Mechanics of Fiber Reinforced Polymer Composite Structures
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Module 5
Micromechanics of Lamina - I
Lecture-10
Introduction and Terminologies

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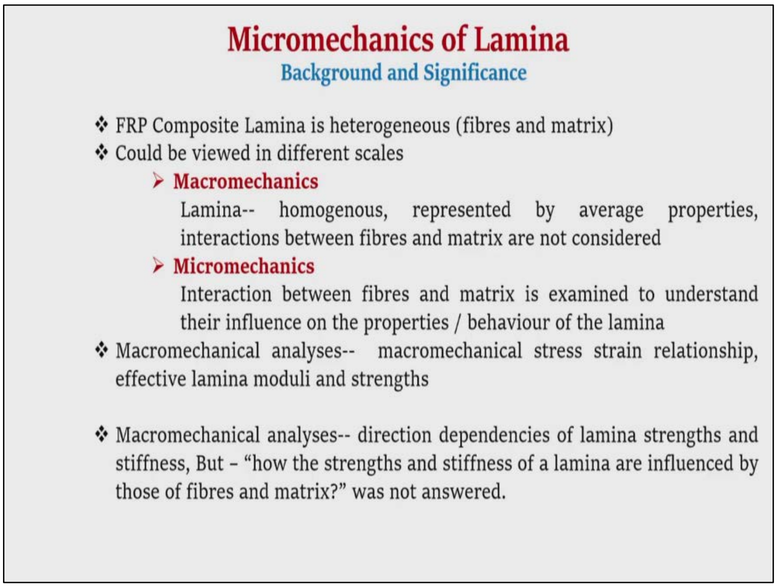

Hello, today we are going to start a new module, the micro mechanics of lamina. Let us first understand the background and significance of the micromechanics of lamina and the importance of studying micromechanics of lamina. Now, a lamina is heterogeneous, consists of fibers and the matrix and it is also orthotropic.

Introduction:

Therefore, the mechanics of lamina could actually be studied in two different scales namely the macromechanics of lamina and micromechanics of lamina. So, in the previous module we have studied the macromechanics in details wherein a lamina is represented by means of its average properties that means average elastic constants and average strength properties and we understood how to develop the stress and relationship based on those average stiffnesses.

Then we discussed the strength failure theories based on the average strength properties, but in macromechanics of lamina, the interactions of the fiber and the matrix have never been addressed. So, in micromechanics, these interactions of fiber and the matrix is actually taken into account. Like macromechanical analysis, we have duly considered the direction dependencies of the strength and the stiffnesses. But one important question that how these strengths and the stiffnesses of a lamina are influenced by the corresponding strengths and stiffnesses of the constituents fibers and the matrix is actually not answered in macromechanics of lamina.

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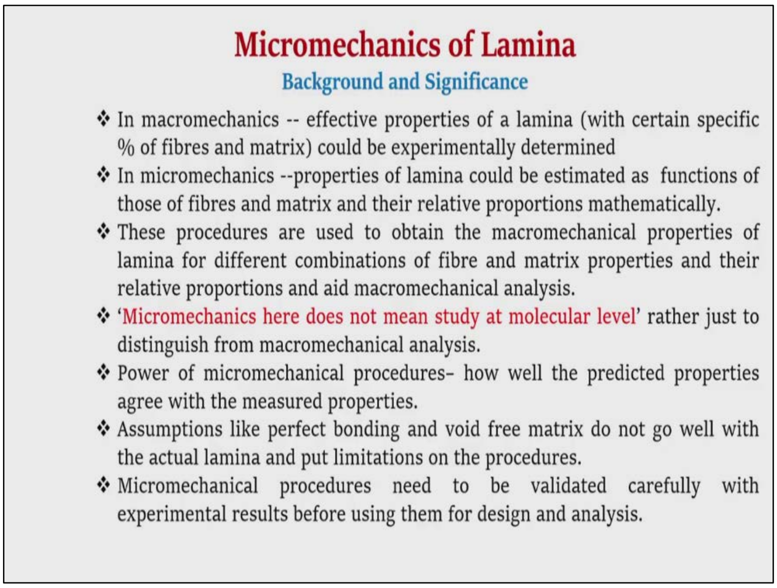

So, in micromechanics this is addressed, that means in micromechanics, we try to estimate the properties of a lamina in terms of the properties of the constituent fibers and the matrix. For example, suppose in macromechanics, we have a glass epoxy lamina where the glass fibers are 60% and 40% is epoxy. We can test the lamina it in a uniaxial tensile testing and find out the stresses and strains. Plotting the stresses and strains we can determine the Young's modulus that is the longitudinal Young's modulus and we can also determine the longitudinal tensile strength of the particular lamina. Now that is true for a given lamina where the percentage of glass fiber is 60% and that of epoxy is 40%. Suppose we want to know if the relative proportions are different that means suppose the glass fiber 50% and epoxy percent is 50%.

Well we can again make another lamina, test it and get the longitudinal Young's modulus as well as the ultimate tensile strength. Therefore, for each different relative proportion of the constituents we can measure, but this is time consuming. Similarly, we can do for the other properties also. In micromechanical analysis certain relations are developed by which we can determine the properties of lamina in terms of the properties of the fibers and the matrix and their relative proportion. These properties are subsequently used for the macromechanical analysis of the laminate. Therefore, in a way these micromechanical procedures actually add to the macromechanical analysis of composites.

Note that this micromechanical analysis of lamina has nothing to do with that the study at the molecular level or at the micro level. It is actually just to distinguish between the macromechanical analysis where the interactions of the fibers and the matrix are not considered but in micromechanical analysis of lamina these interactions are considered. That is the micromechanical analysis of lamina and it has nothing to do with at the micromechanical scales. Now when these relationships in micromechanical procedures are developed by which the properties of a lamina could be determined in terms of the fiber and matrix properties and the relative proportions, the efficacies of those established relations are decided by how well those predicted properties do agree with the experimentally measured properties. Now given the fact that there are key assumptions made in micromechanical analysis, which many times do not match with the actual lamina when tested and therefore put limitations on the micromechanical predictions. For example, the assumption of perfect bonding, it is assumed there is a perfect bonding bonding between the fibers and the matrix also it is assumed that the matrix is void free.

Therefore, before using these micromechanical procedures for design and analysis of composite laminates, they should be carefully validated with the experimentally measured values and then only those relations could be actually used with confidence.

