Mechanics of Fibre Reinforced Polymer Composite Structures

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Lecture – 25
Failure Analysis of Laminates

Hello and welcome to the first lecture of module 9. In the last module, the macro mechanical analysis of laminates was discussed wherein first the constitutive equation of a laminate was developed using classical lamination theory in terms of ABBD matrix relating the force and moment resultants (due to mechanical, thermal and hygroscopic) to the mid surface strains and curvatures of laminate. From the mid surface strains and curvatures, the global strains and stresses in each lamina could be determined. From the global stresses and strains in each lamina, the materials axes stresses and strains could be determined in each lamina. Determination of hygrothermal residual stresses in each lamina due temperature achange and moisture concentration change was also discussed. Those stresses could be superposed to obtain the total stresses in each lamina in the material axes.

Now, once the stresses and strains in each lamina of laminate are determined it is possible to assess the failure or safety of each lamina in a laminate using the strength failure theories of lamina discussed in macro mechanics of a lamina. Recalling that, there are two types of theories viz. the independent theories and the interactive theories, appropriate failure theories, need to be applied to assess the safety or failure of a lamina. Laminate being made made of several laminae stacked together, the failure or safety of a laminate is decided by the failure or safety of each constituting laminae.

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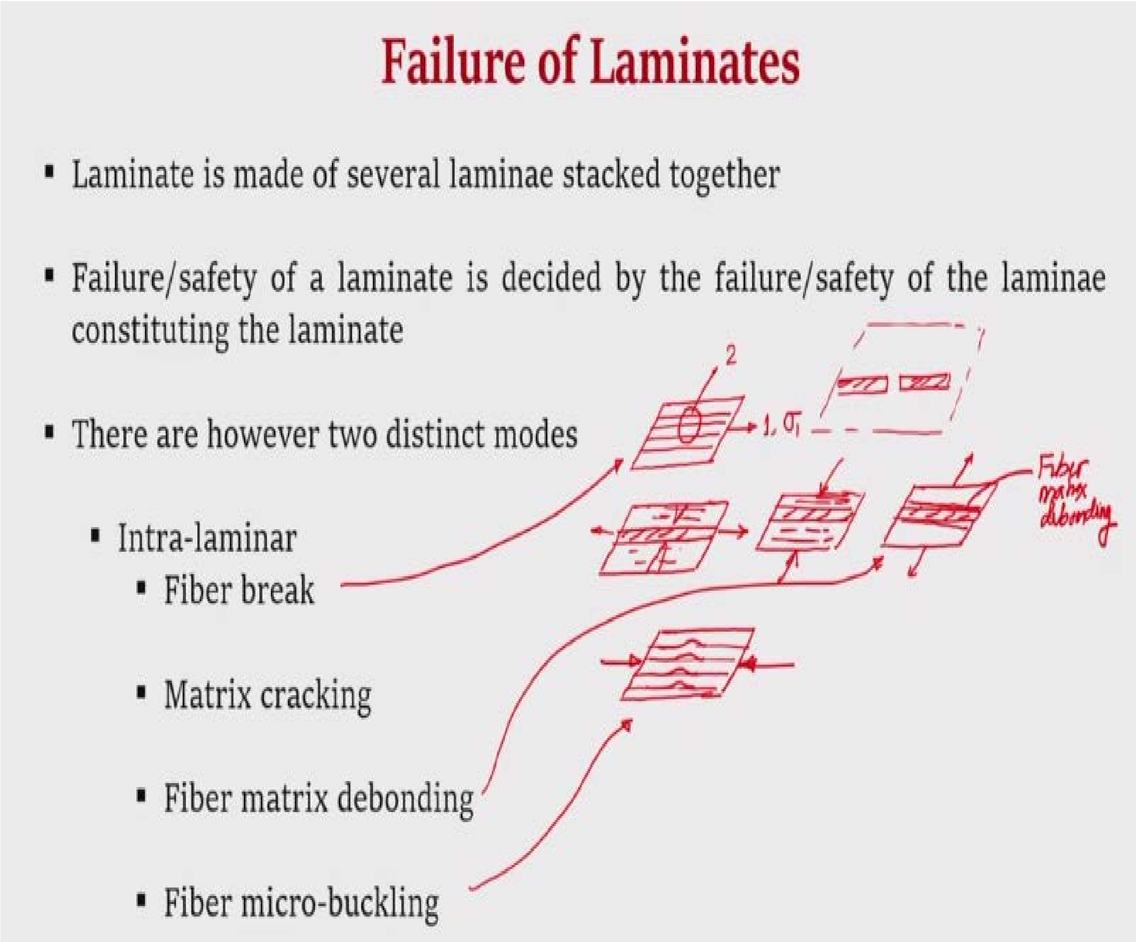
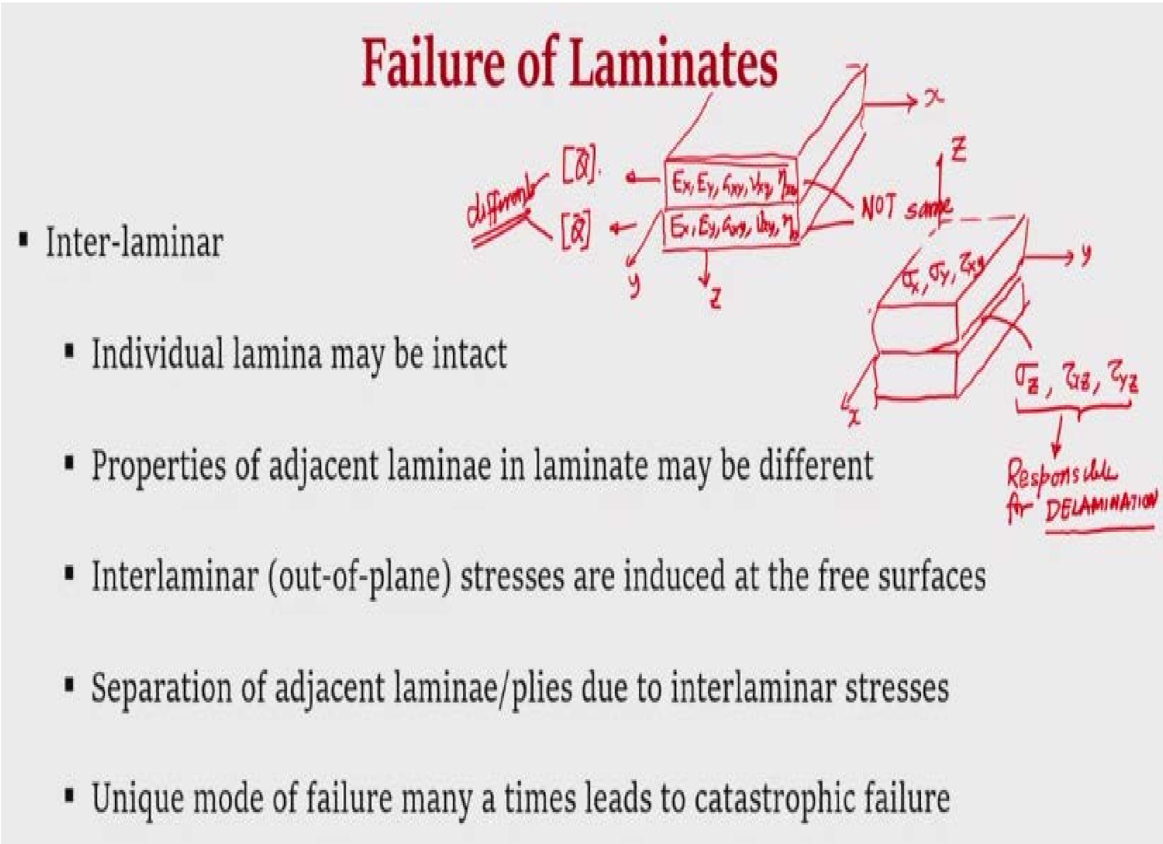


