Mechanics of Fibre Reinforced Polymer Composite Structures
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Module - 9
Failure Analysis of Laminates
Lecture - 26
Failure Analysis under Combined Loading

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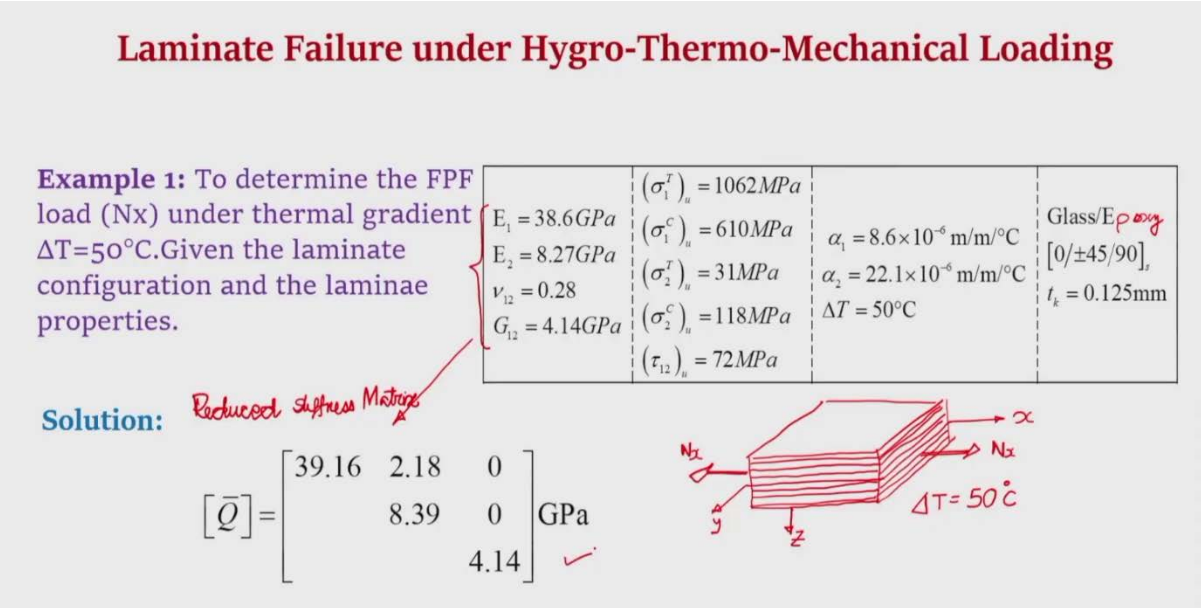

We have been discussing the failure analysis of laminate and in our last class we discussed how to calculate the first ply failure load of a laminate. We understood that with the help of an example and in continuation to that in this lecture we shall discuss the analysis of laminate under hygro-thermo-mechanical loading. That means what happens if a laminate is simultaneously subjected to mechanical load, thermal load as well as hygroscopic load.

So, we will try to understand this with the help of an example. In our previous lectures, we have understood how the hygrothermal residual stresses are actually induced in different lamina of a laminate and in our last lecture we understood how to determine the first ply failure load of a laminate subjected to mechanical loading. In this lecture, we will try to see when a laminate is actually subjected to both hygro-thermo-mechanical loading, how do we carry out the failure analysis.

So, we have taken an example to determine the first ply failure load of a laminate with a given the stacking sequence under thermal gradient of 50°C. This is a glass/epoxy laminate with the stacking sequence of $[0/\pm45/90]_s$, This is a quasi-isotropic laminate.

So, altogether there are 8 layers in this laminate and it is a symmetric laminate. From our last lecture, we know how to determine the first ply failure load $\underline{N_x}$.

So, here we need to determine the first ply failure load $\underline{N_x}$ when this laminate is experiencing a $\Delta T = 50℃$. So, we will approach this problem as two individual problems. First, we will find out the first ply failure load without having any thermal gradient or temperature gradient ΔT. And then we will see separately what happens if there is only ⊗T and then we will analyse how the first ply failure load gets influenced because of the presence of ⊗T.

First, given the lamina properties that is the engineering constants E_1, E_2, ν_{12}, G_{12}, we can calculate the reduced stiffness matrix, using the formulae in terms of engineering constants as

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}};\ Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}};$$
$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}};$$
$$Q_{66} = G_{12}$$

So, for unidirectional glass-epoxy lamina, this is the reduced stiffness matrix [Q] is

$$\left[\bar{Q}\right]_{0^\circ} = \begin{bmatrix} 39.16 & 2.18 & 0 \\ & 8.39 & 0 \\ & & 4.14 \end{bmatrix} \text{GPa}.$$

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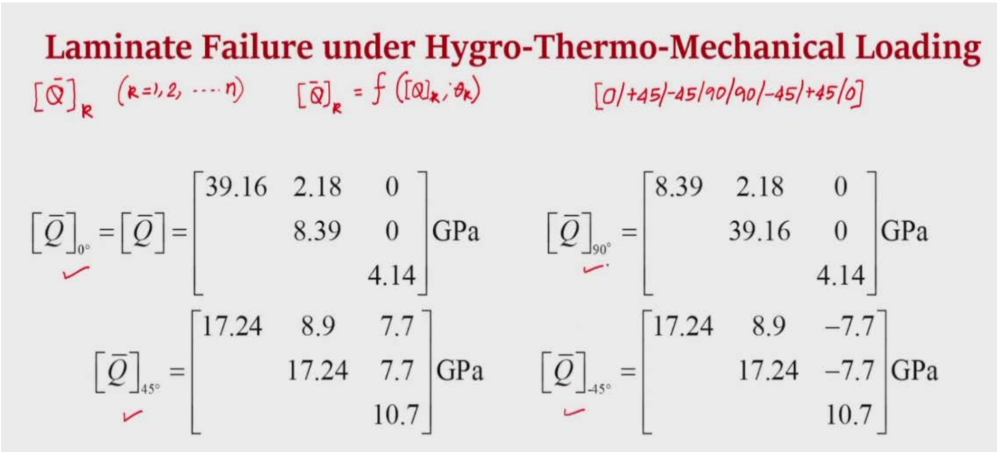

