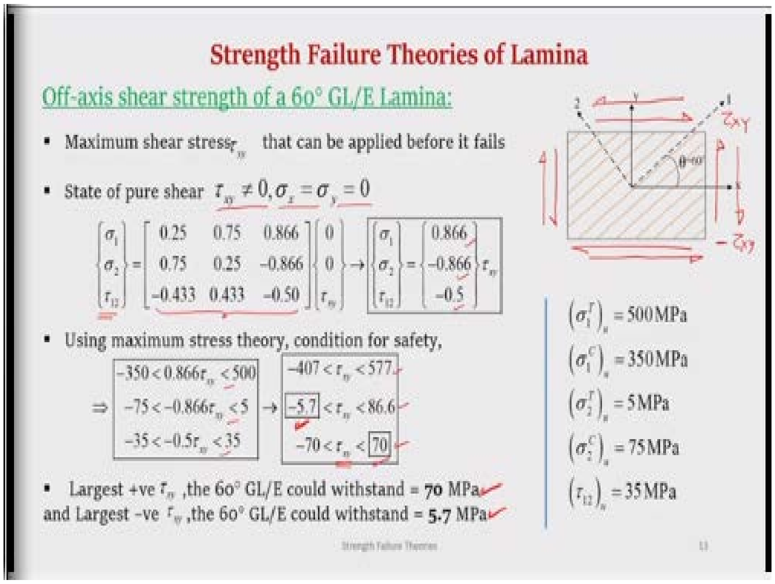

Again, we put these in the maximum stress theory the condition for safety are

$$\boxed{\begin{array}{c} -350 < 0.866\tau_{xy} < 500 \\ -75 < -0.866\tau_{xy} < 5 \\ -35 < -0.5\tau_{xy} < 35 \end{array}}$$

None of this condition could be violated. If any one of this condition is violated then it fails. Now rearranging this we get this; these are the conditions.

$$\boxed{\begin{array}{c} -407 < \tau_{xy} < 577 \\ \boxed{-5.7} < \tau_{xy} < 86.6 \\ -70 < \tau_{xy} < \boxed{70} \end{array}}$$

If τ_{xy} is positive, the maximum value of τ_{xy} = 70 MPa. But suppose τ_{xy} is negative, the maximum value of τ_{xy} = −5.7 MPa.

So, here we get two conditions, first, if τ_{xy} is positive the maximum value it can have is 70 MPa.

On the other hand, if τ_{xy} is negative the maximum value it can have is 5.7 MPa.

Now this positive and negative is as shown in the Fig. So, if we try to find out what is the off-axis shear strength of an angle lamina, we need to specify with reference to the sign of the shear stress, if it is positive in this case, it is 70 MPa, if it is negative, it is 5.7 MPa. Therefore, the sign of off-axis shear stress is very very important.

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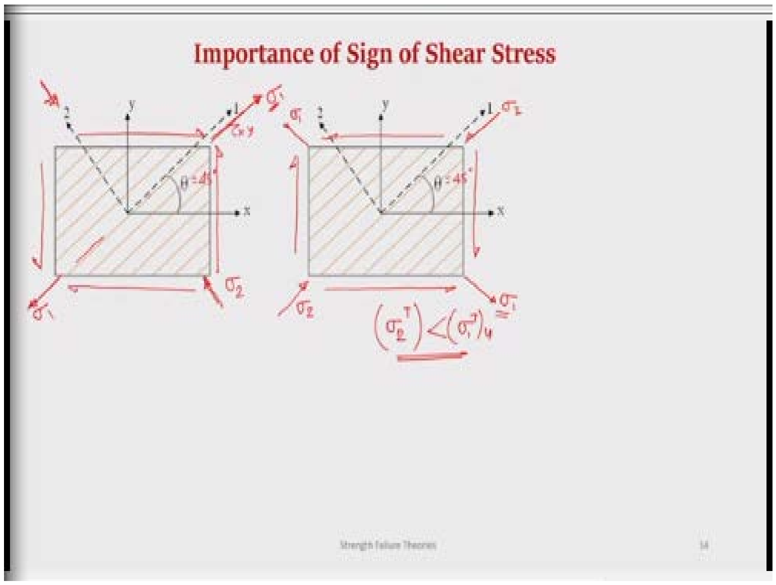

Now, why it happens could be understood from the Fig. above. In case of a 45° lamina subjected to positive shear stress τ_{xy}. This a pure shear it is equivalent to equal and opposite principal stresses in a plane 45°. So, now if this lamina is subjected to τ_{xy}, it translates to that it has a tensile stress along direction 1 and compression stress along direction 2.

But suppose this shear stress is in the opposite direction, in that case this translates to a negative stress along 1 and appositive stress along 2. The transverse tensile strength is much less compared to transverse compression as well as longitudinal tension. Therefore, in the second case of negative shear, the failure will be because of the transverse tensile and hence the strength is less.

So, for a 60° lamina, in the first case, along the fiber the stress is tensile and transverse to fiber the stress is compressive therefore it can withstand a very high magnitude of stress, compared to the second case of negative shear where the tensile stress is transverse to the fiber where it is very, very weak. Therefore, in this case the off-axis shear strength is much lower compared to the off-axis shear strength when the sign is changed.