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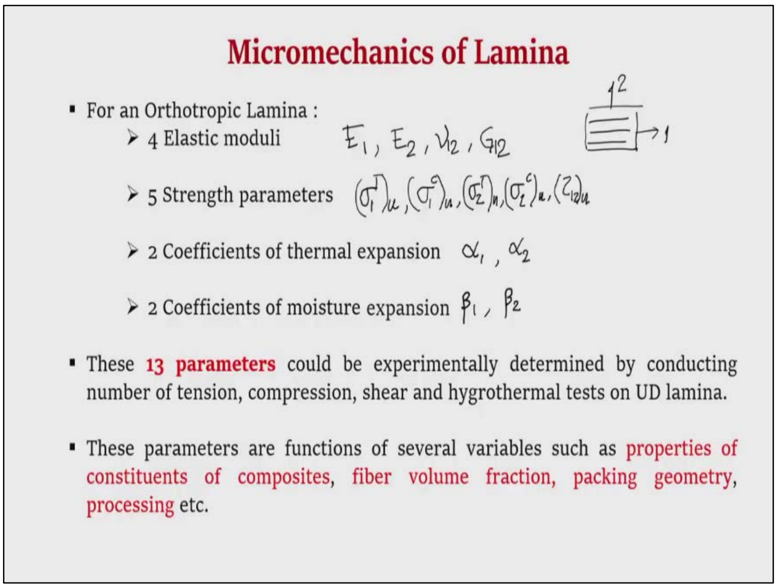

With reference to an orthotropic lamina (1 and 2 are the principal material directions) there are four elastic moduli viz., E_1 the longitudinal Young's modulus, E_2, the transverse Young's modulus, ν_{12}, the in plane Poisson's ratio and G_{12}, the shear modulus.

Similarly, we have five strength parameters as longitudinal tensile strength $(\sigma_1^T)_u$, longitudinal compression strength $(\sigma_1^C)_u$, transverse tensile strength $(\sigma_2^T)_u$ then transverse compression strength $(\sigma_2^C)_u$ and in plane shear strength $(\tau_{12})_u$. They are not equal because the strengths are also direction dependent.

Then we have two coefficients of thermal expansions that is longitudinal coefficient of thermal expansion α_1 and transverse coefficient of thermal expansion as α_2. Then we also have two coefficients of moisture expansion that is β_1, the longitudinal coefficient of moisture expansion and β_2, the transverse coefficient of moisture expansion.

So, altogether there are thirteen parameters in a unidirectional orthotropic lamina and out of that four engineering constants, five strength parameters, two coefficients of thermal expansions and two coefficients of moisture expansions. Now these thirteen parameters could actually be determined by conducting some tests like compression, tension, hygsroscopic, thermal test and we can obtain those parameters.

These are all average properties of an orthotropic lamina. These parameters are actually functions of several variables such as the corresponding properties of the fibers and the matrix and the relative proportion. For example, the longitudinal Young's modulus E_1 of the lamina is definitely influenced by the Young's modulus of the fiber Young's modulus of the matrix and the relative proportion.

If the fiber volume fraction is 50% and matrix is 50%, will get some value of longitudinal Young's modulus. Suppose the fiber is 60% and the matrix is 40% there will be some other value. Therefore, for all these properties of a lamina are decided by the corresponding properties of the fiber and those of the matrix and their relative proportion.

These parameters could be determined by conducting experiments.

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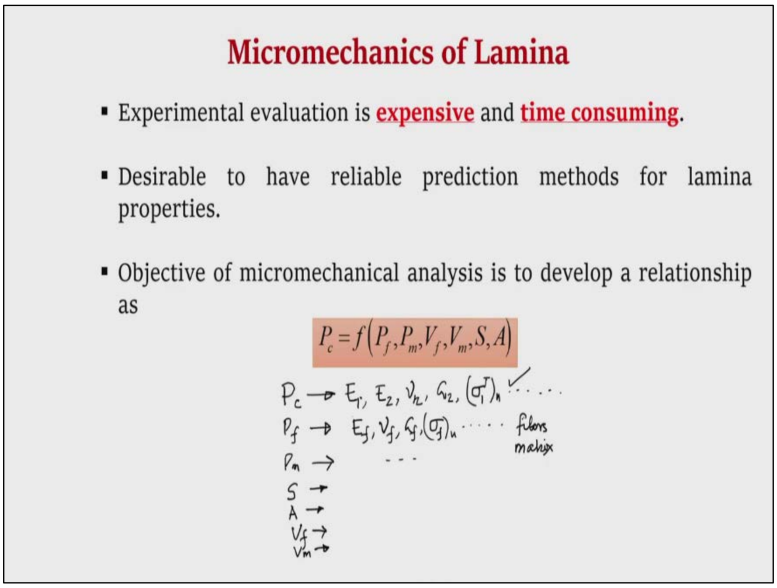

But experiments are actually expensive and time consuming especially more so in the case of orthotropic lamina because the number of parameters are more unlike isotropic materials where we have two engineering constants, Young's modulus Poisson ratio and two strength parameters that means tensile strength and shear strength or in some cases where tensile and compression strings are different three strength parameters. But in the case of an orthotropic material we need four engineering constants E_1 E_2 ν_{12} G_{12} and five strength parameters and then two coefficients of thermal expansion and two coefficients of moisture expansion. So, determination of all these 13 parameters by conducting experiments and that to when the relative proportion of the constituents change is actually time consuming and expensive.

Therefore, it is always desirable to have a reliable prediction method to determine the properties of a lamina in terms of the properties of its constituents and the relative proportion. So, that is how the justification of studying the micro mechanical analysis wherein the objective is actually to develop a relationship as $P_c = f(P_f, P_m, V_f, V_m, S, A)$.

Where P stands for properties, V for volume fraction (suffix, c for composites, f for fiber, m for matrix), S is the shape parameters (shape of the fibers, may be circular or some other shapes) and A is the arrangement (how they are arranged regularly space or not). Properties could be any of these thirteen properties discussed.

So, the objectives of the micromechanical analysis is actually to develop a relationship where the properties of the lamina could actually be obtained as a function of the corresponding properties of the fiber and the matrix and the relative proportion.

