Mechanics of Fiber Reinforced Polymer Composite Structures Prof. Debabrata Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology-Guwahati

Lecture - 29
Interlaminar Stresses- Delamination

Hello and welcome to a new module "Interlaminar stresses in laminates". So, in the last module in macromechanics of laminates the classical lamination theory was discussed in details to obtain ABBD matrix for a laminate. Based on ABBD matrix, special cases of laminate stiffnesses and different types of laminates, symmetric laminate, anti-symmetric laminate, quasi isotropic laminate etc, their importance have been discussed. Then to determination of the stresses and strains in each ply of a laminate has been discussed. From those stresses and strains in each ply, determination of ply failure and then laminate failure using appropriate failure theories were also discussed. Strength of laminate subjected to mechanical or thermal or hygroscopic load or a combination of thermo-hygro-mechanical loading was also understood in light of classical lamination theory. While discussing classical lamination theory, the stresses are considered to be only in plane in each lamina,

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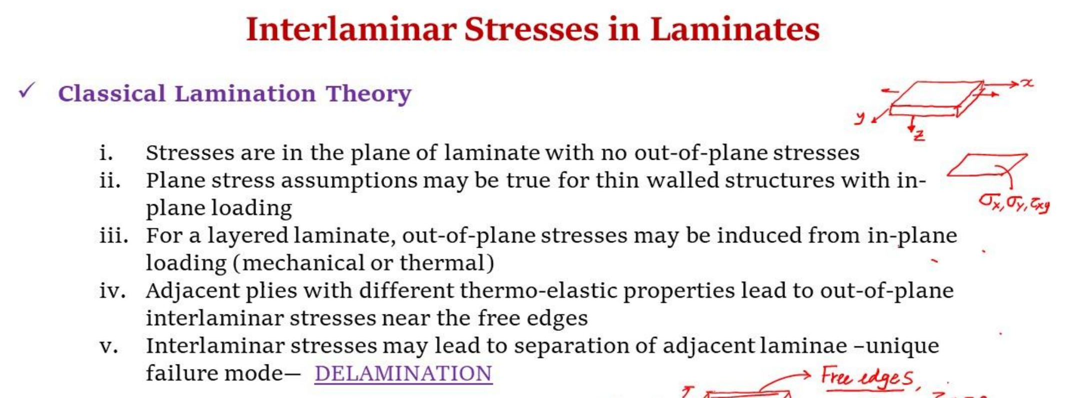

In classical lamination theory one of the key assumptions is that each lamina of the laminate is subjected to only in-plane stresses. With reference to the figure if x-y represent the lateral dimensions of a laminate and z- is the thickness direction, then the lamina the stresses are only σ_x, σ_y and τ_{xy} and no out-of-plane stresses. While this may be true for thin-walled structures within plane loading, but for a layered laminate out-of-plane stress may be induced from in-plane loading. And this is more so because in a laminate the adjacent plies may have different thermo-elastic properties and that might lead to interlaminar stresses especially near the free edge.

These interlaminar stresses actually lead to separation of adjacent laminae or ply which is called delamination. This is very important as it is of the unique mode of failure in laminated composite structures. Therefore, these interlaminar stresses which are actually not considered in classical lamination theory must be addressed and understood. Classical lamination theory does not consider transverse shear also. The transverse shear is neglected because the interface was considered thin and non-shear deformable. The classical lamination theory actually, does not satisfy the stress boundary conditions.

Referring to the Fig. the laminate is subjected to only uniaxial load in x- direction, N_x. That means the edges at x=0 and x=a are subjected to traction. However, the other two edges at y=0 and y=b are actually free edges, ie. free from any load. They are neither subjected to any shear or traction load.

Now when analysing this laminate using CLT, we get σ_x, σ_y and τ_{xy} as shown. Along the width of the laminate, since these two edges are free edges, therefore at these two edges $\tau_{xy} = 0$, $\sigma_y = 0$ following stress boundary conditions. However, classical lamination theory does not predict that. Classical lamination theory gives us a value of τ_{xy}, σ_y, which are constant throughout the lamina. Therefore, it does not satisfy the stress boundary conditions, which is essential for any elasticity problem. These are some of the limitations of the classical lamination theory. But it is observed that because of the adjacent plies may have different thermo-elastic properties, therefore even subjected to in-plane loading there may be out-of-plane stresses especially at the free edges. These out-of-plane stresses are very important as far as the design of the laminate is concerned because they lead to a unique failure mode called delamination which is actually separation of the adjacent plies. Therefore, due considerations must be given to understand this and to incorporate this in design of laminated structure. Therefore, in this lecture, these interlaminar stresses and the mechanics of development of such interlaminar stresses will be developed.

