**Mechanics of Fiber Reinforced Polymer Composite Structures**
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**Module-06**
**Macromechanics of Lamina-II**
**Lecture-16**
**Experimental Evaluation**

Hello and welcome to the 3rd lecture of 6th module, we have been discussing the micromechanics of lamina. In the last few lectures, we understood how the properties of the constituents (fiber and matrix) influence the properties of lamina. We also understood different approaches in micromechanics like mechanics of material approach, elasticity approach, variational approach, semi empirical methods and the mechanics of material approach has been discussed in details.

In the last lecture, using the elasticity approach, we could understand the fiber matrix interaction, the role of the matrix, especially, when a fiber breaks, the fiber micro buckling mode of failure of laminates under compression load. In today's lecture will first solve two problems to determine the lamina properties in terms of the matrix and fiber properties and then we will discuss in brief the experimental determination of the elastic moduli and the strength parameters of lamina. To start with, let us take a simple problem.

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## Experimental Determination of Elastic Moduli and Strengths

Example: In an UD lamina made of Glass/Epoxy, with $E_m$=3GPa, $E_{2f}$=70GPa, $V_f$=0.5, the transverse Young's modulus is 6 GPa. Using Halpin-Tsai relation, determine the transverse Young's modulus of a lamina made of same materials with volume fraction $V_f$=0.65.

Solution Given Epoxy → $E_m = 3$ GPa, Glass Fiber → $E_{2f} = 70$ GPa $\Rightarrow V_f = 0.5 \rightarrow E_2 = 6$ GPa

Halpin Tsai $\dfrac{E_2}{E_m} = \dfrac{1+\xi\eta V_f}{1-\eta V_f}$, $\eta = \dfrac{E_{2f} - E_m}{E_{2f} + \xi E_m}$ and $\xi \Rightarrow$ Reinforcing factor

- Fiber geometry
- Packing "
- Loading condition

Relative Youngs modulus

i) $\dfrac{E_{2f}}{E_m} = 1 \Rightarrow \eta = 0$ ← homogeneous

(ii) $\dfrac{E_{2f}}{E_m} \to \infty \Rightarrow \eta = 1$ ← rigid inclusion

(iii) $\dfrac{E_{2f}}{E_m} = 0 \Rightarrow \eta = \dfrac{1}{\xi}$ ← void

$V_f = 0.5$, $E_2 = 6$ GPa $\Rightarrow \xi = ?$

$\Rightarrow \xi = 1.2 \Rightarrow \eta = 0.91$

for $V_f = 0.65$, $\eta = 0.91$, $\xi = 1.2$

$\dfrac{E_2}{E_m} = 2.74 \Rightarrow E_2 = 2.74\,E_m$
$\Rightarrow E_2 = 8.2$ GPa

**Example:** In an UD lamina made of Glass/Epoxy, with $E_m$=3GPa, $E_{2f}$=70GPa, $V_f$=0.5, the transverse Young's modulus is 6 GPa. Using Halpin-Tsai relation, determine the transverse Young's modulus of a lamina made of same materials with volume fraction $V_f$=0.65.

**Solution:**

Given the Young's modulus of epoxy $E_m$ = 3 GPa and the transverse Young's modulus of glass fiber, $E_{2f}$ = 70 GPa and given that a lamina is made with the fiber volume fraction of $V_f$= 50% that results in the transverse Young's modulus of the composite $E_2$ = 6 GPa. Now using these values in

$$\frac{E_2}{E_m} = \frac{1+\xi\eta V_f}{1-\eta V_f}$$

and $$\eta = \frac{\frac{E_{2f}}{E_m} - 1}{\frac{E_{2f}}{E_m} + \xi}$$ we get, $\xi$ = 1.2 and $\eta$ = 0.91.

Putting these values of $\xi$ = 1.2 and $\eta$ = 0.91 in $\frac{E_2}{E_m} = \frac{1+\xi\eta V_f}{1-\eta V_f}$ , we get corresponding to $V_f$ = **60%, w**e get $\frac{E_2}{E_m} = 2.74$ and therefore, $E_2 = 2.74E_m = 8.2\ \text{GPa}$ . $\xi$ is the reinforcing factor actually depends upon the fiber geometry, packing geometry and of course, the loading condition and this $\eta$ is depends upon the relative Young's modulus of the fiber and the matrix. What is the physical significance?

Suppose $E_{2f}$ / $E_m$ or $E_f$/ $E_m$, = 1, it implies that $\eta$ = 0. This is actually for homogeneous material. Suppose $E_{2f}$ / $E_m$ is very large, say $E_{2f}$ / $E_m$ tends to infinity. In that case $\eta$ = 1. This actually represents rigid inclusions in a medium.

Suppose $E_{2f}$/ $E_m$, or $E_f$/ $E_m$ = 0. It means, if you put that here you get $\eta$ = 1/$\xi$. This actually represents void. That means there is no fiber.

Using Halpin-Tsai equation, we could see that when the fiber volume fraction increases from 0.5 to 0.65, the transverse Young's modulus increases from 6 GPa to 8.2 GPa. Suppose we use the mechanics of material approach for the same.

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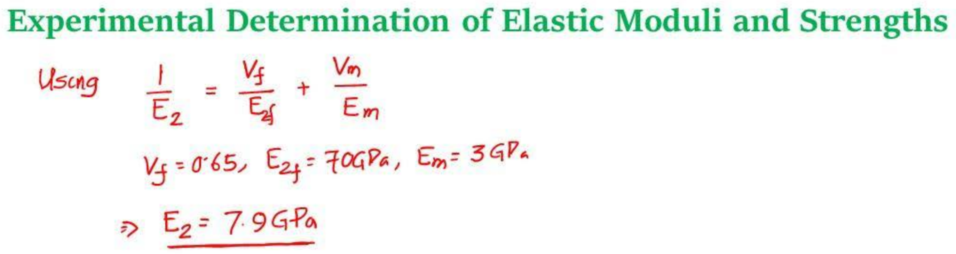


Using mechanics material approach $1/E_2 = V_f/E_{2f} + V_m/E_m$. If we use this put $V_f = 0.65$, $E_{2f} = 70$ GPa, $E_m = 3$ GPa. That gives us $E_2 = 7.9$ GPa. So, there is a difference between what we get using mechanics of material approach and what we get using the Halpin-Tsai relationships.