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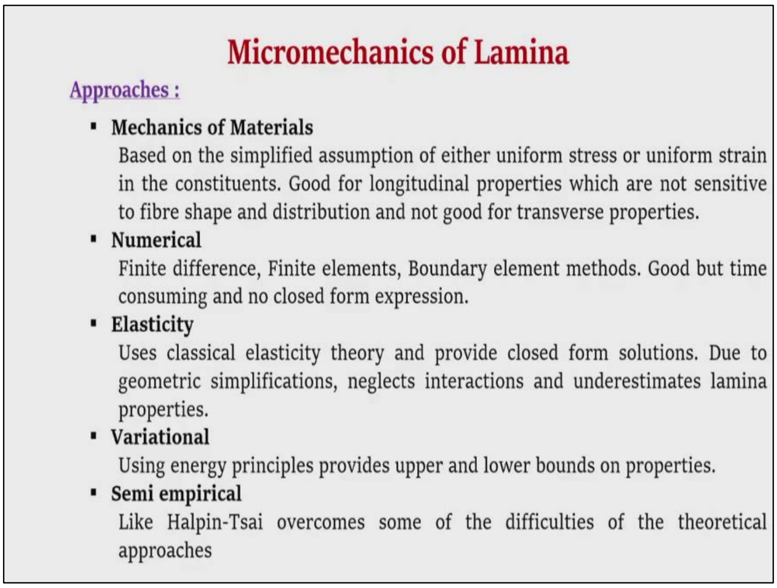

Different Approaches:

In micromechanical analysis, objective remaining the same there are several approaches like

- Mechanics of material approach – based on simplified assumptions of uniform stresses and strains. It is simple and it is reported that this approach provides a good estimate of the longitudinal properties whereas they do not agree well with the experimentally measured values for transverse properties.
- Numerical approach – like finite element method, finite difference method or boundary element methods provide a very good predictions of the properties of lamina in terms of the properties of the fiber and the matrix. However, they do not provide any close form solutions and therefore for each specific case separate numerical simulation is required and it is also time consuming;
- Elasticity approach –based on the classical theory of elasticity where the equations of equilibrium, compatibility conditions and the boundary conditions are used to arrive at a close form solution. However, the geometry is simplified and assumptions are made in terms of geometry and therefore sometimes important interactions between the fibers and the matrix are not considered and they do not agree well with the experimentally measured values.
- Variational approach – is based on the energy principle like principle of minimum potential energy and it provides upper and lower bounds on the properties of the lamina.

- Semi-empirical methods – there are a number of semi-empirical relations and one of the important and very well correlated is Halpin-Tsai method which actually overcomes some of the drawbacks or limitations of the theoretical approaches.

Halpin-Tsai method is many times used to predict the properties of lamina in terms of the properties of the fibers and the matrix and it correlates better with experimental results compared to the theoretical approaches.

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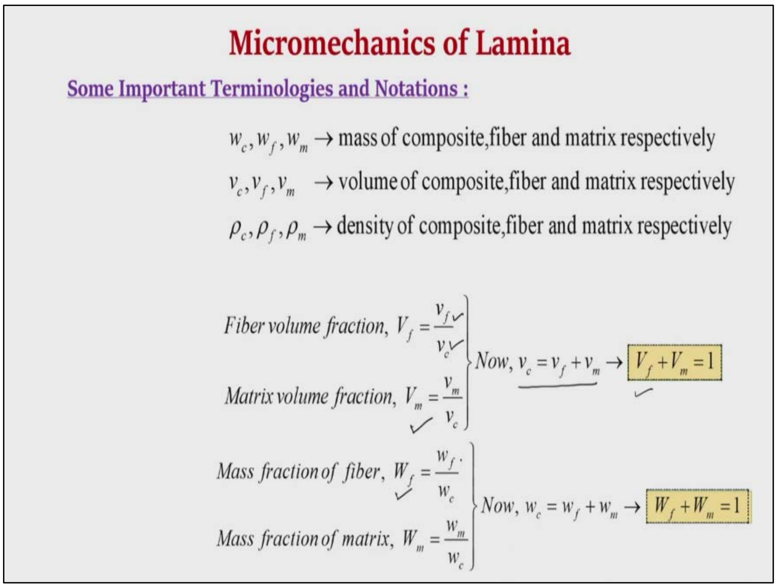

Important Terminologies:

Some of the important terminologies and notations which will be used throughout this micromechanical analysis are discussed below. *w* is the mass of the composite, *v* represents the volume and ρ as usual stands for the density.

$$w_c, w_f, w_m \rightarrow \text{mass of composite,fiber and matrix respectively}$$
$$v_c, v_f, v_m \rightarrow \text{volume of composite,fiber and matrix respectively}$$
$$\rho_c, \rho_f, \rho_m \rightarrow \text{density of composite,fiber and matrix respectively}$$

Then fiber volume fraction and matrix volume fractions are defined as

$$\left.\begin{aligned} &Fiber\ volume\ fraction,\ V_f = \frac{v_f}{v_c} \\ &Matrix\ volume\ fraction,\ V_m = \frac{v_m}{v_c} \end{aligned}\right\} Now,\ v_c = v_f + v_m \rightarrow \boxed{V_f + V_m = 1}$$

Mass fractions are also defined as

$$\left.\begin{aligned} &Mass\ fraction\ of\ fiber,\ W_f = \frac{w_f}{w_c} \\ &Mass\ fraction\ of\ matrix,\ W_m = \frac{w_m}{w_c} \end{aligned}\right\} Now,\ w_c = w_f + w_m \rightarrow \boxed{W_f + W_m = 1}$$

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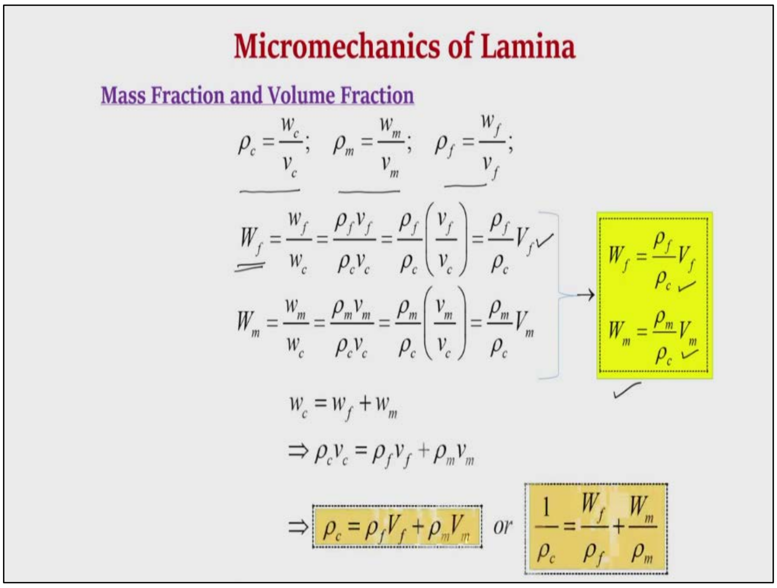