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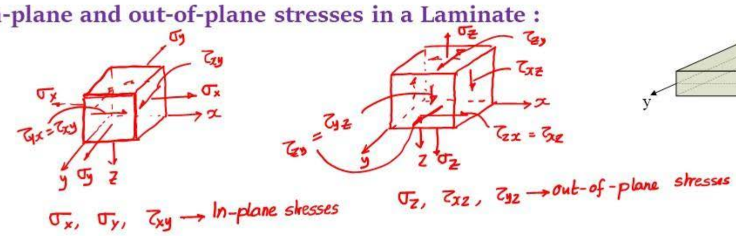
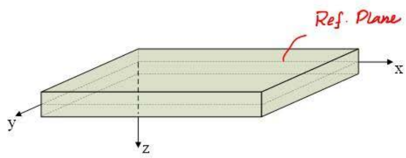
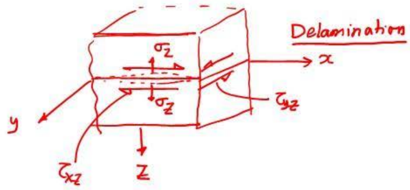
Interlaminar Stresses in Laminates

In-plane and out-of-plane stresses in a Laminate :







Now since we have been discussing on the in-plane and out-of-plane stresses, let us try to understand what exactly are the out-of-plane stresses and in-plane stresses with reference to a laminate. As shown in the Fig., consider this laminate say x, y, z are fixed at the reference plane, which is the mid plane of the laminate. Considering a small volume element from the laminate, x-y is the in-plane. Therefore, σ_x is the in-plane normal stress. Similarly, σ_y is another in-plane normal stress and τ_{xy} is the in plane shear stress. Equality of cross shear tells $\tau_{yx} = \tau_{xy}$. Therefore, the σ_x, σ_y and τ_{xy} are the in-plane stresses.

Then what are the out-of-plane stresses with reference to this coordinate system x-y- z? Again, considering the same small volume element, σ_z is the out-of-plane normal stress which is definitely not in the x-y plane and is perpendicular to the x-y plane. Then we have shear stress in the z plane along x direction, τ_{zx} or x plane z direction, τ_{xz} they are equal because of equality of cross shear. Similarly, shear stress in the z plane and along y direction τ_{zy} or y plane z direction, τ_{yz} are equal due to equality of cross shear. Therefore, these three stresses σ_z, τ_{xz} and τ_{yz} are out of plane stresses. Now with reference to a laminate what the stresses do, the in-plane stresses are responsible for in-plane strains like it could be extension of stretching in that plane. Whereas the out-of-plane stresses, suppose σ_z if it actually acts at the interface it will try to separate two adjacent plies. Similarly, τ_{xz} if it actually acts at an interface it will try to have relative sliding motion between two adjacent plies. Similarly, τ_{yz} will also try to tear one layer above the other. So, the net effect of these out-of-plane interlaminar stresses at the interface are actually to separate the two adjacent laminae. Therefore, this leads to what is called delamination and it is an important mode of failure which needs to be actually considered.

So there are three out-of-plane stresses, one normal and two shear stresses and the effect of those out-of-plane stresses when they act at an interface is to separate two adjacent lamina leading to what is known as delamination. Now Let us try to understand why these out-of-plane stresses do exist.

