Mechanics of Fiber Reinforced Polymer Composite Structures
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Module-07
Elastic Behaviour of Laminates-I
Lecture-17
Laminate-Introduction

Hello and welcome to the first lecture of new module which is elastic behaviour of multi-directional laminate. In the previous modules the macro and micromechanics of lamina have already been covered and in this module, the macromechanical analysis of laminate will be covered.

To start with let us first understand what a laminate is. In one of the previous lectures while introducing fiber reinforced polymer matrix composites, along with some of the important terminologies, like lamina, laminates, etc., laminate was also defined the stacking several unidirectional laminae bonded together to act as a structural element.

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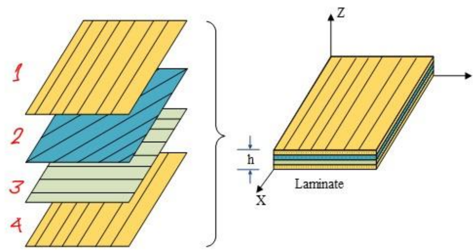
Macromechanical Analysis of Laminates

What is a LAMINATE ?

A LAMINATE is formed by stacking several UD laminae bonded together to act as a single structural elements



As shown in the Fig., there are 4 laminae stacked together to form a laminate and they are perfectly bonded, so as to act as one unit. A lamina is very thin and therefore to provide moderate thickness we stack several of them together to form a laminate. To understand why a laminate is actually required, let us revisit some of the important aspect of lamina what was discussed in micro and macromechanics of lamina.

In macromechanics of lamina, the stress strain behaviour of a lamina was developed considering the lamina to be homogeneous, represented a laminar by average properties. In micromechanics, it was discussed how these average properties are actually obtained as a function of the properties of the constituent’s fiber and the matrix and their relative proportions namely the volume fraction. One important take from micro and macromechanics of lamina is that unlike isotropic materials in an orthotropic lamina the direction dependent properties (like strengths and stiffness, etc) could be controlled by controlling some of the important variables like type of fiber, type of matrix and for a given fiber matrix combination this could be controlled by changing the fiber orientation and fiber volume fraction.

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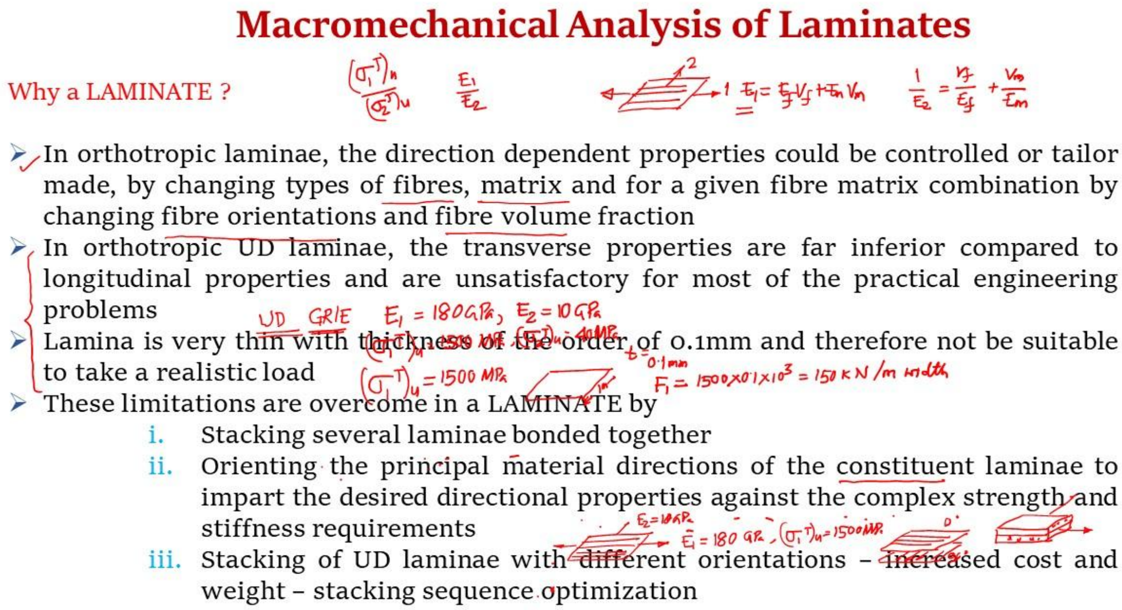