Using the definition of density, the relationship between the mass fraction and volume fraction in terms of density could be established as

$$\rho_c = \frac{w_c}{v_c};\quad \rho_m = \frac{w_m}{v_m};\quad \rho_f = \frac{w_f}{v_f}$$

$$W_f = \frac{w_f}{w_c} = \frac{\rho_f v_f}{\rho_c v_c} = \frac{\rho_f}{\rho_c}\left(\frac{v_f}{v_c}\right) = \frac{\rho_f}{\rho_c} V_f$$

$$W_m = \frac{w_m}{w_c} = \frac{\rho_m v_m}{\rho_c v_c} = \frac{\rho_m}{\rho_c}\left(\frac{v_m}{v_c}\right) = \frac{\rho_m}{\rho_c} V_m$$

$$\rightarrow \boxed{\begin{aligned} W_f &= \frac{\rho_f}{\rho_c} V_f \\ W_m &= \frac{\rho_m}{\rho_c} V_m \end{aligned}}$$

Using these, the density of the composite in terms of the density of the fiber and the density of the matrix could be established as

$$w_c = w_f + w_m$$
$$\Rightarrow \rho_c v_c = \rho_f v_f + \rho_m v_m$$
$$\Rightarrow \boxed{\rho_c = \rho_f V_f + \rho_m V_m} \quad or \quad \boxed{\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}}$$

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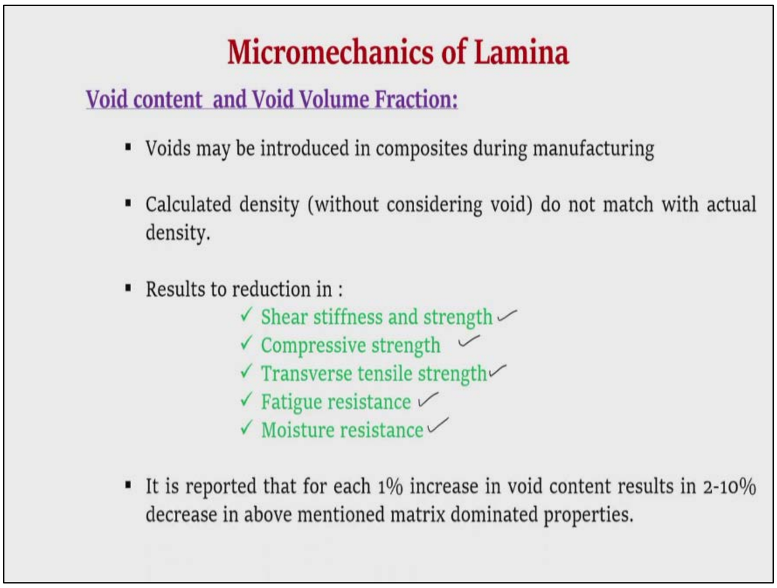

In micromechanical analysis it is assumed that there is no void but because of the manufacturing inaccuracies sometimes there may be a voids in the in the matrix and the presence of void leads to erroneous calculation of density. If the presence of void is not considered and if the volume of the void is not negligible, in that case the density calculated theoretically without considering the void volume will be definitely different from the density which is actually measured experimentally. In addition, the presence of voids results in degradation of some of the matrix dominated properties like shear stiffness and shear strength compressive strength transverse tensile strength fatigue resistance and moisture resistance of the composites and it is reported that for each 1% increase in the void content sometimes the degradation of these matrix dominated properties may vary between 2 - 10%.

Therefore, if the void volume is not negligible in that case it should be taken into account in the micromechanical analysis of the composites. So, therefore it is also important to know the void volume fraction.

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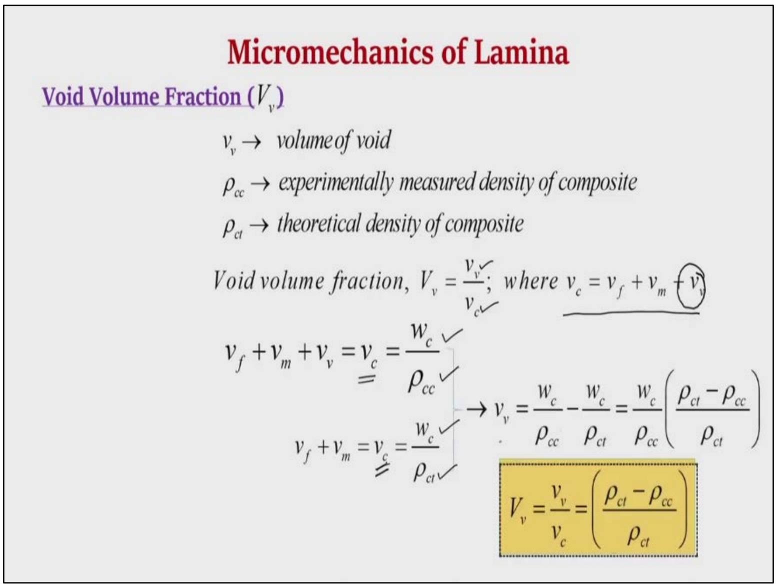

So, analogous to the fiber and matrix volume fraction the void volume fraction is defined as the ratio of the void volume to the volume of the composite as follows.

$$v_v \rightarrow \text{volume of void}$$
$$\rho_{cc} \rightarrow \text{experimentally measured density of composite}$$
$$\rho_{ct} \rightarrow \text{theoretical density of composite}$$

$$\text{Void volume fraction, } V_v = \frac{v_v}{v_c}; \text{ where } v_c = v_f + v_m + v_v$$

In calculating the density,

$$\left.\begin{aligned} v_f + v_m + v_v = v_c = \frac{w_c}{\rho_{cc}} \\ v_f + v_m = v_c = \frac{w_c}{\rho_{ct}} \end{aligned}\right\rangle \rightarrow v_v = \frac{w_c}{\rho_{cc}} - \frac{w_c}{\rho_{ct}} = \frac{w_c}{\rho_{cc}}\left(\frac{\rho_{ct}-\rho_{cc}}{\rho_{ct}}\right)$$

$$\boxed{V_v = \frac{v_v}{v_c} = \left(\frac{\rho_{ct}-\rho_{cc}}{\rho_{ct}}\right)}$$

So, we get a relation where the void volume fraction V_v could be related to the theoretical density and experimentally measured density.