In an *n*-layer laminate, having *n* number of laminae stacked together with each lamina may have different fibre orientations with reference to the laminate analysis axes their strengths will also be different. Now the failure and safety of a laminate is decided by the failure and safety of the each of the constituent laminae, there are two modes of failure viz. **Intralaminar** failure and **Inter-laminar** failure.

In this module the intralaminar failure of laminate will be discussed in determining the strength of a laminate and the inter-laminar failure will aslo be discussed in another module. In intra-laminar failure, a lamina may fail due to several reasons like fiber break, matrix cracking, fiber matrix debonding, fiber micro buckling or a combination of those.

For example, in a lamina experiencing a longitudinal (along the fiber direction) tensile stress though the fibres are very strong and stiff along longitudinal direction, there is a limit to which they can withstand the stress. Now, if the stress in the longitudinal direction exceeds beyond that limit, then there will be fibre break. In macromechanics, it is assumed that all the fibres are of uniform strength, therefore if all the fibres break then the lamina is not in a position to withstand any load and therefore it has failed. As shown in the Fig., under loading, there may be matrix cracks followed by fibre break and the failure of the lamina. Similarly, a lamina experiencing a transverse compression, may lead to compression failure (crushing) of the matrix and a lamina is subjected to transverse tensile load may lead to fibre matrix debonding. In case of a laminate as a whole experiencing a longitudinal compression load, there may be fiber microbuckling under compression and not gross buckling of the laminate.

So, these are the different intralaminar failure that meaning the failure of a lamina which lead to the failure of the laminate. In all these cases under macro mechanical analyse of lamina each lamina is represented by its average properties. The same is true for macrmechanical analysis of a laminate also that means the in discussing the failure of laminate, fibre matrix interactions will not be discussed. Assessing the failure of lamina by means of maximum stress theory, it is possible to assess whether the lamina has failed because of longitudinal tensile, transverse tensile, longitudinal compression, transverse compression or in plane shear which is not possible in the case of interactive theories like Tsai-Hill theory or Tsai-Wu theory. **(Refer Slide Time: 08:29)**



In interlaminar failure of a laminate the two adjacent laminae may have different fibre orientations, therefore with respect to the global axis x, y, their properties are also different that is E_x, E_y, G_{xy}, ν_{xy}, and $\eta_{xy,x}$ for one layer is different the E_x, E_y, G_{xy}, ν_{xy}, and $\eta_{xy,x}$ of the adjacent layer. So, because of this property mismatch of adjacent laminae in a laminate, especially at the free edges there are interlaminar out plane stresses. It is a unique mode of failure in the case of laminated composites which many a times actually leads to catastrophic failure. At the interface, the plies get separated and this progresses under loading and the whole lamina suddenly fails. So, in addition to intralaminar failure, there could be interlaminar failure and the difference here is that in an interlaminar failure two adjacent plies or laminae may be actually intact, but because of the interlaminar normal and shear stresses induced at the interface, the two adjacent plies or laminae may get separated leading to interlaminar failure. So, the failure process in a laminate may be a combination of interlaminar as well as intralaminar failure, but in this particular lecture only intralaminar failure will be discussed and the interlaminar failure in determination of the strength of the laminate will be discussed in another later lecture.

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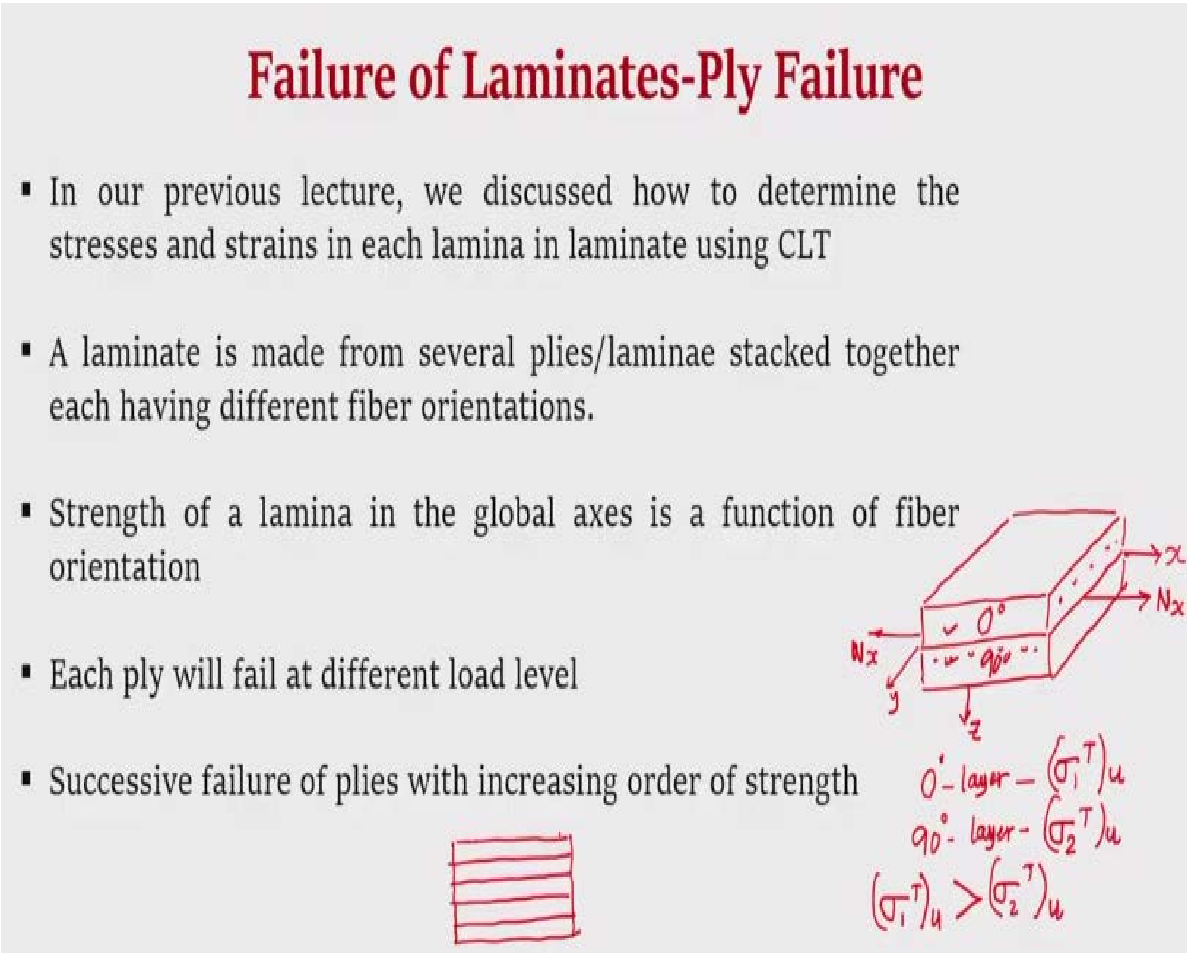

