Mechanics of Fiber Reinforced Polymer Composite Structures
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Lecture-13
Evaluation of Transverse and Shear Strengths

Hello and welcome to the 4th lecture of this module on micro mechanics of lamina. So, in the last lecture we have discussed the determination of longitudinal tensile strength of a lamina wherein it was assumed that the longitudinal tensile strength of a lamina is decided by the fiber failure only.

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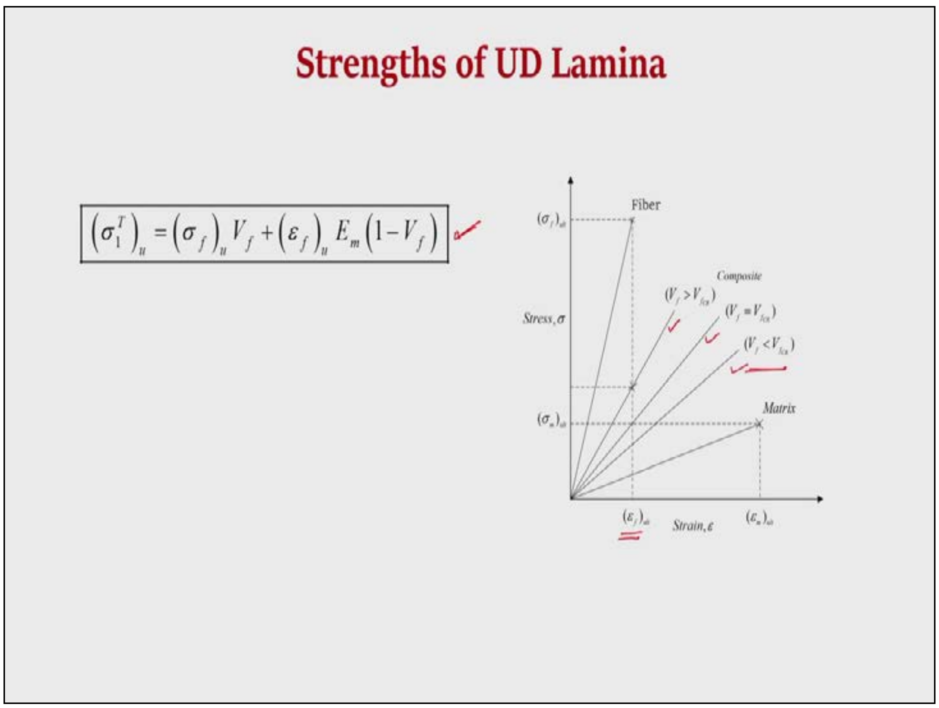

So, when the fibers fail that means when the strain in the fiber reaches the ultimate strain of the fiber then the composite fails and based on that an expression for the longitudinal tensile strength of the composite was derived which is dependent on the strengths and stiffness properties of the fiber and matrix and volume fraction. Therefore, for a given combination of fiber and matrix and for given properties of the fibers and the matrix is the volume fraction which decides what will be the longitudinal tensile strength of a lamina and we understood how the volume fraction actually influences the longitudinal tensile strength in terms of critical volume fraction and minimum volume fraction.

We have discussed in the last lecture that the slope of the stress strain curve for the lamina or the composite will be decided by what is the volume fraction for given properties of the fiber and matrix. Higher is the volume fraction V_f , more it will be inclined towards the fiber stress strain curve and lower is the volume fraction more it will be inclined towards the matrix stress strain curve. If the volume fraction is actually more than a critical volume fraction, then it is ensured that the longitudinal tensile strength of the lamina will be higher than that of the matrix. If the volume fraction is less than the critical volume fraction, then the longitudinal tensile strength of the lamina will be less than that of the matrix, and the objective of adding fibers is not served. We have also seen that there is a minimum volume fraction if the volume fraction is below minimum, then even if fibers break, the remaining matrix could actually withstand the load and below minimum volume fraction, increase in fiber volume fraction leads to reduction in composite strength.

In all these, one of the key assumptions was that the fibers are of uniform strength, that means all the fibers in the lamina break or fail at the same time. This assumption does not go well with the practical composites. Therefore, there will be deviations in the predicted strength from the actual longitudinal tensile strength. There are some other assumptions also made in the determination of the strength parameters as well as stiffness parameters using the micromechanics approach. Because some of the assumptions do not go well with the practical situations, there will be deviations.

Therefore, it is important to understand the factors which actually influence the strengths of a composite or a lamina in particular.