Next, the reduced transform stiffness matrix $\left[\underline{Q}\right]$ for all the layers are evaluated using

$$
\begin{aligned}
\overline{Q}_{11} &= Q_{11}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 + Q_{22}s^4 \\
\overline{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)c^2s^2 + Q_{12}\left(s^4 + c^4\right) \\
\overline{Q}_{22} &= Q_{11}s^4 + 2\left(Q_{12} + 2Q_{66}\right)c^2s^2 + Q_{22}c^4 \\
\overline{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)s\ c^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s^3c \\
\overline{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)s^3c\ - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s\ c^3 \\
\overline{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)c^2s^2 + Q_{66}\left(s^4 + c^4\right)
\end{aligned}
$$

In this case, there are 8 layers, therefore for all the 8 layers or for all 8 lamina we determine reduced transform stiffness matrix $\left[\underline{Q}\right]_k$ (k=1,2,3 …, 8).

$$
\left[\overline{Q}\right]_{0^\circ} = \begin{bmatrix} 39.16 & 2.18 & 0 \\ & 8.39 & 0 \\ & & 4.14 \end{bmatrix} \text{GPa} \qquad \left[\overline{Q}\right]_{90^\circ} = \begin{bmatrix} 8.39 & 2.18 & 0 \\ & 39.16 & 0 \\ & & 4.14 \end{bmatrix} \text{GPa}
$$

$$
\left[\overline{Q}\right]_{45^\circ} = \begin{bmatrix} 17.24 & 8.9 & 7.7 \\ & 17.24 & 7.7 \\ & & 10.7 \end{bmatrix} \text{GPa} \qquad \left[\overline{Q}\right]_{-45^\circ} = \begin{bmatrix} 17.24 & 8.9 & -7.7 \\ & 17.24 & -7.7 \\ & & 10.7 \end{bmatrix} \text{GPa}
$$

Laminate Failure under Hygro-Thermo-Mechanical Loading

$[\bar{Q}]_k$, z_k, z_{k-1}, $\longrightarrow$ Calculate [A], [B], [D]

$$
[A] = \sum_{k=1}^{n}\left[\overline{Q}\right]_k (z_k - z_{k-1}) = 2\left[\left[\overline{Q}\right]_{0^\circ} + \left[\overline{Q}\right]_{45^\circ} + \left[\overline{Q}\right]_{-45^\circ} + \left[\overline{Q}\right]_{90^\circ}\right] \times 0.125
$$

[B] = 0

[D] =

$$
[A] = \begin{bmatrix} 20.50 & 5.54 & 0 \\ & 20.50 & 0 \\ & & 7.42 \end{bmatrix} \underline{GPa - mm}
$$

$$
[A]^{-1} = \begin{bmatrix} \overset{A_{11}^*}{0.0525} & \underline{-0.0142} & 0 \\ & \underline{0.0525} & 0 \\ & & 0.1347 \end{bmatrix} \frac{1}{GPa - mm}
$$

$$
E_x = \frac{1}{\underline{h} \cdot \underline{A_{11}^*}} = \frac{1}{1 \times 0.0525} = 19.05\ GPa
$$

$\longrightarrow$ Effective Young's modulus in extension

3

Having known the values of z_k and z_{k-1} for each layer, we can now calculate [A], [B] and [D] matrix using the formulae.

$$
[A] = \sum_{k=1}^{n}\left[\overline{Q}\right]_k (z_k - z_{k-1}) = 2\left[\left[\overline{Q}\right]_{0^\circ} + \left[\overline{Q}\right]_{45^\circ} + \left[\overline{Q}\right]_{-45^\circ} + \left[\overline{Q}\right]_{90^\circ}\right] \times 0.125
$$

$$[A]=\begin{bmatrix}20.50 & 5.54 & 0\\ & 20.50 & 0\\ & & 7.42\end{bmatrix} GPa-mm$$

and

$$[A]^{-1}=\begin{bmatrix}0.0525 & -0.0142 & 0\\ & 0.0525 & 0\\ & & 0.1347\end{bmatrix}\frac{1}{GPa-mm}$$

Here, it is a symmetric laminate subjected to only Nx, so [B] and [D] are not required. Having calculated the $[A]^{-1}$ we could determine the effective Young's modulus in extension as $E_x=\dfrac{1}{h\cdot A_{11}^{*}}=\dfrac{1}{1\times 0.0525}=19.05\ GPa$ where h is the thickness of the laminate, A_{11}^{*} is the first element of the inverse of $[A]^{-1}$. Note that actually we can also calculate [B] and [D] also but because it is symmetric we know that [B] will be 0.

Also, since our objective is to determine the first ply failure load when the laminate is subjected to only normal load N_x and it is symmetric therefore [D] is not required. For a general laminate, we will actually calculate all the [A], [B] and [D].