For example, E_1 which is the longitudinal Young's modulus is actually could be expressed as

$$E_1 = E_f V_f + E_m V_m$$

Now changing the fiber and the matrix and their relative proportion, E_1 could be decided as per requirement, though there are limits on the relative projections of fibers and the matrix. Similarly, E_2, the transverse Young's modulus is given by

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

Could also be controlled by choosing the fibers, matrix and their proportions. It was also understood that while the longitudinal properties are actually fiber dominated the transverse properties are matrix dominated. Therefore, one of the major advantages of this kind of composites is that the strength and stiffness requirements could be tailor made.

It was also discussed that the transverse properties of unidirectional lamina are far inferior compared to the longitudinal properties. For example, in a typical unidirectional graphite epoxy

(UD GR/E) lamina, $E_1 \approx 180$ GPa and $E_2 \approx 10$ GPa. Similarly, the longitudinal tensile strength is typically of 1500 MPa and the transverse tensile strength is only 40 MPa. The reason is while the longitudinal Young's modulus and the longitudinal tensile strengths are actually decided by the fiber, the transverse properties are actually decided by the matrix. The fibers are very strong and stiff in the longitudinal direction thereby providing very high values of strength and stiffness in the longitudinal direction. Whereas the matrix properties are inferior compared to that of the fiber and the transverse being influenced by the matrix are poorer. Many a times, in practical engineering problems they are not satisfactory that means these poor values of transverse properties do not suffice requirements of many engineering problems.

Then lamina is very thin and by itself cannot be used to take a realistic load. For example, considering a GR/E lamina with ultimate longitudinal tensile strength of 1500 MPa, a lamina which is say 1 m width and say the thickness of 0.1 mm, the maximum force in the longitudinal direction it can take is only 150 kN (1500 N/mm^2 × 0.1 mm × 10^3 = 150 kN/m). These two limitations are actually overcome in a laminate by stacking several laminae bonded together and then orienting the principal material directions of the constituent lamina to impart the desired directional properties against the complex strength and stiffness.

Say for example say we want a component which is equally strong and stiff in both the longitudinal and transverse direction. So, if we take a lamina suppose you want this to be equally strong and stiff both in the longitudinal and transverse direction. If we take a 0° lamina and a 90° lamina and are perfectly bonded at the interface and they form a 2-layer laminate (generally, laminates will be having a large number of layers, but just for the sake of example suppose we have a 2-layer laminate) where one layer is 0° and the other layer is 90° fiber orientation. Then

by just combining 0° and 90° fiber orientation, we could get a component which is equally stiff and strong in both the directions. Similarly, combinations of different fiber orientation stacked together will suffice the complex strength and stiffness requirements and stacking number of laminae together also provides a reasonable thickness which could be actually used as a component in actual engineering applications unlike a lamina which is very thin.

However, the stacking of unidirectional lamina with different orientations increases the cost and the weight. So, optimizing the stacking sequence is also an important problem where the objectives are to minimize the weight and minimize the cost. So, this is in brief why a laminate is required.

It may be noted that the basic elements in a laminate are nothing but large number of laminae stacked together. Therefore, understanding the behavior of laminae which has been studied in micro and macromechanics is important to understand the overall behaviour of the laminate.

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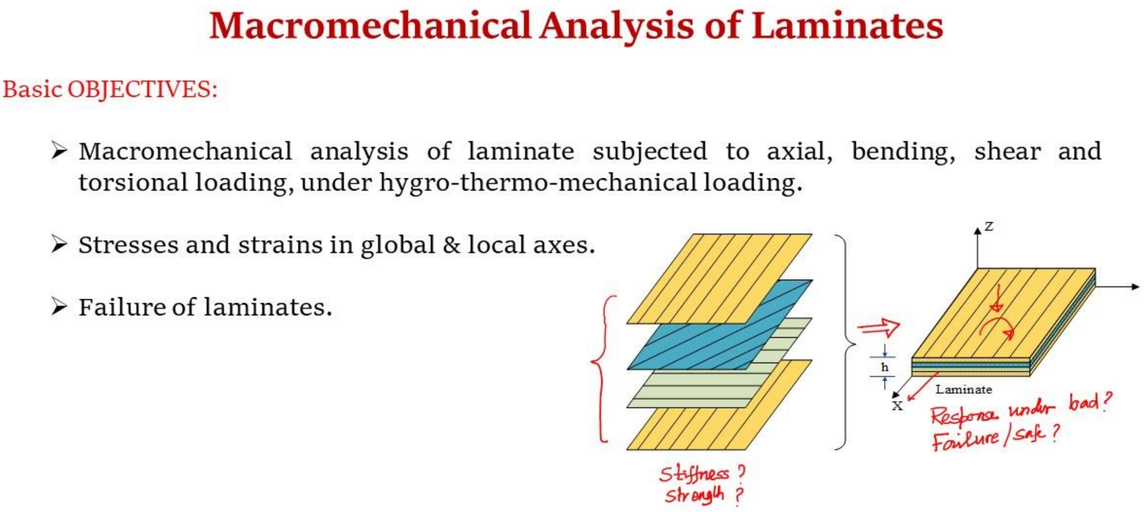