So, because the failure of laminate is actually decided by failure of each ply, now a laminate is actually made of several plies or laminae stacked together and because each lamina may have different fibre orientations, therefore their strengths in the analysis axes are different.

For example suppose a two layer [$0^0/90^0$] laminate is subjected to uniaxial load N_x, it is obvious that the 90^0 lamina will fail first as the transverse tensile strength of a lamina is far lower compared to longitudinal tensile strength. For example, the strengths of UD lamina, for GR/E, $(\sigma_1^T)_u$=1500 MPa and $(\sigma_2^T)_u$=50 MPa. The strength of the 90^0 lamina along x-axis is the transverse tensile strength and hence it fails first.

Similarly, in an n-layer laminate, depending upon the loading one of the lamiae will fail first. Once a lamina fails, that does not necessarily mean that the laminate has failed. The laminate may still capable of withstanding load till all the laminae in the laminate failed. For example, say the load on [$0^0/90^0/0^0$] laminate is such that σ_1 in the 0^0 layers is 200 MPa and σ_2 in the 90^0 layer is 200 MPa. Then, the 90^0 layer will fail ($\sigma_2>(\sigma_2^T)_u$). The fact that the 90^0 layer has failed will degrade the overall stiffness of the laminate. However, the 0^0 layers will still be able to withstand the load till σ_1 in the 0^0 layers reaches $(\sigma_1^T)_u$=1500 MPa. Similarly, in an n-layer laminate first the weakest (depending upon the loading) lamina will fail followed by the next weaker and such successive failure of laminae will continue till the last lamina in the lamina fails. However, at the failure of every lamina stiffness of the laminate degrades. The load at which the first ply in a laminate fails in called the first ply failure (FPF) load and the load the laminate could withstand till the last lamina in the laminate fails is called last ply failure (LPF) load. Most conservative way of determining the strength of lamina is to consider the FPF load as the failure lad of the laminate.

Figure shows the load (N_x) versus strain (ε_x^0) representation of an n-layer laminate subjected to N_x to show the degradation of stiffness after the failure of each ply till the last ply fails. Actual load deformation curve might not show such distinct points of successive ply failure.

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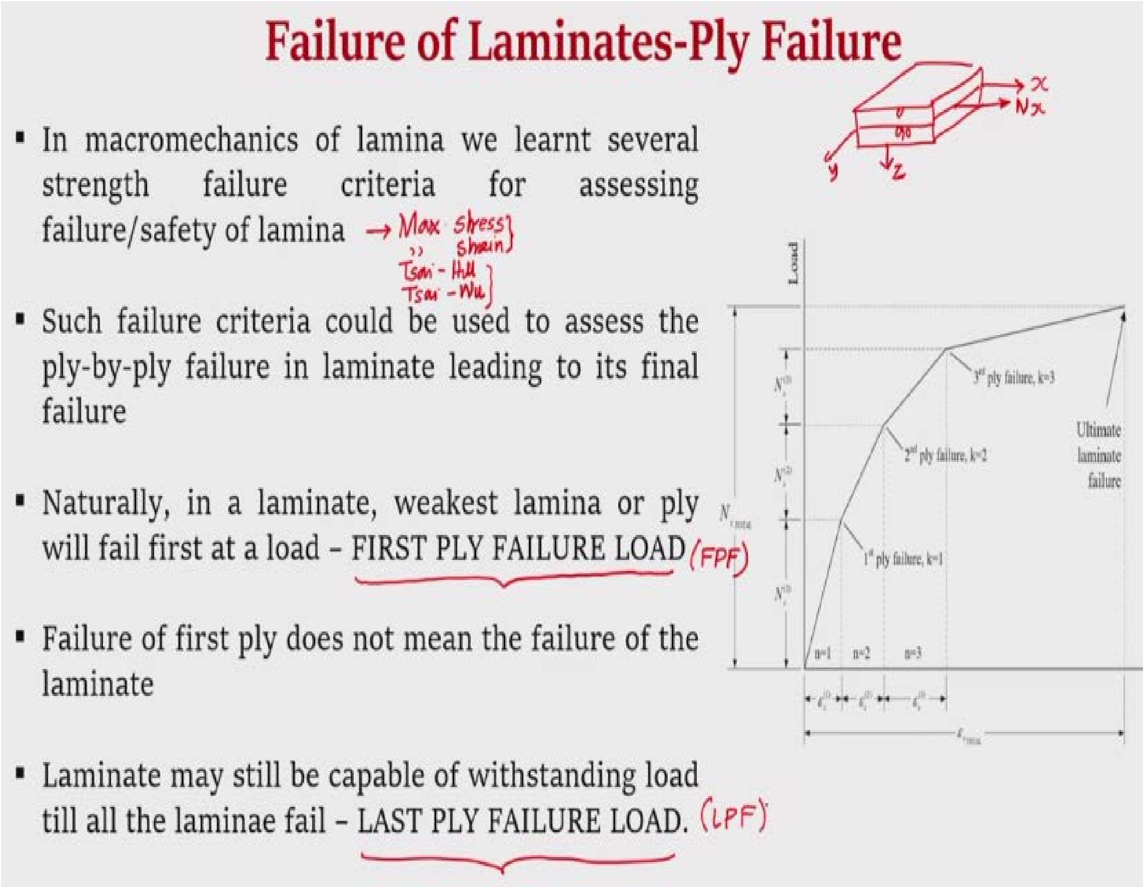

That means, in the analysis of the laminate first the stresses in each lamina is determined and then applying appropriate failure theories the failure or safety of each lamina is assessed. So, naturally in a laminate, the weakest lamina will fail first subjected to load among n lamina whichever is the weakest in this will fail first and that load at which the first lamina fails is known as the first ply failure load, in short it is also written as FPF. However, failure of a ply does not necessarily mean that the whole laminate has failed.