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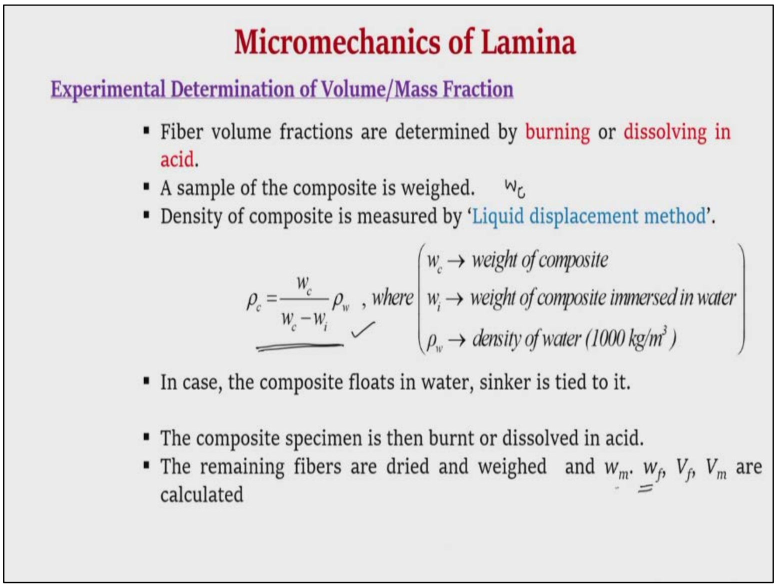

So, how to determine the volume fraction suppose we have a lamina we want to determine what is the volume fraction of the fiber. So, this could be determined using burning or dissolving the lamina in the acid. First a sample of the lamina is weighed and w_c is the weight of the lamina. Then the density of the composite is actually measured by liquid displacement method First, we measure the weight of the composite in air and then we measure the weight of the composite immersed in water and then using the density of water we can find out what is the density of the composite ρ_c. as follows.

$$\rho_c = \frac{w_c}{w_c - w_i}\rho_w \quad , \textit{where} \begin{pmatrix} w_c \rightarrow \textit{ weight of composite} \\ w_i \rightarrow \textit{ weight of composite immersed in water} \\ \rho_w \rightarrow \textit{ density of water (1000 kg/m}^3\textit{)} \end{pmatrix}$$

In case the composite is light and it does not sink in the water therefore in order to determine its weight in the water we provide a sinker and weight of the sinker is actually taken into account while finding our density using this formula. Then the once the density is determined the composite is burnt or dissolved in acid and it is only left with the fibers. The fibers are weighed and the weight of the fiber is w_f. Therefore, if we know what is the weight of the matrix w_m and from these relations we can find out the volume fraction of the fiber and the volume fraction of the matrix. That is how given a sample lamina we could actually determine that the volume fraction of the fiber in the lamina.

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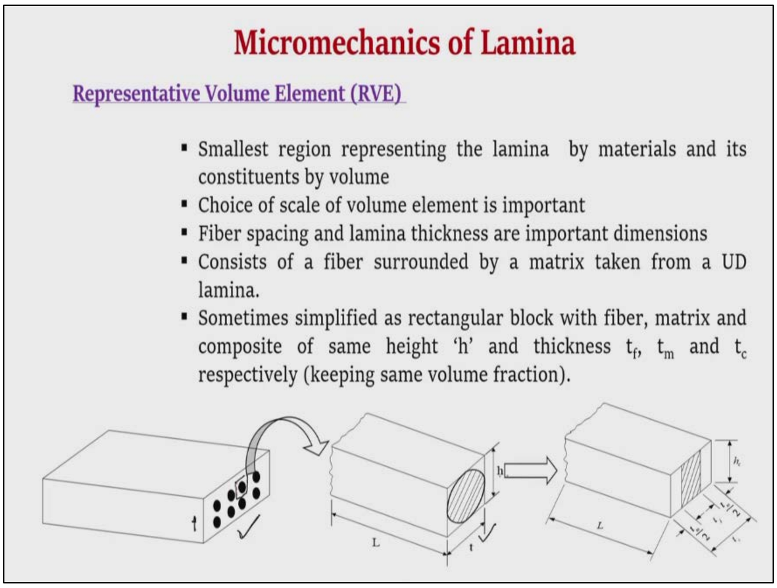

<u>Representative Volume Element</u>

Representative volume element (RVE) is the smallest region representing the lamina by materials and its constituents by volume and is an important concept. It is small volume from the lamina which represents the lamina by its material consisting of both fiber and the matrix with true volume fraction. Suppose the overall volume fraction of the lamina is 60% therefore the representative volume element must be such that the fiber volume to the volume of the representative volume is actually 60%. Now choice of scale of the volume is very important because it must truly represent the lamina. Many a times, one single fiber surrounded by matrix does the purpose but many a times more than one fibers are required depending upon how regularly the fibers are spaced.

Figure shows a RVE for a lamina where the fibers are equally spaced. Fiber spacing and lamina thickness are important dimensions in a representative volume element. Referring to the Fig., suppose in the in the thickness direction, all the layers are similar the fibers are equally spaced and distributed uniformly across the thickness direction.

Therefore, we can take only one fiber surrounded by matrix such that the ratio of the volume of the fiber to the volume of the RVE actually represents the fiber volume fraction of the lamina. Now it is further simplified by considering fiber which is actually of circular cross section to be of rectangular cross section instead of circular without changing the fiber volume fraction.

Therefore, in such a simplified RVE where the fibers are continuous, length of the fiber, length of the matrix and length of the composite is same. Height of the fiber, height of the composite and height of the matrix is also same.

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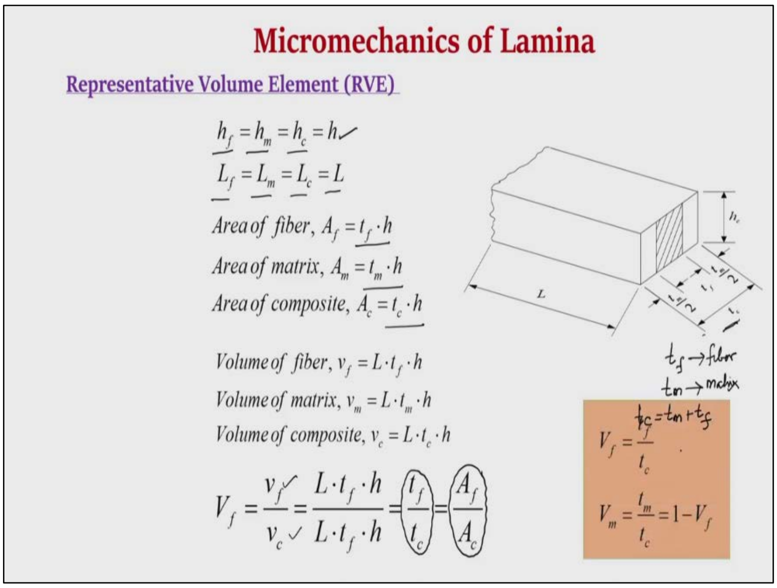