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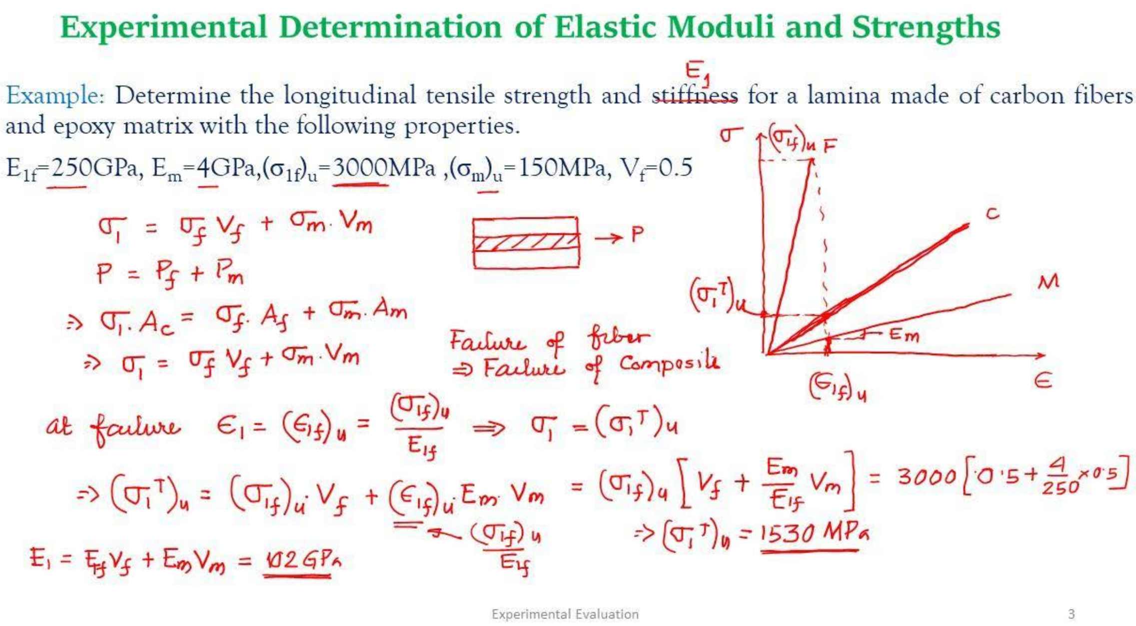


Next, let us take another problem as

**Example:** Determine the longitudinal tensile strength and stiffness for a lamina made of carbon fibers and epoxy matrix with the following properties.

$E_{1f}$=250GPa, $E_m$=4GPa,$(\sigma_{1f})_u$=3000MPa ,$(\sigma_m)_u$=150MPa, $V_f$=0.5

**Solution:**

Solution to this problem is important to understand the failure of a lamina especially longitudinal tensile failure. Here we have to find out the longitudinal tensile strength and longitudinal Young's modulus of lamina made of carbon fibers and epoxy matrix with the following properties as $E_{1f}$=250GPa, $E_m$=4GPa,$(\sigma_{1f})_u$=3000MPa ,$(\sigma_m)_u$=150MPa, $V_f$=0.5.

Fig shows the stress strain curve for the matrix and the stress strain curve for the fiber (not to scale) and the stress strain curve of the composite in between. The slope of stress strain curve of the composite relative to fiber and matrix is decided by the volume fraction. If we keep on increasing the volume fraction, it will be more towards the fiber and if we keep on lowering the volume fraction it will be more towards the matrix. Now if P is load on the composite, it is shared by the fiber ($P_f$) and the matrix ($P_m$) as

$$P = P_f + P_m$$
$$\sigma_1 A_c = \sigma_f A_f + \sigma_m A_m$$

So, the stress in the composite can be written in terms of those in the fiber and the matrx and the volume fractions as

$$\sigma_1 = \sigma_f V_f + \sigma_m V_m$$

Now, we consider that failure of fiber implies the failure of composite or lamina, the strain in the composite at failure is $\varepsilon_1 = (\varepsilon_1^f)_u$ and $(\varepsilon_1^f)_u = (\sigma_1^f)_u / E_{1f}$ following Hooke's law. The strain in the matrix at the failure point is this is $\sigma_m = (\varepsilon_1^f)_u E_m$ . So,

$$\varepsilon_1 = (\varepsilon_{1f})_u = \frac{(\sigma_{1f})_u}{E_{1f}} \Rightarrow \sigma_1 = (\sigma_1^T)_u \; .$$

Therefore, the ultimate longitudinal tensile stress of the lamina is

$$(\sigma_1^T)_u = (\sigma_{1f})_u V_f + (\varepsilon_{1f})_u E_m V_m = (\sigma_{1f})_u [V_f + \frac{E_m}{E_{1f}} V_m] \quad \text{and}$$

$$E_1 = E_f V_f + E_m V_m$$

So, if we put all these values $E_{1f}$=250GPa, $E_m$=4GPa,$(\sigma_{1f})_u$=3000MPa ,$(\sigma_m)_u$=150MPa, $V_f$=0.5, we get $(\sigma_1^T)_u$ = 1530 MPa and $E_1$=102 GPa.