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Laminate Failure under Hygro-Thermo-Mechanical Loading

Use $\begin{Bmatrix}N\\M\end{Bmatrix}=\begin{bmatrix}A & B\\B & D\end{bmatrix}\begin{Bmatrix}\varepsilon^o\\K\end{Bmatrix} \Rightarrow \begin{Bmatrix}\varepsilon^o\\K\end{Bmatrix}=\begin{bmatrix}A & B\\B & D\end{bmatrix}^{-1}\begin{Bmatrix}N\\M\end{Bmatrix}$

In this problem, $N_x = 100\ N/mm$, $N_y = N_{xy} = M_x = M_y = M_{xy} = 0$

$N_x = 100 N/mm \leftarrow$ Considered

$$\begin{Bmatrix}N_x\\0\\0\end{Bmatrix}=\begin{bmatrix}20.50 & 5.54 & 0\\ & 20.50 & 0\\ & & 7.42\end{bmatrix}\begin{Bmatrix}\varepsilon_x^o\\\varepsilon_y^o\\\gamma_{xy}^o\end{Bmatrix}$$

$$\begin{Bmatrix}\varepsilon_x^o\\\varepsilon_y^o\\\gamma_{xy}^o\end{Bmatrix}=\begin{bmatrix}20.50 & 5.54 & 0\\ & 20.50 & 0\\ & & 7.42\end{bmatrix}^{-1}\begin{Bmatrix}100\\0\\0\end{Bmatrix}$$

$$\boxed{\begin{aligned}\varepsilon_x^o &= 5262.3\times10^{-6}\\ \varepsilon_y^o &= -1422.1\times10^{-6}\\ \gamma_{xy}^o &= 0\end{aligned}}$$

N_x $\downarrow$ Symmetric Laminate $\downarrow$ $\begin{Bmatrix}K_x = 0\\K_y = 0\\K_{xy} = 0\end{Bmatrix}$

Having known ABBD matrix and having known the applied force ie. the force resultants and moment resultants we could calculate the mid surface strains and curvatures by taking inverse of ABBD determine the mid surface strains and curvatures as

$$\begin{Bmatrix}N\\M\end{Bmatrix}=\begin{bmatrix}A & B\\B & D\end{bmatrix}\begin{Bmatrix}\varepsilon^o\\K\end{Bmatrix} => \begin{Bmatrix}\varepsilon^o\\K\end{Bmatrix}=\begin{bmatrix}A & B\\B & D\end{bmatrix}^{-1}\begin{Bmatrix}N\\M\end{Bmatrix}$$

Now in this case we have considered N_x = 100 N/mm. In the present case, it is a symmetric laminate only subjected to N_x and hence we could calculate mid surface strains as ([B] is zero and {M} is not applied) from

$$\{N\} = [A]\{\varepsilon^o\} => \{\varepsilon^o\} = [A]^{-1}\{N\}$$

$$N_x = 100\,N/mm$$

$$\begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 20.50 & 5.54 & 0 \\ & 20.50 & 0 \\ & & 7.42 \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} 20.50 & 5.54 & 0 \\ & 20.50 & 0 \\ & & 7.42 \end{bmatrix}^{-1} \begin{Bmatrix} 100 \\ 0 \\ 0 \end{Bmatrix}.$$

Because N_x is the only nonzero, therefore by solving these simultaneous equations we can find out $\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}$. But in general, we will find out the mid surface strains and curvatures using $[ABBD]^{-1}$. In this particular case $K_x = K_y = K_{xy} = 0$ (no curvature) because there is no bending extension coupling (symmetric laminate). Because it is symmetric laminate and only N_x is applied, therefore there is no curvature. So, we calculated the mid surface strains directly.

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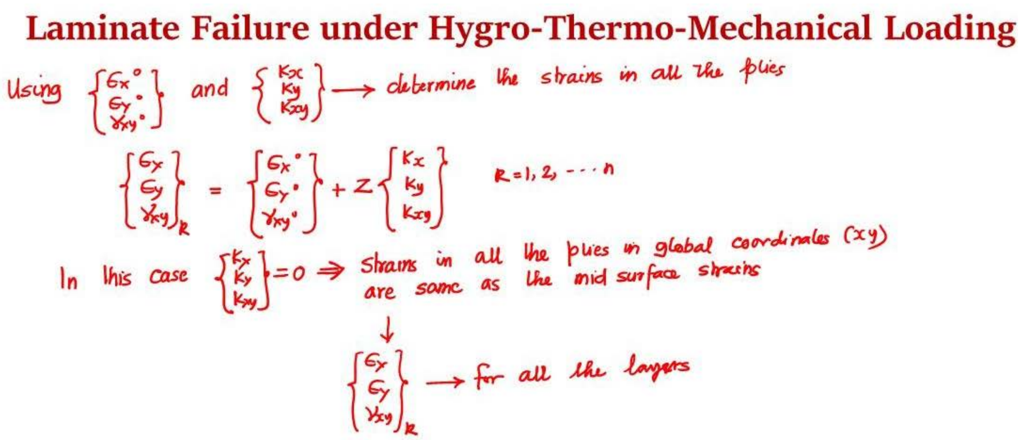

Now, using mid surface strains and curvatures we could determine the strains in all the plies or layers, using

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

In this particular case because {K} = 0 implies that strains in all the layers in global coordinates (x-y) same as the mid surface strains, but in general it may not be so.

So, we can find out actually the strains in all the layers using this formula, in this case n = 8. So, in all the 8 layers we can find out the strains. But in this particular problem because it is a symmetric laminate and only subjected to N_x, there is no curvature and hence the strains [in the global axis x-y] in all the layers are same as that of the mid surface strains. In general strains in all the layers could be obtained from the mid surface strains as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z_k \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \; (k = 1, 2, ..., n)$$

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Laminate Failure under Hygro-Thermo-Mechanical Loading

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & \underline{\underline{sc}} & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix}$$

→ Calculate the material axes strains in all the plies

1	0°
2	45°
3	−45°
4	90°
5	90°
6	−45°
7	45°
8	0°

$$Plies\,1\,\&\,8\,(0°): \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}_{\theta=0°} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \boxed{\begin{matrix} \varepsilon_1 = 5262.3\times10^{-6} \\ \varepsilon_2 = -1422.1\times10^{-6} \\ \gamma_{12} = 0 \end{matrix}} \;;$$

$$Plies\,2\,\&\,7\,(45°): \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}_{\theta=45°} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \boxed{\begin{matrix} \varepsilon_1 = 1920\times10^{-6} \\ \varepsilon_2 = 1920\times10^{-6} \\ \gamma_{12} = -6684.4\times10^{-6} \end{matrix}}$$

6

So, the first layer is 0°, second layer is 45°, third layer is –45°, fourth layer is 90° and fifth layer is again 90°, sixth is –45°, seventh is 45° and eighth is again 0°. So, in each of these layers we know the strain in the analysis axes (x-y).

