Mechanics of Fiber Reinforced Polymer Composite Structures
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Module-08
Elastic Behaviour of Laminates-II
Lecture-23
Hygrothermal Behaviour of Laminates

Hello and welcome. So, the last lecture, effective engineering constants for a laminate namely effective Young's modulus in extension, effective Young's modulus in flexure (both along x- and y- direction), effective shear modulus, effective Poisson's ratio are actually derived in terms of the elements of ABBD matrix only for symmetric laminates, since for unsymmetric laminates it is not possible to decouple the bending and extension response

In today's lecture, hygrothermal stresses in laminates will be discussed. The hygrothermal effects in lamina were discussed in details. A fiber reinforced polymer matrix lamina is actually sensitive to hygrothermal effects and due to the temperature change and moisture absorption the lamina actually experiences strains. If the lamina is actually free that means is if a lamina is subjected to say a temperature change, ΔT or ΔC but is not restrained then it will have strains but it will not experience any stress. But in a laminate the adjacent laminae are actually perfectly bonded and the adjacent lamina may have different fiber orientations and therefore two adjacent laminae will have different coefficient of thermal expansion with respect to laminate x-y axis. Therefore, they are not free but are actually restrained and are not allowed to have their free expansion/contraction.

Therefore, there will be stresses induced in lamina and this stresses are the residual thermal stresses or residual hygroscopic stresses. Now a laminate may experience a temperature change maybe during its fabrication or during service and similarly it could have moisture absorptions during in service and because of that there may be residual hygro-thermal stresses which are induced in the lamina. Therefore, it is important that while analyzing a laminate in terms of determining the stresses in the laminae,these residual hygro-thermal stresses are taken into account.

In micromechanics, even a fiber and the matrix also have different coefficient thermal expansion and therefore there will be residual stresses. But this will not be discussed here because the discussion here is restricted to macomechanics of laminates.

Referring to the figure and considering an n-layer laminate, the strains in the k^{th} (k=1,2,…,n) lamina is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \left[\bar{S}_{ij}\right]_k \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k$$

Now because, the lamina is also subjected to temperature change ΔT and moisture concentration of ΔC, the total strains (in the global x-y) in the k^{th} (k=1,2,…,n) lamina (already discussed, refer to the lecture on hygrothermal stresses in lamina) are given by

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \left[\bar{S}_{ij}\right]_k \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta T + \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \Delta C \qquad (1)$$

As shown in Fig., the outer surface of the k^{th} layer is at a distance of z_{k-1} from the mid-surface and the inner surface is at a distance of z_k from the mid surface of the laminate.

Now in general this ΔT and ΔC will be actually, function of time and location when a laminate is actually subjected to temperature change before it actually reaches the steady state. So, the temperature as well as moisture concentration will be function of time and location. However, in this lecture only the steady state will be considered where the temperature and the moisture concentration in the laminate are independent of time and location.

Now, taking the stress on the other side and taking inverse of the compliance matrix the stresses could be written in terms of the strains. The stresses (in the global x-y) in the k^{th} layer in terms of the mid surface strains and curvatures.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \left[\bar{Q}_{ij}\right]_k \left(\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k - \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k - \Delta C \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \right) \qquad (2)$$

From Classical lamination theory, writing the stress in each layer in terms of mid surface strains and curvatures

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k = \left[\bar{Q}\right]_k\left(\begin{Bmatrix}\varepsilon_x^o\\ \varepsilon_y^o\\ \gamma_{xy}^o\end{Bmatrix} + z\begin{Bmatrix}K_x\\ K_y\\ K_{xy}\end{Bmatrix}\right) - \left[\bar{Q}\right]_k \Delta T\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}_k - \left[\bar{Q}\right]_k \Delta C\begin{Bmatrix}\beta_x\\ \beta_y\\ \beta_{xy}\end{Bmatrix}_k$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k = \left[\bar{Q}_{ij}\right]_k\left[\begin{Bmatrix}\varepsilon_x^o\\ \varepsilon_y^o\\ \gamma_{xy}^o\end{Bmatrix} + z\begin{Bmatrix}K_x\\ K_y\\ K_{xy}\end{Bmatrix}\right] - \left[\bar{Q}_{ij}\right]_k \Delta T\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}_k - \left[\bar{Q}_{ij}\right]_k \Delta C\begin{Bmatrix}\beta_x\\ \beta_y\\ \beta_{xy}\end{Bmatrix}_k \quad (3)$$