The basic objectives of the macromechanical analysis of laminate is that subjected to load (axial load or bending load or shear load or torsional load or it may be in a purely mechanical or maybe combined hygro-thermo-mechanical loading) it is important to know how a laminate responds, meaning what are the stress-strains both in the global and local axis. Because only knowing the stresses and strains in each lamina, appropriate failure criterion to each lamina could be applied to know the safety of the laminae forming the laminate and then take a decision whether the laminate is safe or not.

Therefore, in macromechanical analysis of the laminate, it is important to know the stiffness of the laminate, the strength of the laminate and these are functions of the stiffness and strength of the constitutent laminae. So, these are the basic questions to be answered while discussing the macromechanical analysis of a laminate.

Because a laminate is actually made up of large number of laminae stacked together, therefore, the overall stiffness of the laminate or the strength of the laminate or the hygrothermal properties of the laminate are actually dependent on the corresponding strength, stiffness and properties of the constituent laminae.

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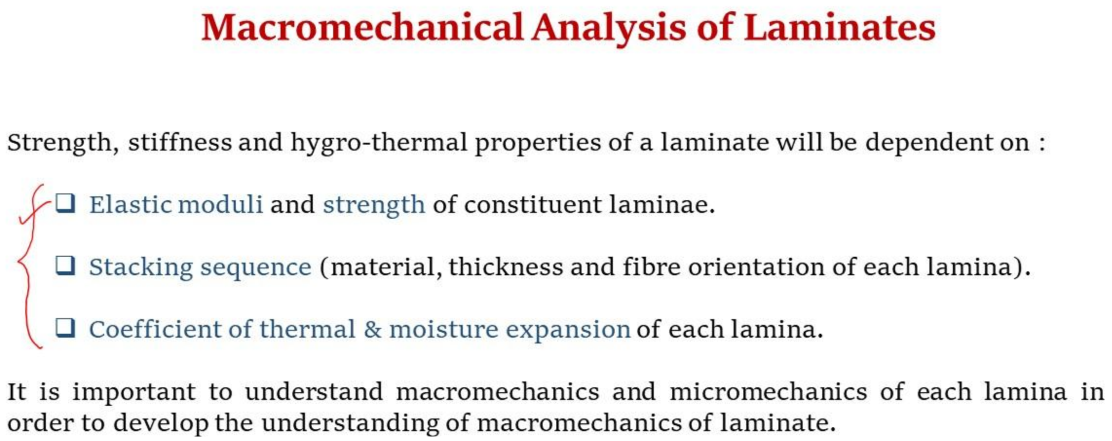

Then stacking sequence of laminae in a laminate means the material of each lamina, thickness of each lamina, the fiber orientation of each lamina and the order in which the laminae are stacked and the strength and stiffness of the laminate is decided by the strength and stiffness of each lamina and how they are stacked. Therefore, it is important that to understand the macromechanical and micromechanical behaviour of the unidirectional lamina to establish the behaviour of a laminate under given loading conditions.

