Mechanics of Fiber Reinforced Polymer Composite Structures

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Module-06
Macromechanics of Lamina-II
Lecture-15
Elasticity Approach

Hello and welcome to the 2nd lecture of the 6th module. We have been discussing micromechanics of lamina and in our last few lectures we actually discussed how to obtain the elastic moduli as well as the strength parameters of fiber composites in terms of the properties of the constituent fiber and the matrix. We also understood that there are different approaches for determination of these elastic moduli as well as the strength parameters.

We also discussed in details the mechanics of material approach for determination of the elastic moduli strength parameters. In the last lecture, we have discussed the determination of hygrothermal parameters for a lamina in terms of those of the fiber and the matrix. In today's lecture we will see in brief the elasticity approach especially to explain the longitudinal tensile strength as well as longitudinal compression strength of a lamina and we will try to discuss in details the role of matrix in longitudinal strength of lamina in particular.

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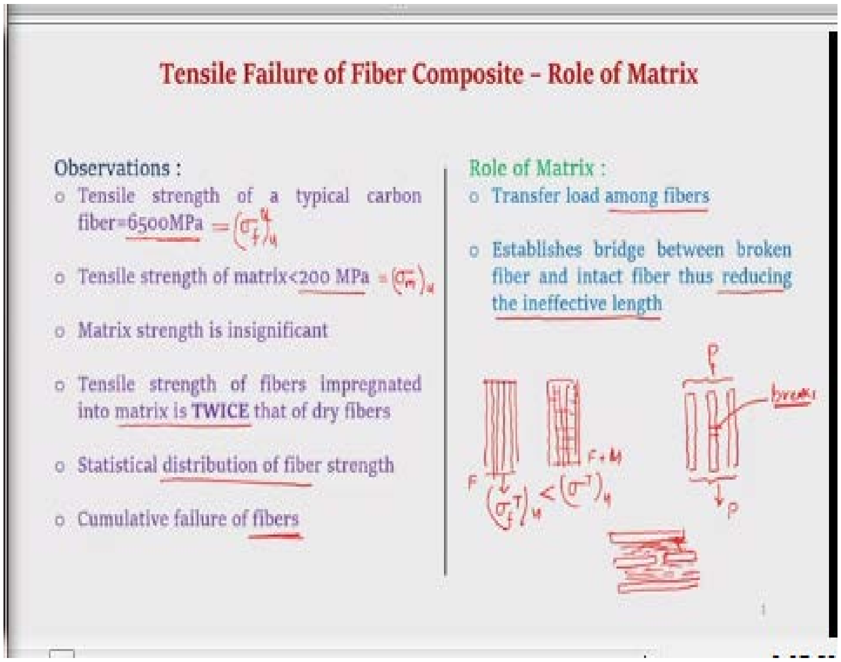

It is known that the longitudinal tensile strength of fibers is far higher compared to that of the matrix. For high strength carbon fibers say the strength is of the order of 6000 MPa and that of the matrix is somewhere of the order of 200 MPa. So, the matrix strength is insignificant in determination of the strength of the composites. Even if we consider say 50% volume fraction of fibers, so apparently the matrix strength is insignificant in determination of the strength of the lamina. However, it was observed that the strength of fibers impregnated into matrix is almost twice that of the dry fibers. Suppose we take only a dry bundle of fibers, say millions of dry of fibers only and no matrix and then we try to find out what is it is tensile strength and then the same bundle of fibers are actually impregnated into matrix and then their strengths are measured. It is observed that the fibers impregnated into matrix shows higher strength compared to that of the dry bundle of fibers.

The reason was attributed to the statistical distribution of fiber strength. That means in our all these mechanics of material approach we have assumed that the fibers are of uniform shape, uniform size and uniform strength, uniform moduli but actually the strengths are statistically distributed. As a result of that it is unlikely that all the fibers will fail at the same time.

Suppose the weakest fiber actually breaks earlier and then it can no longer take part in the load bearing and therefore the load is shared by the remaining fibers and naturally the remaining adjacent fibers are now overloaded. Therefore the chance of the adjacent fiber breaking becomes more, so this is called cumulative fiber breakage.

The weakest fiber breaks thereby overloading the adjacent fibers, so another fiber breaks therefore again overloading the remaining fibers, so this is called cumulative fiber breakage. Because of that even though say the strength of the fiber is estimated to be 6500 MPa it is observed that a dry bundle of fibers actually fails at a stress much lower than that.

Now we know that the role of the matrix is to act as a binder. But in addition to that actually the matrix also transfers load among fibers. Referring to the Fig. when a fiber breaks, if there is no matrix, it cannot take part in loading. But suppose they are bonded by this matrix, the load from the broken fiber is transferred to the nearest adjacent fiber and vice versa.

Therefore even though a particular fiber breaks it is not that it becomes entirely ineffective, a part of it still might take part in load bearing thereby increasing the overall strength of the fiber matrix lamina. In absence of matrix the fiber is completely ineffective but when the matrix is there only a part of the length becomes ineffective in terms of not participating in the load bearing and the remaining part actually takes part in the load bearing thereby increasing the overall strength of the lamina.

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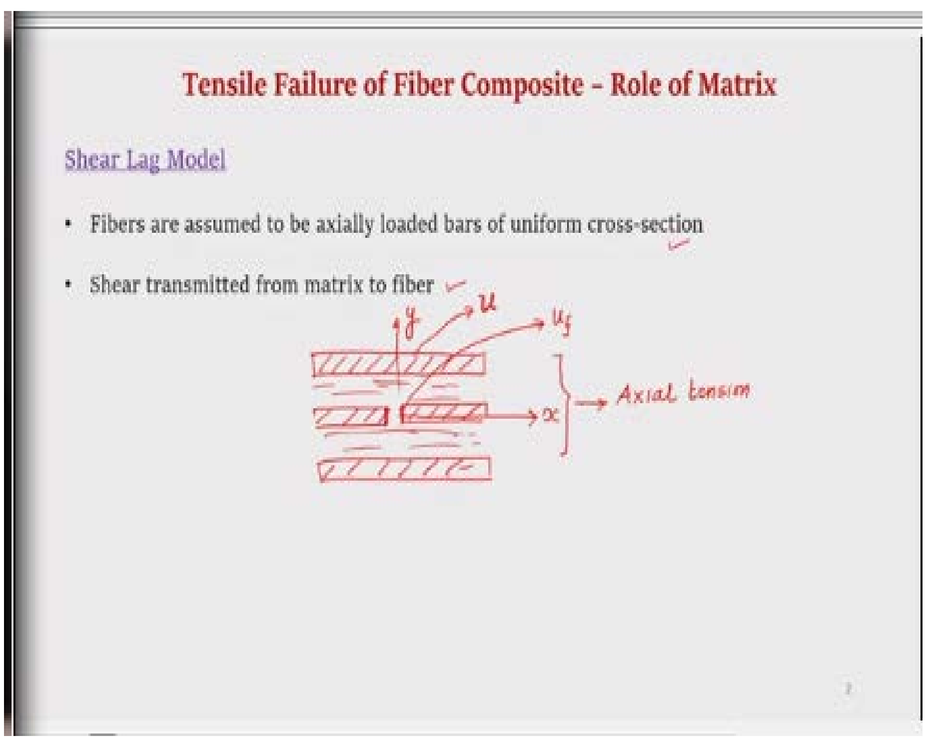

So, this is explained by means of a model called shear lag model. In shear lag model actually the fibers are assumed to be axially loaded and shear is transmitted from the fiber to the matrix. The fibers are idealized as simple axial rod under tension with shear transmitted by the matrix.

As shown, the coordinate system is fixed at the broken fiber and the displacement of this fiber we considered as u_f and suppose the displacement of the adjacent intact fiber is u. We are only considering x- component of displacement when you say displacement.