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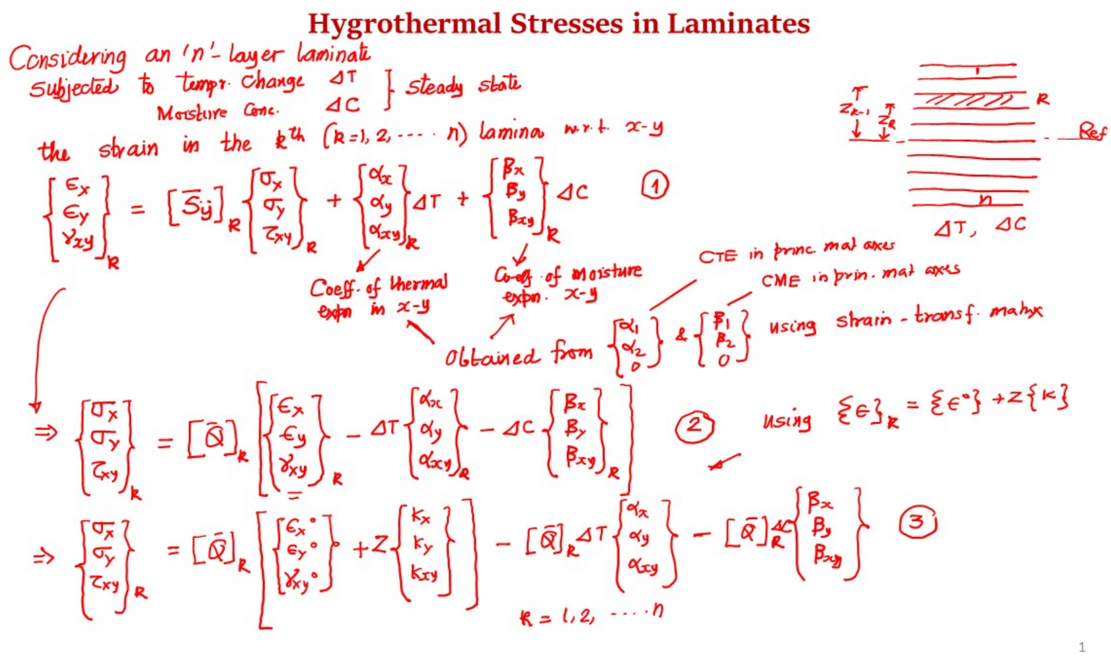

As discussed in the Classical Lamination Theory because the stresses are different in different layers and it is not possible to actually characterize the stresses and strains and therefore the stresses in the layers are actually represented by equivalent force resultant as

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = \int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k dz = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k dz = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[\bar{Q}_{ij}\right]_k\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}_k dz \quad (4)$$

Because the stresses are discontinuous this continuous integration is actually replaced by integrating over the thickness of each lamina where the stresses stress variation is continuous and by summing that over the all the layers as

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix} = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[\bar{Q}\right]_k\left(\begin{Bmatrix}\varepsilon_x^o\\\varepsilon_y^o\\\gamma_{xy}^o\end{Bmatrix}+z\begin{Bmatrix}K_x\\K_y\\K_{xy}\end{Bmatrix}\right)dz-\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[\bar{Q}\right]_k\Delta T\begin{Bmatrix}\alpha_x\\\alpha_y\\\alpha_{xy}\end{Bmatrix}_k dz-\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[\bar{Q}\right]_k\Delta C\begin{Bmatrix}\beta_x\\\beta_y\\\beta_{xy}\end{Bmatrix}_k dz \tag{5}$$

Since for a given lamina [Q] is constant within the thickness of the lamina and also mid surface strains and curvatures are also independent of the z-coordinates, this could be written as

