Mechanics of Fiber Reinforced Polymer Composite Structures

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Module - 3
Macromechanics of Lamina - I
Lecture - 06
Engineering Constants for 2D Lamina

Hello and welcome to the second lecture of this third module, where we have been discussing Macromechanics of lamina.

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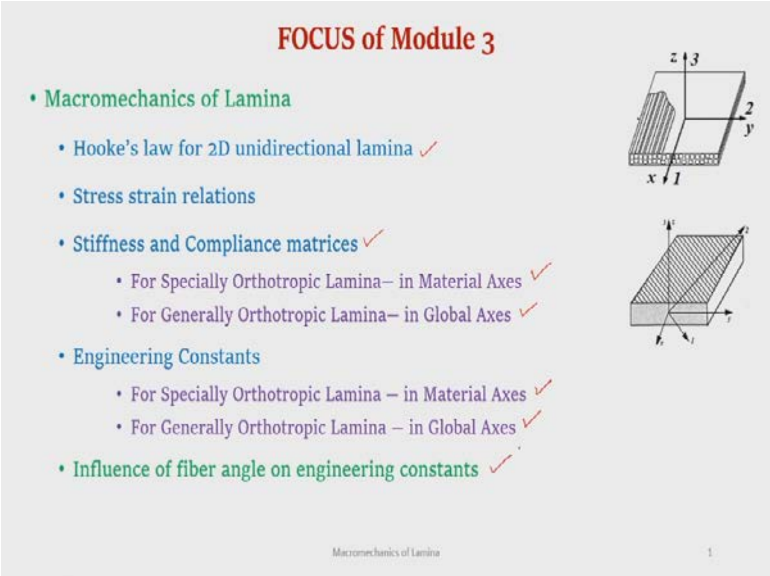

The objectives of this module 3 has been first to understand the stress-strain relationship for a 2-dimensional unidirectional lamina and then to develop the stiffness and compliance matrices for specially orthotropic lamina (where the material axes 1-2-3 coincide with the analysis axes X-Y-Z) as well as for generally orthotropic lamina (where material axes 1-2-3 do not coincide with the analysis axes X-Y-Z). Finally to develop the relationship between the elements of stiffness and compliance matrices and the engineering constants for specially orthotropic lamina as well as for generally orthotropic lamina.

In our last lecture, we discussed Hooke's law and we understood how 3-dimensional generalised Hooke's law could actually be reduced for a 2-dimensional unidirectional lamina. Then the stiffness and compliance matrix both for specially orthotropic lamina as well as for generally orthotropic lamina were developed. Engineering constants for specially orthotropic lamina were also discussed. So, in this lecture the engineering constants for generally orthotropic lamina will be discussed along with the influence of fiber angle on engineering constants.

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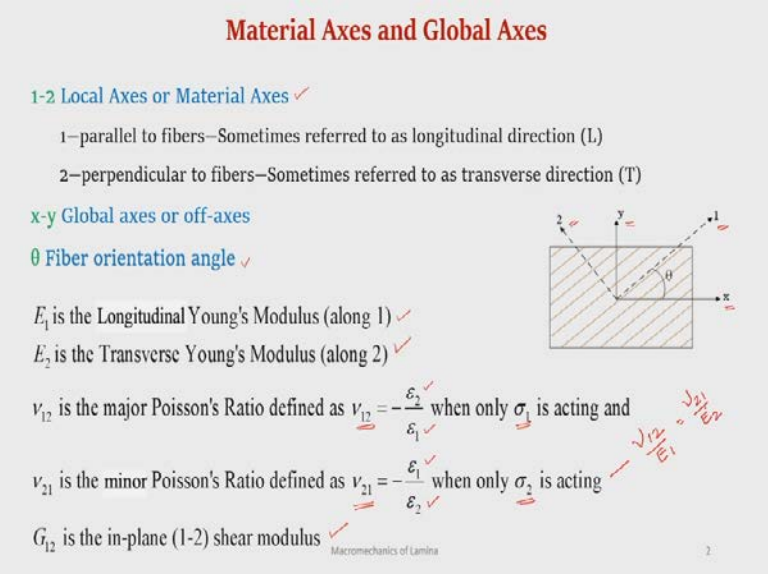

So, just a quick revisit that we understood that there are, suppose parallel to the fiber direction is 1, which is also called the longitudinal direction, and perpendicular to the fiber direction is actually 2, is also known as transverse direction; so, this 1-2 is known as local axes or material axes. And x-y which is actually the global axes; or, the axis which is actually convenient for our analysis is termed as global axes; and the angle the material axes makes with the global axes, say the angle between x and 1, θ is the fiber orientation angle.

Say, this shows a typical lamina where the fibers are oriented at an angle of θ with the x-axis. Now, the Young's modulus; we understood in the last class that we can actually determine the engineering constants, like Young's modulus along direction 1 is known as longitudinal Young's modulus; similarly, Young's modulus along direction 2 is the transverse Young's modulus.

Then, the Poisson's ratio ν_{12} which is nothing but the ratio of the strain along direction 2 to the strain along direction 1 when the stress is applied along direction 1 is the major Poisson's ratio ν_{12}. Similarly, we can define the minor Poisson's ratio ν_{21}. That means, when a stress is applied along direction 2 and the ratio of the strain along direction 1 to the strain along direction 2 is ν_{21}.

And then, we have the G_{12} as the shear modulus in plane 1-2. So, these are the engineering constants, E_1, E_2, ν_{12} and G_{12} are the engineering constants for a lamina with respect to the material axes; of course, ν_{12} and ν_{21} related; therefore, there are actually, ν_{12} by E_1 is equal to;

we have seen that in our last class; therefore, there are actually 4 independent engineering constants, E_1, E_2, ν_{12} and G_{12}, for a lamina in the material axis.

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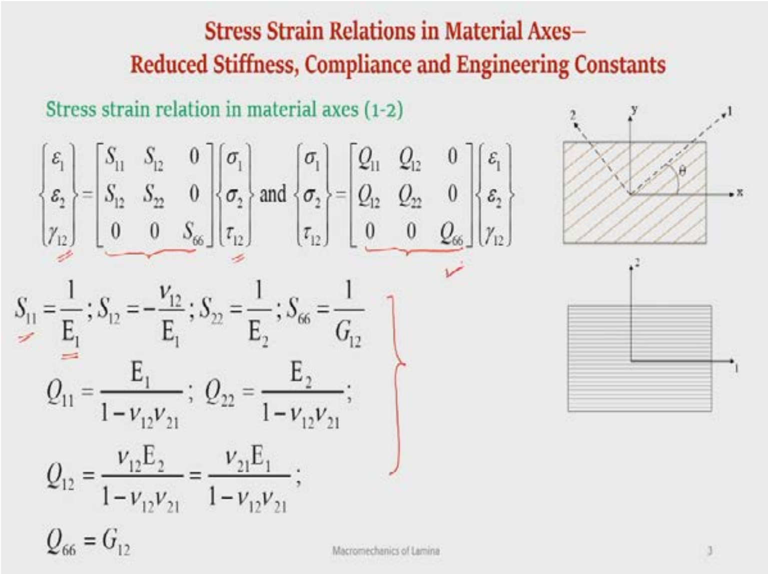

Then, we have developed the stress-strain relationship wherein suppose the stresses in the material axes are related to the corresponding strains and this is the compliance matrix and this is the reduced stiffness matrix; I think we have discussed these things in details in the last class. And then, we could relate the engineering constants in terms of, we could write engineering constant in terms of the elements of the compliance matrix and stiffness matrix like this; that we understood.