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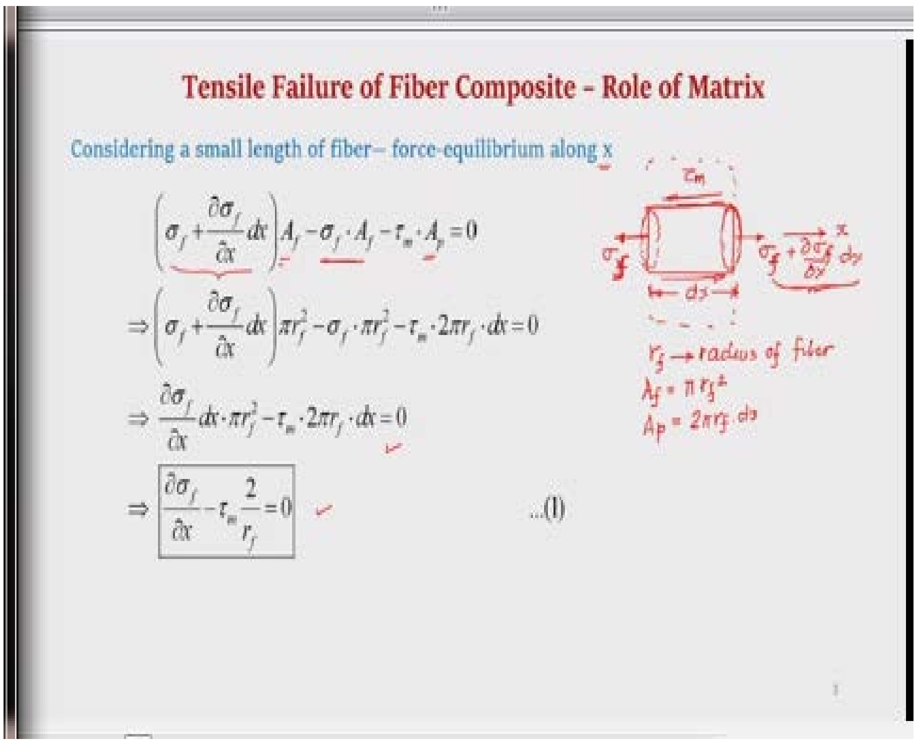


Considering a small length of the fiber, say dx. Because of the tensile load this is experiencing a stress σ_x at x=0 and at a distance dx, this is $\sigma_f + \frac{\partial \sigma_f}{\partial x} dx$. Suppose the shear stress it is experiencing is τ_m. Because there is a difference in the tensile stress at the two ends therefore there will be shear stress at the periphery and here it is actually surrounded by matrix.

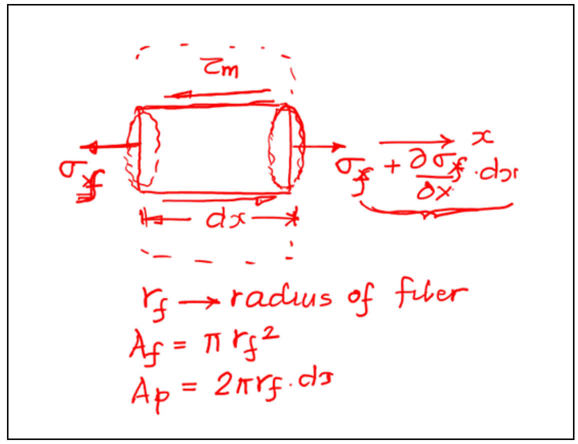
So, considering a small length, the force equilibrium along x is as

$$\left(\sigma_f + \frac{\partial \sigma_f}{\partial x} dx\right) A_f - \sigma_f \cdot A_f - \tau_m \cdot A_p = 0$$

$$\Rightarrow \left(\sigma_f + \frac{\partial \sigma_f}{\partial x} dx\right) \pi r_f^2 - \sigma_f \cdot \pi r_f^2 - \tau_m \cdot 2\pi r_f \cdot dx = 0$$

$$\Rightarrow \frac{\partial \sigma_f}{\partial x} dx \cdot \pi r_f^2 - \tau_m \cdot 2\pi r_f \cdot dx = 0$$

$$\Rightarrow \boxed{\frac{\partial \sigma_f}{\partial x} - \tau_m \frac{2}{r_f} = 0} \qquad \text{...(1)}$$



A_f is the area of the cross section of the fiber, r_f is the radius of fiber. A_p is the entire periphery.

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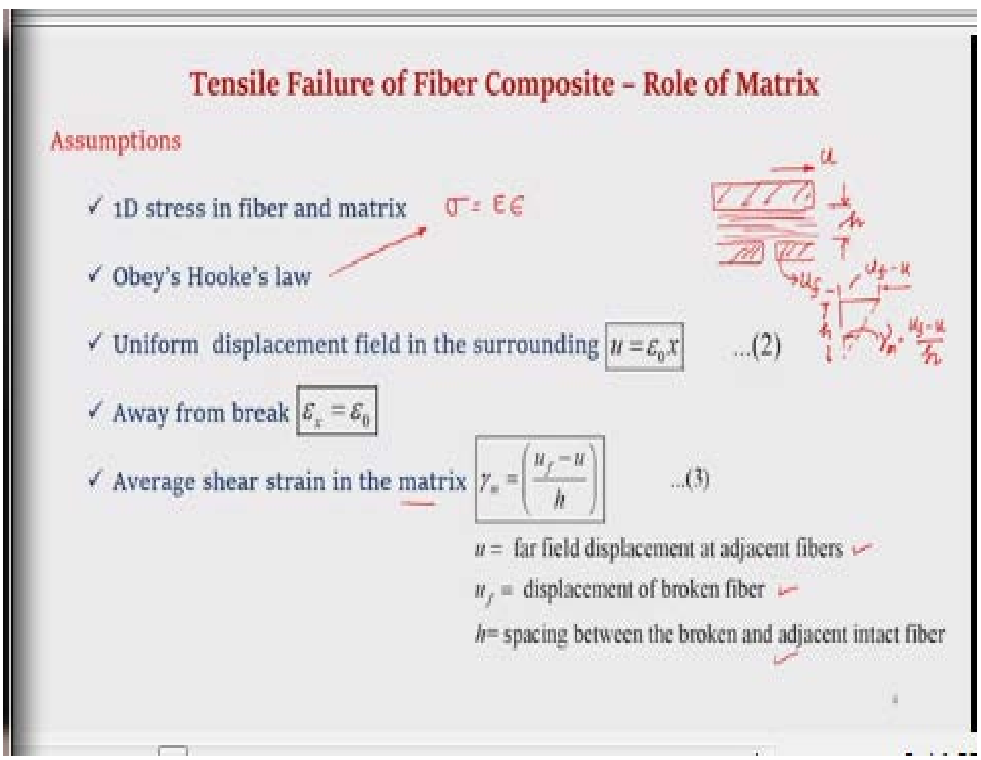
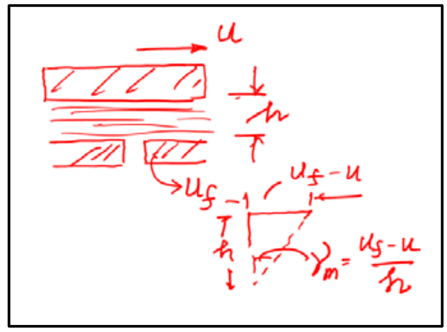


The following assumptions are made:

- 1D stress in fiber and matrix
- Obey's Hooke's law: $\sigma = E\varepsilon$
- Uniform displacement field in the surrounding: $u = \varepsilon_0 x$ (2)

