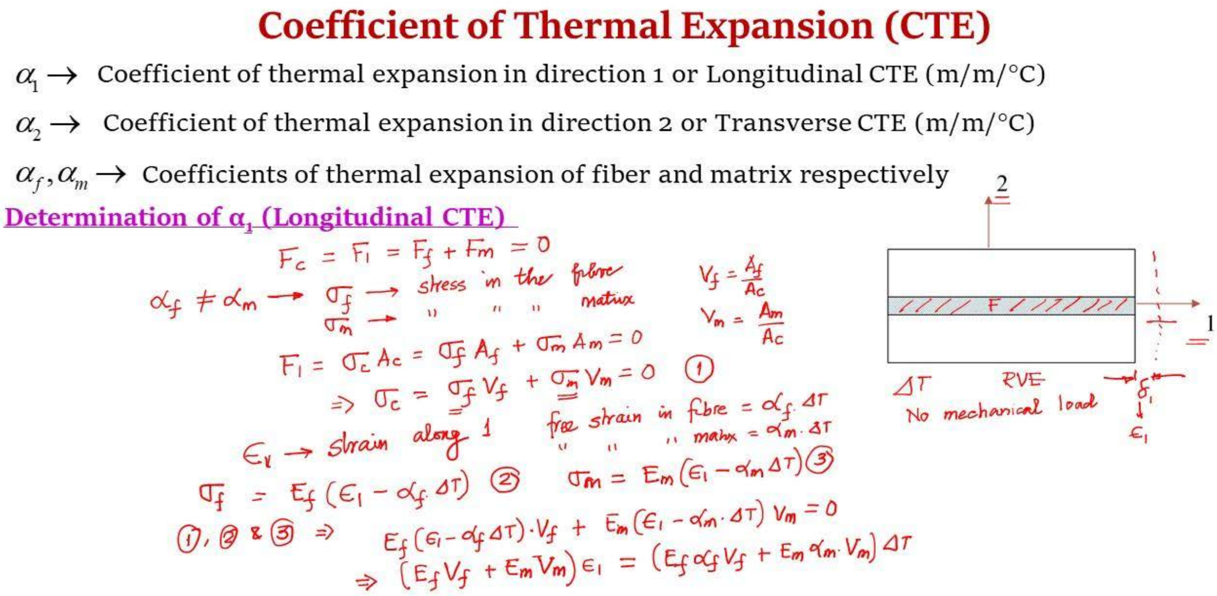
Mechanics of Fiber Reinforced Polymer Composite Structures
Prof. Debabrata Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology-Guwahati

Module-06
Macromechanics of Lamina-II
Lecture-14
Evaluation of Hygrothermal Properties

Hello we have been discussing the determination of stiffness and strength parameters using micromechanical models. In last few lectures the determination of the stiffnesses as well as the strength parameters of a composite using micromechanics model especially with the mechanics of material approach were discussed along with the limitations.

(Refer Slide Time: 01:07)



In this lecture, we will discuss the determination of coefficients of thermal expansion and coefficients of moisture expansion, the parameters responsible for hygrothermal response of a lamina. The importance of the determination of hygrothermal stresses in a lamina was already discussed while discussing the hygrothermal stresses in lamina.

In order to determine the hygrothermal stresses in a laminate we must know the hygrothermal responses of a lamina and in order to understand the hygrothermal response of a lamina coefficient of thermal expansion and coefficient of moisture expansions must be known. So, we shall discuss the determination of coefficients of thermal expansion and coefficients of moisture expansion using micromechanics model.

In an orthotropic lamina, coefficient of thermal expansion is also direction dependent and with reference to the principal material axis (1-2, 1 is the longitudinal axis and 2 is the transverse axis) as shown in Fig., there will be coefficient of thermal expansion along longitudinal direction and coefficient of thermal expansion along the transverse direction and they will be different.

Generally, the coefficient of thermal expansion of this fiber reinforced polymer matrix lamina in the longitudinal direction is much less compared to that in the transverse direction. The reason is, in the longitudinal direction the fiber is having a coefficient of thermal expansion which is much lower than that of the matrix and it puts constraint on the expansion in the longitudinal direction.