That means, here, when we say the stress-strain relationship in material axes, that means, stresses are with respect to the axes 1-2 and the corresponding strains are also with reference to 1-2. And this is how the stress-strains are related.

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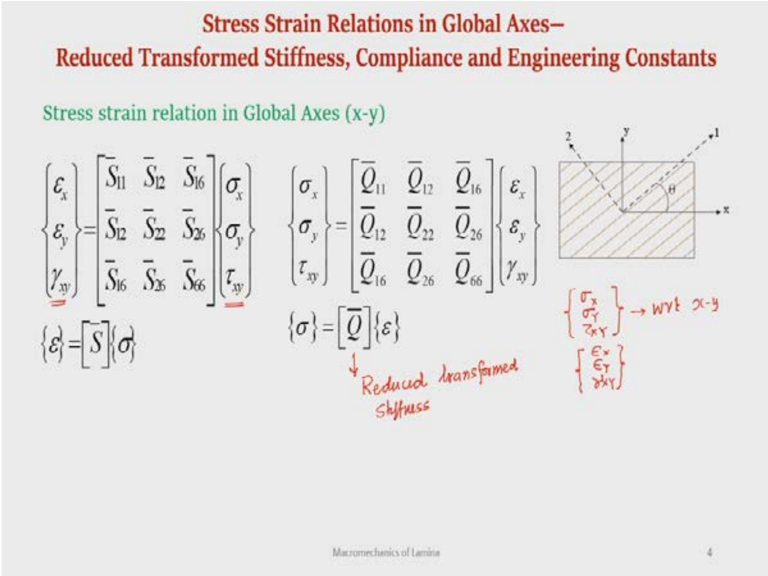

Then, we have also used our knowledge of stress and strain transformation and we could actually obtain the relations between the stresses and strains in the global axes. That means, the stresses and strains with respect to x-y. This stress is with respect to the Cartesian coordinate axes x-y; and similarly, the corresponding strains, ε_x, ε_y, γ_{xy}. And we have obtained the relationship; how did we do so?

Because we have already obtained, we have the relationship between the stresses and strains in material axes. By using the knowledge of stress and strain transformation, we could actually obtain this, and where this $[\bar{Q}]$ is called reduced transformed stiffness; [Q] was reduced stiffness and this is transformed reduced stiffness.

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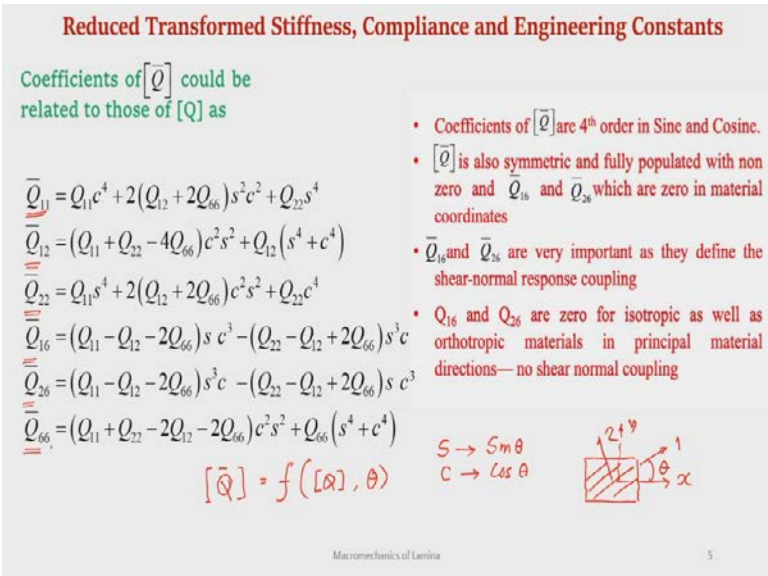

And we have also seen that this $[\underline{Q}]$, that means, the reduced transformed stiffness could actually be expressed, the elements of this $[\underline{Q}]$ matrix could actually be expressed in terms of the elements of [Q] matrix, that means the reduced stiffness matrix and sinθ and cosθ. So, we have seen here that this $[\underline{Q}]$, elements of $[\underline{Q}]$ is actually function of elements of [Q] and θ.

Here, actually s stands for sinθ and c is cosθ. Just for reference, this is 1-2 axes of a lamina and this is the global axis, and this is what is θ. So, we have also established the relationship between the elements of the reduced transformed stiffness matrix to the elements of the stiffness matrix as a function of θ. And we understood that even though there are 6 elements in the reduced transformed stiffness matrix, but they are actually functions of the 4 independent constants, Q 11, Q 12, Q 22, Q 66. They are actually functions of these 4 independent constants and θ.

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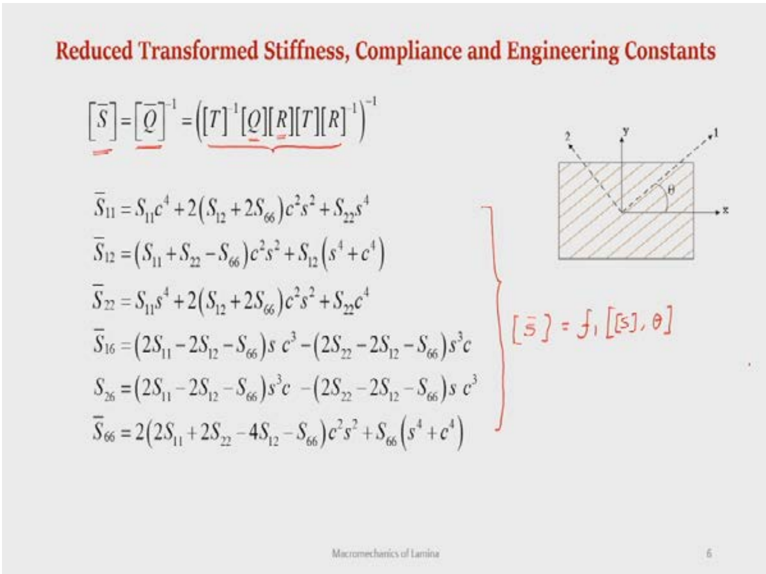

Similarly, since we have the expressions for $[\underline{Q}]$, and if you remember that $[\underline{Q}]$ was actually using the stress and strain transformation; actually, $[\underline{Q}]$ was this, where T is nothing but the stress transformation matrix and [Q] is the reduced stiffness, [R] is just a matrix containing 0, 1 and $\frac{1}{2}$. And therefore, we can take inverse of this and that will be nothing but $[\underline{S}]$ matrix.