- Away from break: $\varepsilon_x = \varepsilon_0$
- Average shear strain in the matrix:



$$\boxed{\gamma_m = \left(\frac{u_f - u}{h}\right)} \qquad ...(3)$$

$u =$ far field displacement at adjacent fibers

$u_f =$ displacement of broken fiber

h= spacing between the broken and adjacent intact fiber

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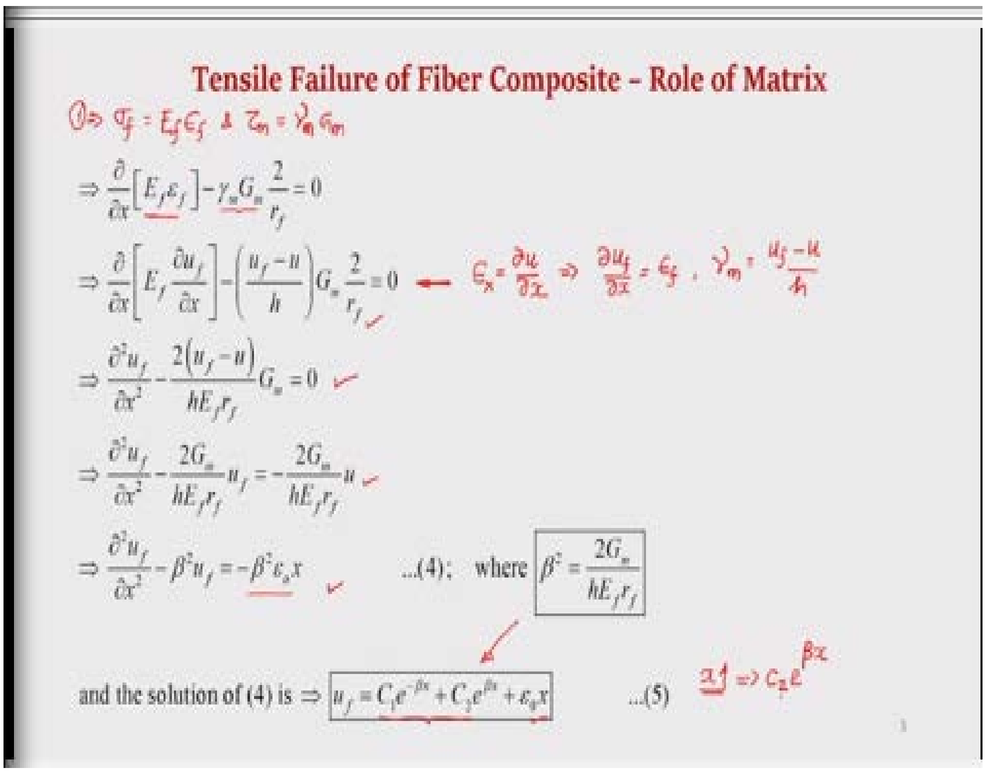

Now

$$\sigma_f = E_f \varepsilon_f \quad \text{and} \quad \tau_m = \gamma_m G_m$$

Therefore, substituting in (1)

$$\Rightarrow \frac{\partial}{\partial x}\left[E_f \varepsilon_f\right] - \gamma_m G_m \frac{2}{r_f} = 0 \qquad (4)$$

Again using the strain displacement relationship $\varepsilon_x = \frac{\partial u}{\partial x}, \; \Rightarrow \frac{\partial u_f}{\partial x} = \varepsilon_f$

and the shear strain in the matrix is $\gamma_m = \frac{u_f - u}{h}$

Therefore (4) becomes

$$\Rightarrow \frac{\partial}{\partial x}\left[E_f \frac{\partial u_f}{\partial x}\right] - \left(\frac{u_f - u}{h}\right) G_m \frac{2}{r_f} = 0$$

$$\Rightarrow \frac{\partial^2 u_f}{\partial x^2} - \frac{2\left(u_f - u\right)}{hE_f r_f} G_m = 0$$

$$\Rightarrow \frac{\partial^2 u_f}{\partial x^2} - \frac{2G_m}{hE_f r_f} u_f = -\frac{2G_m}{hE_f r_f} u$$

$$\Rightarrow \frac{\partial^2 u_f}{\partial x^2} - \beta^2 u_f = -\beta^2 \varepsilon_o x \qquad (5)$$

where $\boxed{\beta^2 = \frac{2G_m}{hE_f r_f}}$ (6)

Equation (5) is a standard second order differential equation and the standard solution for this equation is

$$\boxed{u_f = C_1 e^{-\beta x} + C_2 e^{\beta x} + \varepsilon_0 x} \qquad (7)$$

This solution has got two parts one is for the homogeneous solution $C_1 e^{-\beta x} + C_2 e^{\beta x}$ which is the complementary part an the particular solution which is given by $\varepsilon_0 x$. So, that the displacement field in the broken fiber is obtained as a function of distance x. C_1 and C_2 are the constants which could be determined from the boundary conditions. So, let us see what the boundary conditions are. Now we will appreciate here that in this solution this (7) as x increases the term $C_2 e^{\beta x}$ keeps on increasing therefore for a large value of x this goes to infinity which is not possible.

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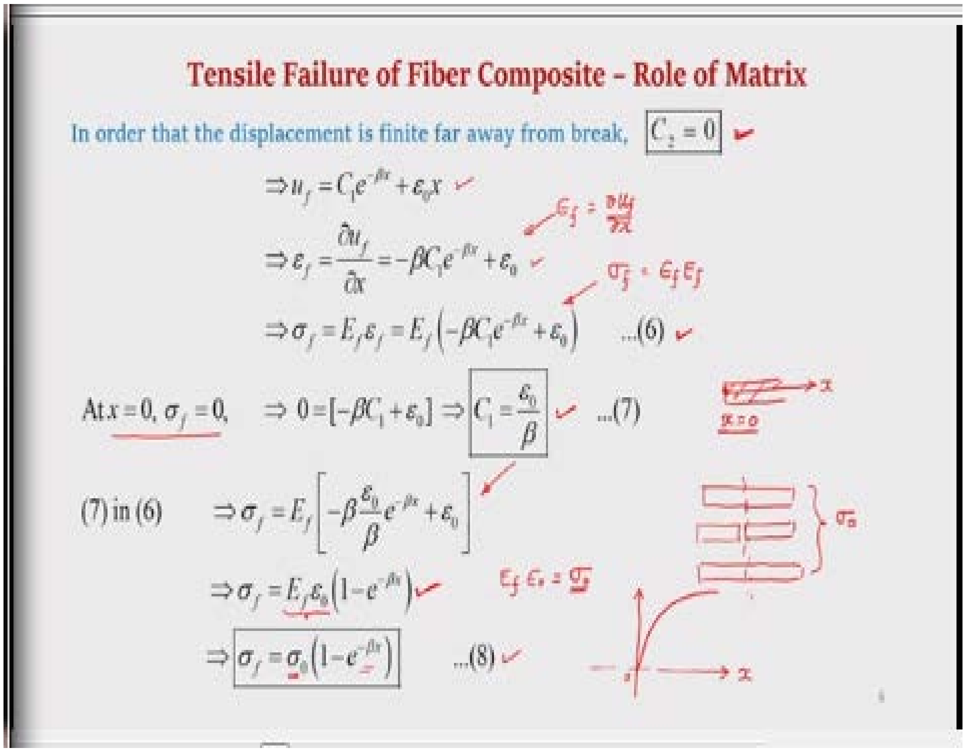

That means in order to ensure that the displacement is finite, $C_2=0$ otherwise we get a solution which is not feasible. Therefore (7) gives

$$u_f = C_1 e^{-\beta x} + \varepsilon_0 x \qquad (8)$$

$$\Rightarrow \varepsilon_f = \frac{\partial u_f}{\partial x} = -\beta C_1 e^{-\beta x} + \varepsilon_0$$

$$\Rightarrow \sigma_f = E_f \varepsilon_f = E_f\left(-\beta C_1 e^{-\beta x} + \varepsilon_0\right) \qquad (9)$$

Now what are the boundary conditions to determine C_1. $C_2=0$ in order to avoid infinite displacement. Now at the break that means at x = 0 it cannot have stress because it is free, there is no constraint.