Considering an RVE as shown in Fig., suppose,

$$\alpha_1 \rightarrow linear\ coefficient\ of\ thermal\ \exp ansion\ in\ direction\ 1/Longitudinal\ CTE$$
$$\alpha_2 \rightarrow linear\ coefficient\ of\ thermal\ \exp ansion\ in\ direction\ 2/Transverse\ CTE$$
$$\alpha_{f,m} \rightarrow CTE\ of\ fiber, matrix\ respectively$$

All the assumptions like perfect bonding, uniform fiber spacing etc which we have discussed also hold good here. Suppose this composite is actually experiencing a temperature change of ΔT, and there is no mechanical load applied. Because the composite is not subjected to any mechanical load therefore there is no net load applied in direction 1.

Even though there is no net load applied, there will be stresses induced in the fiber and the matrix because of the mismatch of coefficient of thermal expansions because $\alpha_f \neq \alpha_m$ which leads to stresses in the fiber and in the matrix, which were already discussed in while studying the hygrothermal stresses.

Because this is perfectly bonded therefore under this ΔT they will have same longitudinal expansion, δ and therefore, they will also have the same strain in along direction 1. But suppose the fiber and the matrix are actually unrestrained, they are free then the expansion of the fiber and the expansion of the matrix would have been different. Therefore, some amount of strain is actually restrained and that leads to the development of stresses. Referring to the RVE in the Fig.,

$$F_1 = 0$$
$$\sigma_c A_c = \sigma_f A_f + \sigma_m A_m = 0$$
$$\sigma_f A_f + \sigma_m A_m = 0$$

So diving by A_c and referring to the RVE, the volume fraction of the fiber ie. $V_f = A_f / A_c$ and the volume fraction of the matrix $V_m = A_m/A_c$, we get

$$\sigma_f V_f + \sigma_m V_m = 0$$

Now let us try to understand what is the stress? So, suppose ε_1 is the strain of the composite strain along 1. Now if the fibers are free then the free strain in fiber would have been $\alpha_f \Delta T$. Similarly free strain in the matrix would have been $\alpha_m \Delta T$. But then they are not allowed to expand freely because they are restrained because the perfect bonding between the fiber and the matrix.

So, the amount of restrain in the fiber is $\varepsilon_1 - \alpha_f \Delta T$. So, multiplied this by the Young's modulus of the fiber will actually give us the stress induced in the fiber as $\sigma_f = E_f (\varepsilon_1 - \alpha_f \Delta T)$. Similarly, the amount of restrain in the matrix is $(\varepsilon_1 - \alpha_m \Delta T)$ and the stress induced in the matrix is $\sigma_m = E_m (\varepsilon_1 - \alpha_m \Delta T)$. Therefore,

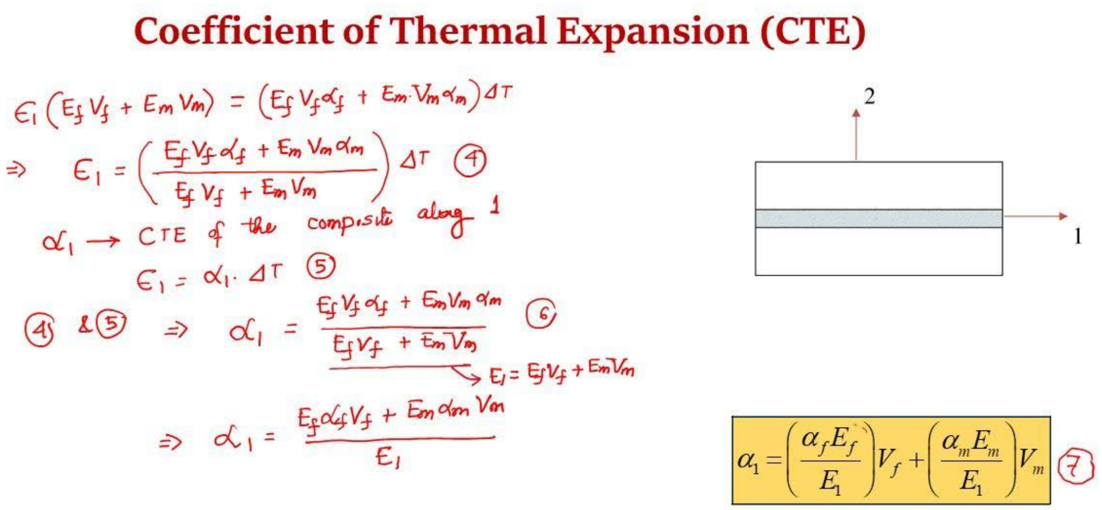
$$\sigma_f = E_f \left(\varepsilon_f - \alpha_f \Delta T \right); \sigma_m = E_m \left(\varepsilon_m - \alpha_m \Delta T \right)$$