As shown in the Fig., heights of the fiber, matrix and the composite are equal and

$$h_f = h_m = h_c = h$$

Lengths of the fiber, matrix and the composite are equal and

$$L_f = L_m = L_c = L$$

and say

$$Area\,of\ fiber,\ A_f = t_f \cdot h$$
$$Area\,of\ matrix,\ A_m = t_m \cdot h$$
$$Area\,of\ composite,\ A_c = t_c \cdot h$$

Now the volume of the fiber, matrix and composite are

$$Volume\,of\ fiber,\ v_f = L \cdot t_f \cdot h$$
$$Volume\,of\ matrix,\ v_m = L \cdot t_m \cdot h$$
$$Volume\,of\ composite,\ v_c = L \cdot t_c \cdot h$$

And the fiber volume fraction is

$$V_f = \frac{v_f}{v_c} = \frac{L \cdot t_f \cdot h}{L \cdot t_f \cdot h} = \frac{t_f}{t_c} = \frac{A_f}{A_c}$$

$$V_f = \frac{t_f}{t_c}$$

The matrix volume fraction is

$$V_m = \frac{t_m}{t_c} = 1 - V_f$$

So, having understood the volume fraction and the RVE, irrespective of what approach is used in micro mechanical analysis there are certain key assumptions which are actually made in micro mechanical analysis as

- Perfect bonding between fibre and matrix – perfect bonding means that at any point of time when the lamina is loaded fibers and matrix do not get deboned from each other.
- Elastic moduli, diameter and space between fibbers are uniform – fibers are uniformly spaced and all the fibers of the same material, same size and shape.
- Fibbers are homogenous, continuous and parallel – fibers are single material homogeneous and they are continuous. No fibers are broken and they are parallel.
- Fibbers and matrix are isotropic and follow Hooke’s law – they are linearly elastic therefore they obey Hooke’s law.
- Fibres possess uniform strength – strength of the fibers are uniform which may not be true actually when we have large number of fibers, the strengths actually vary statistically.
- Composite is free from voids – If there is very small number of voids this is a reasonable assumption. However if the number of the voids are significant which may be because of the manufacturing effects then we have to consider voids.

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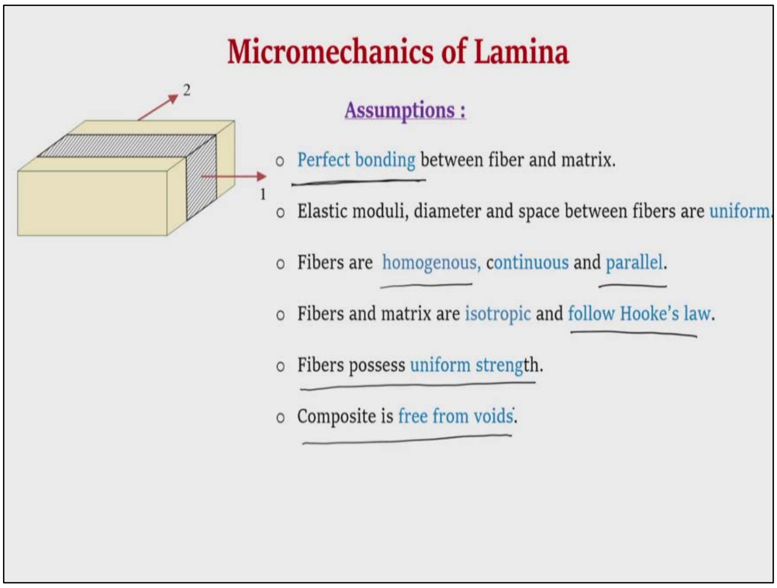

Now having understood the approaches and the key assumptions in micromechanics let discuss how some of these stiffnesses and strengths are actually determined using micromechanics. Let us start with mechanics of material approach which is the most simple among all these approaches.

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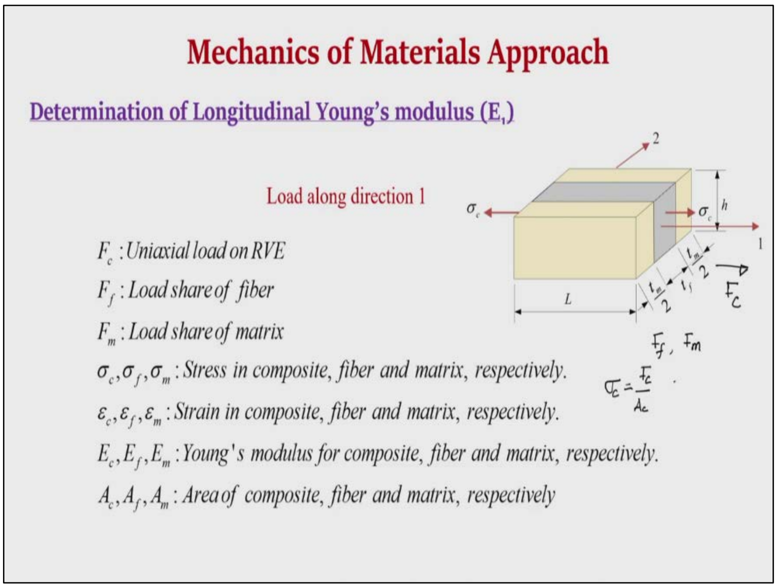

Mechanics of Materials Approach: Determination of longitudinal Young's modulus E_1

Considering an RVE, where a tensile load F_c is applied along principal material direction (1) that is a longitudinal direction 1 as shown in Fig.. and all the assumptions as already mentioned hold good for this analysis. So, if F_c is the total load applied along direction 1, the load will be shared by the fiber (F_f) and the matrix (F_m).

F_c : *Uniaxial load on RVE*

F_f : *Load share of fiber*

F_m : *Load share of matrix*

$\sigma_c, \sigma_f, \sigma_m$: *Stress in composite, fiber and matrix, respectively*

$\varepsilon_c, \varepsilon_f, \varepsilon_m$: *Strain in composite, fiber and matrix, respectively*