Therefore at x = 0 the stress in the broken fiber is zero and (9) gives

$$\text{At } x = 0,\ \sigma_f = 0, \quad \Rightarrow\ 0 = [-\beta C_1 + \varepsilon_0] \Rightarrow \boxed{C_1 = \frac{\varepsilon_0}{\beta}} \qquad (10)$$

So, (10) in (9) gives

$$\Rightarrow \sigma_f = E_f \left[-\beta \frac{\varepsilon_0}{\beta} e^{-\beta x} + \varepsilon_0 \right]$$

$$\Rightarrow \sigma_f = E_f \varepsilon_0 \left(1 - e^{-\beta x}\right)$$

$$\Rightarrow \boxed{\sigma_f = \sigma_0 \left(1 - e^{-\beta x}\right)} \qquad (11)$$

E_f is the Young's modulus of the fiber, ε_o is the far field strain and (11) gives the far field stress. So, the stress in the broken fiber could be expressed as a function of x and it could be clearly seen that σ_f increases with increase in x because as x. So, the variation is as shown in the Fig.

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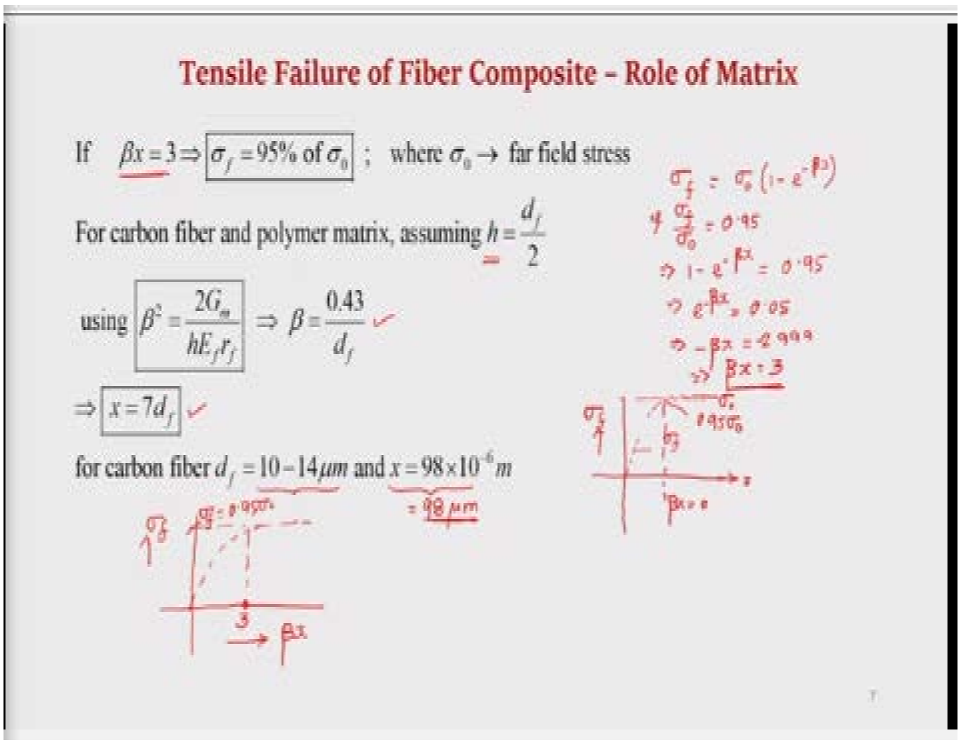

Now if σ_f reaches 95% of the far field stress ie. $\sigma_f = 0.95\sigma_0$, (11) gives $\beta x = 2.99$. Or we can write

$$\text{If} \quad \beta x = 3 \Rightarrow \boxed{\sigma_f = 95\% \text{ of } \sigma_0} \;\; ; \quad \text{where } \sigma_0 \rightarrow \text{ far field stress}$$

That means for a value of $\beta x = 3$, the stress in the broken fiber σ_f reaches 95% of the far field stress σ_o.

Now considering the case of a carbon fiber with a polymer matrix and assuming that the fiber spacing h is equal to the radius of the fiber,

$$h = \frac{d_f}{2}$$

$$\text{using } \boxed{\beta^2 = \frac{2G_m}{hE_f r_f}} \Rightarrow \beta = \frac{0.43}{d_f}$$

$$\Rightarrow \boxed{x = 7d_f}$$

for carbon fiber $d_f = 10 - 14\mu m$ and $x = 98 \times 10^{-6} m$

That means, if a fiber breaks, a portion of the fiber of course becomes ineffective, that means it cannot take part in the load bearing and in the case of a carbon fiber in polymer matrix at a distance of 98 micron, the fiber stress in the broken fiber is 95% that of the nominal stress, that means it starts taking part in the load bearing. So, we understand the role of matrix if we plot βx vs σ_f σ_f is 0.95 of σ_o, at what point where $\beta x = 3$. Therefore the matrix actually helps the broken fiber to be ineffective only over a small length.

That means the broken fiber is ineffective only for a small length and the remaining part actually takes part in the load bearing by the transfer of shear stresses by the matrix ie. by the load transfer of the matrix. That is how the matrix plays an important role in the tensile strength of fiber composites. Especially when considering the statistical distribution of the fiber strength, if a fiber breaks that does not become completely ineffective. The discrepancy in the longitudinal tensile strength of the dry bundle of fibers and the fibers impregnate in to the matrix has thus been explained.

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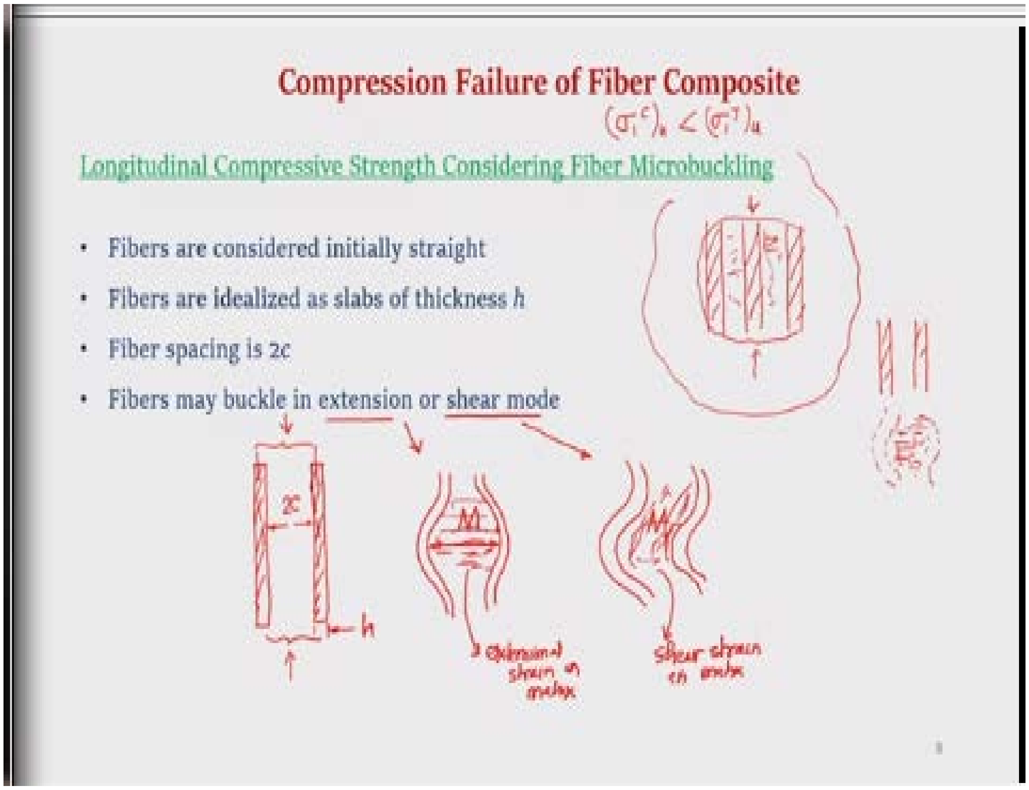

Next we will discuss the longitudinal compression of fiber composite considering fiber microbuckling. The longitudinal compression strength of fiber composite is much less compared to it is tensile strength and one of the reason is that the fibers are slender and therefore even though the composite as a whole does not buckle there may be fiber microbuckling.