$$\varepsilon_f = \varepsilon_m = \varepsilon_1$$

$$\left(E_f \varepsilon_1 - E_f \alpha_f \Delta T \right) V_f + \left(E_m \varepsilon_1 - E_m \alpha_m \Delta T \right) V_m = 0$$

$$\varepsilon_1 \left[E_f V_f + E_m V_m \right] = E_f \alpha_f \Delta T V_f + E_m \alpha_m \Delta T V_m$$

(Refer Slide Time: 11:16)



Therefore,

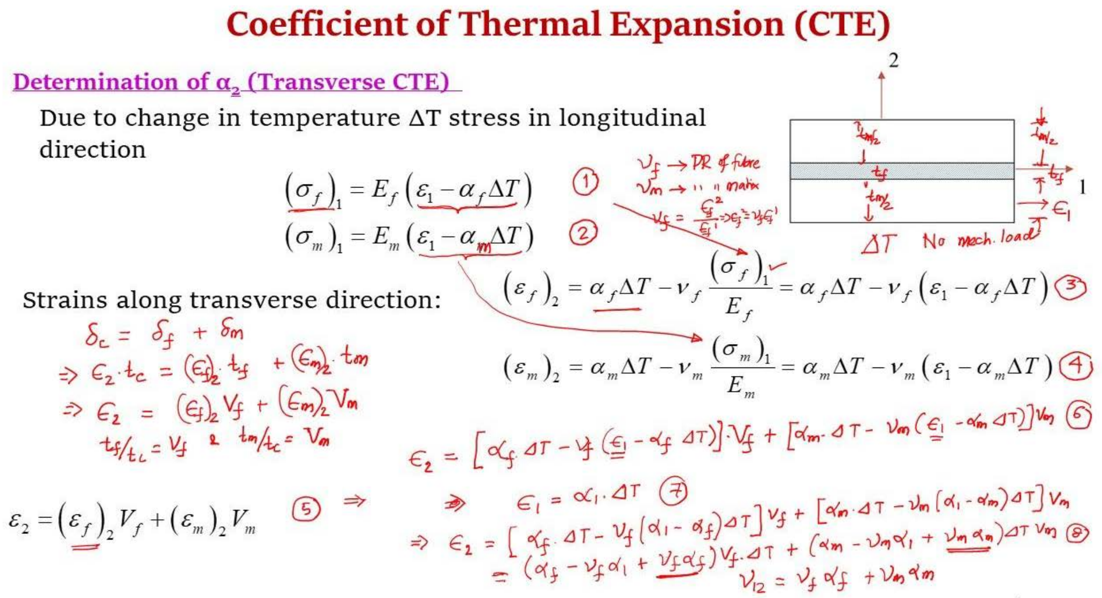
$$\varepsilon_1 = \frac{\left(E_f \alpha_f V_f + E_m \alpha_m V_m \right) \Delta T}{E_f V_f + E_m V_m} = \alpha_1 \Delta T$$

Now, going by the definition of coefficient of thermal expansion along direction 1,

$$\alpha_1 = \frac{\left(E_f \alpha_f V_f + E_m \alpha_m V_m\right)}{E_f V_f + E_m V_m} = \left(\frac{\alpha_f E_f}{E_1}\right) V_f + \left(\frac{\alpha_m E_m}{E_1}\right) V_m \tag{7}$$

It could be seen that this is actually of the form that like this $E_1 = E_f V_f + E_m V_m$ with, $\alpha_f\, E_f/E_1$ as apparent α_f and $\alpha_m\, E_m/E_1$ as apparent α_m So, the longitudinal coefficient of thermal expansion is a function of the coefficient of thermal expansion of the fiber and the matrix as well as the Young's moduli of the fiber and the matrix besides the volume fraction. Next let us see the determination of transverse coefficient of thermal expansion.