Therefore, after the first fly failure, the laminate may still be capable of withstanding load till all the layers or all the laminae have failed which is called the last ply failure load or in short it is called LPF.

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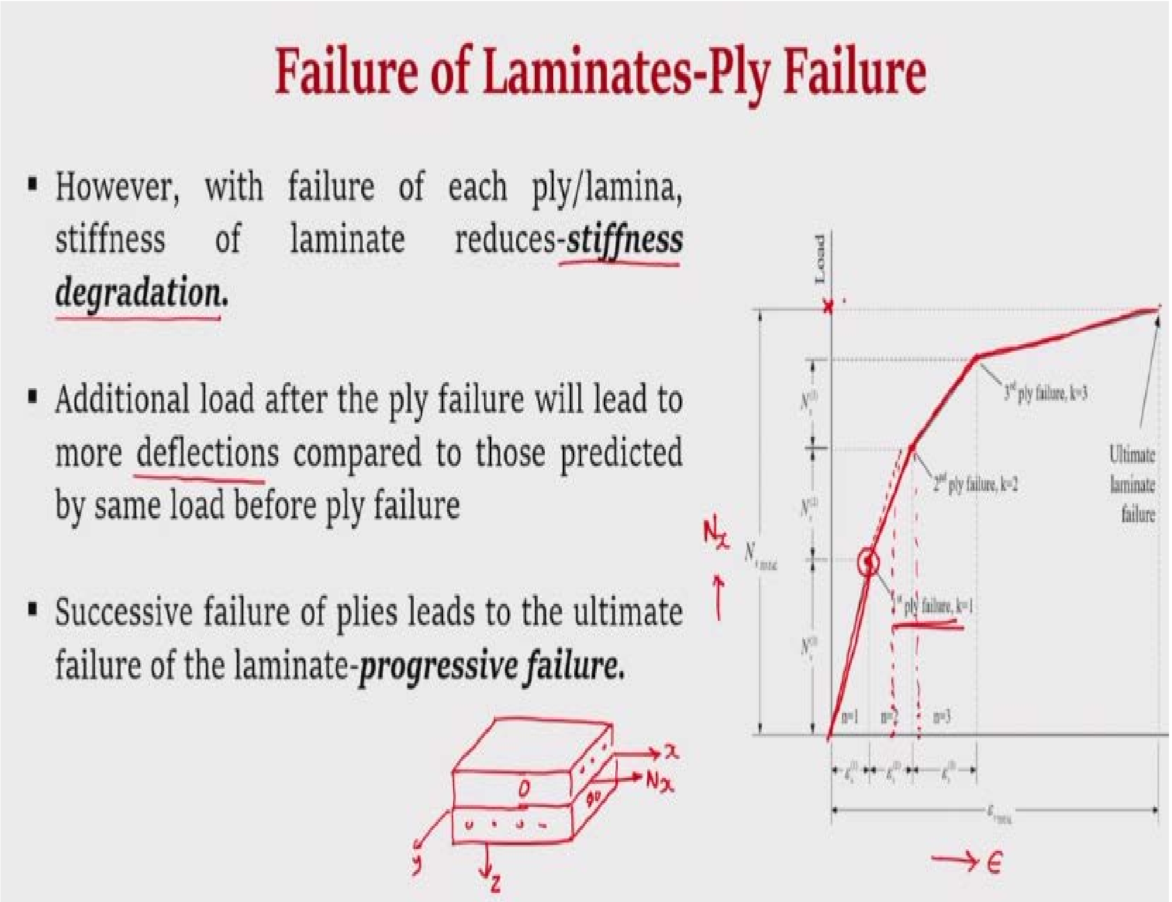

So, the successive failure of plies starting with the first ply failure till all the plies fail lead to the ultimate failure of the laminate. But what happens is that with the failure of each ply even though the first ply failure does not necessarily mean that the laminate has failed, but the failed ply will no more be fully contributing to the load bearing of the laminate. As a result of this, the overall stiffness of the laminate degrades. The overall stiffness of the laminate is in the form of the A B B D matrix. Considering only in-plane load, it is A matrix. The overall stiffness of the laminate will be degraded, the A B D matrix it is a function of $\bar{Q}$ of each layer and the thickness and position of each layer and in case of the failed lamina or ply, it's $\bar{Q}$ may be zero or degraded. Therefore the stiffness of the laminate degrades or the compliance increases. As a result, the deflections under the load after the first ply failure will be more compared to what it would have been before the ply failure. This is clear from the Fig. showing the strain versus N_x.

As show in the Fig at each kink of the strain versus N_x the slope changes, meaning a ply failure and as a result of that the stiffness degrades and therefore the slope decreases, slope of the load deflection line decreases. This is how the first ply fails, then next ply fails and so on till all the plies fail leading to the failure of the laminate. This is the progressive ply failure.