There have been lot of studies by many investigators the longitudinal compression strength is associated with the microbuckling of fibers which leads to the failure.

As shown in the Fig., under the compression load this fiber tends to buckle. But this buckling is actually resisted by the matrix, that means considering two adjacent fibers, suppose the initial state of the fibers is straight and under loading the fibers might try to buckle and the matrix actually experiences strains as the matrix tries to resist that buckling.

Based on such fiber microbuckling the longitudinal compressive strength of the composites has been explained. In this it was assumed that the fibers are initially straight fibers, and subjected to compression. The fibers are idealized as slabs of thickness *h* and width as unity, and the fiber spacing is *2c* (Ref. Fig.)

The fibers might buckle in extension and shear mode. That means under this compression there could be two ways the fibers could buckle viz. the extension mode, where the matrix actually experiences extension, and in shear mode where the two adjacent fibers might actually try to slide over each other and in between the matrix, experience a shear as the matrix tries to resist the buckling (Ref. Fig.). Therefore the fibers could buckle in two distinct modes viz. extension mode and shear mode.

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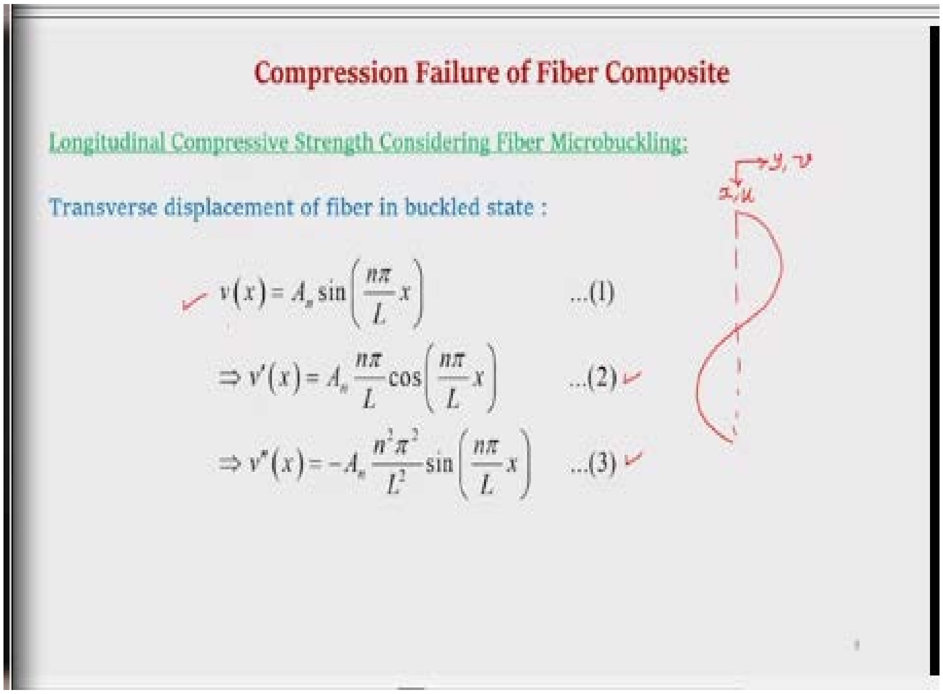

Now considering the fiber buckling, suppose considering the transverse displacement of the fiber to be sinusoidal (ref Fig.). The displacement along x is u and the displacement along y is v. The transverse displacement v is expressed as

$$v(x) = A_n \sin\left(\frac{n\pi}{L}x\right) \qquad ...(1)$$

Taking derivatives

$$\Rightarrow v'(x) = A_n \frac{n\pi}{L}\cos\left(\frac{n\pi}{L}x\right) \qquad ...(2)$$

$$\Rightarrow v''(x) = -A_n \frac{n^2\pi^2}{L^2}\sin\left(\frac{n\pi}{L}x\right) \qquad ...(3)$$

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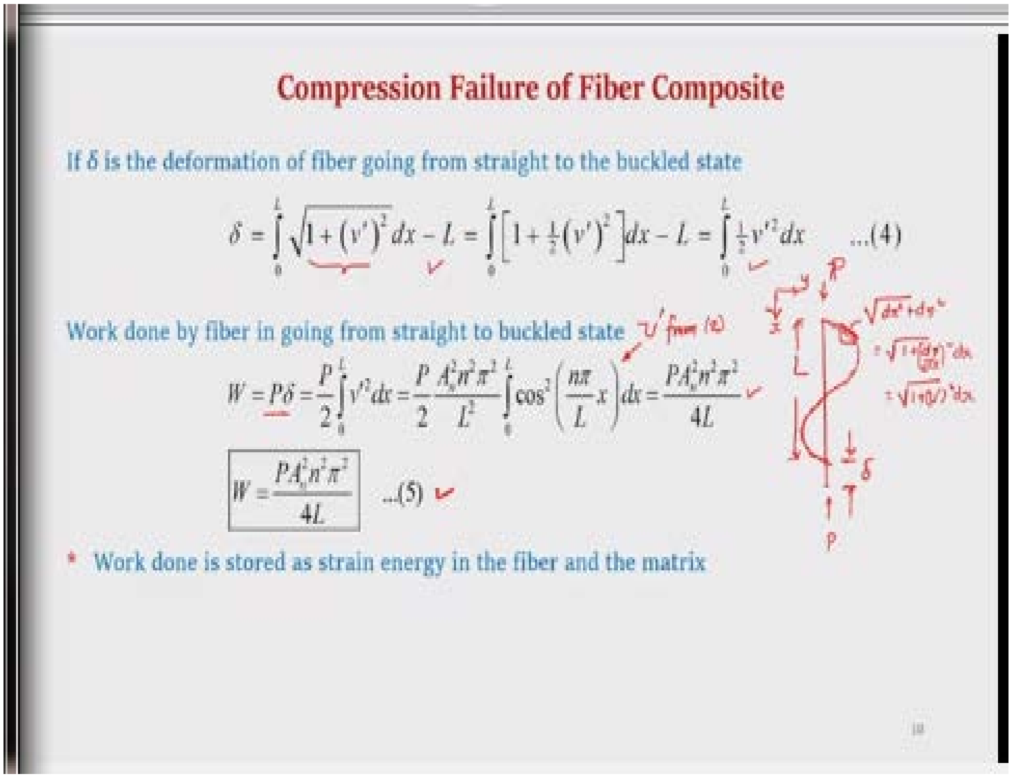

Considering a fiber which is initially straight and subjected to axial compression the axial deformation along x δ. Suppose the length is L after it buckles, so the initial length is given by $\int_0^L \sqrt{dx^2+dy^2}\,dx = \int_0^L \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^L \sqrt{1+(v')^2}\,dx$ and hence

$$\delta = \int_0^L \sqrt{1+(v')^2}\,dx - L = \int_0^L \left[1+\tfrac{1}{2}(v')^2\right]dx - L = \int_0^L \tfrac{1}{2}v'^2\,dx \qquad ...(4)$$

Therefore the work done is

$$W = P\delta = \frac{P}{2}\int_0^L v'^2\,dx = \frac{P}{2}\frac{A_n^2 n^2\pi^2}{L^2}\int_0^L \cos^2\left(\frac{n\pi}{L}x\right)dx = \frac{PA_n^2 n^2\pi^2}{4L}$$

$$\boxed{W = \frac{PA_n^2 n^2\pi^2}{4L}} \qquad ...(5)$$

This total work done is actually stored as strain energy in the fiber as well as in the matrix. Now first let us see that what is the strain energy stored in the fiber.