(Refer Slide Time: 15:27)



Here also the same RVE is considered as shown only subjected to ΔT, no mechanical load. When this RVE of the composite actually experiences a temperature changes of ΔT, the strain in this direction is ε_1 and because of the perfect bonding, the fiber, the matrix and the composite experiences the same strain along 1.

$$\varepsilon_1 = \varepsilon_f = \varepsilon_m \tag{8}$$

and we already discussed while discussing the determination of α_1, the residual stresses in the fiber and the matrix are

$$\left(\sigma_f\right)_1 = E_f\left(\varepsilon_f - \alpha_f \Delta T\right);\left(\sigma_m\right)_1 = E_m\left(\varepsilon_m - \alpha_f \Delta T\right) \tag{9}$$

Now the strains in the direction 2 for the fiber is the direct strain along due to ΔT which is $\alpha_f \Delta T$. In addition, there is strain along direction 2 due to the strain along direction 1 which is the Poisson's effect. Suppose ν_f is the Poisson's ratio of the fiber, then $\varepsilon_f^2 = \nu_f \ \varepsilon_f^1$ and $\varepsilon_f^1 = \sigma_f^1/E_f$,. Similarly, in the direction 2 for the fiber is the direct strain along due to ΔT which is $\alpha_m \Delta T$. In addition, there is strain along direction 2 due to the strain along direction 1 which is the Poisson's effect. Suppose ν_m is the Poisson's ratio of the fiber, *then* $\varepsilon_m^2 = \nu_m \ \varepsilon_m^1$ and $\varepsilon_m^1 = \sigma_m^1/ E_f$,.

So, the transverse strains in the fiber and the matrix that means along direction 2 are

$$\left(\varepsilon_f\right)_2 = \alpha_f \Delta T - \nu_f \frac{\left(\sigma_f\right)_1}{E_f}; \ \left(\varepsilon_m\right)_2 = \alpha_m \Delta T - \nu_m \frac{\left(\sigma_m\right)_1}{E_f} \qquad (10)$$

Total change in in length along the transverse direction is sum of that due to the fiber and the matrix (Ref to the RVE in Fig.)

$$\delta_2 = \delta_f + \delta_m \qquad (11)$$

Expressing in terms of the strains

$$\varepsilon_2 t_c = \varepsilon_f t_f + \varepsilon_m t_m$$
$$\varepsilon_2 = \varepsilon_f \frac{t_f}{t_c} + \frac{t_m}{t_c} \varepsilon_m$$
$$\varepsilon_2 = \varepsilon_f V_f + \varepsilon_m V_m \qquad (12)$$

Taking note of the fact that for this RVE; $t_f / t_c = V_f$ and $t_m / t_c = V_m$. So, we could express the transverse strain in direction 2 in terms of the transverse strain in the fiber and transverse state in the matrix. So, we could express the transverse strain in direction 2 in terms of the transverse strain in the fiber and transverse state in the matrix.

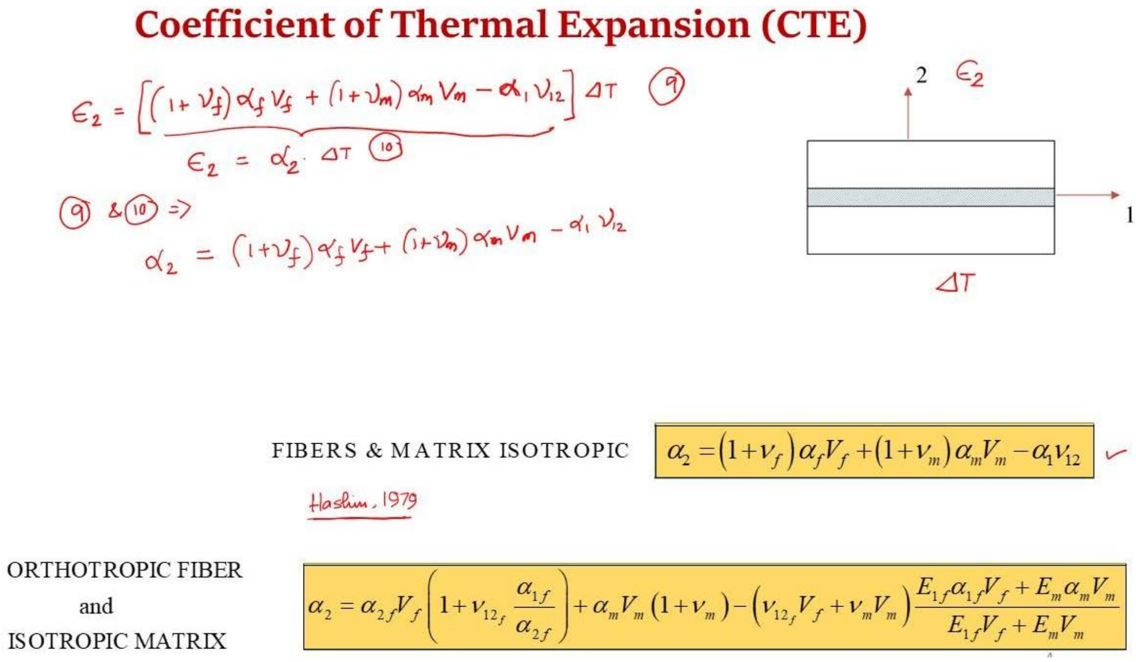
Putting the expressions for transverse strain in the fiber and the matrix from(10) in (12)