So, therefore, we could also obtain the relationship between $[\underline{S}]$, elements of $[\underline{S}]$ as a function of elements of S and θ, similar to that of $[\underline{Q}]$. So, here also there are 6 elements, $\underline{S}_{11}$, $\underline{S}_{12}$, $\underline{S}_{22}$, $\underline{S}_{16}$, $\underline{S}_{26}$, $\underline{S}_{66}$, but actually they are functions of 4 independent constants, S_{11}, S_{12}, S_{22}, S_{66} and θ.

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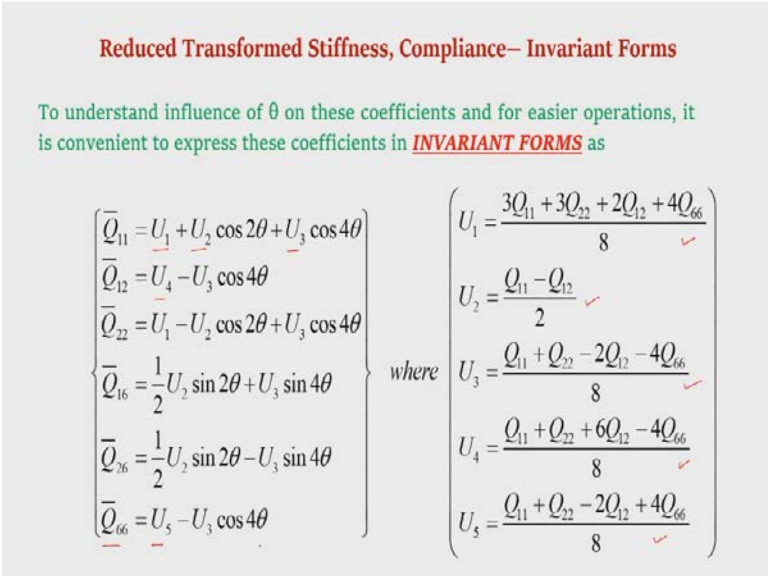

So, now, this reduced transformed stiffness could actually be expressed as in the invariant forms, like we have seen how $\underline{Q}_{11}$ is actually expressed in terms of the 4 independent constants Q_{11}, Q_{12}, Q_{22} and Q_{66}. So, this could be actually written in terms of, in invariant forms. This is the invariant form, where this U_1, U_2, U_3, U_4 and U_5 are actually invariants. When we say invariants, what does it mean?

That means, these are independent of θ. You can see that these are actually the elements of the reduced stiffness matrix in the material axes; therefore, they are independent of θ. Now, expressing this in such invariant form many a times is advantages, because many a times, actually in laminate analysis, we need to perform integration of the stiffness terms. Therefore, if we write in terms of these invariants, it becomes simplified, the analysis becomes simplified. Also, this gives us at first hand idea of how; I mean, we can clearly see how these elements of this reduced transformed stiffness matrix actually vary with θ. We can see; suppose, we want to find out at what angle this $\underline{Q}_{66}$ will be maximum? We can actually find out by taking first derivative with respect to θ and putting it to 0, we can find out for what θ. Similarly, we can see how these variations of this reduced transformed stiffness matrix with θ could be clearly understood by expressing this in the invariant forms.

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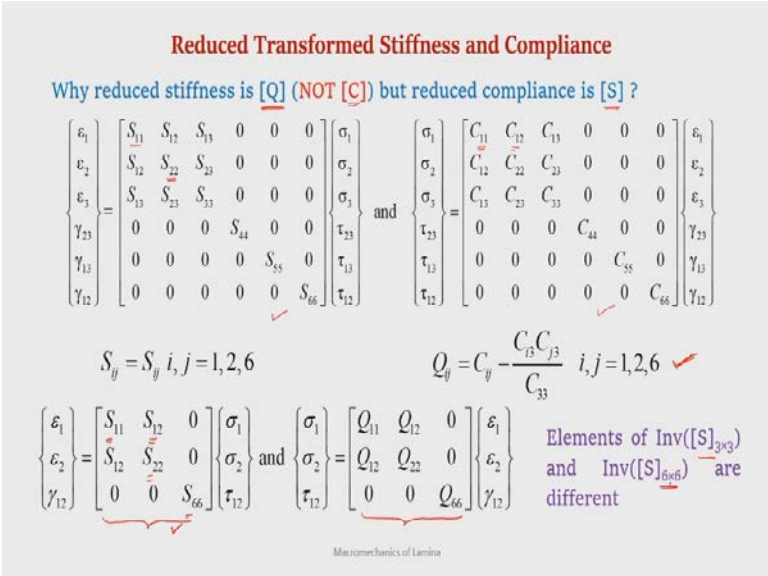

Now, just before we actually go to the engineering constants for a generally orthotropic lamina, just a note that many a times this question comes, the reduced stiffness matrix is actually, or reduced transformed stiffness matrix is actually denoted by Q, not by C; whereas the compliance is S. If you remember, for an orthotropic or in general, for 3-dimensional, the compliance matrix is always denoted by S and the stiffness is by C.

So, when we actually reduce this for a 2-dimensional lamina, the [S] remains still same, we still keep this [S], but this stiffness is made [Q]. The reason is that, actually, these S_{ij} terms, S_{11}, S_{12}, S_{22}, S_{66} could be straightaway taken from this matrix without any change. But the stiffness terms; if you remember how we have actually obtained this [Q], elements of this [Q] matrix, they cannot be taken from this. In fact, they are related like this.

The elements of stiffness matrix are related to the elements of this [C] matrix like this, by this formula. Therefore, it is actually divided by [Q], not by [C], and the reason is that, if you take the inverse of this 3×3 [S] matrix, it will not be same as the inverse of; the elements will get changed when we take the inverse of 6×6 [S] matrix; therefore, they are not same. So, this is just a; I mean, why the reduced stiffness matrix or reduced transformed stiffness matrix is actually denoted by [Q], not by [C], whereas, the compliance matrix is always denoted by [S].

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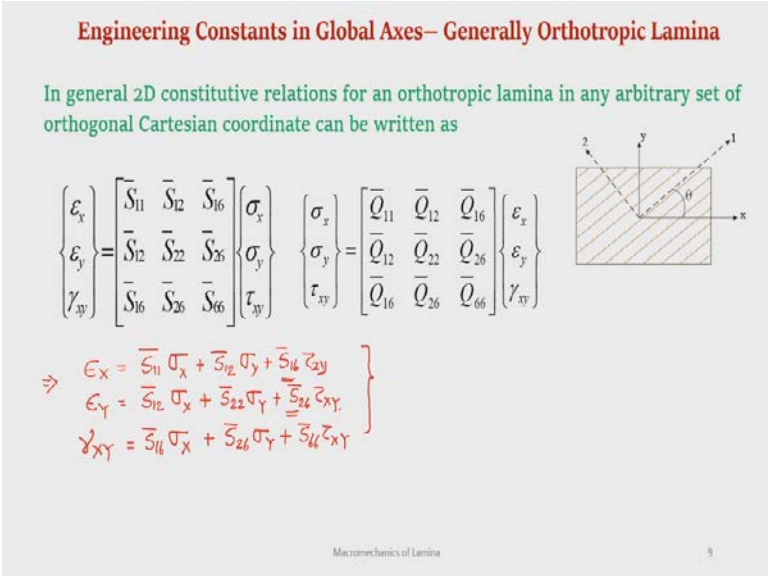

Now, having understood this stress-strain relationship using reduced transformed stiffness and transformed compliance matrix, let us try to understand the the engineering constants with reference to x-y axes for a generally orthotropic angle lamina.