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix} = \sum_{k=1}^{n}\left[\bar{Q}\right]_k\int_{z_{k-1}}^{z_k}dz\begin{Bmatrix}\varepsilon_x^o\\\varepsilon_y^o\\\gamma_{xy}^o\end{Bmatrix}+\sum_{k=1}^{n}\left[\bar{Q}\right]_k\int_{z_{k-1}}^{z_k}zdz\begin{Bmatrix}K_x\\K_y\\K_{xy}\end{Bmatrix}-\Delta T\sum_{k=1}^{n}\left[\bar{Q}\right]_k\int_{z_{k-1}}^{z_k}dz\begin{Bmatrix}\alpha_x\\\alpha_y\\\alpha_{xy}\end{Bmatrix}_k-\Delta C\sum_{k=1}^{n}\left[\bar{Q}\right]_k\int_{z_{k-1}}^{z_k}dz\begin{Bmatrix}\beta_x\\\beta_y\\\beta_{xy}\end{Bmatrix}_k \tag{6}$$

Recalling the definition of **A** and **B** matrices in CLT, this may be written as

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix} = [A]\begin{Bmatrix}\varepsilon_x^o\\\varepsilon_y^o\\\gamma_{xy}^o\end{Bmatrix}+[B]\begin{Bmatrix}K_x\\K_y\\K_{xy}\end{Bmatrix}-\begin{Bmatrix}N_x^T\\N_y^T\\N_{xy}^T\end{Bmatrix}-\begin{Bmatrix}N_x^H\\N_y^H\\N_{xy}^H\end{Bmatrix} \tag{7}$$

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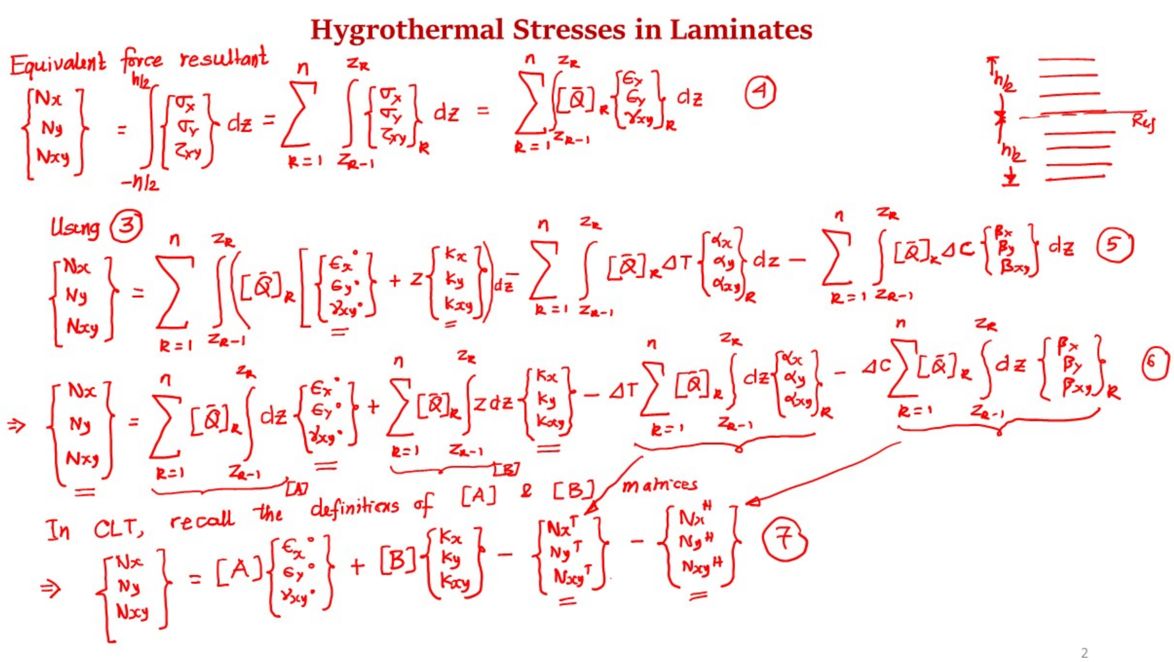

where,

$$\begin{Bmatrix}N_x^T\\N_y^T\\N_{xy}^T\end{Bmatrix} = \Delta T\sum_{k=1}^{n}\left[\bar{Q}\right]_k\begin{Bmatrix}\alpha_x\\\alpha_y\\\alpha_{xy}\end{Bmatrix}_k\left(z_k-z_{k-1}\right) \tag{9}$$

is the equivalent thermal load due to ΔT and

$$\begin{Bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{Bmatrix} = \Delta C \sum_{k=1}^{n} \left[\bar{Q}\right]_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \left(z_k - z_{k-1}\right) \tag{10}$$

is the equivalent hygroscopic load due to ΔC.