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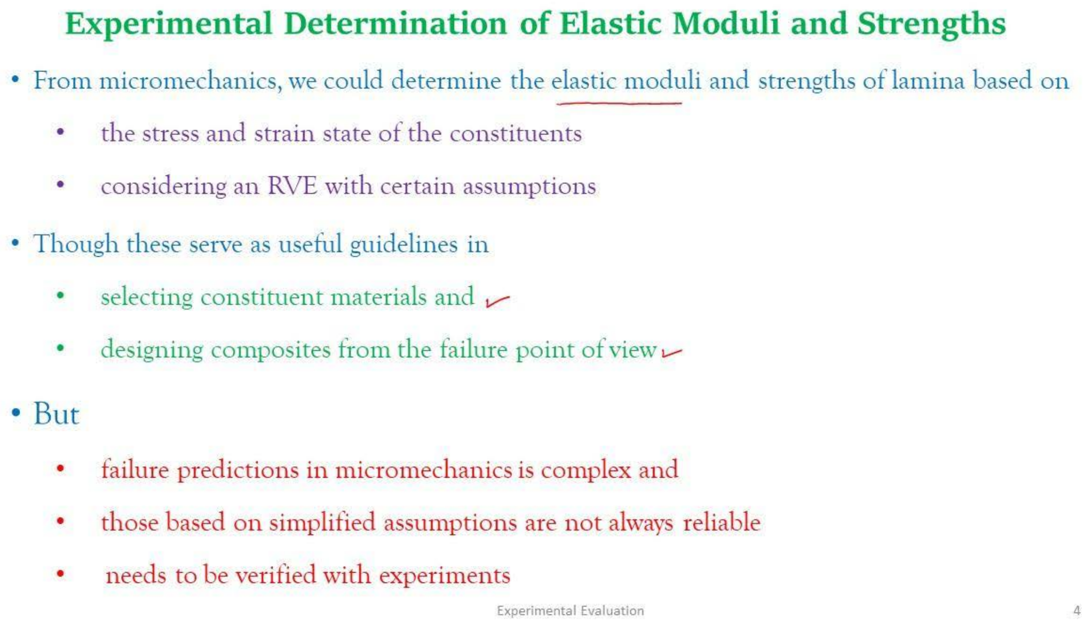


So, from micromechanics we could actually determine the elastic moduli and strength of lamina based on certain stress strain state of the constituents and considering representative volume element with certain assumptions like uniform fiber spacing, fibers are of regular shape and size, perfect bonding, etc.. With those assumptions, we could get certain expressions for elastic moduli of lamina and strengths of lamina in terms of the properties of the fiber and the matrix and volume fraction.

So, these expressions give us an initial guideline to select what kind of the fiber and the matrix need to be selected, with what volume fraction to achieve certain properties of the lamina. However, failure prediction of micromechanics is actually complex, there are interactions between the fiber and the matrix and therefore, those models based on simplified assumptions like the mechanics of material approach are not always reliable. Therefore, they need to be verified with experiments. Therefore, we need experimental determination of this elastic moduli and the strength properties.

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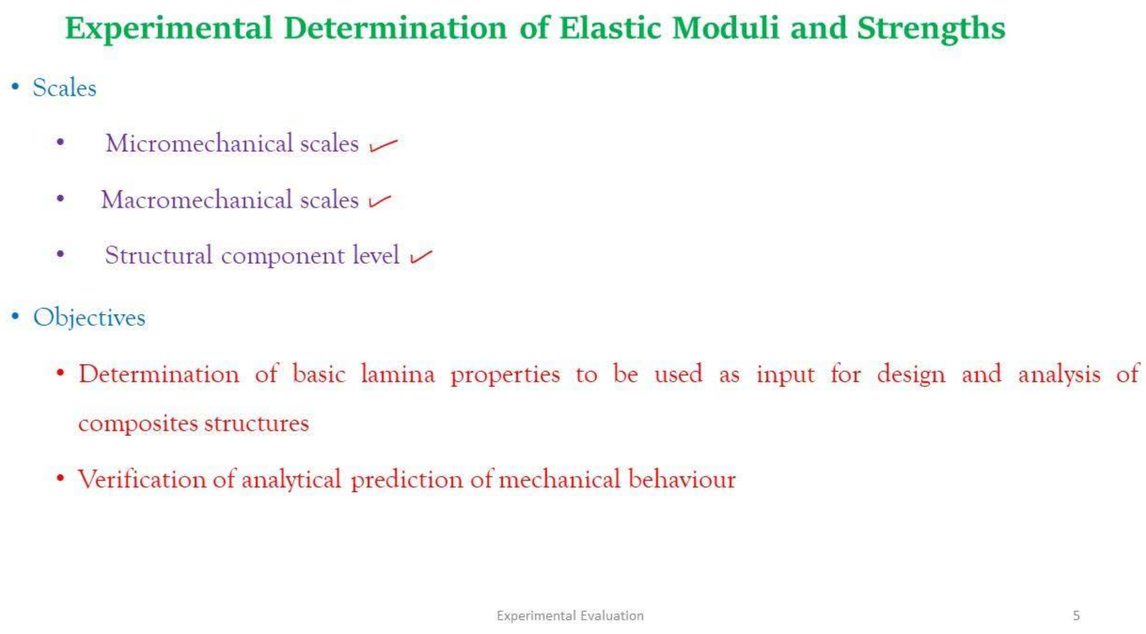


Now, in experimental determination of elastic moduli and strengths, experiments could be conducted at different scales viz. at micromechanical scales, macromechanical scales or at the structural component level. In micromechanical scales basically experiments are performed to determine the constituent properties say the Young's modulus of the fiber, the strength of an individual fiber, the other properties of the constituents and maybe the interface properties. On the other end in macromechanical scale, we find out the strength and elastic moduli of the lamina by conducting tests. The elastic moduli and the strength could be determined at structural component level also.