First, in case 1, we apply only σ_x and if we apply this stress-strain relationship, this leads to a normal strain along x, this ε_x, a normal strain along y, ε_y.

$\underline{Case-I}: \ \sigma_x \neq 0 \quad \sigma_y = 0 \quad \tau_{xy} = 0$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} \rightarrow \boxed{\begin{aligned} \varepsilon_x &= \bar{S}_{11}\sigma_x \\ \varepsilon_y &= \bar{S}_{12}\sigma_x \\ \gamma_{xy} &= \bar{S}_{16}\sigma_x \end{aligned}}$$

$$\mathrm{E}_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\bar{S}_{11}} \qquad \nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\bar{S}_{12}}{\bar{S}_{11}} \qquad \bar{S}_{12} = -\frac{\nu_{xy}}{E_x}$$

$$\eta_{xy,x} = \frac{\gamma_{xy}}{\varepsilon_x} = \frac{\bar{S}_{16}\sigma_x}{\bar{S}_{11}\sigma_x} = \frac{\bar{S}_{16}}{\bar{S}_{11}} \qquad \bar{S}_{16} = \frac{\eta_{xy,x}}{E_x}$$

As could be seen that in addition to the normal strains, it also leads to a shear strain γ_{xy}, (because of non-zero S_{16} and S_{26} terms). Therefore, going by definition,

$$\mathrm{E}_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\bar{S}_{11}}$$

$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\bar{S}_{12}}{\bar{S}_{11}}$$

$$\bar{S}_{12} = -\frac{\nu_{xy}}{E_x}$$

Now, because a normal stress causes a shear strain, that must also be characterized. This is called shear coupling coefficient, denoted as $\eta_{xy,x}$ which decides that if we apply a normal stress along x, what will be the shear strain along x-y. This is as follows:

$$\eta_{xy,x} = \frac{\gamma_{xy}}{\varepsilon_x} = \frac{\bar{S}_{16}\sigma_x}{\bar{S}_{11}\sigma_x} = \frac{\bar{S}_{16}}{\bar{S}_{11}}$$

$$\bar{S}_{16} = \frac{\eta_{xy,x}}{E_x}$$

So, in general, the shear coupling coefficient is $\eta_{ij,i} = \frac{\gamma_{ij}}{\varepsilon_i}$. Where, i,j could be 1, 2 and 6. Thus we have established the Young's modulus along x, Poisson's ratio ν_{xy} and in addition, we have also obtained the shear coupling coefficient in the x-y plane, $\eta_{xy,x}$.

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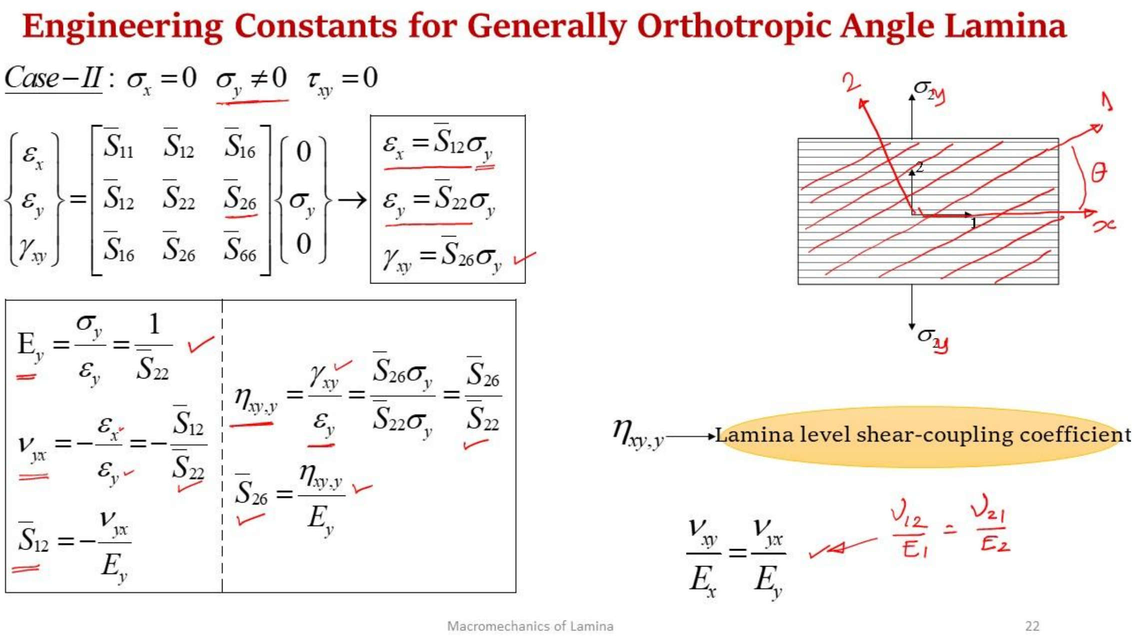

Next, in case 2, we apply only σ_y

$\underline{Case-II}$: $\sigma_x = 0 \quad \sigma_y \neq 0 \quad \tau_{xy} = 0$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ \sigma_y \\ 0 \end{Bmatrix} \rightarrow \boxed{\begin{aligned} \varepsilon_x &= \bar{S}_{12}\sigma_y \\ \varepsilon_y &= \bar{S}_{22}\sigma_y \\ \gamma_{xy} &= \bar{S}_{26}\sigma_y \end{aligned}}$$

So, going by definition, again Young's modulus along y

$$E_y = \frac{\sigma_y}{\varepsilon_y} = \frac{1}{\bar{S}_{22}}$$

$$\nu_{yx} = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{\bar{S}_{12}}{\bar{S}_{22}}$$

$$\bar{S}_{12} = -\frac{\nu_{yx}}{E_y}$$

Again, we have the shear coupling, now it is $\eta_{xy,y}$ defined as

$$\eta_{xy,y} = \frac{\gamma_{xy}}{\varepsilon_y} = \frac{\bar{S}_{26}\sigma_y}{\bar{S}_{22}\sigma_y} = \frac{\bar{S}_{26}}{\bar{S}_{22}}$$

$$\bar{S}_{26} = \frac{\eta_{xy,y}}{E_y}$$

We could also see that $\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}$ is also true for generally orthotropic lamina with reference to x-y.