A closer look in to these expressions shows that these are nothing but some of the forces (per unit length) in each layer due to ΔT and ΔC and could thus be written as

$$\begin{Bmatrix} N_x + N_x^T + N_x^H \\ N_y + N_y^T + N_y^H \\ N_{xy} + N_{xy}^T + N_{xy}^H \end{Bmatrix} = [A] \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + [B] \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} N_x + N_x^T + N_x^H \\ N_y + N_y^T + N_y^H \\ N_{xy} + N_{xy}^T + N_{xy}^H \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ & A_{22} & A_{26} \\ & & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ & & B_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \qquad ...(11)$$

$$\left.\begin{matrix} if\ \ \Delta C = 0 \ \rightarrow \ \{N\}^H = 0 \\ \Delta T = 0 \ \rightarrow \{N\}^T = 0 \end{matrix}\right\rangle \{N\} = [A]\{\varepsilon^o\} + [B]\{K\}$$

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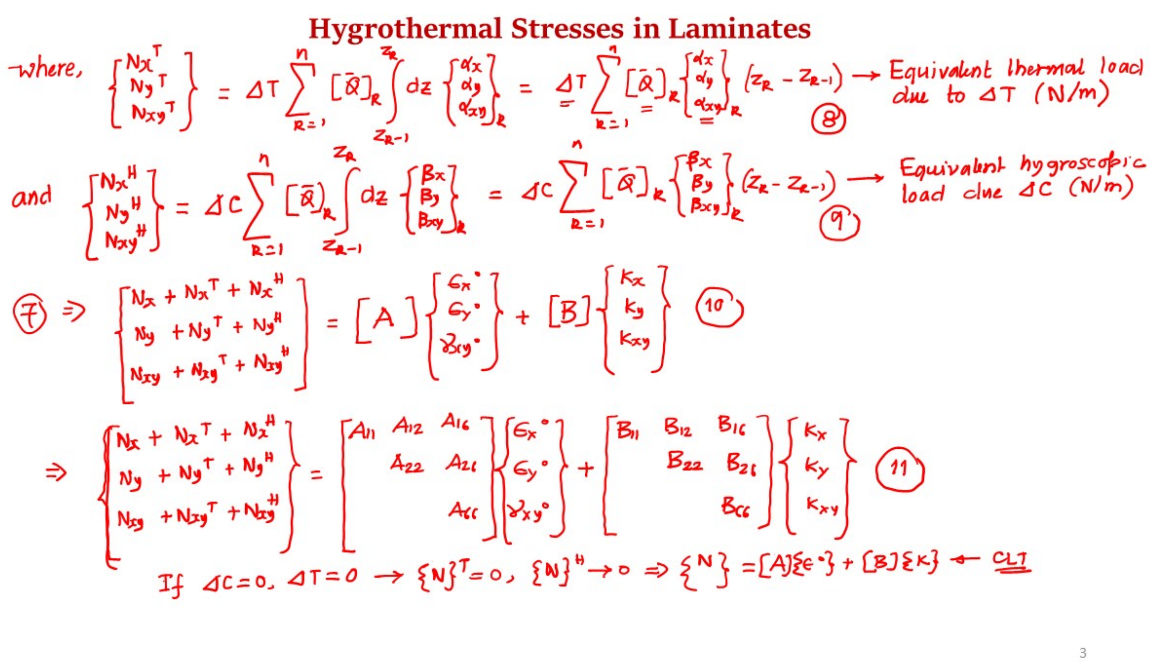

Similarly, the equivalent moment resultant for the n-layer laminate could be obtained by integrating the stresses over the thickness of the laminate as

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} zdz \qquad (12)$$

and using Eqn (3) we get

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \left[\bar{Q}\right] \left(\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} - \Delta C \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} \right) zdz \qquad (13)$$