So, we will discuss in brief the macromechanical scales experiments and the objectives of those experiments are first, to determine the basic lamina properties to be used as input for design and analysis of composite structures. Second, the results are used to verify the prediction of mechanical behaviour. Suppose, we use certain relations to determine the elastic moduli or strength of a lamina we can also verify that how well they actually agree with the experimental results.

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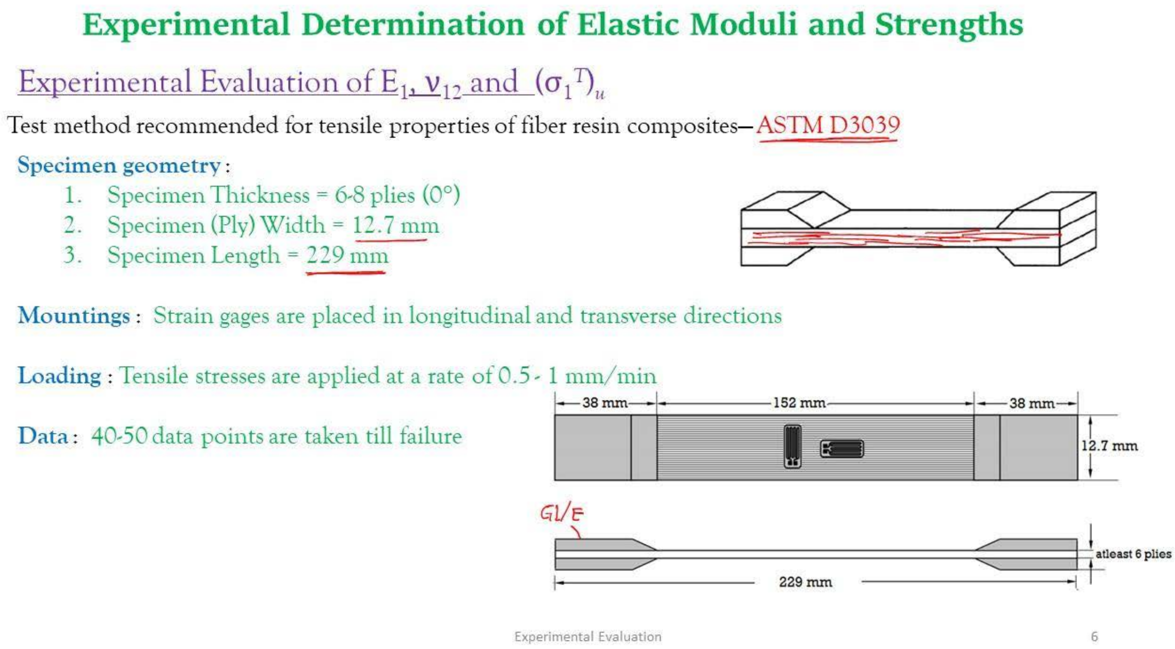
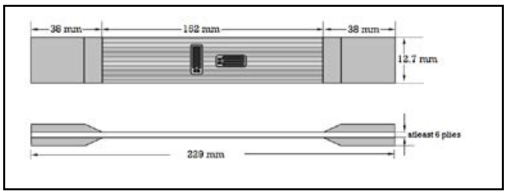



**Experimental Evaluation of $E_1$, $\nu_{12}$ and $(\sigma_1^T)_u$**

Test method recommended for tensile properties of fiber resin composites— ASTM D3039 gives us the full details of this experiment regarding conducting these experiments, the specimen dimensions, loading. All the details are available in ASTM and are not discussed here.




**Specimen geometry** :

1. Specimen Thickness = 6-8 plies (0°)
2. Specimen (Ply) Width = 12.7 mm
3. Specimen Length = 229 mm

**Mountings** : Strain gages are placed in longitudinal and transverse directions

**Loading** : Tensile stresses are applied at a rate of 0.5 - 1 mm/min

**Data** : 40-50 data points are taken till failure

Two strain gauges are mounted, one along the longitudinal direction and another along the transverse direction. One will measure the longitudinal strain and other will measure the transverse strain which is required to determine the Poisson's ratio. The specimen is loaded in UTM and 40 - 50 data for stress and strains are recorded till it fails.

Now, the slope of the $\sigma_1$ - $\varepsilon_1$curve will give us longitudinal Young's modulus, ie $E_1 = \sigma_1/\varepsilon_1$ and the slope of $\varepsilon_2$ - $\varepsilon_1$ gives us the Poisson's ratio ie. $\nu_{12} = -\varepsilon_2 / \varepsilon_1$ . At failure the stress value is the longitudinal tensile strength $(\sigma_1^T)_u$.

Now, observing the failure in a large number of tests for glass/epoxy, it is observed that the failure could be the brittle fracture of the fiber when the volume fraction is less than 40% and in the intermediate volume fraction between 40% to 65% brittle fracture of fiber and associated fiber pullout are also observed as shown in the Fig. At a high-volume fraction, there is fiber matrix de-bonding. So, from a single test, we could obtain the longitudinal Young's modulus $E_1$, major Poisson's ratio $\nu_{12}$ and longitudinal tensile strength $(\sigma_1^T)_u$.