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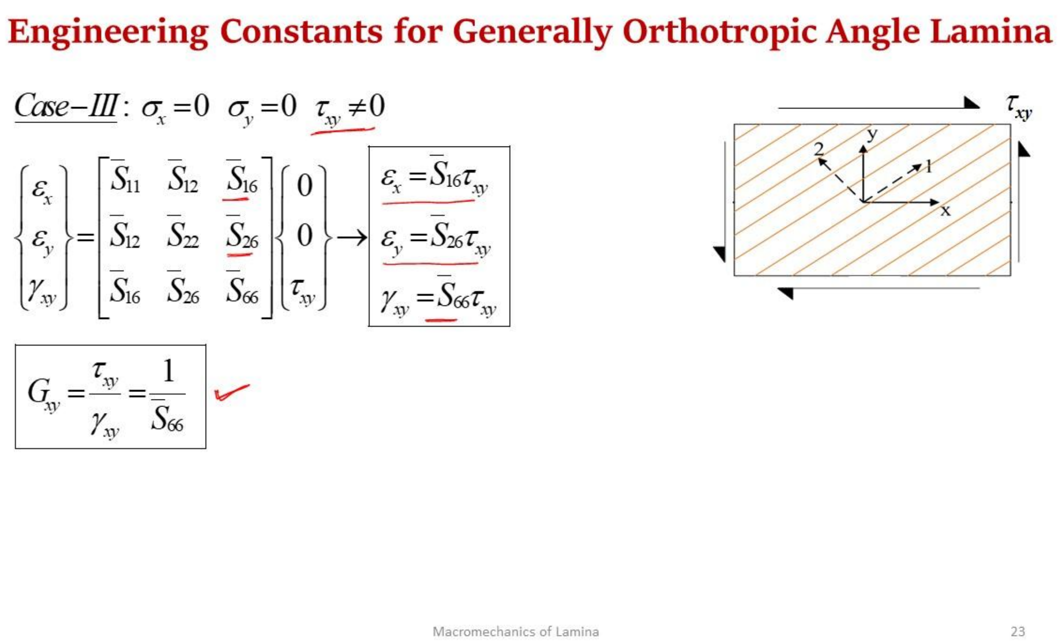

And in case 3, we apply a pure shear τ xy and naturally because the shear coupling coefficient is there, therefore, it leads to a normal strain along x, normal strain along y in addition to the direct shear strain, because of the shear stress. Going by the definition of shear modulus we get

$\underline{Case - III}$: $\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} \neq 0$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \tau_{xy} \end{Bmatrix} \rightarrow \boxed{\begin{matrix} \varepsilon_x = \bar{S}_{16}\tau_{xy} \\ \varepsilon_y = \bar{S}_{26}\tau_{xy} \\ \gamma_{xy} = \bar{S}_{66}\tau_{xy} \end{matrix}}$$

$$\boxed{G_{xy} = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{1}{\bar{S}_{66}}}$$

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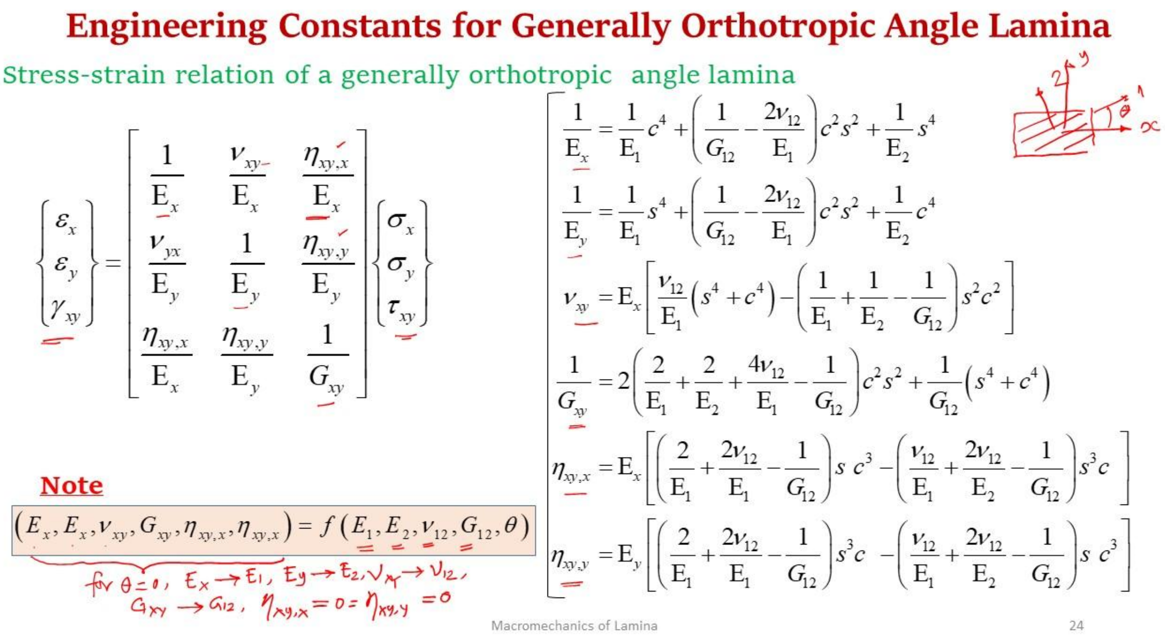

So, in general, the stress-strain relationship in terms of engineering constants for a generally orthotropic angle lamina (where 1-2 do not coincide with x-y are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & \frac{\nu_{xy}}{E_x} & \frac{\eta_{xy,x}}{E_x} \\ \frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & \frac{\eta_{xy,y}}{E_y} \\ \frac{\eta_{xy,x}}{E_x} & \frac{\eta_{xy,y}}{E_y} & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

Again E_x, E_y, ν_{xy}, G_{xy} and $\eta_{xy,x}$ and $\eta_{xy,y}$ could actually be related to E_x, E_y, ν_{xy}, G_{xy} and θ as

$$\frac{1}{E_x} = \frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)c^2 s^2 + \frac{1}{E_2}s^4$$

$$\frac{1}{E_y} = \frac{1}{E_1}s^4 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)c^2 s^2 + \frac{1}{E_2}c^4$$

$$\nu_{xy} = E_x\left[\frac{\nu_{12}}{E_1}\left(s^4 + c^4\right) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)s^2 c^2\right]$$

$$\frac{1}{G_{xy}} = 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)c^2 s^2 + \frac{1}{G_{12}}\left(s^4 + c^4\right)$$

$$\eta_{xy,x} = E_x\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s\,c^3 - \left(\frac{\nu_{12}}{E_1} + \frac{2\nu_{12}}{E_2} - \frac{1}{G_{12}}\right)s^3 c\right]$$

$$\eta_{xy,y} = E_y\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^3 c - \left(\frac{\nu_{12}}{E_1} + \frac{2\nu_{12}}{E_2} - \frac{1}{G_{12}}\right)s\,c^3\right]$$

So, even though there are 6 engineering constants, but actually these 6 engineering constants are nothing but functions of the 4 independent engineering constants and θ. Therefore, even though there is a shear coefficient, it is still orthotropic, but the axes of orthotropy do not coincide with the direction of loading. If we apply load along the direction of orthotropy, it will still show no shear-extension coupling.