Then knowing these strains, we can calculate the material axes strain in all the plies using this strain transformation because we know that θ_k (fiber orientation) for each ply and therefore we know the transformation matrix and using that we could determine the strains in the material axes in all plies. Now, in this particular case there are 8 plies, four above the middle surface and four below it and they are symmetric.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix}_k = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix}_k$$

Therefore, in the first and eighth layer which has 0° layers, the strains are

$$Plies\,1\&8(0°): \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}_{\theta=0°} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \boxed{\begin{matrix} \varepsilon_1 = 5262.3\times10^{-6} \\ \varepsilon_2 = -1422.1\times10^{-6} \\ \gamma_{12} = 0 \end{matrix}} ;$$

Of course, you can see that these are same as xy because 0° means the principal material directions do coincide with the global axis. Then in the 45° layers these are the material axes strains are calculated as

$$Plies\,2\&7(45°): \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}_{\theta=45°} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \boxed{\begin{matrix} \varepsilon_1 = 1920\times10^{-6} \\ \varepsilon_2 = 1920\times10^{-6} \\ \gamma_{12} = -6684.4\times10^{-6} \end{matrix}}$$

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Laminate Failure under Hygro-Thermo-Mechanical Loading

$$Plies\,3\&6(-45°): \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}_{\theta=-45°} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \boxed{\begin{matrix} \varepsilon_1 = 1920\times10^{-6} \\ \varepsilon_2 = 1920\times10^{-6} \\ \gamma_{12} = 6684.4\times10^{-6} \end{matrix}}$$

$$Plies\,4\&5(90°): \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}_{\theta=90°} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \boxed{\begin{matrix} \varepsilon_1 = -1422.1\times10^{-6} \\ \varepsilon_2 = 5262.3\times10^{-6} \\ \gamma_{12} = 0 \end{matrix}}$$

In the −45° (3rd and 7th layers) and in the 90° layers (4th and 5th layers), the material axes strains are calculated as

$$Plies\,3\,\&\,6(-45^\circ):\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \dfrac{\gamma_{12}}{2}\end{Bmatrix}=\begin{bmatrix}c^2 & s^2 & -2sc\\ s^2 & c^2 & 2sc\\ sc & -sc & c^2-s^2\end{bmatrix}_{\theta=-45^\circ}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \dfrac{\gamma_{xy}}{2}\end{Bmatrix}\;\boxed{\begin{matrix}\varepsilon_1=1920\times10^{-6}\\ \varepsilon_2=1920\times10^{-6}\\ \gamma_{12}=6684.4\times10^{-6}\end{matrix}}$$

$$Plies\,4\,\&\,5(90^\circ):\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \dfrac{\gamma_{12}}{2}\end{Bmatrix}=\begin{bmatrix}c^2 & s^2 & -2sc\\ s^2 & c^2 & 2sc\\ sc & -sc & c^2-s^2\end{bmatrix}_{\theta=90^\circ}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \dfrac{\gamma_{xy}}{2}\end{Bmatrix}\;\boxed{\begin{matrix}\varepsilon_1=-1422.1\times10^{-6}\\ \varepsilon_2=5262.3\times10^{-6}\\ \gamma_{12}=0\end{matrix}}$$

As could be seen from these calculations that in the 0° and 90° layers there is no shear strain because these are special orthotropic lamina therefore there is no shear extension coupling. On the other hand for the +45° and −45° layers, there are shear strains even though it is subjected only N_x because it is an angle lamina. So, there are shear strains in +45° and −45° laminae.

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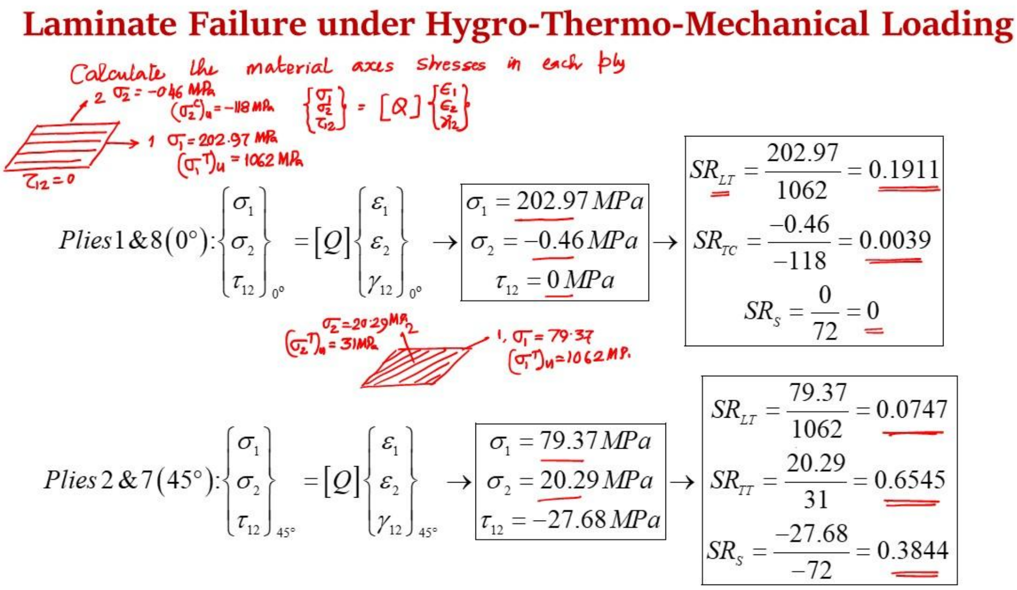