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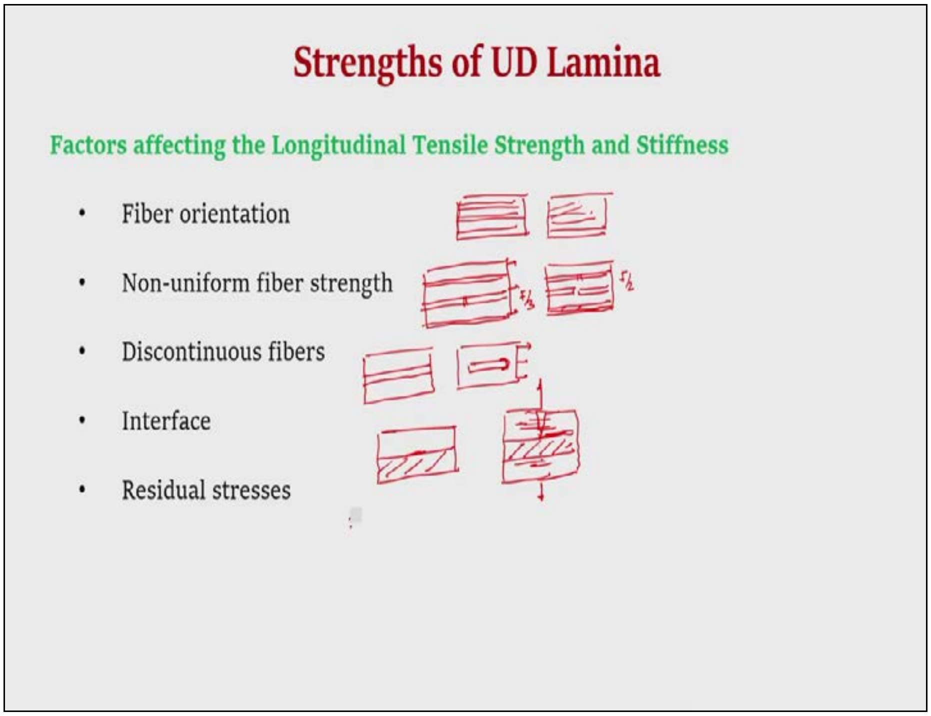

So, some of the important factors influencing the strengths are

Fiber orientation – Fiber are assumed to be perfectly aligned. The fact that the longitudinal strength of the fiber is much higher, in a lamina, the fibers aligned along the direction of loading will provide maximum strength compared to that if some of the fibers are not aligned or say there is misalignment. Misalignment thus affects the strength of the composite. The amount deviation will be decided by the number of fibers which are misaligned and the degree of misalignment.

Non-uniform fiber strength – Strength of the fibers are actually not uniform even though an assumption has been made in the micromechanics model that the strength of the fibers is uniform but it is not so. Actually, the strengths are statistically distributed and therefore affects the longitudinal tensile strength of the composite. Say for example subjected to load suppose the weakest fibers break, then the adjacent intact fibers are over stressed and as a result of that the chances of the failure of those fibers becomes higher. This leads to cumulative fiber break leading to the failure of the lamina. So, this is how the cumulative fiber breakage takes place and the actual tensile strength of the lamina will be less than that has been predicted with the assumptions that the fibers are of uniform strength.

Discontinuous fibers – It was assumed that all the fibers are continuous. Suppose there are discontinuous fibers. Subjected to load, the load on the matrix is transferred to the fiber by the edge of the fiber and there is a stress concentration at that point. If the zone of stress concentration is far less compared to the length of the fiber we can consider that the fibers are of infinite length. Otherwise, for short fibers this has to be taken into account and that influences the longitudinal tensile strength of the composite.

Interface –A perfect interface has been assumed and stronger is the interface, stronger is the composite. If the interface is weak, then the transverse crack, which sometimes gets induced in a composite, propagates to the interface leading to the degradation of the strength of the composite. When a transverse crack grows, it encounters a fiber and because the fibers are very strong, the crack actually propagates through the interface. So, for a weaker interface, it propagates through this fiber matrix interface and then the failure takes place, thereby reducing the overall strength of the composite.

Residual stresses – Residual stresses are induced in the lamina because of two reasons viz. the mismatch in coefficient of thermal expansion and the fabrication temperature is different compared to the operating temperature. Residual stresses thus induced get added to the stresses applied by the mechanical loading whenever composite or lamina is loaded and thereby the strength of the composite is reduced. Therefore, residual stresses should be taken into account while analyzing the strength of a laminated composite.

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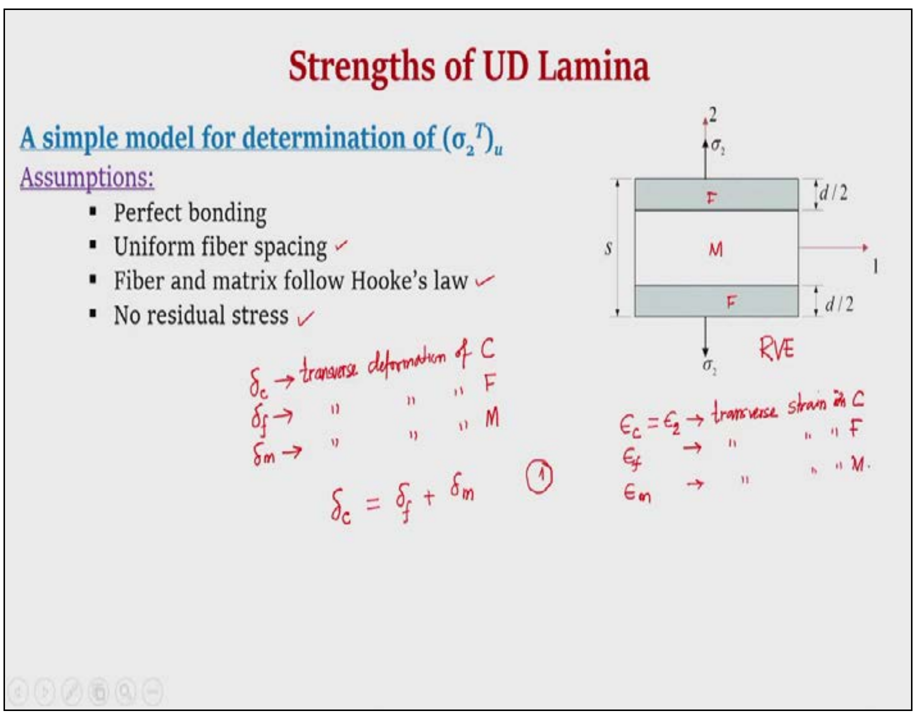