$$\varepsilon_2 = \left(\alpha_f \Delta T - \nu_f \left(\alpha_1 - \alpha_f\right) \Delta T\right) V_f + \left(\alpha_m \Delta T - \nu_m \left(\alpha_1 - \alpha_m\right) \Delta T\right) V_m$$
$$\varepsilon_2 = \left(\left(1 + \nu_f\right) \alpha_f V_f + \left(1 + \nu_m\right) \alpha_m V_m - \alpha_1 \left(\nu_f V_f + \nu_m V_m\right)\right) \Delta T = \alpha_2 \Delta T$$

Now, going by the definition of coefficient of thermal expansion along direction 2 and taking note of the fact that $\nu_{12} = \nu_f \ \alpha_f + \nu_m \ \alpha_m$

$$\alpha_2 = (1+\nu_f)\alpha_f V_f + (1+\nu_m)\alpha_m V_m - \alpha_1 \nu_{12} \qquad (13)$$

(Refer Slide Time: 24:13)



Now in deriving these expressions actually the both fiber and the matrix properties are isotropic. There are works (Hashin in 1979) where the properties of the fiber have been actually considered as orthotropic and the expression for α_2 is obtained as

$$\alpha_2 = \alpha_{2f} V_f \left(1+\nu_{12_f}\frac{\alpha_{1f}}{\alpha_{2f}}\right) + \alpha_m V_m (1+\nu_m) - \left(\nu_{12_f} V_f + \nu_m V_m\right)\frac{E_{1f}\alpha_{1f}V_f + E_m \alpha_m V_m}{E_{1f}V_f + E_m V_m} \qquad (14)$$

Now you will see that α_1 and α_2 actually depends on the α_f , α_m , their volume fraction and elastic properties.

For most of these fiber reinforced polymer matrix composites α_1 is less compared to α_2. This is because in the longitudinal direction the fiber actually restrains the expansion and therefore the α_1 is less. But in the transverse direction it is dominated by the matrix and therefore α_2 is more compared to that of α_1.

(Refer Slide Time: 27:46)

Coefficient of Moisture Expansion (CME)

Determination of β_1, β_2 (Longitudinal and Transverse CME)

$\beta_1 \rightarrow$ Coefficient of moisture expansion in direction 1 or Longitudinal CME (m/m/kg/kg)

$\beta_2 \rightarrow$ Coefficient of moisture expansion in direction 2 or Transverse CME (m/m/kg/kg)

$\beta_f, \beta_m \rightarrow$ Coefficients of moisture expansion of fiber and matrix respectively

$\Delta C_c, \Delta C_f, \Delta C_m \rightarrow$ Moisture concentration in composite, fiber, matrix respectively (kg/kg)

$\varepsilon_1 \rightarrow$ Strain along direction 1

Stress in the fiber, $\sigma_f = E_f\left(\varepsilon_1 - \beta_f \Delta C_f\right)$ ①

Stress in the matrix, $\sigma_m = E_m\left(\varepsilon_1 - \beta_m \Delta C_m\right)$ ②