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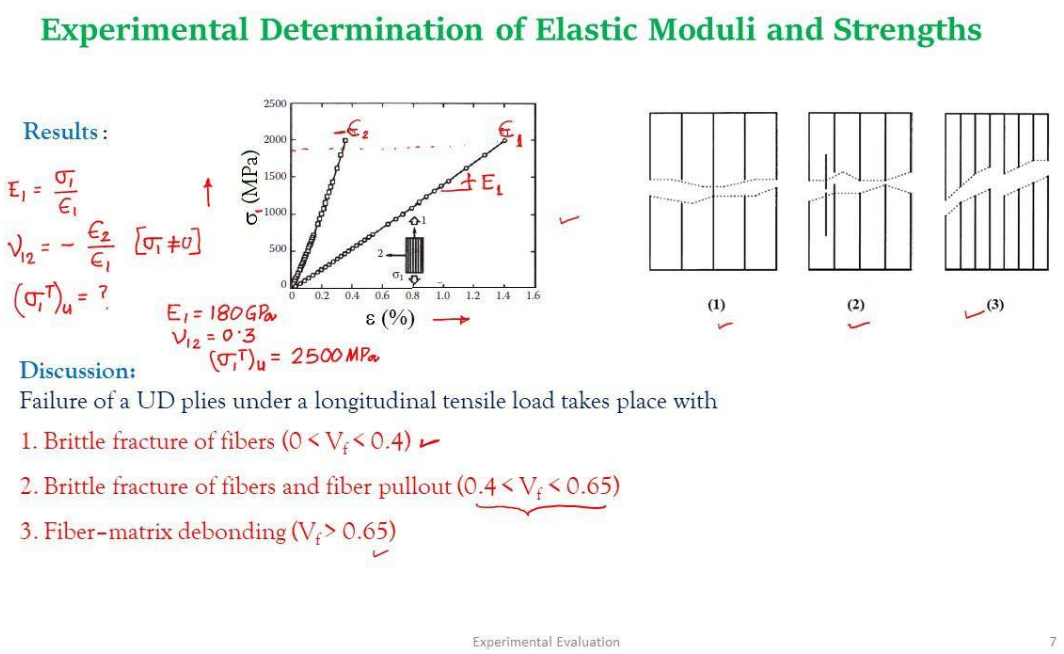
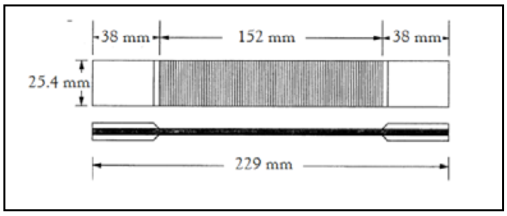



## <u>Experimental Evaluation of $E_2$, $\nu_{21}$ and $(\sigma_2^T)_u$</u>

The procedure for finding the $(\sigma_2^T)_u$ is the same as for finding the $(\sigma_1^T)_u$ . Only the specimen dimensions differ. Compared to the longitudinal tensile testing specimen, the width of the specimen is double and the thickness of the specimen is also more, 8 -16 plies of 90° because we are trying to find out the transverse properties




**Specimen geometry** :

1. Specimen Thickness: 8-16 plies (90°)
2. Specimen (Ply) Width = 25.4 mm

3. Specimen Length = 229 mm

**Mountings** : Strain gages are placed in longitudinal and transverse directions

**Loading** : Tensile stresses are applied at a rate of 0.5 - 1 mm/min

**Data** : 40-50 data points are taken till failure

From this test, the transverse Young's modulus $E_2$, minor Poisson's ratio $\nu_{21}$ and the transverse tensile strength $(\sigma_2^T)_u$ could be obtained.

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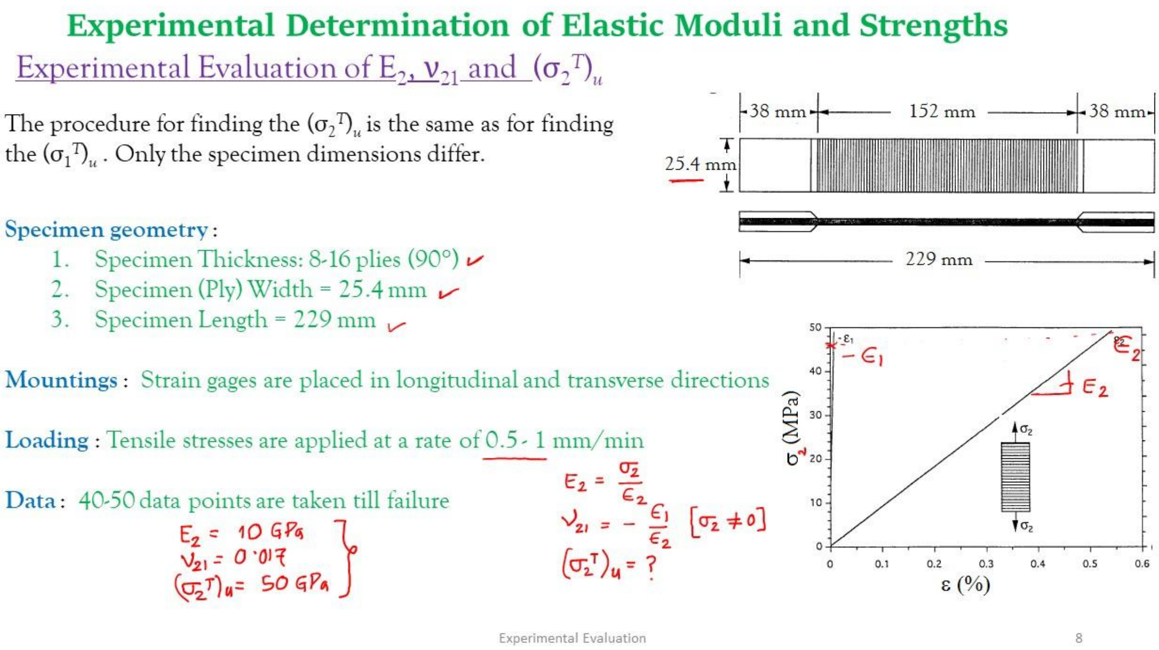


Again, for a typical graphite epoxy lamina we get say $E_2 = 10$ GPa, $\nu_{21} = 0.017$ and ultimate transverse tensile strength $(\sigma_2^T)_u = 50$ GPa. The transverse properties predicted by the mechanics of materials approach do not agree well with the experiments. This is due to the fact that, in addition to the properties of the fiber and the matrix, it is also decided by the bond strength between the fiber and the matrix. Also, presence of void drastically changes the transverse properties and during manufacturing if there is a residual stress that also influences the transverse strength and modulus.