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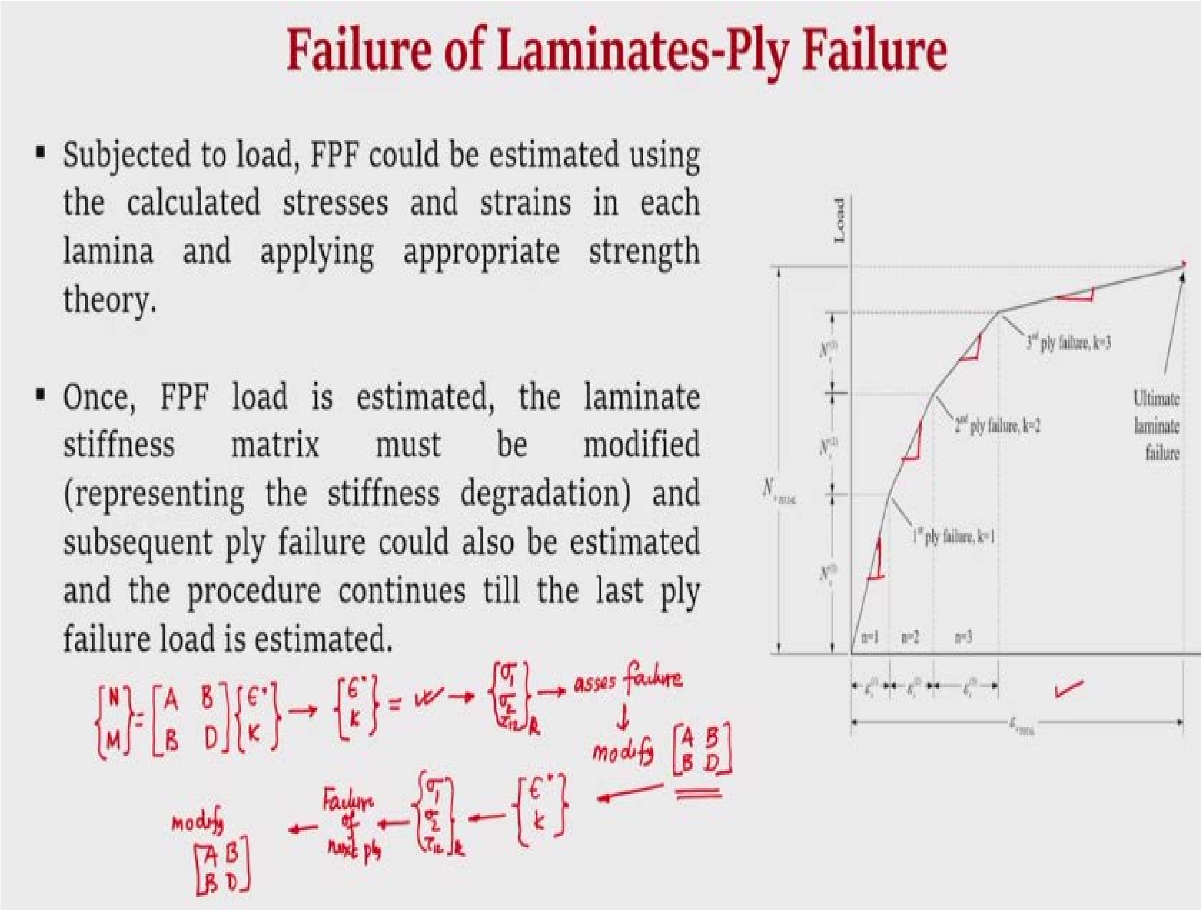

Therefore, we could determine the first ply failure load and the last ply failure load of a laminate. There are two approaches in determination of last ply failure load. As mentioned earlier, once a ply fails, its contribution to the overall load bearing is either completely absent or significantly reduces and this needs to be taken into account for further analysis of the laminate to capture successive ply failure. There are two approaches to consider this viz.

- **Complete Degradation**– where the stiffness of the failed ply is made zero to recalculate the stiffness of the laminate. This is a conservative approach even though depending upon the applied load and the failed lamina, the lamina may still contribute in load bearing.
- **Partial Degradation** – where the stiffness of the ply is not completely made zero but depending upon the lamina and the mode failure some of the components of the stiffness matrix are made zero. It may not be always possible to know the mode. Say for example if a 90^0 lamina failed due to transverse tension, it is essentially a matrix failure and hence it will not be able to carry any tensile load in the transverse direction or in-pane shear. However, this 90^0 layer may still contribute in tensile load bearing along the longitudinal direction as the fibres are still intact. Therefore rather than

making all the elements of the stiffness zero, only those components which are matrix dominated (say E_2 and G_{12}) are made zero to recalculate the degraded stiffness of the failed lamina and hence that of the laminate.

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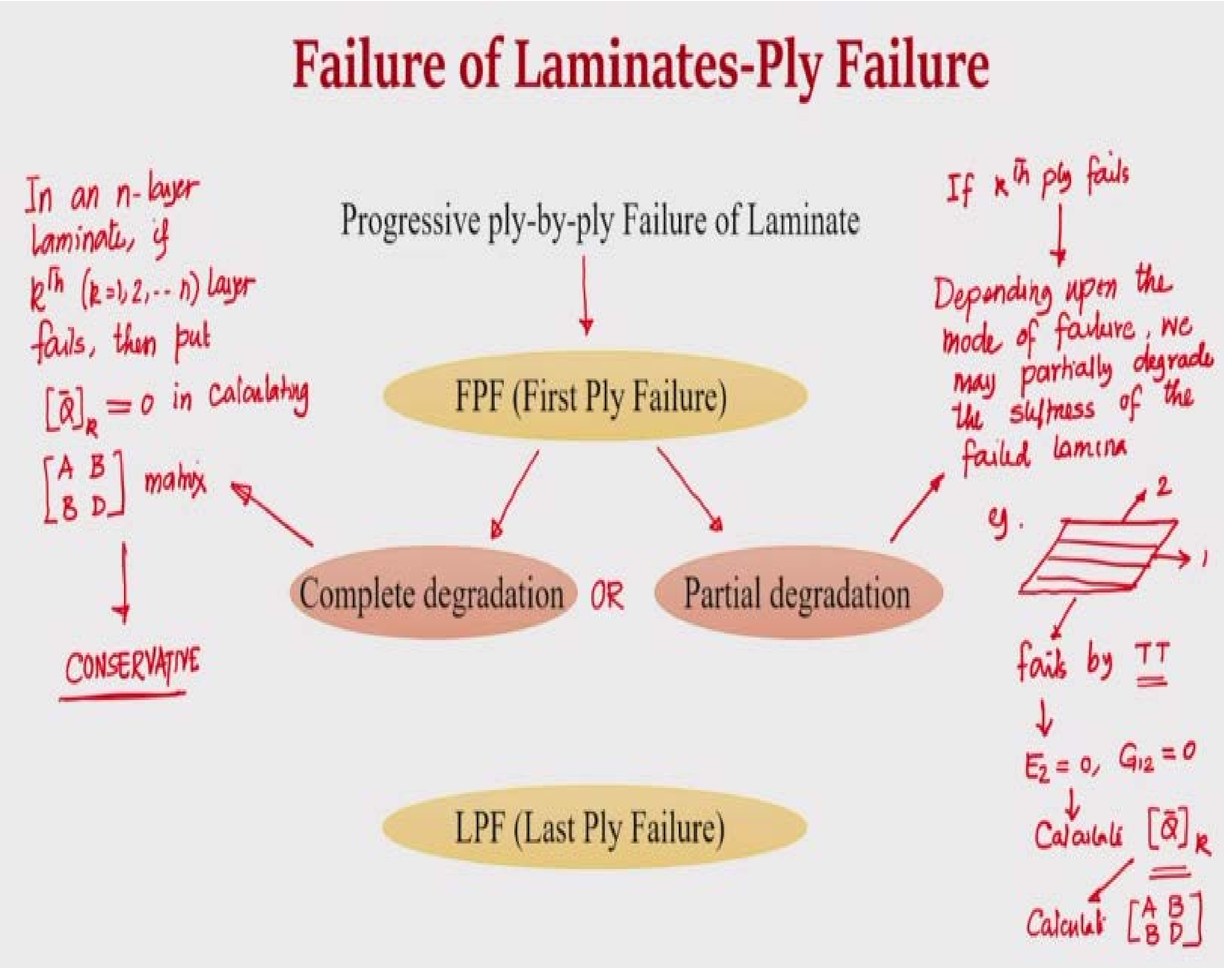