Replacing the integration over the thickness by sum over all the layers and integrating over the thickness of each layer

$$\begin{aligned} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} &= \left[\sum_{k=1}^{n} \left[\bar{Q}\right]_k \int_{z_{k-1}}^{z_k} zdz \right] \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \left[\sum_{k=1}^{n} \left[\bar{Q}\right]_k \int_{z_{k-1}}^{z_k} z^2 dz \right] \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \\ &- \left[\Delta T \sum_{k=1}^{n} \left[\bar{Q}\right]_k \int_{z_{k-1}}^{z_k} zdz \right] \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} - \left[\Delta C \sum_{k=1}^{n} \left[\bar{Q}\right]_k \int_{z_{k-1}}^{z_k} zdz \right] \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} \end{aligned} \qquad (14)$$

Recalling the definition of **B** and **D** matrices in CLT, this may be written as

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [B] \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + [D] \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} - \begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} - \begin{Bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{Bmatrix} \qquad (15)$$

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Hygrothermal Stresses in Laminates

Equivalent moment resultant

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z\,dz \quad (12)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k z\,dz \quad (13) \Rightarrow \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\bar{Q}]_k \left[\begin{Bmatrix} \epsilon_x^\circ \\ \epsilon_y^\circ \\ \gamma_{xy}^\circ \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k - \Delta C \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \right] z\,dz \quad (14)$$

$$(14) \Rightarrow \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \underbrace{\sum_{k=1}^{n} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} z\,dz}_{[B]} \begin{Bmatrix} \epsilon_x^\circ \\ \epsilon_y^\circ \\ \gamma_{xy}^\circ \end{Bmatrix} + \underbrace{\sum_{k=1}^{n} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} z^2\,dz}_{[D]} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} - \Delta T \sum_{k=1}^{n} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} z\,dz \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k - \Delta C \sum_{k=1}^{n} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} z\,dz \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k$$

Recalling the definitions of [B] and [D] matrices (14) =>

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [B] \begin{Bmatrix} \epsilon_x^\circ \\ \epsilon_y^\circ \\ \gamma_{xy}^\circ \end{Bmatrix} + [D] \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} - \begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} - \begin{Bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{Bmatrix} \quad (15)$$

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where,

$$\begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} = \frac{\Delta T}{2} \sum_{k=1}^{n} \left[\bar{Q}\right]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \left(z_k^2 - z_{k-1}^2\right) \tag{16}$$

is the equivalent thermal moment due to ΔT and

$$\begin{Bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{Bmatrix} = \frac{\Delta C}{2} \sum_{k=1}^{n} \left[\bar{Q}\right]_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \left(z_k^2 - z_{k-1}^2\right) \tag{17}$$

is the equivalent hygroscopic moment due to ΔC. Eqn (15) Could be written as

$$\begin{Bmatrix} M_x + M_x^T + M_x^H \\ M_y + M_y^T + M_y^H \\ M_{xy} + M_{xy}^T + M_{xy}^H \end{Bmatrix} = [B] \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + [D] \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \tag{18}$$

Combining the force resultant and moment resultants (11) and(18)

$$\begin{Bmatrix} N + N^T + N^H \\ M + M^T + M^H \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ K \end{Bmatrix} \tag{19}$$

or

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ K \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} - \begin{Bmatrix} N^H \\ M^H \end{Bmatrix} \tag{20}$$

Thus the relationship between equivalent force and moment resultants and the mid surface strains and curvatures in a laminate subjected to mechanical, thermal and hygroscopic loads has been established. These equations could be used to determine the mid surface strains and curvatures in a laminate which experiences mechanical load, temperature gradient ΔT and moisture absorption ΔC and subsequently to determine residual hygrothermal stresses which we shall discuss next.