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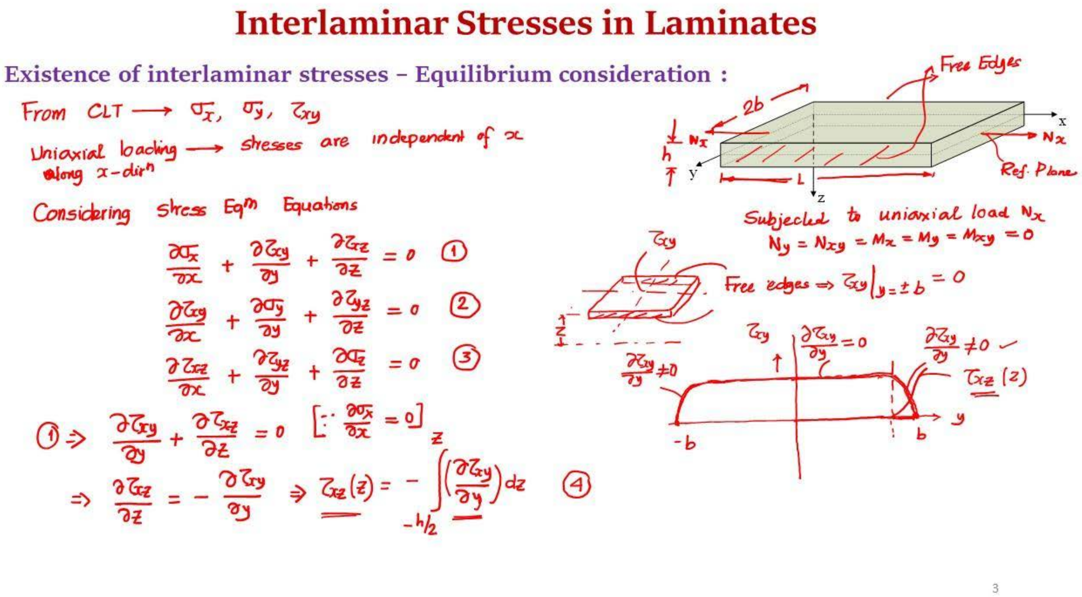

So, to understand this later we will start with equilibrium equations, okay. Referring to the Fig. the width of the laminate is say *2b*. The length of the laminate is say *L* and the thickness is *h*.

Now considering a laminate subjected to only N_x, from classical lamination theory, only σ_x, σ_y and τ_{xy} could be determined and it does not yield the other three out-of-plane stresses. However, we know that the stresses induced must satisfy the stress equilibrium equations.

Since it is uniaxial loading, it may be assumed that stresses are independent of x. Therefore, the stresses are independent of x. Now considering stress equilibrium equations in the absence of body forces,

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}=0 \ (1) \\ \frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}=0 \ (2) \\ \frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=0 \ (3) \end{aligned}\right\}$$

$$\frac{\partial \sigma_x}{\partial x}, \frac{\partial \tau_{xy}}{\partial x}, \frac{\partial \tau_{xz}}{\partial x}$$

Now since the laminate is subjected to uniaxial load and it is assumed that the stresses are independent of x, therefore $\frac{\partial \sigma_x}{\partial x}, \frac{\partial \tau_{xy}}{\partial x}, \frac{\partial \tau_{xz}}{\partial x}$ are zero and it follows as

$$\underline{Variation\, of\ \sigma_x\ along\ x=0};\ \frac{\partial \sigma_x}{\partial x}=0$$
$$\rightarrow \frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}=0$$
$$\rightarrow \frac{\partial \tau_{xz}}{\partial z}=-\frac{\partial \tau_{xy}}{\partial y}$$
$$\rightarrow \tau_{xz}(z)=-\int_{-t/2}^{z}\frac{\partial \tau_{xy}}{\partial y}dz \quad (4)$$
$$\underline{Variation\, of\ \tau_{xy}\ along\ x=0};\ \frac{\partial \tau_{xy}}{\partial x}=0$$
$$\rightarrow \frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}=0$$
$$\rightarrow \frac{\partial \tau_{yz}}{\partial z}=-\frac{\partial \sigma_y}{\partial y}$$
$$\rightarrow \tau_{yz}(z)=-\int_{-t/2}^{z}\frac{\partial \sigma_y}{\partial y}dz \quad (5)$$
$$\underline{Variation\, of\ \tau_{xz}\ along\ x=0};\frac{\partial \tau_{xz}}{\partial x}=0$$
$$\rightarrow \frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=0$$
$$\rightarrow \frac{\partial \sigma_z}{\partial z}=-\frac{\partial \tau_{yz}}{\partial y}$$
$$\rightarrow \sigma_z(z)=-\int_{-t/2}^{z}\frac{\partial \tau_{yz}}{\partial y}dz \quad (6)$$