Therefore, from the material axes strains we can now calculate the stresses in the material axes in each ply by multiplying the material axis strain with the reduce stiffness matrix as follows

$$Plies\,1\,\&\,8(0^\circ):\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}_{0^\circ}=[Q]\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}_{0^\circ}\rightarrow\boxed{\begin{matrix}\sigma_1=202.97\,MPa\\ \sigma_2=-0.46\,MPa\\ \tau_{12}=0\,MPa\end{matrix}}$$

$$SR_{LT} = \frac{202.97}{1062} = 0.1911$$
$$SR_{TC} = \frac{-0.46}{-118} = 0.0039$$
$$SR_{S} = \frac{0}{72} = 0$$

$$Plies\,2\,\&\,7\left(45°\right):\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}_{45°} = \left[Q\right]\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}_{45°} \rightarrow \boxed{\begin{matrix}\sigma_1 = 79.37\,MPa\\ \sigma_2 = 20.29\,MPa\\ \tau_{12} = -27.68\,MPa\end{matrix}}$$

$$SR_{LT} = \frac{79.37}{1062} = 0.0747$$
$$SR_{TT} = \frac{20.29}{31} = 0.6545$$
$$SR_{S} = \frac{-27.68}{-72} = 0.3844$$

$$Plies\,3\,\&\,6\left(-45°\right):\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}_{-45°} = \left[Q\right]\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}_{-45°} \rightarrow \boxed{\begin{matrix}\sigma_1 = 79.37\,MPa\\ \sigma_2 = 20.29\,MPa\\ \tau_{12} = 27.68\,MPa\end{matrix}}$$

$$SR_{LT} = \frac{79.37}{1062} = 0.0747$$
$$SR_{TT} = \frac{20.29}{31} = 0.6545$$
$$SR_{S} = \frac{27.68}{72} = 0.3844$$

$$Plies\,4\,\&\,5\left(90°\right):\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}_{90°} = \left[Q\right]\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}_{90°} \rightarrow \boxed{\begin{matrix}\sigma_1 = -44.22\,MPa\\ \sigma_2 = 41.05\,MPa\\ \tau_{12} = 0\,MPa\end{matrix}}$$

$$SR_{LC} = \frac{-44.22}{-610} = 0.0725$$
$$SR_{TT} = \frac{41.05}{31} = 1.3242$$
$$SR_{S} = \frac{0}{72} = 0$$

Having calculated the materials axes strains in each lamina we know that a lamina could fail in five probable failure modes like longitudinal tensile (LT), transverse tensile (TT), longitudinal compressive (LC), transverse compressive (TC) and in-plane shear (S). Comparing these stresses with the corresponding strengths will tell us in which mode the particular lamina might fail. Therefore, we also evaluated the strength ratio for each mode as shown.

What is the significance of strength ratio? If the strength ratio is 1, it means the lamina will fail in that particular mode. It is important to note that the strength ratio is obtained by dividing the stress by the corresponding strength (which is decided by the sign of the stress).

(Refer Slide Time: 23:48)

Laminate Failure under Hygro-Thermo-Mechanical Loading

$$Plies\,3\,\&\,6(-45°): \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_{-45°} = [Q] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_{-45°} \rightarrow \boxed{\begin{matrix} \sigma_1 = 79.37\,MPa \\ \sigma_2 = 20.29\,MPa \\ \tau_{12} = 27.68\,MPa \end{matrix}} \rightarrow \boxed{\begin{matrix} SR_{LT} = \dfrac{79.37}{1062} = 0.0747 \\ SR_{TT} = \dfrac{20.29}{31} = 0.6545 \\ SR_S = \dfrac{27.68}{72} = 0.3844 \end{matrix}}$$

$$Plies\,4\,\&\,5(90°): \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_{90°} = [Q] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_{90°} \rightarrow \boxed{\begin{matrix} \sigma_1 = -44.22\,MPa \\ \sigma_2 = 41.05\,MPa \\ \tau_{12} = 0\,MPa \end{matrix}} \rightarrow \boxed{\begin{matrix} SR_{LC} = \dfrac{-44.22}{-610} = 0.0725 \\ SR_{TT} = \dfrac{41.05}{31} = 1.3242 \\ SR_S = \dfrac{0}{72} = 0 \end{matrix}}$$

(Refer Slide Time: 24:07)

Laminate Failure under Hygro-Thermo-Mechanical Loading

Ply	σ_1	σ_2	τ_{12}	SR_L	SR_T	SR_S	Failure Mode
1(0°)	202.97	-0.46	0	0.1911(LT)	0.0039(TC)	0	LT
2(45°)	79.37	20.29	-27.68	0.0747(LT)	0.6545(TT)	0.3844(S)	TT
3(-45°)	79.37	20.29	27.68	0.0747(LT)	0.6545(TT)	0.3844(S)	TT
4(90°)	-44.22	41.05	0	0.0725(LC)	1.3242(TT)	0	TT
5(90°)							
6(-45)							
7(45°)							
8(0°)							

Corresponding to $N_x = 100\ N/mm$

N_x at which the 90° plies will fail (SR=1) in TT

$$N_x = \frac{100}{1.3242} N/mm = 75.52\,N/mm \leftarrow \text{FPF load}$$

Now, having known the strength ratios we now tabulate the strength ratios for all the plies. Please see that there are actually 8 plies, but we have tabulated only 4, you can tabulate the other 4 also. Because it is symmetric it will be exactly same. The stresses will also be exactly same. Six is –45°. Seven is 45° and eight is 0°, so exactly same. The stresses will be exactly the same and so will be the strength ratios.

Now what we have done here is we have first plotted $_1$, $_2$, $_{12}$ and the corresponding strength ratios in longitudinal, in transverse and in shear and for this 0° layer we have compared all the strength ratios and you could see that among these three strength ratios longitudinal, transverse and shear this is the highest among these three. Therefore, if the 0° layer fails it will fail in this mode longitudinal tensile.

The mode corresponding to the highest strength ratio will be the first mode to reach failure.