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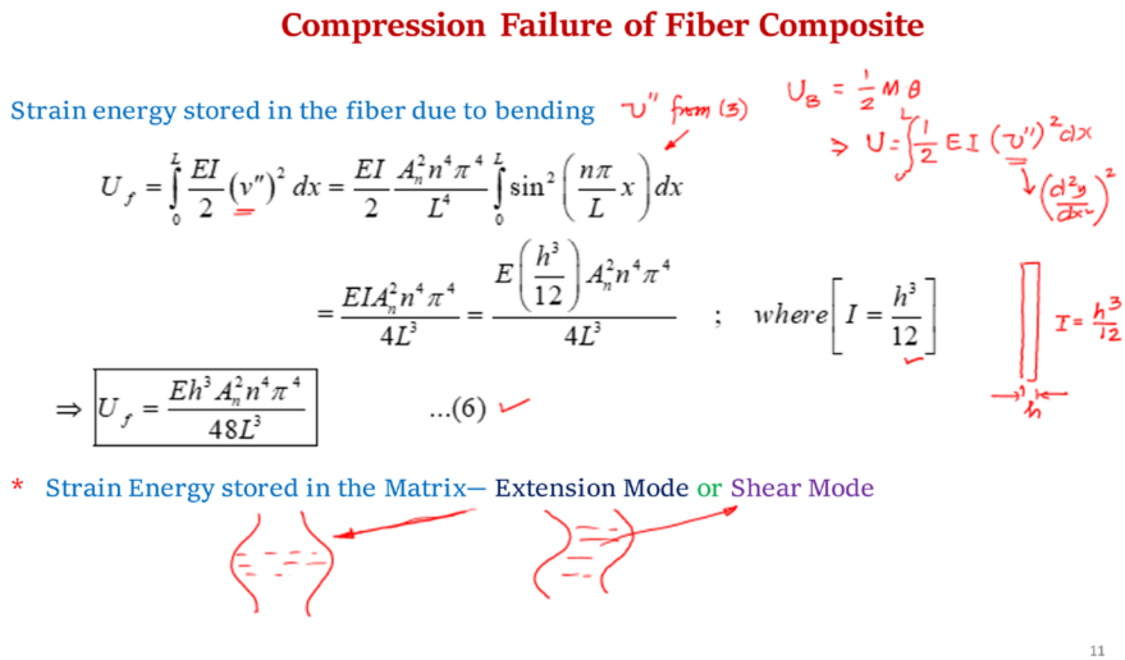

The fiber actually bends therefore we can find out the strain energy of fiber using the bending energy. Using the strain energy due to bending from the theory of pure bending the strain energy could be expressed

$$U_f = \int_0^L \frac{EI}{2}(v'')^2 dx = \frac{EI}{2}\frac{A_n^2 n^4 \pi^4}{L^4}\int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx = \frac{EIA_n^2 n^4 \pi^4}{4L^3}$$

Considering the fiber idealized as a slab of thickness h and unit width therefore the second moment is $I = \frac{h^3}{12}$ and

$$U_f = \frac{EIA_n^2 n^4 \pi^4}{4L^3} = \frac{E\left(\frac{h^3}{12}\right)A_n^2 n^4 \pi^4}{4L^3} \quad ; \quad where\left[I = \frac{h^3}{12}\right]$$

$$\Rightarrow \boxed{U_f = \frac{Eh^3 A_n^2 n^4 \pi^4}{48L^3}} \qquad \text{...(6)}$$

So, we have used this here and we get the expressions for strain energy stored in the fiber due to bending. So, out of the total work done a part of this is stored as a strain energy in the fiber because the bending of the fiber. And the part of it is actually stored in the strain energy of the matrix. Depending upon the mode the matrix could experience extensional strain leading to extension mode, or it could be shear mode where the matrix will be experiencing shear strain. So, let us discuss out one by one.

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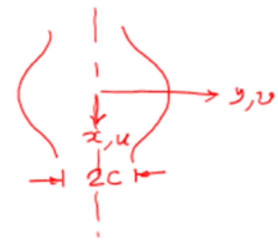
Compression Failure of Fiber Composite

Strain Energy stored in the Matrix– Extension Mode



$$\varepsilon_y = \frac{v(x)|_c - v(x)|_{-c}}{2c} = \frac{1}{c} A_n \sin\left(\frac{n\pi}{L} x\right) \qquad ...(7)$$

$v(x)|_c = -v(x)|_{-c}$

Extensional strain energy stored in the matrix,

$$U_m = \frac{1}{2}\int_0^L E_m \varepsilon_y^2 dv = \frac{1}{2}\int_0^L E_m \varepsilon_y^2 2c dx \qquad ...(8)$$

$$\Rightarrow \boxed{U_m = \frac{E_m L}{2c} A_n^2} \qquad ...(9)$$

$U = \int_V \frac{1}{2} \sigma \epsilon \, dv$

$= \frac{1}{2}\int E \epsilon \cdot \epsilon \cdot dv$

$= \frac{1}{2}\int_V \epsilon^2 E \, dv$

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Say suppose we want to find out the strain energy stored in the matrix in extension mode. So, referring to Fig., the strain along *y* is nothing but

$$\varepsilon_y = \frac{v(x)|_c - v(x)|_{-c}}{2c} = \frac{1}{c} A_n \sin\left(\frac{n\pi}{L} x\right) \qquad ...(7)$$

Therefore the strain energy stored in the extension mode is

$$U = \int_V \frac{1}{2} \sigma \varepsilon dv$$
$$= \frac{1}{2}\int_V E \varepsilon \varepsilon dv$$
$$= \frac{1}{2}\int_V \varepsilon^2 E dv$$

So, in this case, so this could be written as

$$U_m = \frac{1}{2}\int_0^L E_m \varepsilon_y^2 dv = \frac{1}{2}\int_0^L E_m \varepsilon_y^2 2cdx \qquad ...(8)$$

$$\Rightarrow \boxed{U_m = \frac{E_m L}{2c} A_n^2} \qquad ...(9)$$

(9) gives the strain energy stored in the matrix purely because of extensional strain.