In the last lecture, longitudinal tensile strength of a lamina was discussed. Now, let us discuss the transverse tensile strength. Unlike the longitudinal tensile strength which is almost entirely decided by the fiber properties, the transverse tensile strength is actually influenced by many other factors like bond strength of the interface presence of voids etc.

It is observed that the transverse tensile strength of a lamina or a composite is actually less than that of the matrix tensile strength means addition of fiber actually have a negative effect on the transverse tensile strength. Now a simplest way to determine the transverse tensile strength is to assume that that transverse tensile strength is due to the failure of the matrix.

Considering a simple RVE as shown in the Fig. having fiber diameter *d* and the fiber spacing as *s* and the stress applied in the direction 2 is σ_2. Suppose the transverse displacement of the composite, fiber and matrix are δ_c, δ_f and δ_m respectively.

With usual assumptions of perfect bonding, uniform fiber spacing, both fiber, matrix and the composites follow Hooke's law that is linear elastic and no residual stress.

$$\text{Now, } \delta_c = \delta_f + \delta_m \qquad (1)$$

Now suppose ε_c , ε_f and ε_m are the transverse strains (along 2) in the composite, fiber and matrix respectively.

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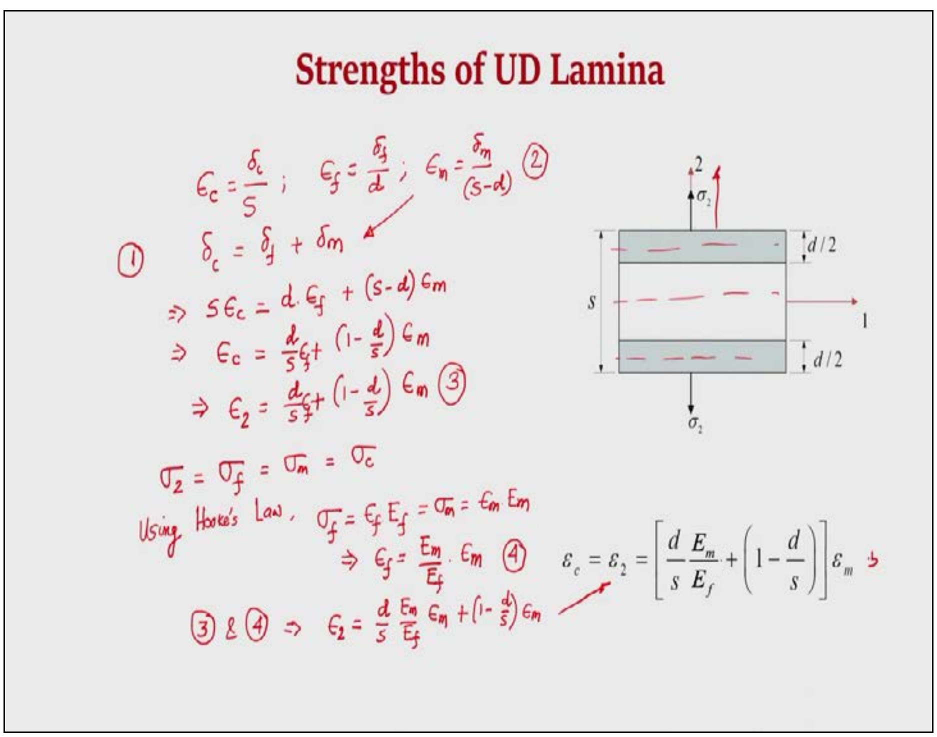

Now looking at this RVE and going by the definition of the strain we can write the expression for strain ε_c

$$\varepsilon_c = \frac{\delta_c}{s};\ \varepsilon_f = \frac{\delta_f}{d};\ \varepsilon_m = \frac{\delta_m}{s-d} \qquad (2)$$

$$\text{So, } \delta_c = \varepsilon_c s;\ \delta_f = \varepsilon_f d;\ \text{and } \delta_m = \varepsilon_m (s-d)$$

$$\delta_c = \delta_f + \delta_m$$

$$\Rightarrow s\varepsilon_c = d\varepsilon_f + (s-d)\varepsilon_m$$

$$\Rightarrow \varepsilon_c = \frac{d}{s}\varepsilon_f + \left[1 - \frac{d}{s}\right]\varepsilon_m$$

$$\Rightarrow \varepsilon_2 = \frac{d}{s}\varepsilon_f + \left[1 - \frac{d}{s}\right]\varepsilon_m \qquad (3)$$