Let us take an example to show the steps in determination of the first ply failure load of a laminate.. In the last module examples were solved to show the steps in determination of material axes stresses and strains in each lamina in a laminate. Therefore in this example those steps will not be repeated and the students are advised to go through those problems again. Here, only the application of failure theories to determine the first ply failure load will be discussed.

Example 1: In the first problem a Glass/epoxy [0/90] laminate as shown in the Fig. is subjected to only Nx ≠ 0 (Ny=Nxy=Mx=My=Mxy=0). Properties of the UD Glass/epoxy lamina are given as

$$\begin{aligned} E_1 &= 38.6\,GPa \\ E_2 &= 8.27\,GPa \\ \nu_{12} &= 0.28 \\ G_{12} &= 4.14\,GPa \end{aligned} \qquad [0/90] \quad t_k = 5\text{mm}$$

FPF load Nx is to be determined.

<u>Solution:</u>

Here the laminate is a [0/90]. Glass/epoxy subjected to Nx = 100 N/mm and Nx = N_y= N_{xy}=0. Similarly M_x= M_y= M_{xy}=0. The laminate configuration is given that is the number of layers in the laminate, properties (E_1, E_2, ν_{12}, G_{12}) of each layer, geometry of each lamina meaning the thickness (5 mm) of the lamina as well as the positions with reference to the mid surface of the laminate that is the stacking sequence is known.

Note that this is not a symmetric laminate and is an unsymmetric laminate. In this problem, intentionally an unsymmetric laminate has been considered to keep the problem general in nature that is all the three matrices A, B and D exist. In addition, only a two layer laminate is considered for the ease of analysis. However the procedure is same for a general n-layer laminate.

<u>Step 1:</u> Reduced stiffness matrix $[Q]$ for UD Glass/epoxy lamina has been calculated as

$$\text{From } E_1 = 38.6GPa, E_2 = 8.27GPa, \nu_{12} = 0.28, G_{12} = 4.14GPa \text{ using}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}};\ Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}; Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}; Q_{66} = G_{12},$$

$$[Q] = \begin{bmatrix} 39.16 & 2.18 & 0 \\ & 8.39 & 0 \\ & & 4.14 \end{bmatrix} \text{GPa}$$

<u>Step 2:</u> From $[Q]$ and knowing fiber orientation angle θ for each lamina, elements of reduced transformed stiffness matrix $[\bar{Q}]$ for each lamina are calculated using the following which is discussed in details in macromechanics of lamina.

$$\left\{ \begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}s^4 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})c^2s^2 + Q_{12}(s^4 + c^4) \\ \bar{Q}_{22} &= Q_{11}s^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}c^4 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})s\,c^3 - (Q_{22} - Q_{12} - 2Q_{66})s^3c \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})s^3c - (Q_{22} - Q_{12} - 2Q_{66})s\,c^3 \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})c^2s^2 + Q_{66}(s^4 + c^4) \end{aligned} \right\}$$

And the reduced transformed stiffness matrices for each layer are

$$\left[\bar{Q}\right]_{0^\circ} = \left[\bar{Q}\right] = \begin{bmatrix} 39.16 & 2.18 & 0 \\ & 8.39 & 0 \\ & & 4.14 \end{bmatrix} \text{GPa} ;$$

$$\left[\bar{Q}\right]_{90^\circ} = \begin{bmatrix} 8.39 & 2.18 & 0 \\ & 39.16 & 0 \\ & & 4.14 \end{bmatrix} \text{GPa}$$

So, [A] is calculated, we can calculate $[A]^{-1}$ and the effective Young's modulus of the laminate along X- direction as $E_x = 1/(h \times A_{11}^{*})$ where h is the thickness of the laminate and A_{11}^{*} is the first element of $[A]^{-1}$. In this case it comes out to be 75.3 GPa. So, this is the effective extensional Young's modulus of the laminate.

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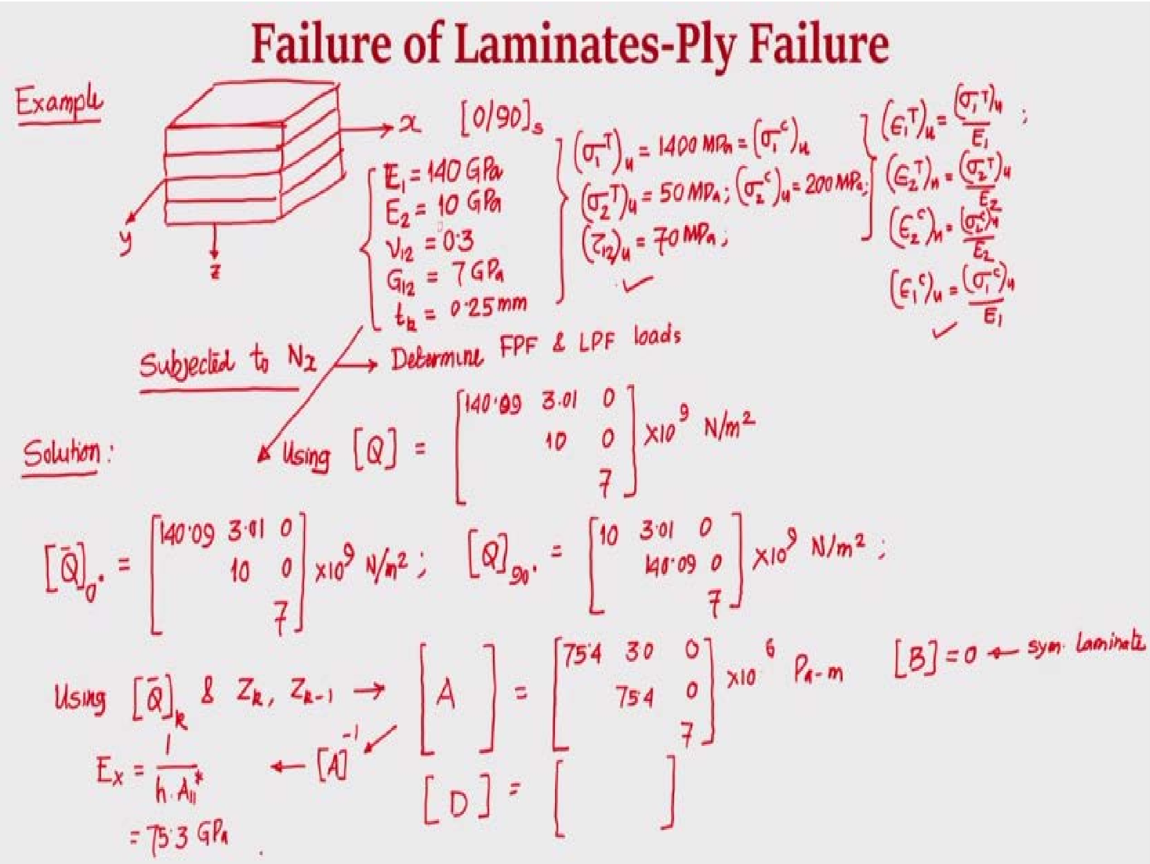