Free strain exp^n of fibre $= \beta_f \cdot \Delta C_f$

" " " Matrix $= \beta_m \cdot \Delta C_m$

$F_1 = F_f + F_m = 0$

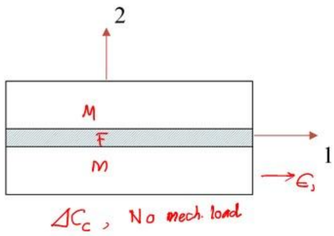
$$\sigma_f V_f + \sigma_m V_m = 0 \quad ③$$

①, ② & ③ $\Rightarrow E_f\left(\epsilon_1 - \beta_f \cdot \Delta C_f\right) \cdot V_f + E_m\left(\epsilon_1 - \beta_m \Delta C_m\right) V_m = 0$

$\Rightarrow \left(E_f V_f + E_m V_m\right) \epsilon_1 = E_f \cdot \beta_f \cdot \Delta C_f \cdot V_f + E_m \beta_m \Delta C_m \cdot V_m$

$$\Rightarrow \epsilon_1 = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m \cdot V_m}{E_f V_f + E_m V_m} = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m \cdot V_m}{E_1} \quad ④$$

$$= \left(\frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m}{E_1 \cdot \Delta C_c}\right) \Delta C_c \quad ⑤$$



5

in the hygrothermal stress analysis of lamina, we have also discussed the importance of determination of hygroscopic stresses in a polymer matrix composite where the matrix absorbs moisture and therefore it experiences strain. Because most of the fibers, actually inorganic fibers are insensitive to moistures therefore there will be residual stresses and these residual stresses should be taken into account while analyzing or while designing the components made of this fiber reinforced laminated composites.

So, let us see how we determine the coefficient of moisture expansion similar to the coefficient of thermal expansion. The coefficients of moisture expansion in composites are also direction dependent, and they are different in the longitudinal direction and in the transverse direction. Let us see how to determine the coefficient of moisture expansion. Suppose,

$\beta_1 \rightarrow linear\ coefficient\ of\ thermal\ \exp ansion\ in\ direction\ 1 / Longitudinal\ CME$

$\beta_2 \rightarrow linear\ coefficient\ of\ thermal\ \exp ansion\ in\ direction\ 2 / Transverse\ CME$

$\beta_{f,m} \rightarrow CME\ of\ fiber, matrix\ respectively$

$\Delta C_{f,m} \rightarrow moisture\ concentration\ in\ the\ fiber, matrix\ \left(kg/kg\right)$

Analogous to the thermal expansion, since the lamina (RVE as shown) is only experiencing ΔC and no mechanical load, But residual stresses will be induced in the fiber and the matrix due to the difference between the free expansion and the constrained expansion due to perfect bonding and the residual stresses are $\sigma_f = E_f\left(\varepsilon_f - \beta_f \Delta C_f\right); \sigma_m = E_m\left(\varepsilon_m - \beta_m \Delta C_m\right)$. Therefore

$$F_1 = 0$$
$$\sigma_c A_c = \sigma_f A_f + \sigma_m A_m = 0$$
$$\sigma_f V_f + \sigma_m V_m = 0$$

Now,

$$\sigma_f = E_f\left(\varepsilon_f - \beta_f \Delta C_f\right); \sigma_m = E_m\left(\varepsilon_m - \beta_m \Delta C_m\right)$$

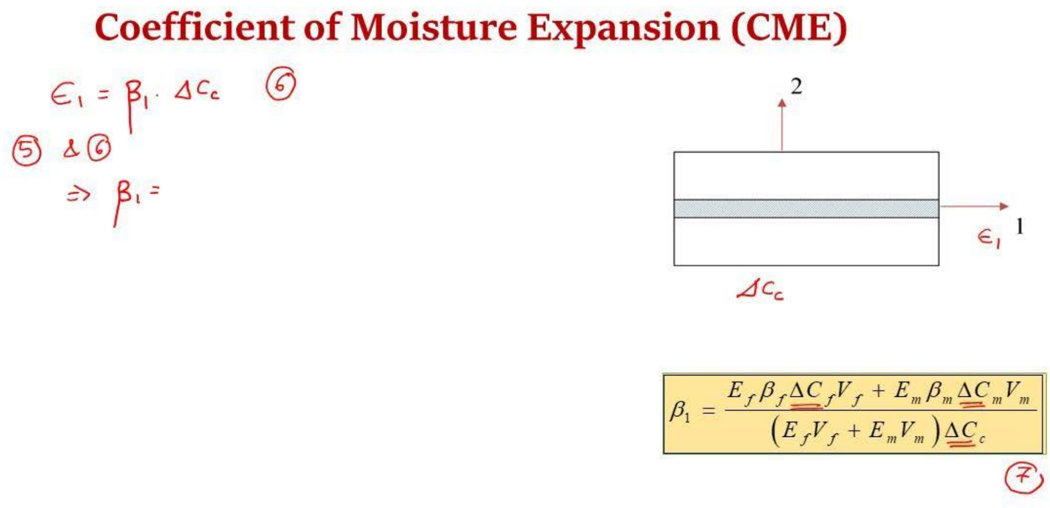
Using $\varepsilon_f = \varepsilon_m = \varepsilon_1$ due to perfect bonding