So, what we have learnt in this lecture is that, we have obtained the stress-strain relationship for lamina, both with reference to the material axes 1-2 as well as with reference to the analysis axes x-y which may not coincide with the material axes.

We understood that, 4 independent engineering elastic constants are required to characterise a lamina and those could be related to the measurable engineering constants like E_1, E_2, ν_{12}, G_{12}.

While the stress-strain relationship for a generally orthotropic lamina, (where the analysis axes or loading axes do not coincide with the material axes), there are 6 constants in the compliance and reduced transformed stiffness matrix. However, those 6 constants are actually functions of the 4 independent elastic constants for a specially orthotropic lamina and θ.

Now, if you put θ=0, we will get back the same; like, if we put theta is equal to 0, this E_x, E_y, ν_{xy}, G_{xy} and $\eta_{xy,x}$ and $\eta_{xy,y}$ will be nothing but E_1, E_2, ν_{12}, G_{12} and 0. So, basically, for an orthotropic lamina, we need 4 independent elastic constants and that there are 4 engineering constants.

Now, for a given E_1, E_2, ν_{12}, G_{12}, for a general lamina, these engineering constants (in x-y) vary with θ in the global axes. It is important to understand say for example what will be the value of E_x at a particular θ. We have discussed that if θ = 0, then $E_x = E_1$ and when θ = 90˚, $E_x = E_2$ and when θ varies between 0˚ to 90˚, there will be different values of E_x.

So, these engineering constants vary with θ, and how they will vary, of course, decided by the values of E_1, E_2, G_{12}, ν_{12}. Therefore, the nature of variation depends on the values of the engineering constants in material axes. Now, in order to understand the variation of these engineering constants with θ, they could actually be plotted with θ to see how they actually vary. First let us see how E_x varies.

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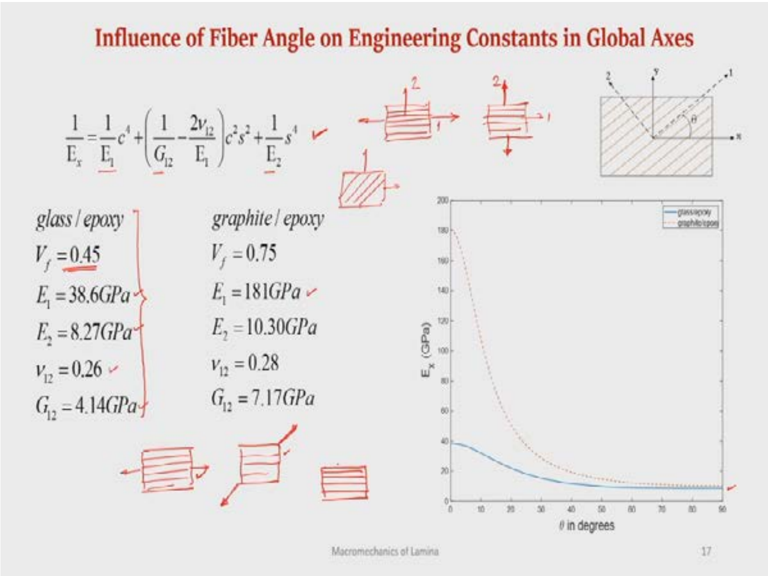

Using the expression of E_x in terms of E_1, E_2, G_{12}, ν_{12} and θ, E_x has been plotted for different values of θ between 0˚ to 90˚ , for two kinds of lamina viz. glass epoxy for a volume fraction of 45% and a graphite epoxy for a volume fraction of 75% as shown in Fig.

It could be clearly seen from these figures that at θ = 0˚, the E_x is nothing but E_1. At θ = 90˚, this E_x is nothing but E_2. Now, naturally, E_x is maximum when θ is equal to 0˚. It is understood, because, if we take a lamina and load it in the longitudinal direction, the load is actually borne by the fibers which are very stiff in the longitudinal direction. Therefore, the stiffness is maximum. On the other hand, if we load it in the transverse direction, the fibers do not carry the load, it is actually the matrix, therefore, it is minimum. So, E_2 is much less compared to E_1 and between θ = 0˚ and θ = 90˚ the value of E_x varies between E_1 is and E_2.

This is due to the fact that for any other angle say for θ = 45˚ maybe the fibers take a part of the load and not the full and the matrix takes a part of it and therefore, it is in between.

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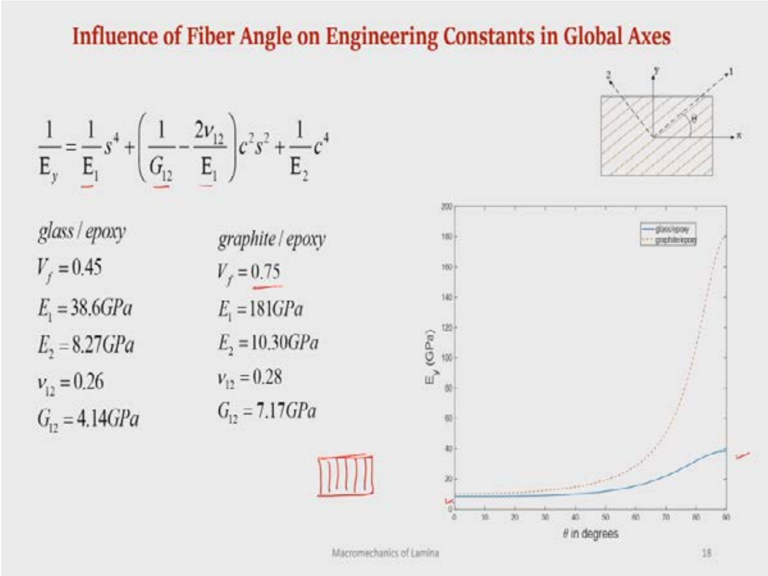

Similarly, we can see the variation of E_y, at θ = 0˚, $E_y = E_2$, at θ = 90˚, $E_y = E_1$; and in between, it varies. Figure shows the variation of E_y for glass epoxy and graphite epoxy.