Now,

$$\sigma_2 = \sigma_f = \sigma_m$$

Because this is the stress is in this direction 2, so, uniform stress, of course with the assumption that the fibers are equally spaced in the thickness direction. So, stresses are same in the transverse direction in the in the fiber matrix and the composite. Now with the assumptions that that the fiber, matrix and the composite follow Hooke's law.

$$\sigma_f = \varepsilon_f E_f = \sigma_m = \varepsilon_m E_m$$

$$\varepsilon_f = \frac{E_m}{E_f}\varepsilon_m \qquad (4)$$

Now putting Eq. (4) in Eq. (3)

$$\Rightarrow \varepsilon_2 = \frac{d}{s}\frac{E_m}{E_f}\varepsilon_m + \left[1 - \frac{d}{s}\right]\varepsilon_m$$

$$\Rightarrow \varepsilon_c = \varepsilon_2 = \frac{d}{s}\frac{E_m}{E_f}\varepsilon_m + \left[1 - \frac{d}{s}\right]\varepsilon_m \qquad (5)$$

So, we could obtain an expression for transverse strain in the composite in terms of the transverse strain in the matrix the Young's moduli of the matrix and the fiber and the relative spacing *d* /*s* which is relative dimensions of the fibers and the matrix.

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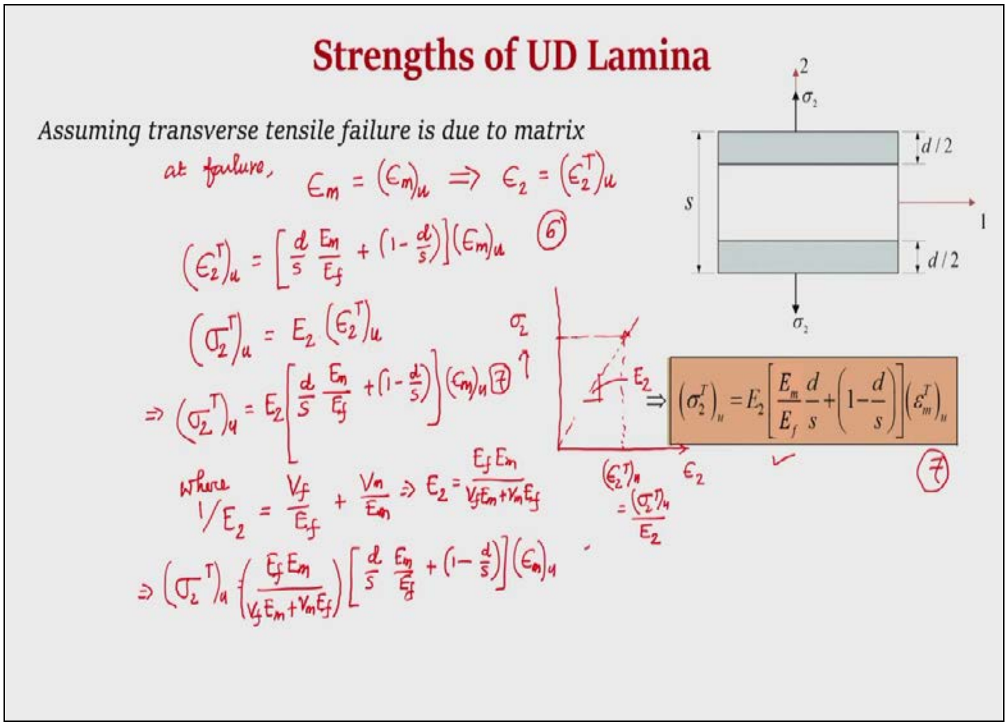

Now with the assumption that the transverse failure of the composite is due to the failure of the matrix meaning that at failure, the strain in the matrix ε_m is equal to ultimate strength of the matrix. So, from the last equation that means equation number 5 we can now write we can replace ε_m by $(\varepsilon_m)_u$ and ε_2 as $(\varepsilon_2^T)_u$.

Therefore

$$(\varepsilon_2^T)_u = \left(\frac{d}{s}\frac{E_m}{E_f} + \left[1 - \frac{d}{s}\right]\right)(\varepsilon_m)_u \qquad (6)$$

Therefore, again the composite also always Hooks law. Therefore we can write the transverse tensile strength of the composite is nothing but $E_2(\varepsilon_2^T)_u$.

Therefore

$$(\sigma_2^T)_u = E_2\left(\frac{d}{s}\frac{E_m}{E_f} + \left[1 - \frac{d}{s}\right]\right)(\varepsilon_m)_u \qquad (7)$$

$$\text{where,} \quad \frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$\Rightarrow E_2 = \frac{E_f E_m}{V_f E_m + V_m E_f}$$

$$(\sigma_2^T)_u = \frac{E_f E_m}{V_f E_m + V_m E_f}\left(\frac{d}{s}\frac{E_m}{E_f} + \left[1 - \frac{d}{s}\right]\right)(\varepsilon_m)_u \quad (8)$$

So, this is the expression for transverse tensile strength of a lamina in terms of the strength of the matrix and the Young's moduli of the fiber in the matrix and the volume fraction (*d/s*). This is a simple model where it was assumed that the transverse tensile failure of the lamina is because of the strain in the matrix reaching the ultimate strain of the matrix.