$$\left(E_f \varepsilon_1 - E_f \beta_f \Delta C_f\right)V_f + \left(E_m \varepsilon_1 - E_m \beta_m \Delta C_m\right)V_m = 0$$
$$\varepsilon_1\left[E_f V_f + E_m V_m\right] = E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m$$
$$\varepsilon_1 = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m}{E_f V_f + E_m V_m} = \beta_1 \Delta C_c$$

And going by the definition of coefficient of moisture expansion along 1,

$$\beta_1 = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m}{\left(E_f V_f + E_m V_m\right)\Delta C_c} \tag{15}$$

(Refer Slide Time: 36:04)



Now this ΔC_c could be written in terms of ΔC_f and ΔC_m as follows

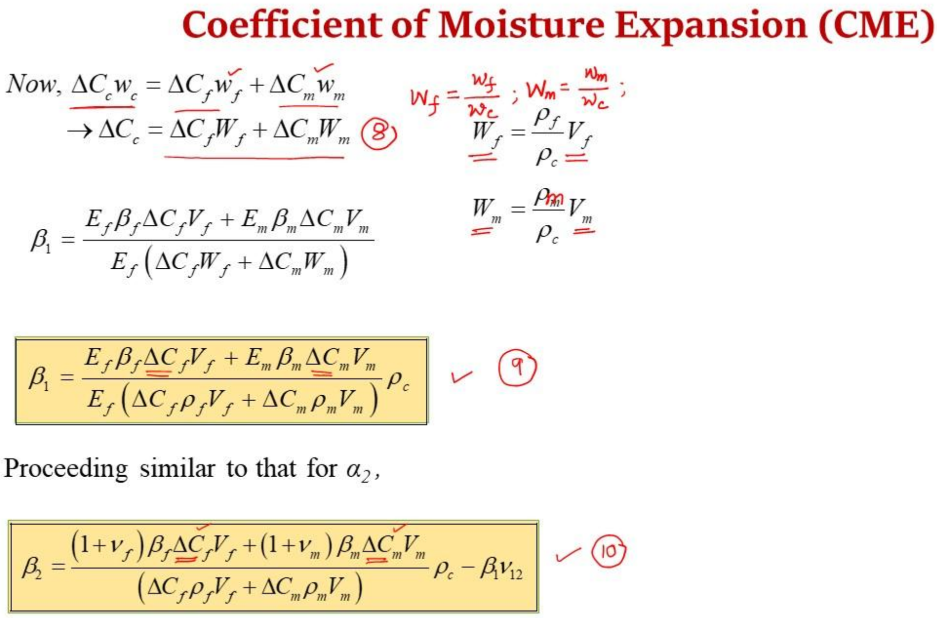
$$\beta_1 = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m}{\left(E_f V_f + E_m V_m\right)\Delta C_c} \tag{15}$$

$$now\ \Delta C_c w_c = \Delta C_f w_f + \Delta C_m w_m \rightarrow \Delta C_c = \Delta C_f W_f + \Delta C_m W_m$$

$$\beta_1 = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m}{E_f \left(\Delta C_f W_f + \Delta C_m W_m\right)} \qquad W_f = \frac{\rho_f}{\rho_c} V_f \qquad W_m = \frac{\rho_{fm}}{\rho_c} V_m$$

$$\boxed{\beta_1 = \frac{E_f \beta_f \Delta C_f V_f + E_m \beta_m \Delta C_m V_m}{E_f \left(\Delta C_f \rho_f V_f + \Delta C_m \rho_m V_m\right)} \rho_c} \tag{16}$$