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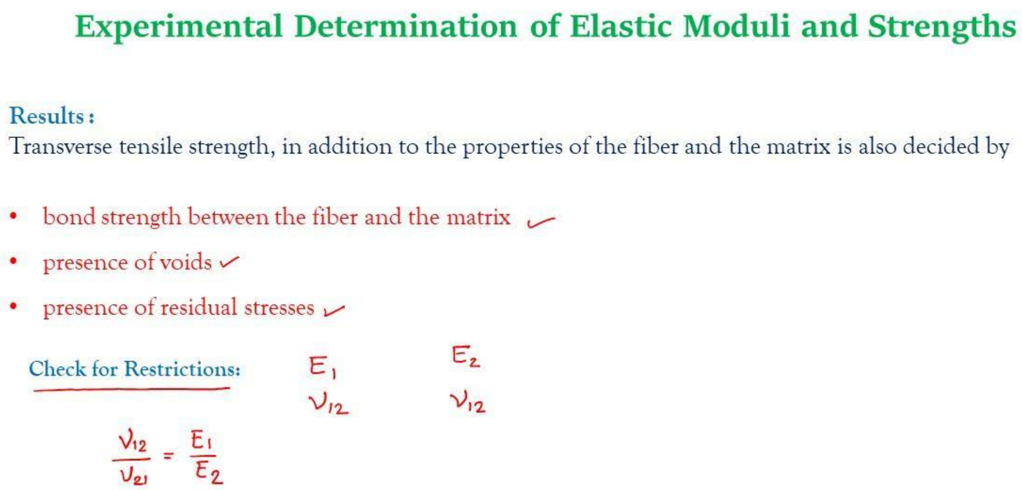


Knowing that $\nu_{12}/\nu_{21} = E_1/E_2$, we could check that whether this followed or not. Similarly, other restrictions discussed in macromechanical analysis of lamina could be checked after we determine the properties so these results could be used with confidence.

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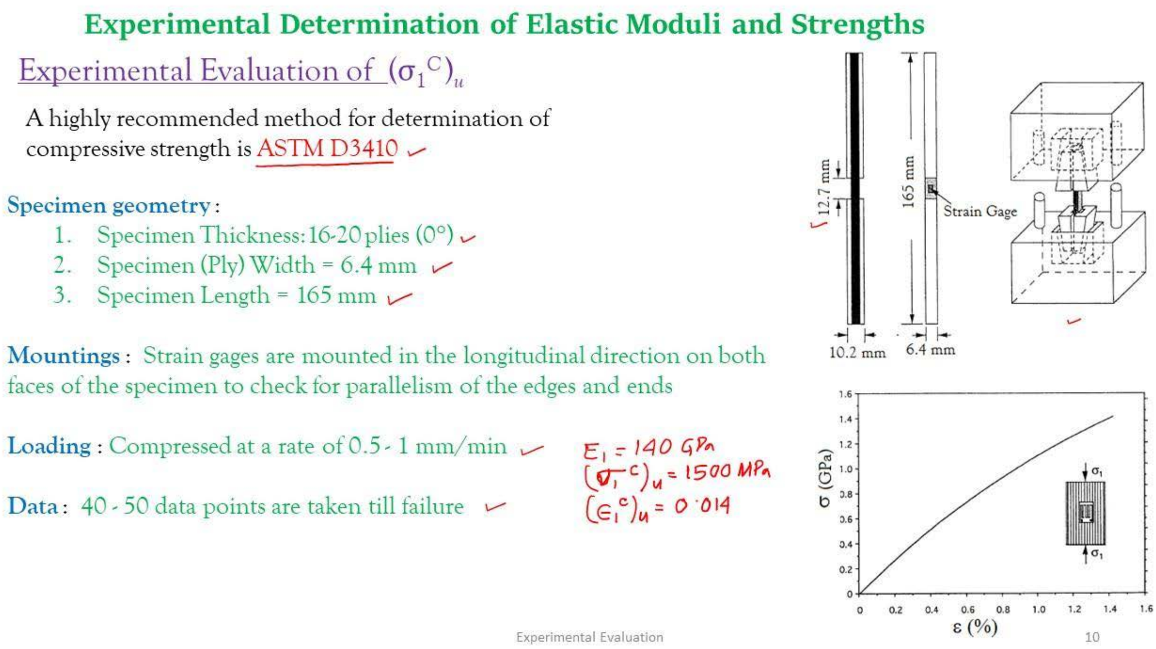


## <u>Experimental Evaluation of $(\sigma_1^C)_u$</u>

Again, one of the most recommended methods is following ASTM D3410. Full details of this could be found in ASTM D3410.

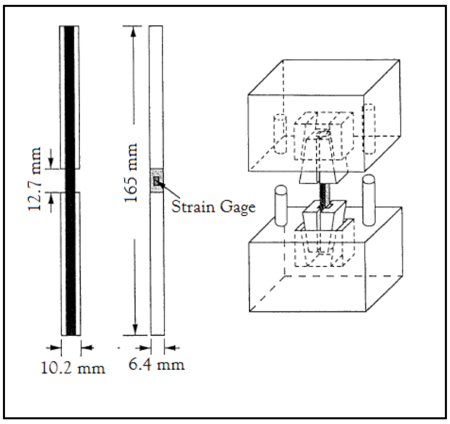
**Specimen geometry** :

1. Specimen Thickness:16-20 plies (0°)
2. Specimen (Ply) Width = 6.4 mm
3. Specimen Length = 165 mm

**Mountings** : Strain gages are mounted in the longitudinal direction on both faces of the specimen to check for parallelism of the edges and ends

**Loading** : Compressed at a rate of 0.5 - 1 mm/min

**Data** : 40 - 50 data points are taken till failure




In this case the specimen is thicker with 16 – 20, 0° plies because we are trying to find out the longitudinal compression strength. The plies width is 6.4 mm, specimen length is 165 mm, but except 12.7 mm the most of the length of the specimen are actually gripped to put constraints on almost on the major portion of the length of the specimen to eliminate buckling, otherwise without sufficient constraint it might buckle. Strain gauges are mounted in the longitudinal direction on both faces of the specimen to ensure that they are parallel which is ensured by identical readings of the strain gages on both the faces.