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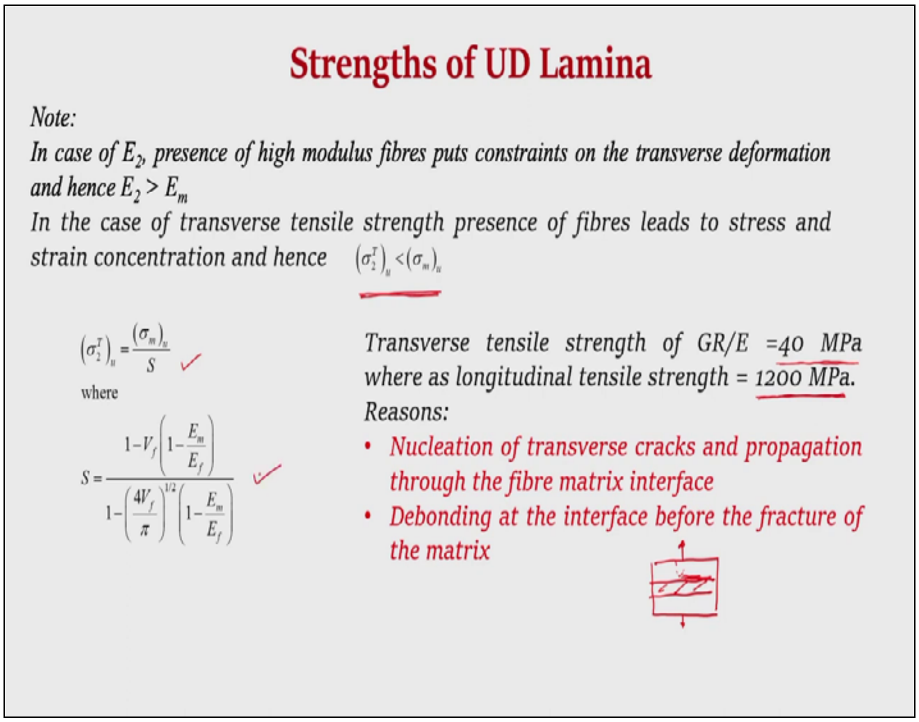

Now as you have discussed earlier see the; now if you see that in the case of E_1 or in the case of $(\sigma_1^T)_u$ actually the presence of fibers led to the improvement in the longitudinal modulus as well as longitudinal tensile strength. Of course if the volume fraction is kept above certain critical volume fraction that leads to the improvement in the longitudinal tensile strength. Also, E_1 that is the longitudinal Young's modulus is of the composite is more than that of the matrix by adding the high modulus fibers.

Even in case of E_2, the transverse modulus of the composite is more than that of the matrix because the presence of the high modulus fibers actually puts constraints in the transverse deformation thereby increasing the transverse modulus. But in the case of transverse tensile strength actually the presence of the fibers leads to induction of stress concentration in the matrix and thus the strength of the composite is actually the transverse tensile strength of the composite is actually less than that of the matrix. So, the presence of the fibers in the case of transverse tensile strength is having a negative effect. For example, for graphite epoxy the transverse tensile strength is only 40 MPa whereas the longitudinal tensile strength is 1200 MPa this is the reason is because under transverse loading the cracks actually nucleate in the transverse direction and it propagates through the fiber matrix interface.

So, the net result is that the transverse strength of the composite is less than that of the matrix and it is sometimes written as

$$(\sigma_2^T)_u = \frac{(\sigma_m)_u}{s}$$

where s is strength reduction factor given by

$$s = \frac{1 - V_f\left(1 - \frac{E_m}{E_f}\right)}{1 - \left(\frac{4V_f}{\pi}\right)^{1/2}\left(1 - \frac{E_m}{E_f}\right)}$$

Now for some values it could be maybe 2 that means the strength of the transverse tensile strength of the composite is half that of the matrix.

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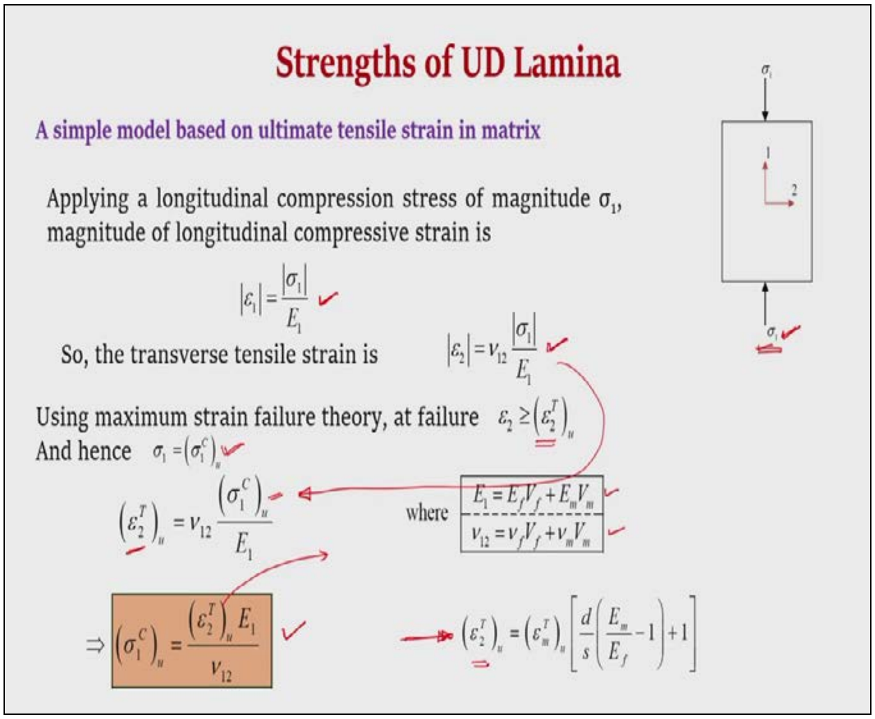