Now, applying maximum stress theory we can assess the failure or safety of each lamina where for each lamina, the material axes stresses are compared with the corresponding strengths. Note that the corresponding strengths will be as per the sign of the stresses. For example, in the present case, for the 0^0 layer, σ_1 = 186 MPa. Now, because σ_1 is positive the corresponding strength is actually $(\sigma_1{}^T)_u$ and is 1400 MPa. Had it been negative, the corresponding strength would have been $(\sigma_1{}^C)_u$ Now, for a given lamina the different modes of failure are longitudinal tension (LT), longitudinal compression (LC), transverse tensile (TT), transverse compression (TC) or in-plane shear (S).

How far it is away from the failure in a particular mode is decided by the ratio of the stress to the corresponding strength, called the strength ratio. Strength ratio for LT is σ_1 / $(\sigma_1^T)_u$. In this particular case SR= 186 / 1400 = 0.13.

Then the transverse stress σ_2 0^0 layer has been computed to be σ_2 = 3.4 MPa. Again because it is positive therefore the corresponding strength is $(\sigma_2^T)_u$ = 50 MPa. So, how far it is from the failure in this TT mode is decided by the strength ratio = σ_2 / $(\sigma_2^T)_u$ = 0.068 in this case. Check the calculations again.

The shear stress induced 0^0 layer is zero, because it is specially orthotropic laminate, each layer is a specially orthotropic layer and it is only subjected to in-plane normal load therefore there is no shear strain. Had it been an angle lamina, subjected to N_x there would have been τ_{12}. But for the sake of completeness, we will write the strength ratio is τ_{12} / $(\tau_{12})_u$ and in this case it is 0. Therefore, there is no chance of failure in shear.

Similarly, for 90^0 layer, σ_1 = –3.4 MPa. Now, because it is negative, the corresponding strength is $(\sigma_1^C)_u$ and while determining the strength ratio in the LC mode, we take the absolute value. So, the strength ratio is 3.4 / 1400 = 0.002. Take note of the fact that strength ratio is the ratio of the absolute values. Similarly, σ_2 is 13 MPa in the transverse direction, the corresponding strength is $(\sigma_2^T)_u$ = 50 MPa and the strength ratio in TT is 13 / 50= 0.26 and τ_{12} is 0 and the SR = 0.

We now tabulate the stresses in each ply and the strength ratio (SR) values corresponding to each stress to assess which is the most probable mode of failure.

The objective is to determine at what N_x the first ply fails. Corresponding to N_x = 100 kN/m, the stresses in each ply are tabulated along with SR corresponding to each mode. So, none of these plies have failed under this load but the highest value of Sr is 0.26 corresponding to the TT mode for the 90^0 layer. But if we keep on increasing this load N_x naturally the stresses will also increase linearly because this is a linear elastic analysis and as a result the strength ratio will also increase.

Now, strength ratio SR = 1 means it fails. Now, if we keep on increasing N_x the load 90^0 layer will fail first due to transverse tensile. In this case, it is 0.26 , so for failure of 90^0 layer in transverse tensile that is for S R = 1 corresponding N_x will be for N_x = 100 / 0.26 kN/m = 378 kN/m. So, this is the first ply failure load and the first ply to fail is the 90^0 layer in transverse tensile (TT) mode.

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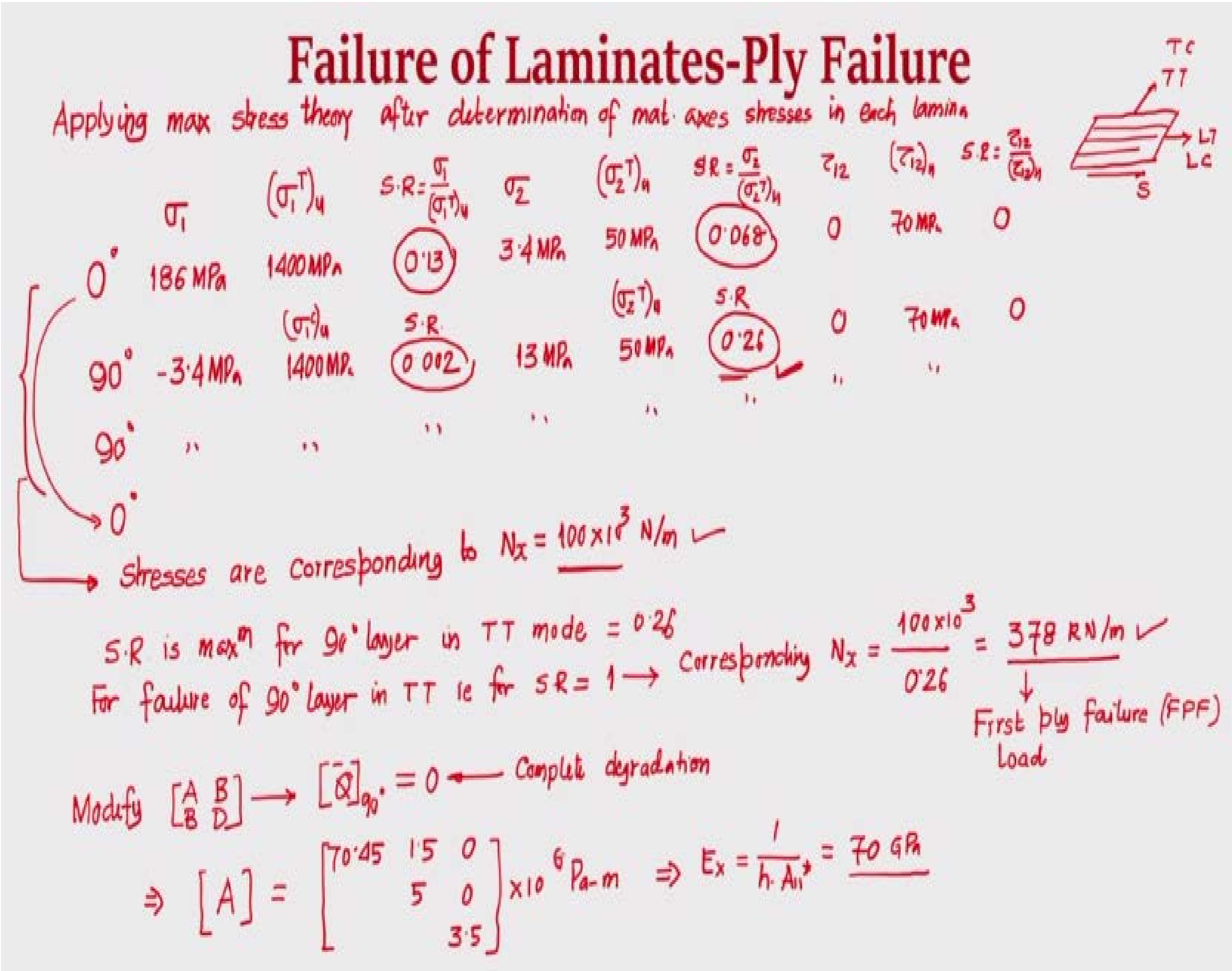