So, for a typical graphite epoxy we get $E_1$ = 140 GPa. Then $(\sigma_1^c)_u$ =1500 MPa and $(\varepsilon_1^c)_u$ = 0.014. So, this is a typical for a graphite epoxy you obtain from longitudinal compression.

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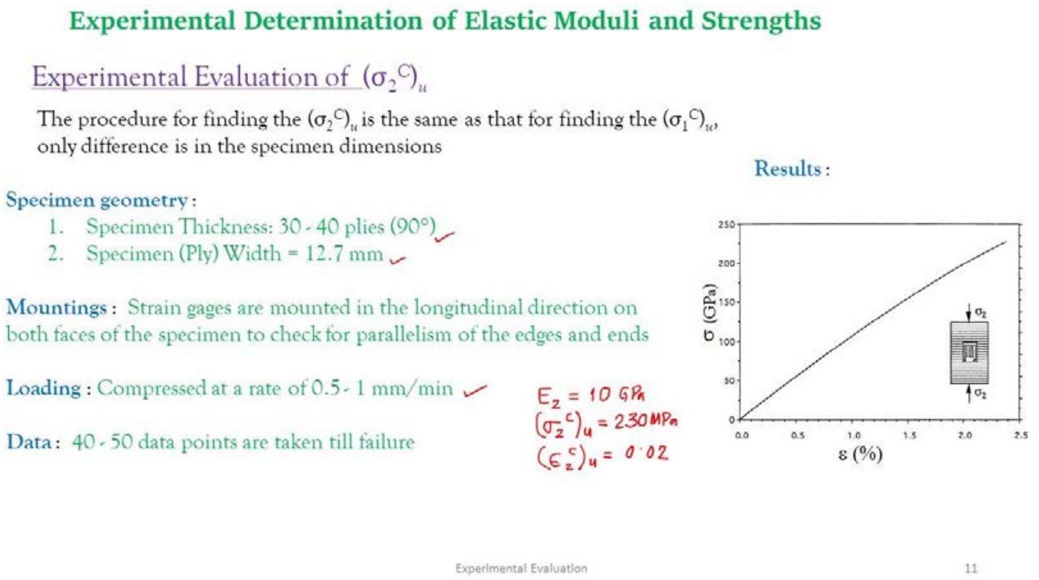


## <u>Experimental Evaluation of</u> $(\sigma_2^C)_u$

Similarly, we can also have the transverse compression, the procedure is same again the specimen is little different. So, here the thickness is 30 – 40, 90° plies and the ply width is also little more it is 12.7 mm. Strain gauges are mounted in the longitudinal direction on both faces to check for parallelism and till failure 40 to 50 data points are taken.

**Specimen geometry** :

1. Specimen Thickness: 30 - 40 plies (90°)
2. Specimen (Ply) Width = 12.7 mm

**Mountings** : Strain gages are mounted in the longitudinal direction on both faces of the specimen to check for parallelism of the edges and ends

**Loading** : Compressed at a rate of 0.5 - 1 mm/min

**Data** : 40 - 50 data points are taken till failure

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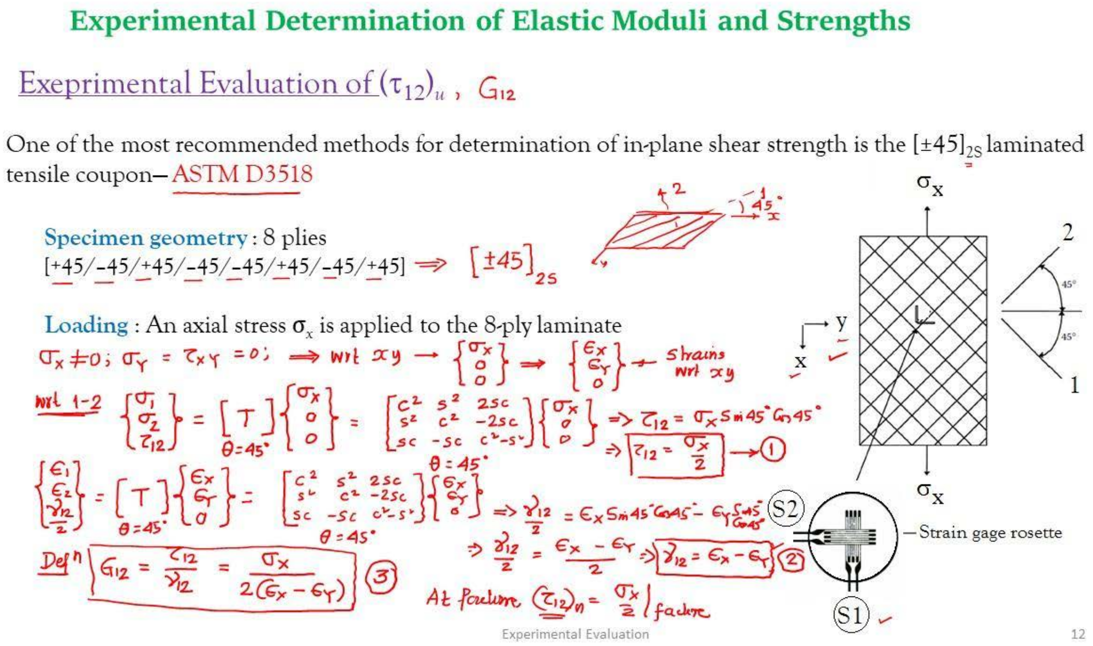
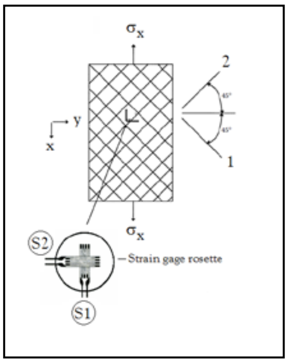