E_c, E_f, E_m : *Young's modulus for composite, fiber and matrix, respectively*

A_c, A_f, A_m : *Area of composite, fiber and matrix, respectively*

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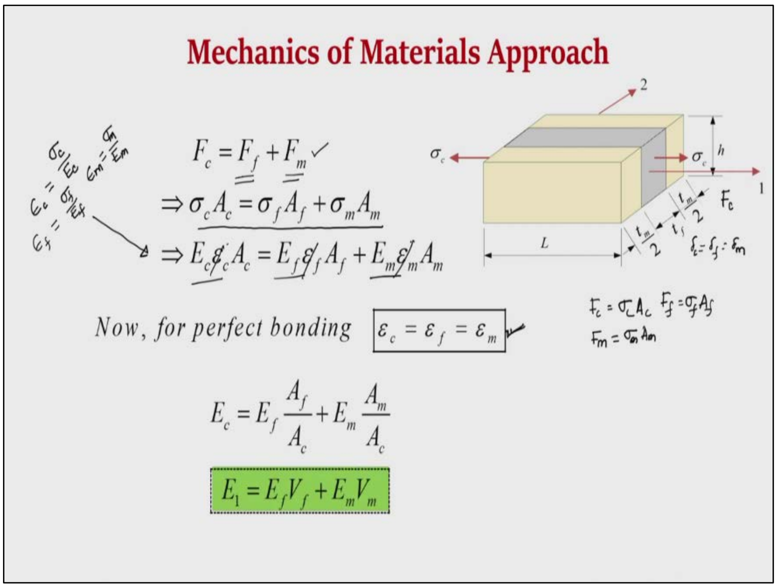

Now,

$$F_c = F_f + F_m$$

$$\Rightarrow \sigma_c A_c = \sigma_f A_f + \sigma_m A_m$$

Using Hooke's law,

$$\Rightarrow E_c \varepsilon_c A_c = E_f \varepsilon_f A_f + E_m \varepsilon_m A_m$$

Because of perfect bonding, under this load the extension along 1 is same in the matrix, fiber and composites ie. $\delta_c = \delta_f = \delta_m$. Now because their initial lengths (L) are also same, therefore the strains are same.

$$Now,\ for\ perfect\ bonding \quad \boxed{\varepsilon_c = \varepsilon_f = \varepsilon_m}$$

$$E_c = E_f \frac{A_f}{A_c} + E_m \frac{A_m}{A_c}$$

Now, for this simplified representative volume element $A_f / A_c = V_f$, the volume fraction of the fiber and $A_m / A_c = V_m$, the volume fraction of the matrix. E_c is nothing but in this case the Young's modulus of the composite along direction 1 because the load is applied along direction 1 in this case. Therefore,

$$\boxed{E_1 = E_f V_f + E_m V_m}$$

So, a relation between the longitudinal Young's modulus (E_1) of the composite lamina in terms of the Young's modulus of the fiber (E_f) and the Young's modulus of the matrix (E_m) and the relative proportion could be established and it could be clearly seen that as the volume fraction increases, E_1 increases. Now, in general the Young's modulus of fiber is far higher compared to that of the matrix. So, the change in Young's modulus of the matrix does not have much influence on E_1 . Change in Young's modulus of the matrix actually is insignificant until or unless the fiber volume fraction is very near to zero. Therefore, E_m does not influence E_1 and therefore E_1 is actually a fiber dominated property.

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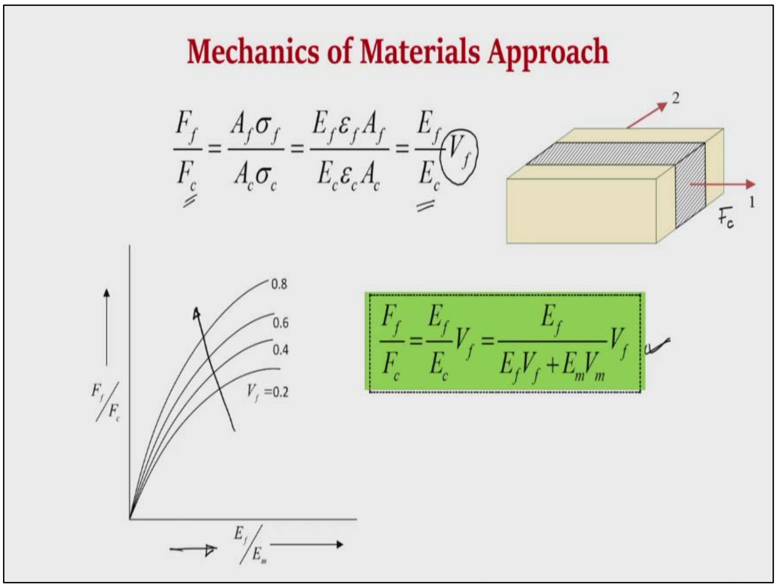

Fibers are very strong and stiff therefore the proportion of the load carried by the fiber to the load total load F_f / F_c could be written as

$$\frac{F_f}{F_c} = \frac{A_f \sigma_f}{A_c \sigma_c} = \frac{E_f \varepsilon_f A_f}{E_c \varepsilon_c A_c} = \frac{E_f}{E_c} V_f$$

$$\boxed{\frac{F_f}{F_c} = \frac{E_f}{E_c} V_f = \frac{E_f}{E_f V_f + E_m V_m} V_f}$$

Thus, F_f / F_c is a function of E_f / E_c and the volume fraction V_f. Naturally for a given Young's modulus of fiber if the volume fraction is more the more load is carried by the fibers.

Figure shows the plot between F_f / F_c and E_f / E_m for different V_f and it could be seen that for a given ratio of E_f / E_m it increases with V_f and for a given V_f, as E_f increases, more load is carried by the fibers.

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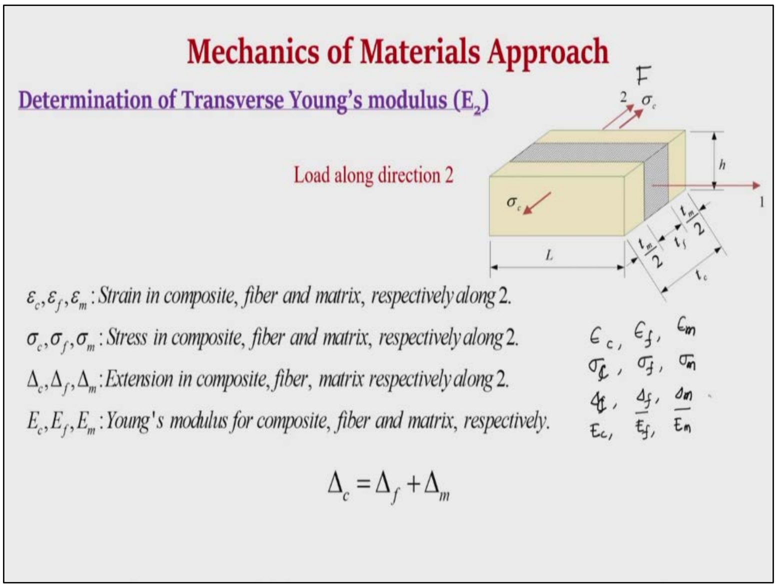