Next is a longitudinal compressive strength of a composite. Now the models which are actually used for tensile strength may not be suitable for longitudinal compressive strength. Because under compression the failure modes are different there are different possible failure modes like transverse tensile, shear failure, fiber micro buckling.

Depending upon the volume fraction, if the volume fraction is very low the fiber micro buckling may take place even at a much lower level of stress whereas with a practical volume fraction of say maybe forty percent the fiber micro buckling many times actually is preceded by the fiber matrix debonding. Referring to the Fig., fiber micro buckling may be in the extension mode or in shear mode. Fiber micro buckling and shear failure will be addressed later where there are localized buckling of the fibers and not gross buckling of the laminate.

In the transverse tensile mode, when the composite is subjected to compression loading, the matrix will experience tensile strain in the lateral direction and due to this tensile strain in the lateral direction the matrix might fail or there may be fiber matrix debonding.

A simple model based on the ultimate tensile strain of the matrix is shown in the Fig. where a lamina is subjected to a compression stress σ_1 in the direction 1 which is the principal material direction and naturally because of that there will be transverse strain.

$$|\varepsilon_1| = \frac{|\sigma_1|}{E_1}$$

So, due to Poisson's effect, the transverse tensile strain is

$$|\varepsilon_2| = \nu_{12}\frac{|\sigma_1|}{E_1}$$

Using maximum strain failure theory, at failure, $\varepsilon_2 \geq (\varepsilon_2^T)_u$ and hence $\sigma_1 \geq (\sigma_1^C)_u$

$$(\varepsilon_2^T)_u = \nu_{12}\frac{(\sigma_1^C)_u}{E_1}$$

$$(\sigma_1^C)_u = E_1\frac{(\varepsilon_2^T)_u}{\nu_{12}}$$

where $E_1 = E_f V_f + E_m V_m$; $\nu_{12} = \nu_f V_f + \nu_m V_m$ and $(\varepsilon_2^T)_u = (\varepsilon_m^T)_u \left(\frac{d}{s}\left[\frac{E_m}{E_f} - 1 \right] + 1 \right)$

So, this is a simple model, however this model does not go well with the experimental results. Therefore a more rigorous model considering the fiber micro backlink has to be considered.

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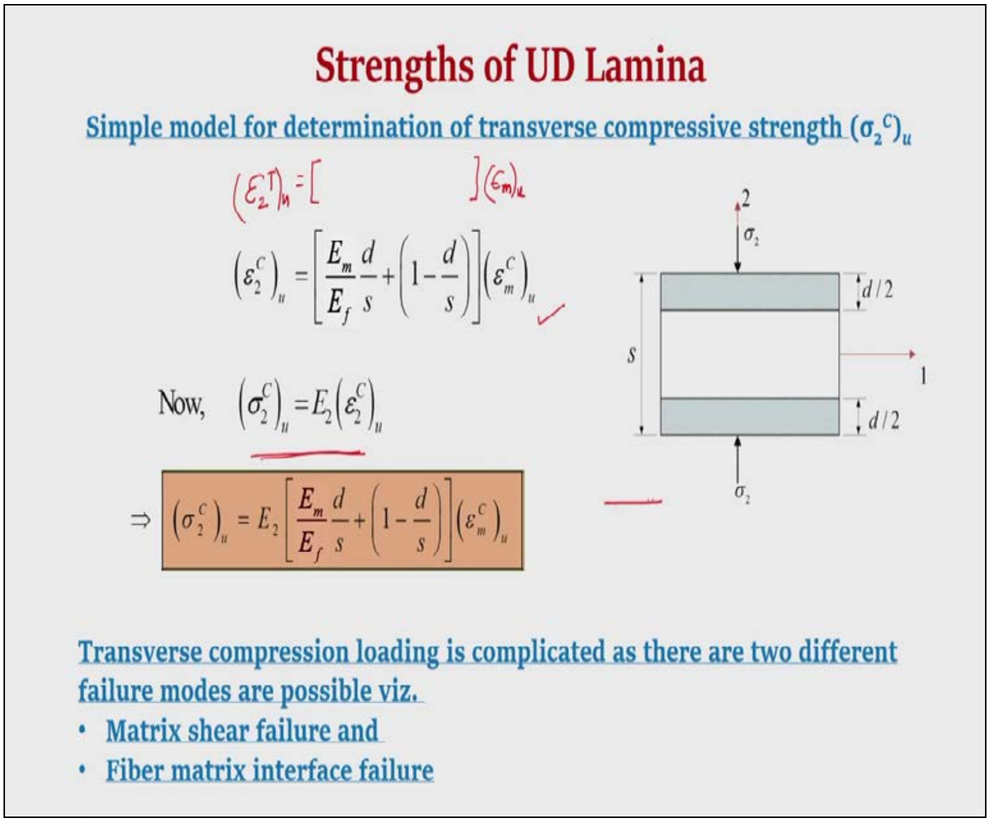