From (4), it could be seen that interlaminar shear stress τ_{xz} at any z will exist provided there is a gradient of τ_{xy} with y,ie. $\frac{\partial \tau_{xy}}{\partial y}\neq 0$. If $\frac{\partial \tau_{xy}}{\partial y}=0$ there will be no τ_{xz}. So, the given lamina we can find out what is τ_{xy}. Now whatever is the value of τ_{xy}, at the two edges τ_{xy} must be 0. That is the stress boundary condition because no load is applied on those edges. Therefore, $\tau_{xy}\big|_{y=\pm b}=0$. Now we find out τ_{xy} using classical lamination theory and there is a $\frac{\partial \tau_{xy}}{\partial y}$ at the region near the free edges which is not zero leading the to existence of τ_{xz}. Therefore, even though there is no τ_{xz} here, because at this region there is a $\frac{\partial \tau_{xy}}{\partial y}$, therefore there is τ_{xz} as a function of z near the free edge. So, this is precisely the reason why at the free edge there will be out-of-plane shear stress τ_{xz}. Now for $\frac{\partial \tau_{xy}}{\partial y}$ to exist there must be τ_{xy}. If τ_{xy} itself is not there then that $\frac{\partial \tau_{xy}}{\partial y}$ is not there. So, mathematically we can show that at the free edge there will be τ_{xz}.

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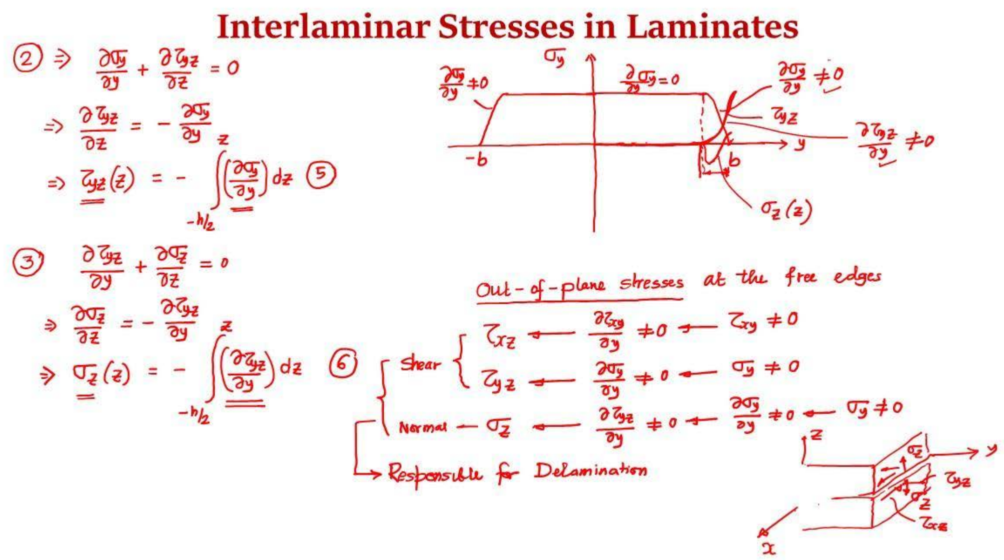

Similarly from (5), it is clear that if there exists a gradient of σ_y with respect to y, $\frac{\partial \sigma_y}{\partial y} \neq 0$ that leads to τ_{yz} at a particular z. Now let us try to understand what this $\frac{\partial \sigma_y}{\partial y}$ means. Again, for a given laminate, from classical lamination theory if find out σ_y. But we know that the two free edges at y = ±b must be free from stresses and they must satisfy the stress boundary condition. Therefore, at y = ±b, $\sigma_y = 0$.

So that means, at this region near the free edge there is a nonzero stress gradient $\frac{\partial \sigma_y}{\partial y} \neq 0$