**Exeprimental Evaluation of $(\tau_{12})_u$**

One of the most recommended method for determination of in-plane shear strength is using a $[\pm 45]_{2S}$ meaning that there are [± 45] lamina stacked in a sequence like this +45/−45. '2' means this sequence is repeated twice and S means, it is symmetric. So, the laminate is +45/−45/+45/−45/−45/+45/−45/+45 following ASTM D3518.




**Specimen geometry** : 8 plies

[+45/–45/+45/–45/–45/+45/–45/+45]

**Loading** : An axial stress $\sigma_x$ is applied to the 8-ply laminate

That means it is only subjected to $\sigma_x$.

The objective is to determine $\tau_{12}$ that is ultimate shear strength as well as $G_{12}$. The state of stress with respect to X-Y, is $\{\sigma_x\ 0\ 0\}$. This will lead to strains $\varepsilon_x$, $\varepsilon_y$. There will be no shear strain because it is ±45. Now, 1 and 2 are the principle material direction. So, we can write the stresses in and the strains with reference to the material axis, (1-2), $\sigma_1$, $\sigma_2$, $\tau_{12}$ using the stress transformation matrix as.

$$\sigma_x \neq 0;\ \sigma_y = \tau_{xy} = 0$$

$$\Rightarrow \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{Bmatrix}; \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \left[T_{45^\circ}\right] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \Rightarrow \tau_{12} = \frac{\sigma_x}{2} \qquad (1)$$

$$\Rightarrow \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} = \left[T_{45^\circ}\right] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{0}{2} \end{Bmatrix} \Rightarrow \gamma_{12} = \varepsilon_x - \varepsilon_y \qquad (2)$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{\sigma_x}{2(\varepsilon_x - \varepsilon_y)} \qquad (3)$$

So, from this what we get is $\tau_{12} = \sigma_x /2$ and $\gamma_{12} = \varepsilon_x - \varepsilon_y$. That means applying stress $\sigma_x$ , we know what is $\tau_{12}$ and from the two strain gauge readings we get $\varepsilon_x$ and $\varepsilon_y$ and hence $\gamma_{12}$ and using $G_{12} = \tau_{12} / \gamma_{12} = \sigma_x /(2 (\varepsilon_x - \varepsilon_y))$. So, knowing $\sigma_x$, $\varepsilon_x$ and $\varepsilon_y$, we can find out what is $G_{12}$.

So, this ±45° coupon is actually loaded till failure and $(\tau_{12})_u$ is nothing but $\sigma_x/2$ at failure. So, we get $(\tau_{12})_u$ and $G_{12}$.

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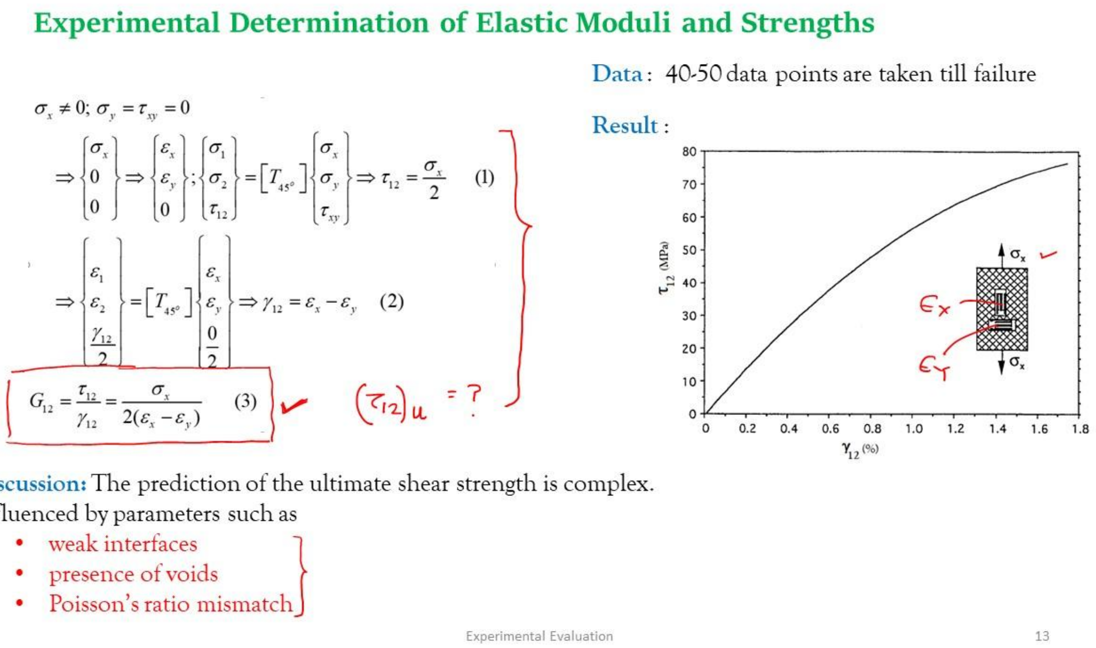


The transverse and shear properties estimated from the mechanics of material approach do not agree well with the experimental observations. The reasons are that in addition to the properties of the constituent materials like the shear modulus of the fiber and the shear modulus of the matrix, it is also influenced by weak interface, presence of voids, Poisson's ratio mismatch between the fiber and the matrix etc.