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Hygrothermal Stresses in Laminates

where
$$\begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} = \Delta T \sum_{k=1}^{n} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} z\,dz \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k = \frac{\Delta T}{2} \sum_{k=1}^{n} [\bar{Q}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (z_k^2 - z_{k-1}^2) \rightarrow \text{Equivalent thermal moment due to } \Delta T \text{ (N-m/m)} \quad (16)$$

and
$$\begin{Bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{Bmatrix} = \Delta C \sum_{k=1}^{n} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} z\,dz \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k = \frac{\Delta C}{2} \sum_{k=1}^{n} [\bar{Q}]_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k (z_k^2 - z_{k-1}^2) \rightarrow \text{Equivalent hygroscopic moment due to } \Delta C \text{ (N-m/m)} \quad (17)$$

$$(15) \Rightarrow \begin{Bmatrix} M_x + M_x^T + M_x^H \\ M_y + M_y^T + M_y^H \\ M_{xy} + M_{xy}^T + M_{xy}^H \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ & & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ & & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (18)$$

$$(11)\ \&\ (18) \Rightarrow \begin{Bmatrix} N + N^T + N^H \\ M + M^T + M^H \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ \kappa \end{Bmatrix} \quad (19)$$

$$\Rightarrow \begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ \kappa \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} - \begin{Bmatrix} N^H \\ M^H \end{Bmatrix} \quad (20)$$

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Eqn (19) could be used to compute the hygrothermal residual stress in laminate due ΔT and ΔC. Let us consider an n –layer laminate subjected to only ΔT. Therefore, corresponding to ΔT, we could immediately calculate the equivalent $\boldsymbol{N^T}$ and $\boldsymbol{M^T}$ and corresponding to ΔC, we could calculate $\boldsymbol{N^H}$ and $\boldsymbol{M^H}$. If the laminate is subjected to any applied mechanical load then, $\boldsymbol{N}$ and $\boldsymbol{M}$ are given. Now, for the given laminate, first we compute the ABBD matrix for the laminate following the definition as discussed in CLT. Next we use Eqn (19) as follows

$$\begin{Bmatrix} \varepsilon^o \\ K \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} N + N^T + N^H \\ M + M^T + M^H \end{Bmatrix} \quad (21)$$

to determine the mid surface strains $\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}$ and curvatures $\begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$.

Knowing the mid surface strains and curvatures, strains at k^{th} (k=1,2,…,n) layer could be determined as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z_k \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \quad (22)$$

where z_k is the distance of the middle surface of the k^{th} layer from the mid surface.

Given the coefficient of thermal expansion values $\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}_k$ and coefficient of moisture expansion values $\begin{Bmatrix} \beta_2 \\ \beta_2 \\ 0 \end{Bmatrix}_k$ of the k^{th} layer, $\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k$ and $\begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k$ could be obtained by using the strain transformation matrix.

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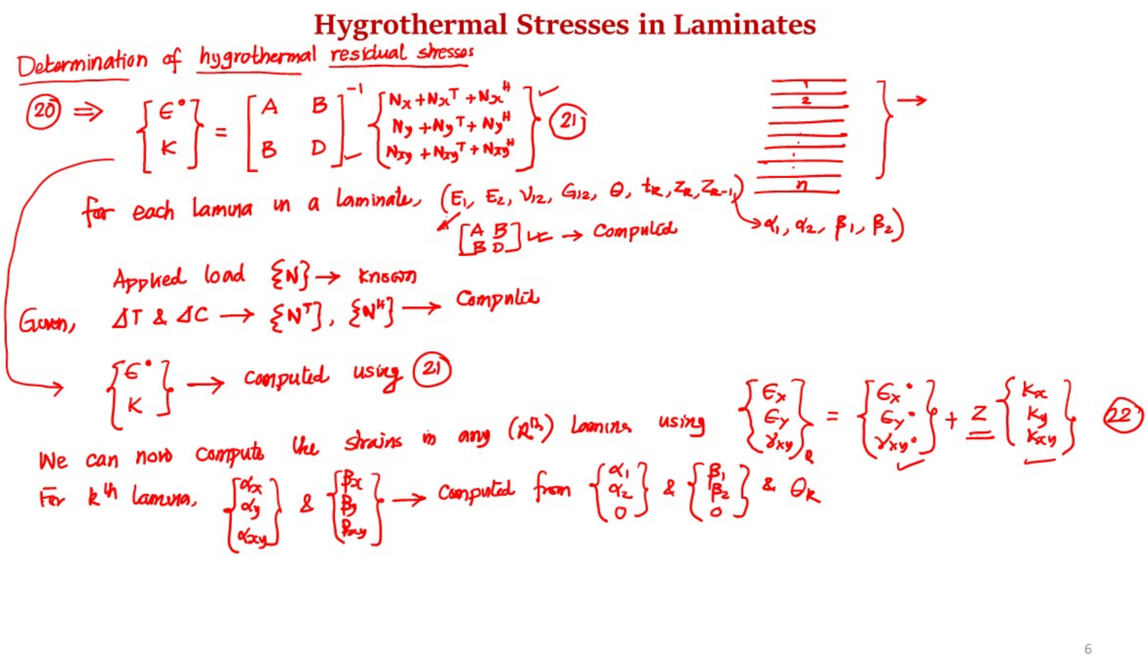