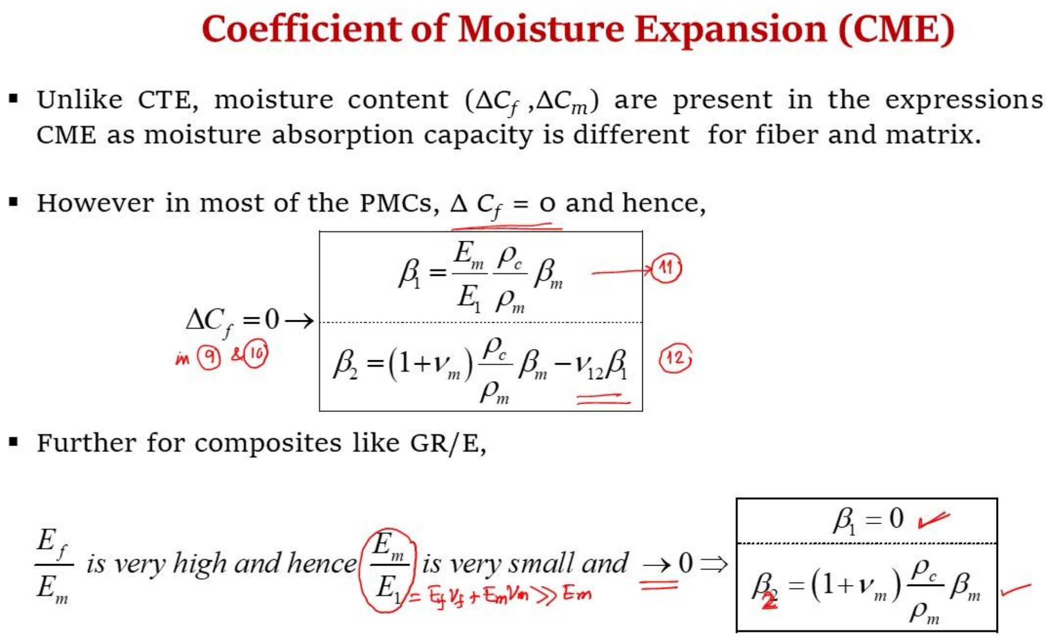
(Refer Slide Time: 37:28)



This is the expression for the coefficient of moisture expansion along direction 1. Similarly, we can also determine the coefficient of moisture expansion in direction 2 (β_2) analogous to determination of α_2 and we obtain

$$\boxed{\beta_2 = \frac{\left(1+\nu_f\right)\beta_f \Delta C_f V_f + \left(1+\nu_m\right)\beta_m \Delta C_m V_m}{\left(\Delta C_f \rho_f V_f + \Delta C_m \rho_m V_m\right)} \rho_c - \beta_1 \nu_{12}} \tag{17}$$

(Refer Slide Time: 40:13)



Note that unlike CTEs (α_1 and α_2), in the expressions for CMEs (β_1 and β_2) contains the moisture concentration terms (ΔC_f and ΔC_m) as the moisture absorptions for the fiber and the matrix are different. However, for most of the polymer matrix composites, the inorganic fibers are are almost insensitive to moisture and hence $\Delta C_f = 0$ and putting $\Delta C_f = 0$ in (17), we get

$$\Delta C_f = 0 \rightarrow \boxed{\begin{array}{c} \beta_1 = \dfrac{E_m}{E_1}\dfrac{\rho_c}{\rho_m}\beta_m \\ \hdashline \beta_2 = (1+\nu_m)\dfrac{\rho_c}{\rho_m}\beta_m \end{array}} \qquad (18)$$

Now, since, $\frac{E_f}{E_m} \simeq \infty$, and $E_1 = E_f V_f + E_m V_{m1}$ therefore, $\frac{E_m}{E_1} \rightarrow 0$ and hence

$$\boxed{\begin{array}{c} \beta_1 = 0 \\ \hdashline \beta_2 = (1+\nu_m)\dfrac{\rho_c}{\rho_m}\beta_m \end{array}} \qquad (19)$$

So, this is much more simplified and do not contain ΔC_f and ΔC_m. So, this is the expressions for coefficient of moisture expansion in the longitudinal as well as in the transverse direction.