We have arbitrarily taken a load N_x = 100 N/mm and corresponding to this the highest strength ratio is 1.3242 which corresponds to TT mode of failure of the 90° plies. Therefore, N_x at which the 90° plies will fail, meaning SR = 1 in transverse tensile is obtained as Nx at failure = 100/1.3242= 75.52 N/mm and this is the first ply failure load.

Among these 8 plies two 90°s plies will fail first in transverse tensile and therefore this is the first ply failure load. This we have done earlier also and it is straightforward. Now, our problem here was to determine the first ply failure load when this particular laminate is actually experiencing a ⊗T = 50°C.

(Refer Slide Time: 28:24)

Laminate Failure under Hygro-Thermo-Mechanical Loading

$\Delta T = 50^\circ C$

$$\{\alpha\}_{XY} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \dfrac{\alpha_{xy}}{2} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \leftarrow \text{for each ply } \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}$$

$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{0^\circ} = \begin{Bmatrix} 8.6\times10^{-6} \\ 22.1\times10^{-6} \\ 0 \end{Bmatrix} \text{m/m/}^\circ\text{C}$	$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{90^\circ} = \begin{Bmatrix} 22.1\times10^{-6} \\ 8.6\times10^{-6} \\ 0 \end{Bmatrix} \text{m/m/}^\circ\text{C}$
$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{45^\circ} = \begin{Bmatrix} 15.35 \\ 15.35 \\ -13.50 \end{Bmatrix} \times10^{-6}\text{m/m/}^\circ\text{C}$	$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{-45^\circ} = \begin{Bmatrix} 15.35 \\ 15.35 \\ 13.50 \end{Bmatrix} \times10^{-6}\text{m/m/}^\circ\text{C}$

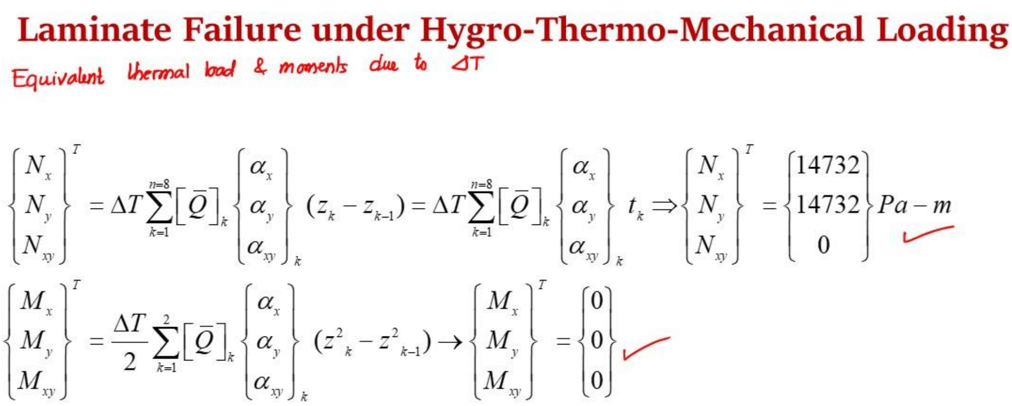
Therefore, what we do is now we take this as a separate problem. This laminate is subjected to $\Delta T = 50^\circ\text{C}$ and therefore we would like to determine due to this $\Delta T = 50^\circ\text{C}$ what are the residual stresses induced in each ply of this lamina. We have done it earlier. So, following the same procedure first we determine for each ply the coefficient of thermal expansion in the global axis.

How we know that the coefficient of thermal expansion in the material axes and they follow the same transformation as that of strain we could obtain the coefficient thermal expansions in x-y for each ply (0°, 45°, –45° and 90°) as

$$\{\alpha\}_{XY} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \dfrac{\alpha_{xy}}{2} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}$$

$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{0^\circ} = \begin{Bmatrix} 8.6\times10^{-6} \\ 22.1\times10^{-6} \\ 0 \end{Bmatrix}$ m/m/°C	$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{90^\circ} = \begin{Bmatrix} 22.1\times10^{-6} \\ 8.6\times10^{-6} \\ 0 \end{Bmatrix}$ m/m/°C
$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{45^\circ} = \begin{Bmatrix} 15.35 \\ 15.35 \\ -13.50 \end{Bmatrix}\times10^{-6}$ m/m/°C	$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{-45^\circ} = \begin{Bmatrix} 15.35 \\ 15.35 \\ 13.50 \end{Bmatrix}\times10^{-6}$ m/m/°C

(Refer Slide Time: 29:40)



So, having known this 〈x, 〈y, 〈xy for each layer we could now determine the equivalent thermal load and moments due to $\Delta T = 50$°C. Now it is a symmetric laminate, therefore there will be no moment but just for the sake of completeness we have shown both equivalent thermal load and moment (though moment comes out to be 0 anyway).

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}^T = \Delta T \sum_{k=1}^{n=8} [\bar{Q}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (z_k - z_{k-1}) = \Delta T \sum_{k=1}^{n=8} [\bar{Q}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k t_k \Rightarrow \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}^T = \begin{Bmatrix} 14732 \\ 14732 \\ 0 \end{Bmatrix} Pa-m$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T = \frac{\Delta T}{2} \sum_{k=1}^{2} [\bar{Q}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k ({z^2}_k - {z^2}_{k-1}) \rightarrow \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

(Refer Slide Time: 30:33)

Laminate Failure under Hygro-Thermo-Mechanical Loading

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} \rightarrow \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}^{-1} \begin{Bmatrix} 14732 \\ 14732 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} \\ \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{Bmatrix} 5.67\times10^{-4} \\ 5.67\times10^{-4} \\ \underline{0} \end{Bmatrix} m/m \\ \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} 1/m \end{Bmatrix}$$