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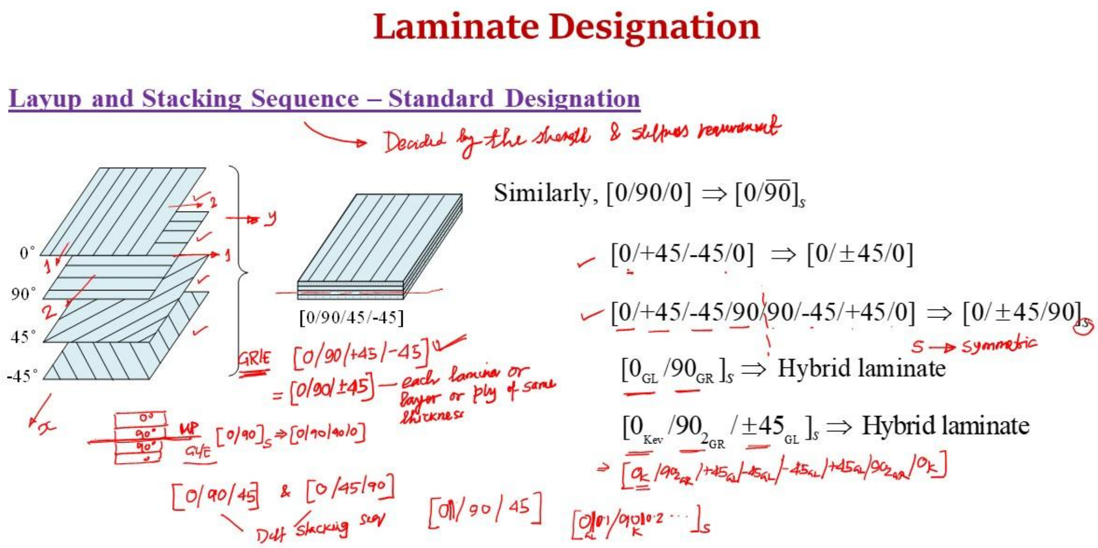

Let us first understand how a laminate is actually designated or specified. There are standard way of designating a laminate layup and stacking sequence. If four laminae say made of graphite/epoxy with fiber orientations 0°, 90°, +45°, -45° are stacked together, then it is designated as [0°/90°/+45°/-45°] graphite/epoxy (GR/E) laminate. Again say

$[0_4°/90_2°/+45°/-45°]$ means four 0° layers, two 90° layers followed by one +45° and one −45° layer. These fiber orientation angles are with reference to the laminate x-axis as shown.

Until or unless mentioned, the layers/laminae are of the same material and each layer/lamina is of same thickness. Sometimes, a laminate may have layers which are symmetrically stacked with reference to the mid surface, for example the laminate [0°/90°/+45°/−45°/−45°/+45°/90°/0°] and is designated as $[0°/90°/+45°/-45°]_s$ where the suffix 's' stands for symmetric laminate. The significance of such laminate will be discussed in details later in this course.

Laminates may be single material laminate, where all the layers are made of same material say a $[0°/90°]_s$ glass epoxy laminate. There are examples where the layers are made from more than one material. For example $[0°_{Kev/E}/90_{Gl/E}°]_s$ consists of in this case it is 0° kevlar epoxy and 90° glass epoxy layers to form a hybrid laminate.

A hybrid laminate may be a requirement. Say many a times the outer layer is made with kevlar epoxy an impact load is anticipated as Kevlar epoxy has better impact properties. Note that layup and stacking sequence are different. Two laminates, [0/90/45] and [0/45/90] have same layup but their stacking sequences are different.

Note that the stacking sequence of a laminate is decided by the strength and stiffness requirement whether it will be [0/90/0] or [0/+45/-45/90/0] is decided by the strength and stiffness requirements.

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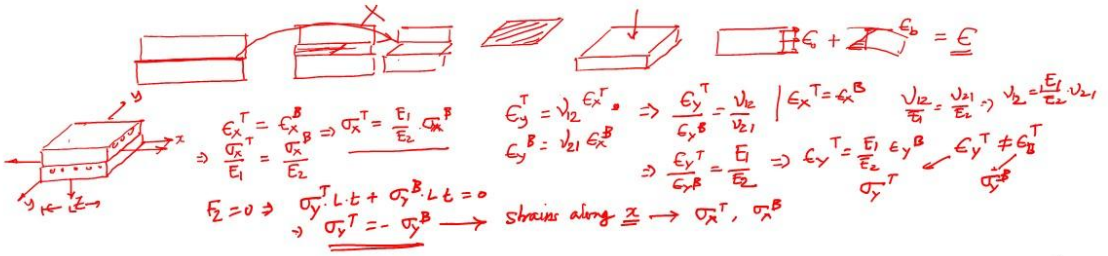
Classical Lamination Theory (CLT)