. Equation (5) tells that if there is $\frac{\partial \sigma_y}{\partial y}$ which is nonzero that leads to τ_{yz}. Therefore, there will be τ_{yz}. So, at the two free edges there will be interlaminar shear stress τ_{yz} due to $\frac{\partial \sigma_y}{\partial y} \neq 0$. Now for that there should be σ_y. If there is no σ_y there is no $\frac{\partial \sigma_y}{\partial y}$. Therefore, away from free edge there is no τ_{yz}. But near the free edge where there is a steep gradient of $\frac{\partial \sigma_y}{\partial y}$ there is τ_{yz}. So, near the free edge there are interlaminar shear stresses because there are stress gradients of $\frac{\partial \tau_{xy}}{\partial y}$ and $\frac{\partial \sigma_y}{\partial y}$ exists. Away from that because there is no stress gradient, therefore there is no interlaminar shear stress. These are the free edge interlaminar shear stresse. So now let us turn to third equilibrium equation. Now this equation (6) shows that if there is a nonzero $\frac{\partial \tau_{yz}}{\partial y}$ that will lead to a σ_z. Now what is $\frac{\partial \tau_{yz}}{\partial y}$, we have already seen from (5) that in the region near the free edges, there is a $\frac{\partial \tau_{yz}}{\partial y}$. Because τ_{yz} increases from 0 to maximum value in this region. Therefore, we can see that in this region $\frac{\partial \tau_{yz}}{\partial y}$ exists. Therefore, that leads to a σ_z. So, we could now understand from the equilibrium equations, why three out-of-plane interlaminar stresses, one normal stress and two interlaminar shear stresses. Now what are the reasons? τ_{xz} is induced because of the existence of $\frac{\partial \tau_{xy}}{\partial y}$, τ_{yz} is induced because of the existence of $\frac{\partial \sigma_y}{\partial y}$, and σ_z is induced because of the existence of $\frac{\partial \tau_{yz}}{\partial y}$.

So, having understood that why these interlaminar stresses are actually induced, now let us see that how this actually happens.

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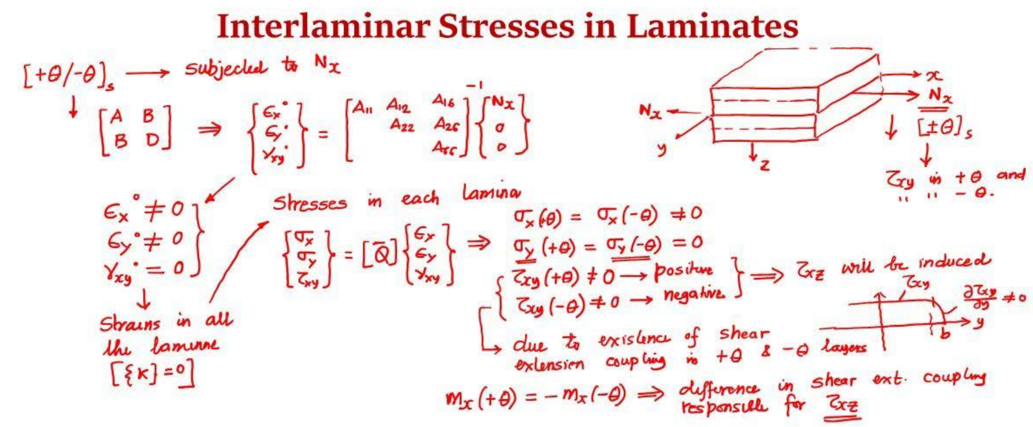
Mathematically using equilibrium equations we could see that interlaminar stresses are actually induced. Now suppose, we consider a symmetric angle ply laminate subjected to only N_x, Two adjacent laminae ($+\theta$ and $-\theta$) are shown.

Therefore, for this laminate, ABBD matrix could be obtained. Now because it is a symmetric laminate subjected to only in-plane load, we can relate the axial, in-plane force to the in-plane strains (decoupled) as.

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ & A_{22}^* & A_{26}^* \\ & & A_{66}^* \end{bmatrix} \begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} \rightarrow \boxed{\begin{matrix} \varepsilon_x^o \neq 0 \\ \varepsilon_y^o \neq 0 \\ \gamma_{xy}^o = 0 \end{matrix}}$$

Now from this could see that subjected to N_x there will be ε_x^o, there will be ε_y^o, but there will be no γ_{xy}^o because $[+\theta/-\theta]_s$ is a balanced laminate. Now these mid surface strains will also be the strains in all the layers because there are no curvatures. Now from the strains, we can find out the stresses σ_x, σ_y, τ_{xy} in each lamina by multiplying the strains with the corresponding reduced transform stiffness matrix for that lamina as shown below.