Mid surface strains due to $\underline{\underline{\Delta T}}$

Curvature due to ΔT

So, once we know the equivalent thermal forces and moments, we can now use force moment and strain curvature relationship using ABBD matrix and from this putting the values of equivalent thermal loads N_x, N_y and N_{xy} we could calculate the mid surface strains and curvatures. Iit is a symmetric laminate therefore curvature will be zero, though for the sake of completeness we have shown it here as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} \rightarrow \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}^{-1} \begin{Bmatrix} 14732 \\ 14732 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} \\ \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{Bmatrix} 5.67\times10^{-4} \\ 5.67\times10^{-4} \\ 0 \end{Bmatrix} m/m \\ \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} 1/m \end{Bmatrix}$$

(Refer Slide Time: 31:26)

Laminate Failure under Hygro-Thermo-Mechanical Loading

Strain in plies in X-Y coordinates, *for* $\Delta T = 50°\text{C}$,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \rightarrow$$

Ply	1	2	3	4
θ	$0°$	$45°$	$-45°$	$90°$
ε_x	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}
ε_y	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}
γ_{xy}	0	0	0	0

Free thermal strains in plies in X-Y coordinates,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^T = \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_\theta \rightarrow$$

Ply	1	2	3	4
θ	$0°$	$45°$	$-45°$	$90°$
ε_x^T	4.3×10^{-4}	7.67×10^{-4}	7.67×10^{-4}	1.1×10^{-3}
ε_y^T	1.1×10^{-3}	7.67×10^{-4}	7.67×10^{-4}	4.3×10^{-4}
γ_{xy}^T	0	-6.75×10^{-4}	-6.75×10^{-4}	0

Having determined the mid surface strains due to ⊗T we now determine the strains in all the layers using this formula as

Strain in plies in X-Y coordinates, *for* $\Delta T = 50°\text{C}$,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \rightarrow$$

Ply	1	2	3	4
θ	$0°$	$45°$	$-45°$	$90°$
ε_x	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}
ε_y	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}	5.67×10^{-4}
γ_{xy}	0	0	0	0

Because there is no curvature, therefore four strains in all the layers in the global axes (x-y) will be same as that of the mid surface strains. Strains in all the layers in the global axis xy due to $\Delta T = 50℃$ is listed here. It is obvious for 0° and 90° layers there is no shear strain because there is no shear extension coupling. But for the 45° and –45° there is ©xy. As could be seen that in the mid surface strains due to ⊗T there is no ©xy. The reason is that the laminate is actually a quasi-isotropic laminate, therefore subjected to N_x there is no ©xy. Now knowing the mid surface strains we could find out the strains in all the layers. Because the curvature is 0, therefore the strains in all the layers are same as that of the mid surface strains.

So, we have tabulated the strains in all the layers in the global axis x-y as

Free thermal strains in plies in X-Y coordinates,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^T = \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_\theta \rightarrow$$

Ply	1	2	3	4
θ	0°	45°	−45°	90°
ε_x^T	4.3×10^{-4}	7.67×10^{-4}	7.67×10^{-4}	1.1×10^{-3}
ε_y^T	1.1×10^{-3}	7.67×10^{-4}	7.67×10^{-4}	4.3×10^{-4}
γ_{xy}^T	0	-6.75×10^{-4}	-6.75×10^{-4}	0

That means all the layers will experience the same strains in x and y directions, but suppose each of these individual layers are free and are actually subjected to ⊗T, they would have experienced different strains (called free thermal strain) because there is no constraint, but now all the adjacent layers are actually constrained due to perfect bonding with the adjacent layers and are not allowed to experience free thermal strain. The difference between the free thermal strain and the common strain is nothing but the residual strain in each layer. We have discussed this earlier also. Therefore, the free thermal strains in X-Y coordinates are actually calculated as ⊗T 〈 as shown above.

(Refer Slide Time: 33:59)

Laminate Failure under Hygro-Thermo-Mechanical Loading

Residual strains in plies in X-Y coordinates,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^R = \underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta} - \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^T \rightarrow$$

Ply	1	2	3	4
θ	0°	45°	-45°	90°
ε_x^R	1.37×10^{-4}	-2×10^{-4}	-2×10^{-4}	-5.33×10^{-4}
ε_y^R	-5.33×10^{-4}	-2×10^{-4}	-2×10^{-4}	1.37×10^{-4}
γ_{xy}^R	0	6.75×10^{-4}	6.75×10^{-4}	0

$\therefore$ Residual stresses in plies in X-Y coordinates in MPa,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_\theta^R = \left[\bar{Q}\right]_\theta \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^R \rightarrow$$

Ply	1	2	3	4
θ	0°	45°	−45°	90°
σ_x^R	4.21	0	0	-4.21
σ_y^R	-4.21	0	0	4.21
τ_{xy}^R	0	4.21	-4.21	0

And the difference between the free thermal strain and the common strain is the residual strains. Therefore, the residual strain in each ply is now obtained by subtracting the free thermal strains from the common strains as tabulated here.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^R = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta - \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^T \rightarrow$$

Ply	1	2	3	4
θ	0°	45°	-45°	90°
ε_x^R	1.37×10^{-4}	-2×10^{-4}	-2×10^{-4}	-5.33×10^{-4}
ε_y^R	-5.33×10^{-4}	-2×10^{-4}	-2×10^{-4}	1.37×10^{-4}
γ_{xy}^R	0	6.75×10^{-4}	6.75×10^{-4}	0

Then, once we have the residual strains, we can now calculate the residual stresses in each ply. So, the residual stresses in global coordinate (x-y) are calculated by multiplying the residual strains with the reduced transform stiffness matrix for that particular ply as shown and tabulated below.