Next let us discuss the determination of transverse compressive strength. So, again a simple model analogous to the transverse tensile strength could be used. It is similar to transverse tensile strength but we have replaced the tension by compression and the same expression which was derived for the transverse tensile strength is used where $(\varepsilon_2^T)_u$ is replaced by $(\varepsilon_2^C)_u$ and $(\varepsilon_m)_u$ is replaced by $(\varepsilon_m^C)_u$ in the following expression

$$(\varepsilon_2^T)_u = \left(\frac{E_m}{E_f}\frac{d}{s} + \left[1 - \frac{d}{s}\right] \right)(\varepsilon_m)_u$$

we get

$$(\varepsilon_2^C)_u = \left(\frac{E_m}{E_f}\frac{d}{s} + \left[1 - \frac{d}{s}\right] \right)(\varepsilon_m^C)_u$$

and following Hooke's law (multiplying this by E_2) we get the expression for transverse compression strength as

$$(\sigma_2^C)_u = E_2(\varepsilon_2^C)_u$$

$$\Rightarrow (\sigma_2^C)_u = E_2\left(\frac{E_m}{E_f}\frac{d}{s}+\left[1-\frac{d}{s}\right]\right)(\varepsilon_m^C)_u$$

So, from the analysis of some compression failure specimen it could be observed that many times it is because the matrix shear failure or fiber matrix interface failure under this there may be failure at the interface of the fiber and the matrix and it might fail. Therefore, the simple model does not work well for determination of the transverse compression strength. However, this gives us an idea of how the transverse compression strength is actually influenced by the fiber properties and the matrix properties and the relative volume fractions.

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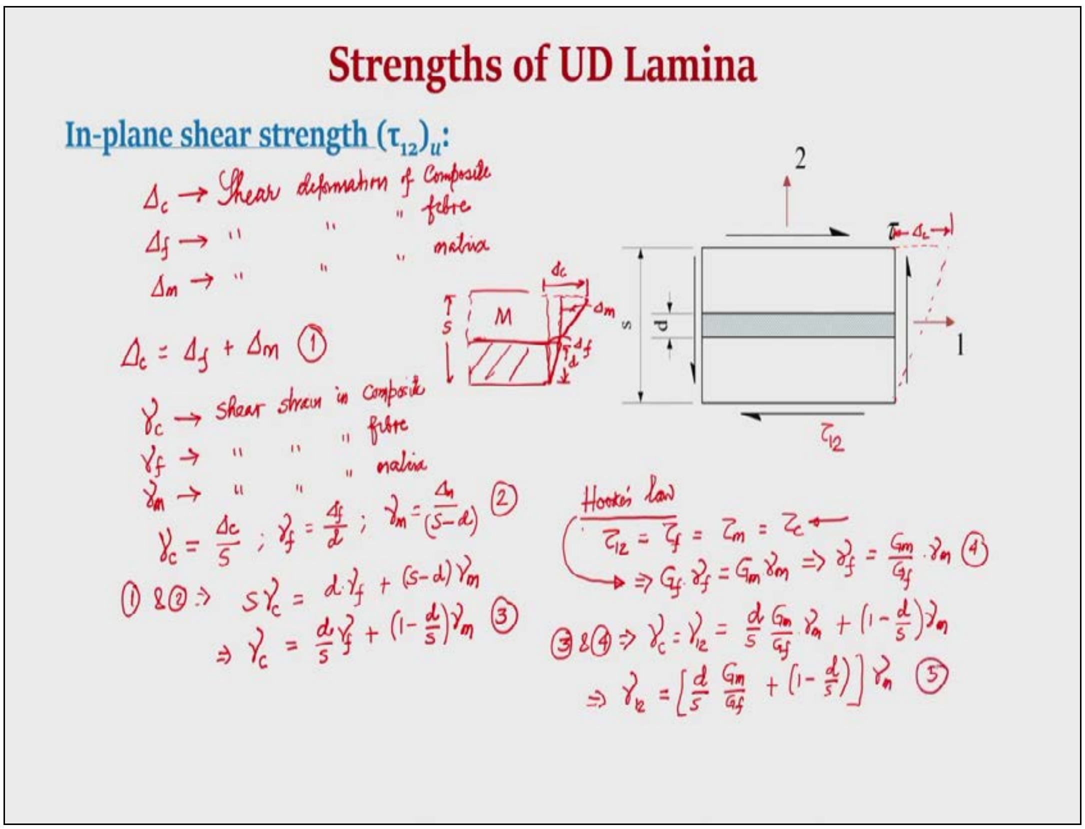