Next the free thermal strains and free hygroscopic strains of the k^{th} layer are calculated as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^T = \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \quad \text{and} \quad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^H = \Delta C \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \tag{23}$$

and the residual thermal strains in the k^{th} layer are computed as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^R = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k - \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^T \tag{24}$$

and the residual hygroscopic strains in the k^{th} layer are computed as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^R = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k - \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^H \tag{25}$$

From this residual thermal strain in k^{th} layer in the global axes, we could calculate the residual thermal stresses in the global axes as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k^R = \left[\bar{Q}\right]_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^R \tag{25}$$

The residual thermal stresses in the k^{th} layer in the material axes are then calculated as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k^R = \left[T\right]_k \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k^R = \left[T\right]_k \left[\bar{Q}\right]_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k^R \tag{26}$$

This way we can determine the residual thermal stresses in all the layers of the laminate.
Proceeding in the same way, we could also determine the residual hygroscopic stresses in all the layers.

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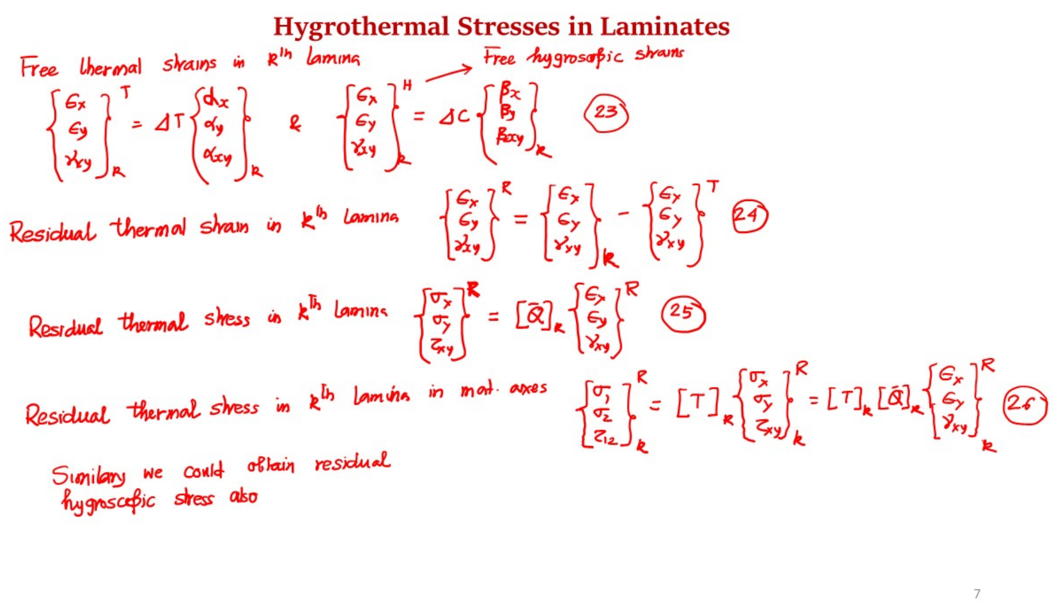

So, once we get the residual thermal stress and the residual hygroscopic stress; these are the additional stresses in a lamina besides the stresses induced because of the applied mechanical load and this needs to be added to the lamina to get the actual total stress in a lamina. Then based on those stresses only the assessment should be made whether a lamina is safe or a lamina fails.