- Perfect bonding of laminae
- Bond is infinitesimally thin and NOT shear deformable
- Laminae can not slip over each other
- Behaviour of UD laminae subjected to in-plane loading are utilized to establish the behaviour of layered laminate
- UD laminae were subjected to only in-plane load,, the laminate may be subjected to both in-plane and transverse load
- In-plane strain and strain due to transverse load are superposed



6

This knowledge of macromechanical analysis of lamina and micromechanical analysis of lamina, the stress strain behaviour of unidirectional for specially orthotropic or for angle lamina (where the material axis are oriented differently with respect to analysis axis or global axis) and the strength failure theories for such lamina are actually combined to establish the average stiffness or the response of a laminate under loading in classical lamination theory.

Now in the laminate we always assume that it is perfectly bonded. And that means if we consider two successive layers or lamina or ply they are perfectly bonded and the interface is infinitely thin and not shear deformable, meaning, that the adjacent layers cannot slip over each other.

Before going to classical lamination theory let us understand that how actually the laminate stress analysis could be done.

For example considering a simple two layer laminate with one 0° layer and one 90° layer with x-y-z fixed at the mid surface as shown. Suppose the laminate is subjected to a load along x-direction. Then the total load will be shared by the 0° and 90° and that will be proportional to their stiffnesses. Because they are perfectly bonded therefore the strain along x- of the top layer and the strain along x- of the bottom layer must be same. Therefore,

$\varepsilon_x^T = \varepsilon_x^B$ and using Hooke's law,

$$\frac{\sigma_x^T}{E_1} = \frac{\sigma_x^B}{E_2} \Rightarrow \sigma_x^T = \frac{E_1}{E_2}\sigma_x^B$$

This is the relationship between the stress along x direction of the top and the bottom layer. Now there will be transverse strains along y due to Poisson's effect and are

$$\left.\begin{aligned} \varepsilon_y^T &= \nu_{12}\varepsilon_x^T \\ \varepsilon_y^B &= \nu_{21}\varepsilon_x^B \end{aligned}\right\} \text{ which implies}$$

$$\frac{\varepsilon_y^T}{\varepsilon_y^B} = \frac{\nu_{12}}{\nu_{21}} = \frac{E_1}{E_2} \text{ and hence}$$

$$\varepsilon_y^T = \frac{E_1}{E_2}\varepsilon_y^B \rightarrow \boxed{\varepsilon_y^T \neq \varepsilon_y^B}$$

Therefore, the strains in the direction y- are not same in the top and the bottom layer. Bottom and top layers are perfectly bonded, but if the strains are not same, therefore there will be stresses, this leads to the stress in the y direction of the top layer and the stress in the y direction of the bottom layer. But the net force in the y direction is zero. Since the net force along y-direction is zero, so

$$F_2 = 0 \rightarrow \sigma_y^T.L.t + \sigma_y^B.L.t = 0 \rightarrow \boxed{\sigma_y^T = -\sigma_y^B}$$

So, because of this stress there will be strains along x- and those strains again will not be equal that will lead to stresses. These stresses need to be added to the applied stresses and this keeps on continuing till we converge.

Considering a multilayer laminate having different orientations, it will be a complicated analysis. Classical lamination theory, provides a simpler way combining equilibrium equations the compatibility of the laminate as a whole. Classical lamination Theory (CLT) is based on some important assumptions as**(Refer Slide Time: 37:51)**

Classical Lamination Theory (CLT)

Assumptions:

1. Each layer is homogeneous and orthotropic.
2. Laminate is thin and its lateral dimension is much larger compared to its thickness and the laminate is loaded in its plane only i.e the laminae are in the plane stress state ($\sigma_z = \tau_{xz} = \tau_{yz} = 0$).
3. All displacements are small compared to the laminate thickness .
4. Displacements are continuous throughout the laminate.
5. In plane displacements vary linearly along the thickness of laminate i.e are linear function of z.
6. Transverse shear strains γ_{xz} and γ_{yz} are negligible. This along with assumption 5 implies that a straight line perpendicular to the middle surface remains straight and perpendicular after deformation.
7. Strain displacement and stress-strain relations are linear.
8. ϵ_z is negligible compared to ϵ_x and ϵ_y.