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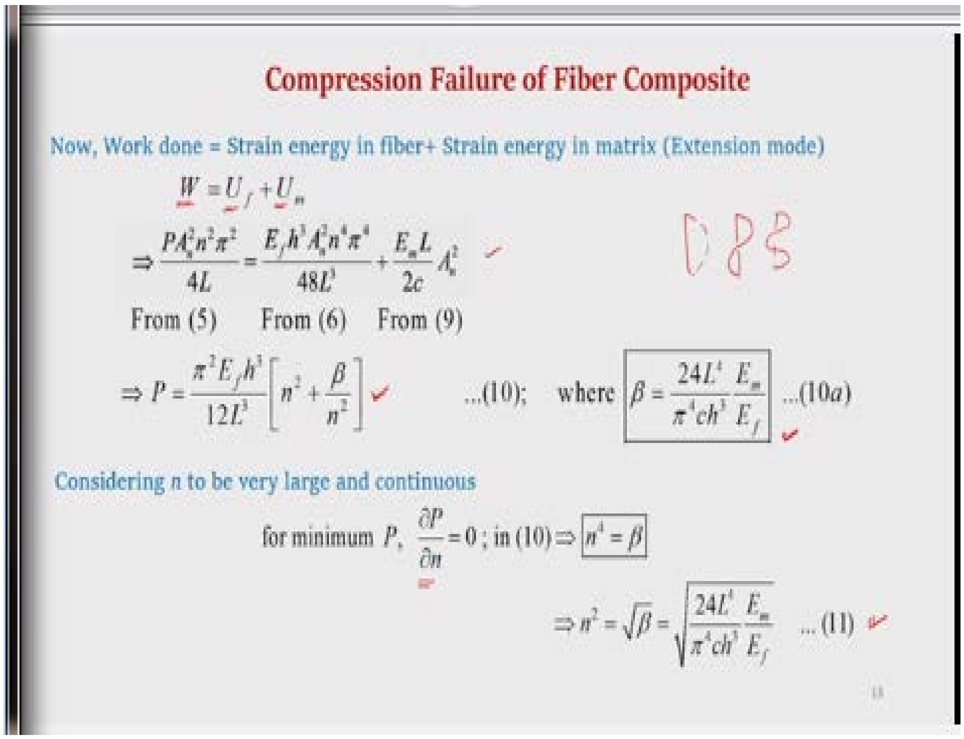

Now we have the work done from equation number (5), we have the strain energy stored in the fiber from equation number (6), strain energy stored in the matrix because of extension from equation number (9). Equating the work done to the strain energy stored as

$$W = U_f + U_m$$

$$\Rightarrow \underset{\text{From (5)}}{\frac{PA_n^2 n^2 \pi^2}{4L}} = \underset{\text{From (6)}}{\frac{Eh^3 A_n^2 n^4 \pi^4}{48L^3}} + \underset{\text{From (9)}}{\frac{E_m L}{2c} A_n^2}$$

we get an expression for P as

$$\Rightarrow P = \frac{\pi^2 E_f h^3}{12L^3}\left[n^2 + \frac{\beta}{n^2}\right] \qquad ...(10);$$

and where β is defined as

$$\boxed{\beta = \frac{24L^3}{\pi^4 ch^3}\frac{E_m}{E_f}} \quad ...(10a)$$

Now n (n = 1, 2, 3, ….) represents the modes of buckling. Now the minimum value of P at which it will buckle is a function of n. For large n, considering n to be continuous, taking the first derivative with respect to n and putting it to 0 and we get this condition

$$\text{for minimum } P, \ \frac{\partial P}{\partial n} = 0 \text{ ; in (10)} \Rightarrow \boxed{n^4 = \beta}$$

$$\Rightarrow n^2 = \sqrt{\beta} = \sqrt{\frac{24L^3}{\pi^4 ch^3}\frac{E_m}{E_f}} \quad ... (11)$$

Putting (11) in 10) we get the expressions for P as this.

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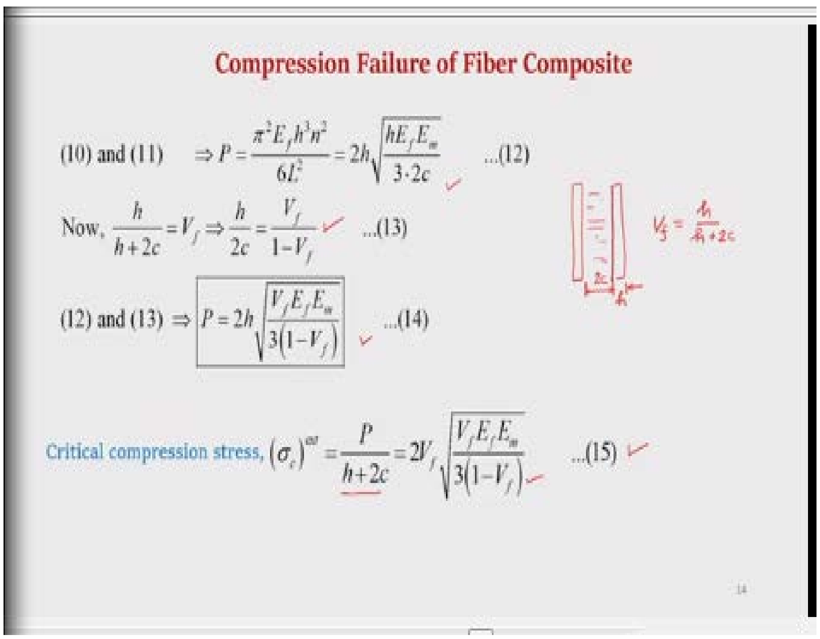

$$\text{(10) and (11)} \qquad \Rightarrow P = \frac{\pi^2 E_f h^3 n^2}{6L^2} = 2h\sqrt{\frac{hE_f E_m}{3\cdot 2c}} \qquad ...(12)$$

Now we have considered this as the two adjacent fibers separated by matrix, distance is 2c and this is h. For unit width and for a given length the volume fraction of fiber is nothing but

$$\text{Now, } \frac{h}{h+2c} = V_f \Rightarrow \frac{h}{2c} = \frac{V_f}{1-V_f} \qquad ...(13)$$

Putting (13) in (12), we get the expression for minimum load at which microbuckling will take place as

$$\text{(12) and (13)} \Rightarrow \boxed{P = 2h\sqrt{\frac{V_f E_f E_m}{3(1-V_f)}}} \qquad ...(14)$$

Diving the critical load by the area (h+$2c$), the critical compression stress is

$$(\sigma_c)^{ext} = \frac{P}{h+2c} = 2V_f\sqrt{\frac{V_f E_f E_m}{3(1-V_f)}} \qquad ...(15)$$

So, we get this as the expression for the critical compression stress at which the fiber microbuckling will take place and this is a function of Young's modulus of the fiber, Young's modulus of the matrix and the volume fraction. This is because it is due to extensional mode, in the same way we can also see what happens in the shear mode.

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Compression Failure of Fiber Composite

Shear Mode :

Assuming only shear stress in the matrix

$$\boxed{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}} \qquad ...(16)$$

from ⑧,

$$\text{where, } \frac{\partial v}{\partial x} = A_n \frac{n\pi}{L}\cos\left(\frac{n\pi}{L}x\right) \quad ...(17) \quad \text{and} \quad \frac{\partial u}{\partial y} = \frac{u(c)-u(-c)}{2c} \quad ...(18)$$

$$\text{Now, } u(c) = -u(-c) = \frac{h}{2}\frac{\partial v}{\partial x} = \frac{h}{2}A_n\frac{n\pi}{L}\cos\left(\frac{n\pi}{L}x\right) \quad ...(19)$$

$$\text{(18) and (19)} \Rightarrow \frac{\partial u}{\partial y} = \frac{h}{2c}A_n\frac{n\pi}{L}\cos\left(\frac{n\pi}{L}x\right) \quad ...(20)$$

$$\text{So, } \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left(1+\frac{h}{2c}\right)A_n\frac{n\pi}{L}\cos\left(\frac{n\pi}{L}x\right) \quad ...(21)$$