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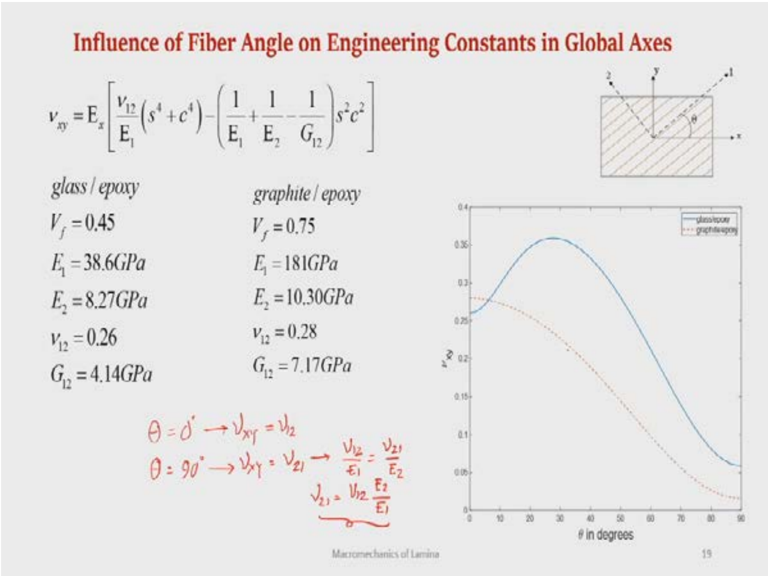

Similarly, using the expression for ν_{xy} in terms of E_1, E_2, G_{12}, ν_{12} and θ, the variation of ν_{xy}, the Poisson's ratio (in x-y) with θ could be plotted as shown. It could be clearly seen that θ = 0°, $\nu_{xy} = \nu_{12}$ and at θ = 90˚, $\nu_{xy} = \nu_{21}$. We could check this using the relation $\nu_{12} / E_1 = \nu_{21} / E_2$; therefore, you can find out ν_{21} is equal to ν_{12} E_2 by E_1. As shown in the Fig., the variation of ν_{12} with θ is more pronounced for graphite epoxy and it is maximum at around 35°, whereas for glass epoxy, it is monotonically decreasing, the nature of variation is decided by the values of E_1, E_2, ν_{12} and G_{12}.

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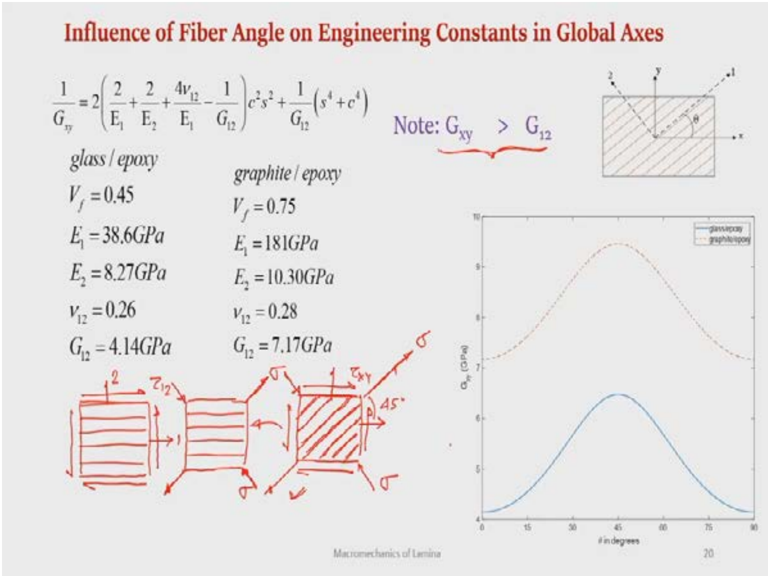

In the same way, using the expression for G_{xy} in terms of E_1, E_2, G_{12}, ν_{12} and θ, the same trend for both glass epoxy and graphite epoxy has been obtained. Here also, it could be clearly seen that $\theta = 0^o$, $G_{xy}= G_{12}$ and at $\theta = 90^\circ$, $G_{xy}= G_{21}$ and it varies at intermediate angles between 0^o and 90°. In both the cases of glass epoxy and graphite epoxy, G_{xy} is maximum at 45°. G_{xy} being the shear modulus in the x-y plane, it actually quantifies the resistance to shear. Now, it is important to note that G_{xy} is always greater than G_{12} and is maximum at 45°. This could be explained as follows. Suppose in a lamina, subjected to pure shear τ_{xy} in x-y which is equivalent to equal and opposite normal stresses at an angle of 45°. Therefore in a 45°lamina, the fibers along the 45°direction carries the normal stress and hence provides maximum resistance to shear. On the other hand, in 0° lamina subjected to pure shear τ_{12} the normal stress along the 45° direction is actually resisted by the matrix and hence shear resistance is minimum That is why G_{xy} is maximum at 45°.Therefore, G_{xy} is greater than G_{12}; this is the reason.

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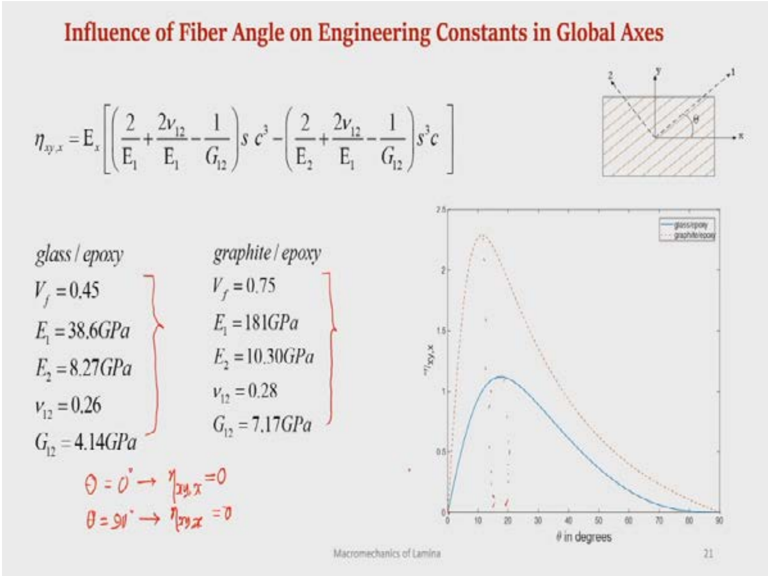

Similarly, using the expression for $\eta_{xy,x}$ in terms of E_1, E_2, G_{12}, ν_{12} and θ, we could plot the variation of shear coupling coefficient with. It is clear that that at $\theta = 0^\circ$, $\eta_{xy,x} = 0$ and at $\theta = 90^\circ$ also, $\eta_{xy,x} = 0$. This is because there is no shear-extension coupling in the material axes ie. at $\theta = 0^\circ$ and $\theta = 90^\circ$. But in between, for glass epoxy, it is maximum somewhere maybe around 20˚, again, the nature of the trend will be decided by what are the values of the E_1, E_2, G_{12}, ν_{12}. In the present case the variation is plotted for glass epoxy and graphite epoxy.

So, having understood these variations, we can clearly see that in case of engineering constants in global axes, in addition to E_x, E_y, G_{xy} and ν_{xy}, there is also a shear-extension coupling coefficient $\eta_{xy,x}$ which needs to be defined because there is shear-extension coupling present. This is because of the existence of non-zero $\underline{S}_{12}$ and $\underline{S}_{26}$ terms. However, these S_{16} and S_{26} and similarly, Q_{16} and Q_{26} terms are always 0 in the material axes when we define the stress-strain relationship in material axes.