$\therefore$ Residual stresses in plies in X-Y coordinates in MPa,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_\theta^R = \left[\bar{Q}\right]_\theta \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_\theta^R \rightarrow$$

Ply	1	2	3	4
θ	0°	45°	–45°	90°
σ_x^R	4.21	0	0	-4.21
σ_y^R	-4.21	0	0	4.21
τ_{xy}^R	0	4.21	-4.21	0

(Refer Slide Time: 34:46)

Laminate Failure under Hygro-Thermo-Mechanical Loading

$\therefore$ Residual stresses in plies in materials axes (1-2 coordinates) in MPa,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_\theta^R = [T]_\theta \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_\theta^R \rightarrow$$

Ply	1	2	3	4
θ	0°	45°	–45°	90°
σ_1^R	4.21	4.21	4.21	4.21
σ_2^R	-4.21	-4.21	-4.21	-4.21
τ_{12}^R	0	0	0	0

due to ΔT

And once we have the residual stresses, we could now calculate the residual stresses in the materials axes using the stress transformation in each ply as follows.

$\therefore$ Residual stresses in plies in materials axes (1-2 coordinates) in MPa,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_\theta^R = [T]_\theta \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_\theta^R \rightarrow 2$$

Ply	1	2	3	4
θ	0°	45°	–45°	90°
σ_1^R	4.21	4.21	4.21	4.21
σ_2^R	-4.21	-4.21	-4.21	-4.21
τ_{12}^R	0	0	0	0

These residual stresses in the material axes of each ply are solely due to ⊗T. Now, because of ⊗T, we could obtain the residual stresses. Now our problem was to determine the first ply failure load or rather to understand the influence of ⊗T on the first ply failure load.

(Refer Slide Time: 35:31)

Laminate Failure under Hygro-Thermo-Mechanical Loading

For $\Delta T = 50°\text{C}$, Residual thermal stress in transverse direction of 90° ply $= -4.21 MPa$

Total stress in the transverse direction in 90° ply $\sigma_2 = \sigma_2(\text{due to Nx}) + \sigma_2^R(\text{Residual thermal})$

$$= 41.1 + (-4.21) = 36.89 MPa$$

$\sigma_2^R \rightarrow$ constant for ΔT= 50°C and does not change with Nx

$\sigma_2(\text{due to } N_x) \rightarrow$ varies linearly with N_x

−4.21

Now, the condition for failure is $\sigma_2(\text{due to } N_x) + \underbrace{\sigma_2^R(\text{Residual thermal})}_{\Delta T = 50°C} = (\sigma_2^T)_u = 31$ MPa

$\Rightarrow \boxed{\sigma_2(\text{due to } N_x) = 35.21 MPa}$

Corresponding to N_x=100N/mm, $\sigma_2 = 41.1 MPa$

FPF

$\Rightarrow$ FPF load at which $\sigma_2(\text{due to } N_x)$ will be $35.21 MPa$ is $\boxed{N_x = \frac{100}{\left(\frac{41.1}{35.21}\right)} = 85.67\, N/mm}$

Now, if you remember when we have taken 100 N/mm as N_x what were the stresses? The possible mode of failure was the transverse tensile in the 90° layer. The 90° layer would have failed first in the transverse tensile mode and the stress 90° layer was 41.1 MPa. So, suppose in addition to this N_x =100 N/mm, this laminate is now also experiencing $\Delta T = 50°C$, then there is an additional stress of 4.21 MPa in the 90° layer but this is compressive and

Total stress in the transverse direction in 90° ply

$$\sigma_2 = \sigma_2(\text{due to Nx}) + \sigma_2^R(\text{Residual thermal}) = 41.1 + (-4.21) = 36.89 MPa.$$

So, there is a net decrease in the total stress in the transverse direction of the 90° layer. Now, to determine the first ply failure load, there are two components, one is ʃ2 due to N_x, another is ʃ2 because of ⊗T; which is the residual stress. Now, this residual thermal stress is due to $\Delta T = 50°C$ and it does not change with N_x, it is independent of N_x. On the other hand, the ʃ2 due to N_x varies linearly with N_x. Because of 100 it is 41, if you make it 200 it will be 82…. Therefore, the condition for failure is that the total stress ʃ2 due to N_x and due to residual thermal is equal to the $(\sigma_2^T)_u$. that is the ultimate transverse tensile stress which is 31 MPa. Therefore, since four $\Delta T = 50°C$ this is constant, therefore the ʃ2 due to N_x required for this failure condition to occur is calculates as

$\sigma_2^R \rightarrow$ constant for ΔT= 50°C and does not change with Nx

σ_2 (due to N_x) $\rightarrow$ varies linearly with N_x

Now, the condition for failure is $\sigma_2(\text{due to } N_x) + \sigma_2^R(\text{Residual thermal}) = \left(\sigma_2^T\right)_u = 31$

$\Rightarrow \boxed{\sigma_2(\text{due to } N_x) = 35.21 MPa}$

Corresponding to N_x=100N/mm, $\sigma_2 = 41.1 MPa$

$\Rightarrow$ FPF load at which σ_2 (due to N_x) will be $35.21 MPa$ is $\boxed{N_x = \frac{100}{\left(\frac{41.1}{35.21}\right)} = 85.67\ N/mm}$

Therefore, this is the first ply failure load. Now, we can clearly see here that because the presence of ⊗T, the first ply failure load has increased from 75.5 N/mm to 85.67 N/mm.

Because the $\Delta T = 50℃$ actually leads to a compressive residual stress in the 90° ply in transverse direction, therefore this actually opposes the failure in the transverse tensile direction and therefore there is an increase. Suppose, we would have made $\Delta T = -50℃$ then what would have happened? Then in that case the residual thermal stress in the in the 90° layer would have been tensile and that would have been added and that would have led to reduced first ply failure load.

You may try that putting $\Delta T = 50℃$ what is the first ply failure load? Therefore, what we understand is that the residual thermal stress does influence the first ply failure load. Similarly, we can also calculate the last ply by failure load and we can see the influence of the residual thermal stress. In the same manner, we can also see how the residual hygroscopic stress also influences the first ply filler load.

Now, doing it manually is tedious, therefore you can just write a small code to determine the first ply failure load under only mechanical loading or under combined mechanical and thermal loading or under combined mechanical, thermal and hygroscopic loading.