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Referring to the Fig., in shear mode two adjacent fibers, separated by a distance 2c actually slide over each other. Therefore under shear, the points on the fibers move along x- direction (as shown) by a distance $\frac{h}{2}\frac{dv}{dx}$ which is nothing but u at c. Now assuming pure shear, now using the strain displacement relationship, shear strain is

$$\boxed{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}} \quad ...(16)$$

We already have the expressions for

$\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ as

$$\frac{\partial v}{\partial x} = A_n \frac{n\pi}{L} \cos\left(\frac{n\pi}{L}x\right) \quad ...(17) \qquad \text{and} \quad \frac{\partial u}{\partial y} = \frac{u(c) - u(-c)}{2c} \qquad ...(18)$$

Putting these in the expression for this shear strain (16), we get an expression for the shear strain as

$$\text{Now, } u(c) = -u(-c) = \frac{h}{2}\frac{\partial v}{\partial x} = \frac{h}{2} A_n \frac{n\pi}{L} \cos\left(\frac{n\pi}{L}x\right) \qquad ...(19)$$

$$(18) \text{ and } (19) \quad \Rightarrow \frac{\partial u}{\partial y} = \frac{h}{2c} A_n \frac{n\pi}{L} \cos\left(\frac{n\pi}{L}x\right) \qquad ...(20)$$

$$\text{So, } \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left(1 + \frac{h}{2c}\right) A_n \frac{n\pi}{L} \cos\left(\frac{n\pi}{L}x\right) \quad ...(21)$$

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Compression Failure of Fiber Composite

Strain energy stored in the matrix due to shear

$U_s = \int \frac{1}{2} \gamma_m \tau_m dv = \frac{1}{2} \int \gamma_m \gamma_m G_m dv$

$$U_m = \frac{1}{2}\int_0^L G_m \gamma_m^2 dv = G_m c\left(1 + \frac{h}{2c}\right)^2 \frac{A_n^2 n^2 \pi^2}{2L} \quad ; \quad \text{using } \boxed{\gamma_m = \gamma_{xy}} \text{ from (21)}$$

$$\Rightarrow \boxed{U_m = G_m c\left(1 + \frac{h}{2c}\right)^2 \frac{A_n^2 n^2 \pi^2}{2L}} \qquad ...(22)$$

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So, once we put this expression for the shear strain in the strain energy stored because of pure shear, we get

$$U_s = \int \frac{1}{2} \gamma_m \tau_m dv = \frac{1}{2} \int \gamma_m \gamma_m G_m dv$$

Using the stress strain relationship of the matrix, the strain energy stored in the matrix due to shear is

$$U_m = \frac{1}{2}\int_0^L G_m \gamma_m^2 dv = G_m c\left(1+\frac{h}{2c}\right)^2 \frac{A_n^2 n^2 \pi^2}{2L} \quad ; \quad \text{using } \boxed{\gamma_m = \gamma_{xy}} \text{ from (21)}$$

$$\Rightarrow \boxed{U_m = G_m c\left(1+\frac{h}{2c}\right)^2 \frac{A_n^2 n^2 \pi^2}{2L}} \qquad ...(22)$$

(22) gives the expression for the strain energy stored in the matrix entirely because of the shear because in this case the matrix is experiencing shear.

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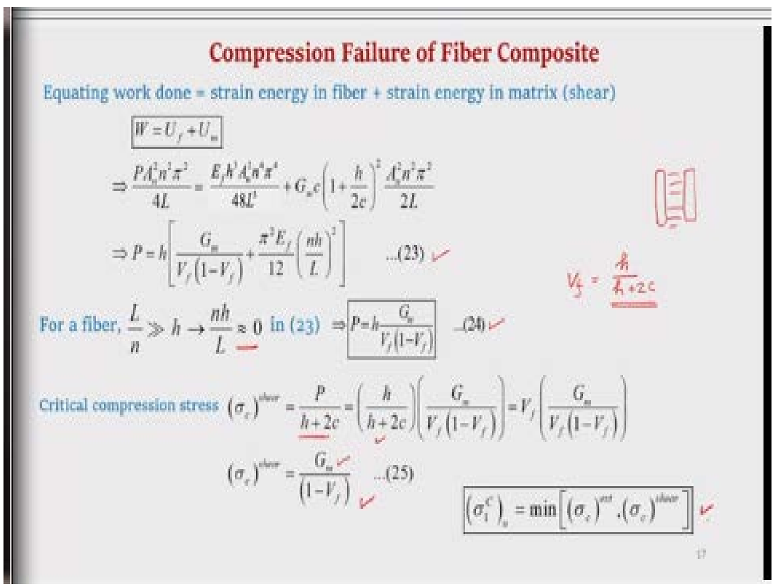

And now again equating the work done to the strain energies stored in the fiber (bending) and in the matrix (shear),

$$\boxed{W = U_f + U_m}$$

$$\Rightarrow \frac{PA_n^2 n^2 \pi^2}{4L} = \frac{Eh^3 A_n^2 n^4 \pi^4}{48L^3} + G_m c\left(1+\frac{h}{2c}\right)^2 \frac{A_n^2 n^2 \pi^2}{2L}$$

$$\Rightarrow P = h\left[\frac{G_m}{V_f(1-V_f)} + \frac{\pi^2 E_f}{12}\left(\frac{nh}{L}\right)^2\right] \qquad ...(23)$$

Now because the fiber is slender, its length is far higher compared to its thickness and $\frac{L}{n} \gg h \rightarrow \frac{nh}{L} \approx 0$,. Therefore putting this in equation (23), we get the expression for critical buckling load at which the fiber microbuckling takes place in the shear mode as

$$\Rightarrow \boxed{P = h\frac{G_m}{V_f\left(1-V_f\right)}} \quad ...(24)$$

Therefore the critical compression stress at which microbuckling in the shear mode takes place.

Now using $V_f = \frac{h}{h+2c}$, the critical compression stress at which fiber microbuckling in shear mode will take place is given by

$$\left(\sigma_c\right)^{shear} = \frac{P}{h+2c} = \left(\frac{h}{h+2c}\right)\left(\frac{G_m}{V_f\left(1-V_f\right)}\right) = V_f\left(\frac{G_m}{V_f\left(1-V_f\right)}\right)$$

$$\left(\sigma_c\right)^{shear} = \frac{G_m}{\left(1-V_f\right)} \quad ...(25)$$

(25) gives the compression strength of a lamina based on shear mode fiber micro buckling and is purely a function of the shear modulus of the matrix and the volume fraction. Considering extension and shear mode whichever is minimum is considered to be the critical compression stress at which fiber microbuckling will take place and that decides what is the longitudinal compression strength of a fiber composites. The compression strength of a lamina is given by

$$\boxed{\left(\sigma_1^C\right)_u = \min\left[\left(\sigma_c\right)^{ext}, \left(\sigma_c\right)^{shear}\right]} \quad (26)$$

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Compression Failure of Fiber Composite

Over estimates → Longitudenal Compression strength

Gl/E — 600-1000 MPa ← expt
- 8700 → extension
- 2200 → shear

Reasons —
- Irregular fiber spacing
- Imperfection in bonding
- Filer mis allignment
- Poisson's ratio mismatch bet^{n} fiber & matrix

It is observed that considering the fiber microbuckling it over estimates the longitudinal compression strength. It does not agree well with the experiment. Experimentally it is observed that the strength obtained using this is much higher compared to the experimentally observed strength. Say for example in a typical say glass epoxy, the experimentally observed compression strength is anything around 600 to 1000 MPa. From this fiber microbuckling we get 8700 MPa considering extension mode and 2200 considering shear mode. Therefore they do not agree well rather they over estimates. The reasons are that the assumptions of regular fiber spacing, perfect bonding, perfectly aligned fibers besides the Poisson's ratio mismatch..

So, these are the reasons due to which it does not correlate well with the experiments rather it overestimates. However, we understood that how the fiber and matrix actually interacts and more importantly we understood the role of matrix in determining both the longitudinal tensile strength as well as the longitudinal compression strength in a fiber matrix composites.