After the first fly fails, the stiffness (ABBD) matrix is modified to evaluate the second ply (in this case the last ply) failure load. Note that when we say the 90^0 ply has failed, out of these 4 plies both the 90^0 plies have failed. So, we are left with two 90^0 plies.

The modified ABBD is calculated by putting $[\bar{Q}]$ for 90^0 plies = 0 means we are using complete degradation. Once we have the modified [A] matrix, we could find the degraded Ex as $E_x = 1/(h \times A_{11}^*)$ where h is the thickness of the laminate and A_{11}^* is the first element of the modified $[A]^{-1}$ is equal to 70 GPa compared to 75.5 GPa before degradation.

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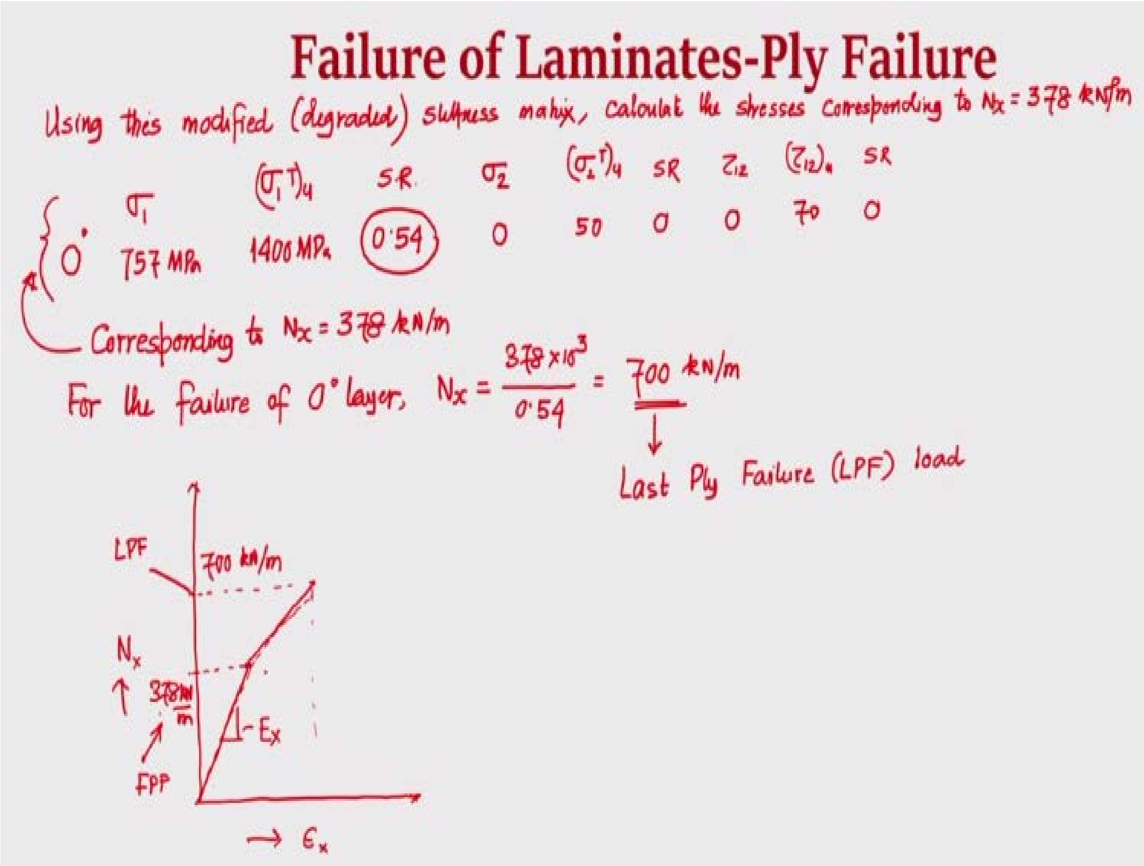

Use this modified A matrix, the stresses in the laminae are calculated and tabulated along with the SR corresponding to N_x = 378 kN/m at which the first ply has failed and the stresses along with the SR have been tabulated as shown and the highest SR corresponds to LT is 0.54.

Of course, in this case a simple especially orthotropic symmetric laminates has been taken, and solved it manually just understand how we apply failure theories to estimate the first ply and last ply failure. There are two things now, it could be any n-layer laminate same procedure will follow. Second thing we have applied maximum stress theory, it is not necessarily. So, we can apply maximum strain theory or we can apply Tsai-Hill theory or Tsai-Wu theory and the same thing we follow.

This is the FPF and this is the last ply failure load. So, please note that in actual practice the load deflection curve of laminas subjected to uniaxial load will not show such distinct points where the slope changes, but it will be more or less kind of; it will not be that distinct. The reason is that in this case we have assumed that in the calculation at that particular strain all the plies are failed.