Next let us discuss the in plane shear strength. Again as shown in the Fig. considering an subjected to pure shear it is subjected to pure shear τ_{12}. Now, suppose Δ_c , Δ_f and Δ_m are the shear deformation of the composite, fiber and the matrix respectively due to τ_{12}. Therefore

$$\Delta_c = \Delta_f + \Delta_m \qquad (1)$$

Now, suppose, γ_c, γ_f and γ_m are the shear strains the composite, fiber and the matrix respectively. By definition of shear strain (Ref Fig.)

$$\gamma_c = \frac{\Delta_c}{s};\ \gamma_f = \frac{\Delta_f}{d};\ \gamma_m = \frac{\Delta_m}{(s-d)} \qquad (2)$$

From Eqn. (1) and (2) $\Rightarrow\ s\gamma_c = d\gamma_f + (s-d)\gamma_m$

$$\Rightarrow\ \gamma_c = \frac{d}{s}\gamma_f + \left(1-\frac{d}{s}\right)\gamma_m \qquad (3)$$

Now, the shear stress in the composite, fiber and the matrix are same ie. $\tau_{12} = \tau_f = \tau_m = \tau_c$ and using Hooke's law,

$$\Rightarrow\ G_f\gamma_f = G_m\gamma_m$$

$$\Rightarrow\ \gamma_f = \frac{G_m}{G_f}\gamma_m \qquad (4)$$

From Eqn. (3) and (4) $\Rightarrow\ \gamma_c = \gamma_{12} = \frac{d}{s}\frac{G_m}{G_f}\gamma_m + \left(1-\frac{d}{s}\right)\gamma_m$

$$\Rightarrow\ \gamma_{12} = \left[\frac{d}{s}\frac{G_m}{G_f} + \left(1-\frac{d}{s}\right)\right]\gamma_m \qquad (5)$$

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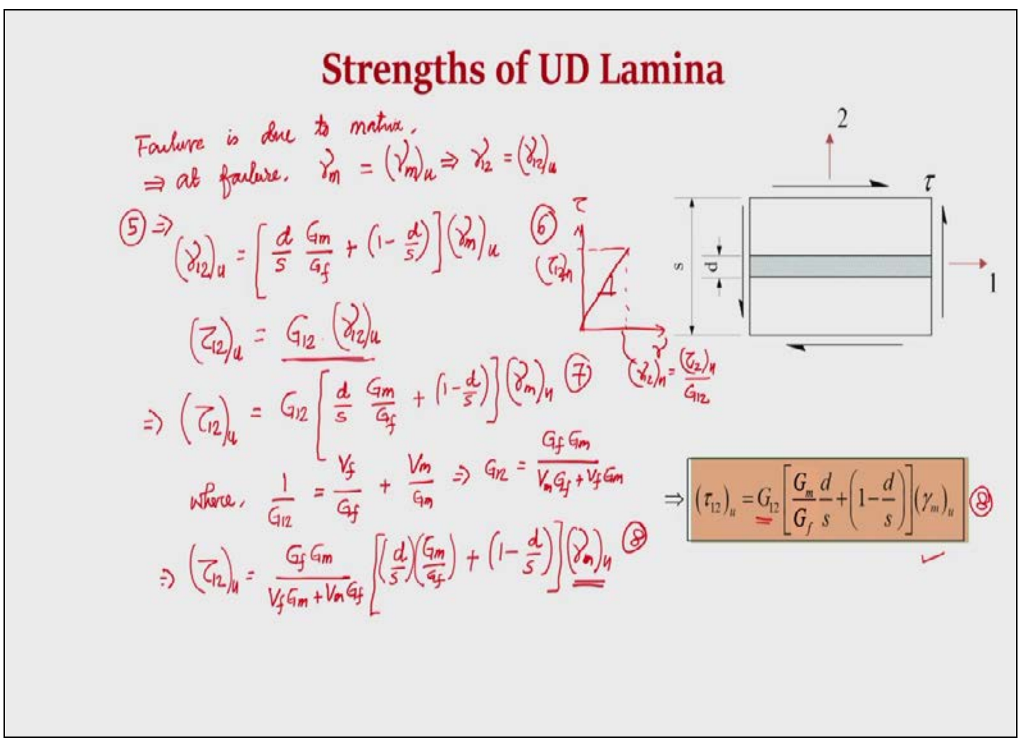

Now considering that the failure is due to matrix meaning at failure at failure, the shear strain in the matrix reaches the ultimate shear strain of the matrix.

At failure, $\gamma_m = (\gamma_m)_u \Rightarrow \gamma_{12} = (\gamma_{12})_u$

Substituting in Eqn. (5) $\Rightarrow$
$$(\gamma_{12})_u = \left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_m)_u \qquad (6)$$

and using Hooke's law,

$$(\tau_{12})_u = G_{12}(\gamma_{12})_u$$

$$\Rightarrow \quad (\tau_{12})_u = G_{12}\left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_m)_u \qquad (7)$$

Where, $\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \quad \Rightarrow \quad G_{12} = \frac{G_f G_m}{V_m G_f + V_f G_m}$

$$\Rightarrow \quad (\tau_{12})_u = \frac{G_f G_m}{V_m G_f + V_f G_m}\left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_m)_u \qquad (8)$$

So, this is the ultimate shear strength of the composite in terms of the ultimate shear strain of the matrix and other properties and dimensions.