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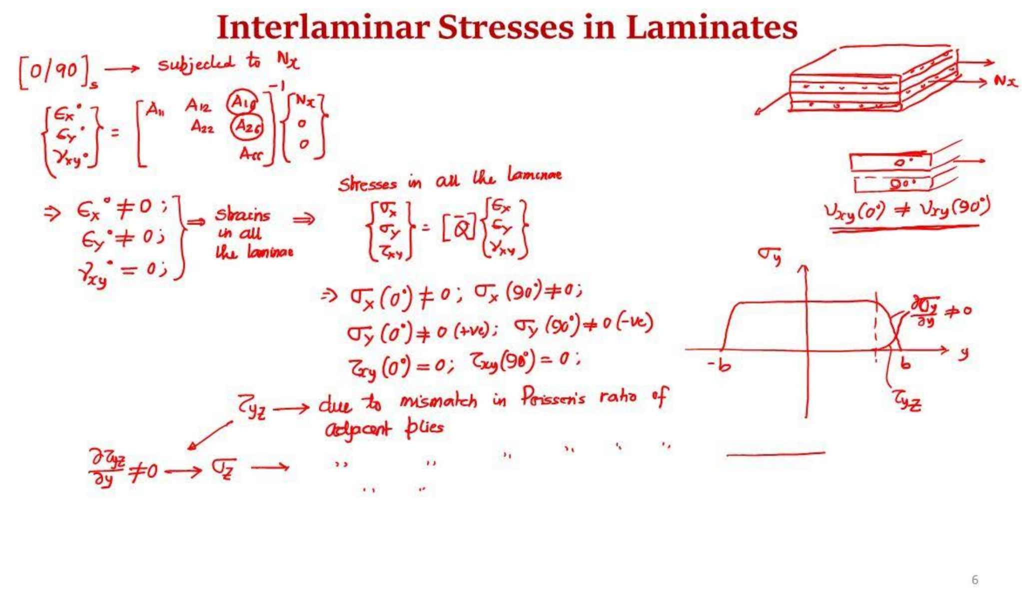
Therefore, there will be, a normal stress σ_x at $+\theta$ and $-\theta$ layer and they are equal. There will be no normal stress in the y direction $\sigma_y=0$ in both $+\theta$ and $-\theta$ layers. But, due to existence of lamina level shear extension coupling [Q_{16} and Q_{26}], there will be shear stress τ_{xy}, in both the layers ie. $\tau_{xy} \neq 0$ in both $+\theta$ and $-\theta$ layer but $\tau_{xy}(+\theta)$ is positive and $\tau_{xy}(-\theta)$ is negative. Even though there is no mid surface shear strain, but there will be shear stresses in each $+\theta$ and $-\theta$ layers.

In each layer there will be shear stresses corresponding to the normal strains. So, there is τ_{xy}. Now because there is τ_{xy}, therefore at the free edges τ_{xy} has to drop down to 0. Therefore, there will be $\frac{\partial \tau_{xy}}{\partial y}$. Now why this τ_{xy}, nonzero τ_{xy} is developed because there is a shear extension coupling. This is due to existence of shear extension coupling in + θ and $-\theta$ layers and the shear extension coupling of + θ is actually opposite to that in $-\theta$. Therefore, there is a difference in share extension coupling, and this difference in responsible for τ_{xz}. Similarly considering a specially orthotropic laminate [0/90]s subjected to only Nx as shown. Now because it is a symmetric laminate subjected to only in-plane load, we can relate the axial, in-plane force to the in-plane strains (decoupled) as.

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & 0 \\ & A_{22}^* & 0 \\ & & A_{66}^* \end{bmatrix} \begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} \rightarrow \boxed{\begin{matrix} \varepsilon_x^o \neq 0 \\ \varepsilon_y^o \neq 0 \\ \gamma_{xy}^o = 0 \end{matrix}}$$

Now from this could see that subjected to N_x there will be ε_x^o, there will be ε_y^o, but there will be no γ_{xy}^o because it is a specially orthotropic laminate and A_{16} and A_{26} are zero Now these mid surface strains will also be the strains in all the layers because there are no curvatures. Now from the strains, we can find out the stresses σ_x, σ_y, τ_{xy} in each lamina by multiplying the strains with the corresponding reduced transform stiffness matrix for that lamina as shown below.**(Refer Slide Time: 50:57)**



Therefore, there will be, σ_x in both 0° and 90°layers. Also, σ_y will be there for 0° layer and 90° layer, but for 0° layer this will be positive and for 90° layer it will be negative and there is no τ_{xy} since there is no lamina level shear extension coupling (Q_{16} = Q_{26}=0).