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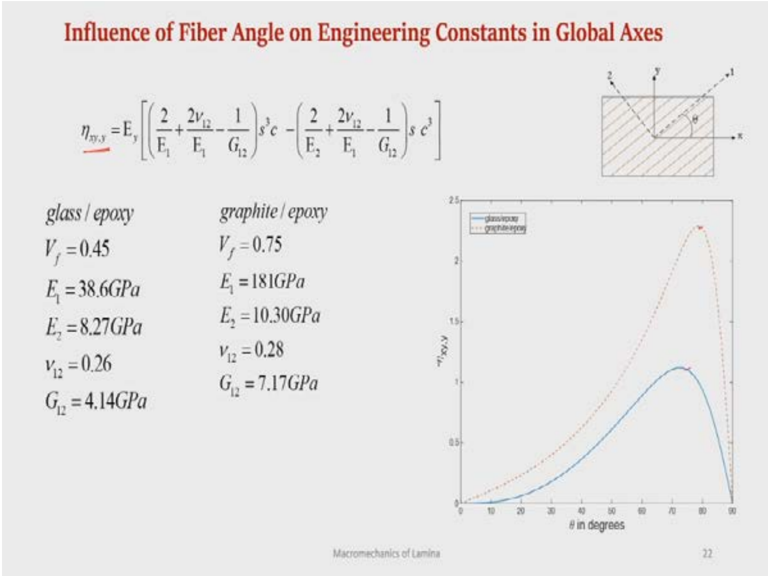

Similarly, we could plot the variation of $\eta_{xy,y}$ with θ using the expression in terms of E_1, E_2, G_{12} and ν_{12}. Again, for be 0 at θ = 0˚ and θ = 90˚ the value is zero and it is maximum value and the corresponding at θ at which it is maximum depends upon the values of E_1, E_2, G_{12} and ν_{12}.

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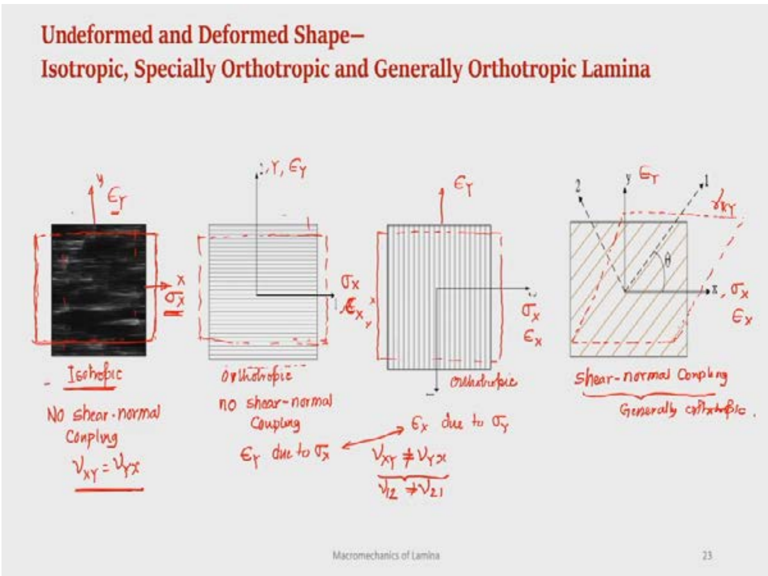

Now, to understand the implication of these engineering constants we consider three different specimen of identical dimensions as shown in Fig. First one is made of isotropic material, second one is a specially orthotropic lamina and the third one is an angle lamina (generally orthotropic). Now, suppose we apply a load in the x- direction (as shown) say stress, σ_x. The specimen made of isotropic material will extend along x and there will be contraction along y. So, there will be strain along x, ε_x and along y, ε_y (Poisson's) due to σ_x. There is no shear-extension coupling.

In the case of specially orthotropic (1-2 coincide with x-y), subjected to σ_x, there will be strains along x, ε_x and there is strain along y, ε_y (Poisson's) and because it is specially orthotropic and there is no shear-extension coupling, because the material axes, x and y coincides with 1 and 2.

However, when it comes to this generally orthotropic lamina, subjected to σ_x only, in addition to having strain ε_x and ε_y, it will also have shear strain γ_{xy} and the deformed shape is as shown due to the shear extension coupling.

Then, what is the difference between isotropic and orthotropic is that, in case of isotropic, whether we apply stress along x or stress along y, the strains in the other direction will be same which is not true for orthotropic. That is ε_y (Poisson's) due to σ_x and ε_x (Poisson's) due to σ_y are same in the case of orthotropic but in the case of an orthotropic material, ε_y (Poisson's) due to σ_x and ε_x (Poisson's) due to σ_y are NOT same since $\nu_{12} \neq \nu_{21}$.

This is the difference between orthotropic and isotropic. In both orthotropic and isotropic, there is no shear extension coupling, but in orthotropic materials, even in the material axes ν_{xy} is not equal to ν_{yx}, that is, ν_{12} is not equal to ν_{21} but it is same in isotropic materials.

So, we can see that in case of a generally orthotropic lamina, even if we apply a normal stress, that leads to shear strain and vice versa. Now, the question is, then what is the importance of this angle lamina?

This is very important in the sense that, say for example, if we have understood the variations of this E_x, E_y, you can clearly see E_x is highest, that means, is equal to E_1 when θ is equal to 0˚. Therefore, suppose we want an object which should be very stiff against longitudinal loading, then our choice will be a lamina whose longitudinal axis actually coincides with the global axis. So, θ is equal to 0˚; but it will be poor in the transverse direction ie. in the y direction.

Suppose we also want that it should have substantial stiffness along the y direction, then we have to take a lamina, whose fibers are oriented in angle of 90˚, the loading axis. Suppose we want both, it should be stiff in the x as well as y direction, then, naturally we have to take a combination of these two lamina, one 0˚, another 90˚ , but both these 0˚ and 90˚ lamina are poor against shear.

Therefore, suppose we want a component which should be, we should have sufficient resistance to shear, then we must have a very high value of shear modulus. Then, we know that at 45˚, the shear modulus is maximum, so, we must take a lamina which is having a fiber orientation of 45˚. Suppose, we want a component which should have the sufficient stiffness along x, sufficient stiffness along y, it should also have in-plane shear stiffness, then we must have a lamina which is 0˚, which is 90˚ as well as 45˚.

That is what the idea of making a laminate. Therefore, angle lamina is also important. Depending upon the stiffness requirement, we need different fiber orientation of the lamina. Even the strength requirements also dictate that. So, depending on the stiffness requirements, we need to have a combination of 0˚, 90˚ or other angle lamina.

But what happens is, as soon as we introduce angle lamina, there will be shear-extension coupling. As long as it is 0 and 90˚, there is no shear-extension coupling, but as soon as it is deviating from 0 and 90˚; suppose, at 45˚ there is a shear-extension coupling. So, again, that has to be addressed in the design. It is very important to understand the stress-strain relationships or the behaviour of a lamina subjected to load to understand the behaviour of a laminate which is composed of number of laminas stacked together.

Therefore, understanding the variation of these properties in the global axes is very important. We will discuss these things in details when we actually go for analysis of laminate.