Because there is σ_y, therefore there is $\frac{\partial \sigma_y}{\partial y}$. When they are loaded along x, because of the Poisson's ratio, there will be contraction in the y- direction. This contraction in the y- direction for the 0° layer and for the 90° layer are different because their Poisson's ratios are different. Now, ν_{xy} (0°) ≠ ν_{xy}(90°).

$$\nu_{xy}\left(0^{\circ}\right)=\nu_{yx}\left(90^{\circ}\right)\rightarrow\nu_{xy}\left(0^{\circ}\right)\neq\nu_{xy}\left(90^{\circ}\right)\rightarrow\boxed{\Delta\nu_{xy}=\nu_{xy}\left(0^{\circ}\right)-\nu_{xy}\left(90^{\circ}\right)}$$

Because they are not equal, therefore they are not allowed to contract by the same amount in the lateral direction and that leads to existence of σ_y. So therefore, $\frac{\partial \sigma_y}{\partial y}$ exists. And because of this there will be τ_{yz}. Therefore the reason for existence of σ_y is due to the mismatch in Poisson's ratios between the adjacent layers. So τ_{yz} is due to the mismatch in Poisson's ratio of adjacent plies. So, we understood from equilibrium equations that why τ_{xz} and τ_{yz} are developed. τ_{xz} is because of the mismatch in the shear extension coupling coefficient and τ_{yz} is because of the mismatch of the Poisson's ratio.

Similarly, because τ_{yz} exists, $\frac{\partial \tau_{yz}}{\partial y}$ exists leading to the existence of σ_z. So, σ_z is also due to mismatch in Poisson's ratio of adjacent plies. And they are at the free edge, okay. If it is not free edge then at that edge the stresses will not drop down to 0 okay. So, it is there in the free edge. Now free edge may not be always at the edges of the laminate. Suppose you have a laminate, I can show you some of the free edges.

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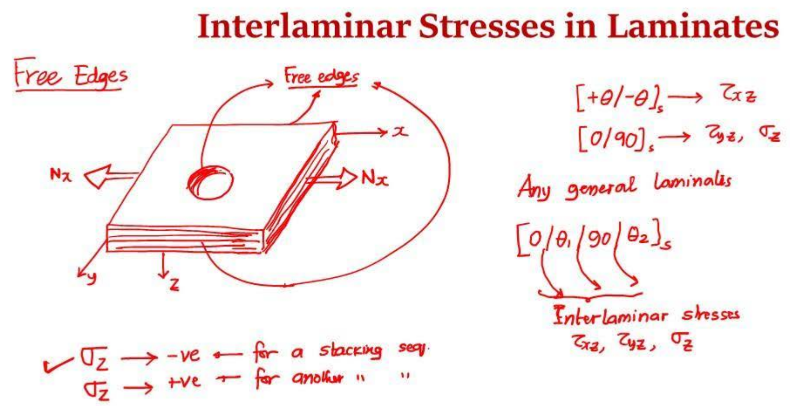

Suppose in a laminate there is a hole. This also forms a free edge, an edge which is free, which is not subjected to any load or traction. So, this phenomenon of development of interlaminar out-of-plane stresses will also be seen at this free edge of the hole along with the free edges. Now for the sake of explanation an angle laminate was considered to showt how τ_{xz} is developed and a cross ply laminate has been considered to show how τ_{yz} and σ_z are developed.

Now for any general laminate, all these three stresses may exist at the free edges. For example considering a $[0/\theta_1/90/\theta_2]_s$, at each interface, between 0° and θ_1 there will be, between θ_1 and 90°, between 90° and θ_2 there will be interlaminar stresses. So, all these interlaminar stresses shear stresses τ_{xz} and τ_{yz} and normal stress σ_z, may simultaneously exist at the free edges. So, depending upon what is the magnitude of these interlaminar stresses, delamination will initiate at a particular interface. Therefore, it is very important to understand how the interlaminar stresses are induced which might lead to the failure of the laminate. Also, it is decided by the stacking sequence, because at a particular interface between two adjacent laminae whether interlaminar stresses will be there or not. If it is there, what will be the magnitude is actually decided by the stacking sequence. Therefore, by changing the stacking sequence we can change the interlaminar stresses. For example, if σ_z is negative for a stacking sequence and for another sequence suppose σ_z is positive so we will prefer the negative σ_z because positive σ_z is actually leads to delamination whereas negative σ_z does not have any influence on the delamination. So, it is important to understand the influence of stacking sequence on this interlaminar stresses which are responsible for delamination.