Mechanics of Materials Approach: Determination of longitudinal Young's modulus E_2

Considering the same RVE, but the load is applied along direction 2 that means it is the transverse direction. So, subjected to a tensile load along direction 2 suppose,

$\varepsilon_c, \varepsilon_f, \varepsilon_m$: *Strain in composite, fiber and matrix, respectively*

$\sigma_c, \sigma_f, \sigma_m$: *Stress in composite, fiber and matrix, respectively*

$\Delta_c, \Delta_f, \Delta_m$: *Transverse extension in composite, fiber, matrix respectively*

E_c, E_f, E_m : *Young's modulus for composite, fiber and matrix, respectively*

t_c, t_f, t_m : *Thickness of composite, fiber and matrix, respectively*

We apply tensile load along 2 therefore the extension in the directions 2 for the composites fiber and the matrix are

$$\Delta_c = \Delta_f + \Delta_m$$

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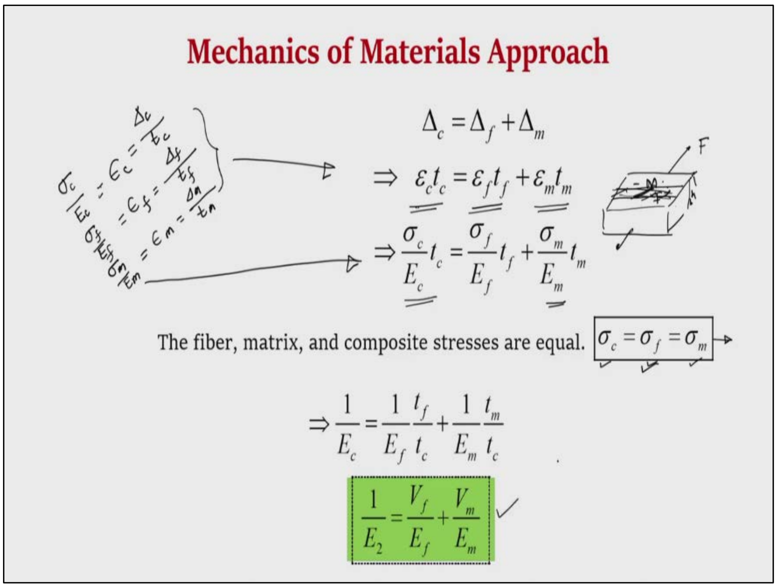

So, now we can write that because it is loaded in the direction 2. So, an ε c is the strain. So, by the definition of the strain along direction 2 strain along direction 2 of the composite is nothing but the change in length by initial length. Similarly, for the mat fiber it is the change in length Δ_f that means the extension along direction 2 to the initial length similarly for the matrix it is change in length around direction 2 by the initial length along direction 2.

Referring to the fig., and using the definition of strains $\Delta_c = \varepsilon_c t_c$, $\Delta_f = \varepsilon_f t_f$ and $\Delta_m = \varepsilon_m t_m$.

$$\Delta_c = \Delta_f + \Delta_m$$
$$\Rightarrow \quad \varepsilon_c t_c = \varepsilon_f t_f + \varepsilon_m t_m$$

Now again we the assumption that fiber matrix and the composites all obey actually Hooke's law therefore the strains could be written in terms of the stresses as

$$\Rightarrow \frac{\sigma_c}{E_c} t_c = \frac{\sigma_f}{E_f} t_f + \frac{\sigma_m}{E_m} t_m$$

Now along direction 2 the stresses are considered to be equal ie. $\boxed{\sigma_c = \sigma_f = \sigma_m}$ This is reasonable only when the fibers are regularly spaced along the thickness. If the fibers are not regularly spaced along the thickness in that case it will not be same and the stresses will be shared by the fiber and the matrix.

$$\Rightarrow \frac{1}{E_c} = \frac{1}{E_f}\frac{t_f}{t_c} + \frac{1}{E_m}\frac{t_m}{t_c}$$

Now this E_c is nothing but E_2 because this is the Young's modulus of the lamina in the direction 2.Referring to the RVE in Fig., $t_f / t_c = V_f$, the volume fraction of the fiber and $t_m / t_c = V_m$, the volume fraction of the matrix. So,

$$\boxed{\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}}$$

This is the relation between the transverse Young's modulus of the lamina in terms of the Young's modulus of the fiber and the Young's modulus of the matrix and the relative proportion. So, using mechanics of material approach we could establish the relationship where the longitudinal Young's Modulus and transverse Young's modulus in terms of the Young's modulus of the fiber and the Young's modulus of the matrix and the relative proportion.

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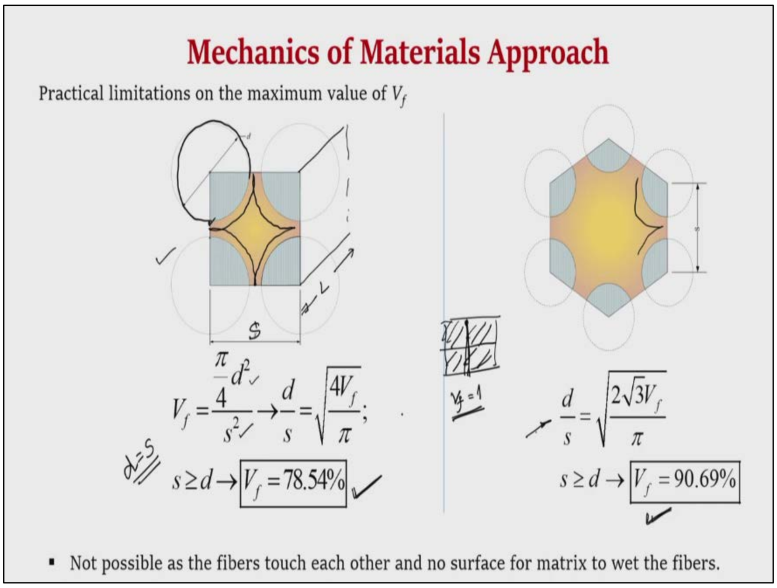

There are practical limitations on the maximum value of volume fraction. Now depending upon the packing arrangement of the fibers suppose the fibers are regularly spaced in a square array as shown in Fig. Suppose in a square array '*s*' is the fiber spacing and '*d*' is the diameter of the fiber. So, in an RVE of square where length of the fiber length of the matrix and the length of the composites are same, the fiber volume fraction is

$$V_f = \frac{\frac{\pi}{4}d^2}{s^2} \rightarrow \frac{d}{s} = \sqrt{\frac{4V_f}{\pi}};$$

In the limit, the two adjacent fibers may touch each other when $d = s$ and the maximum fiber volume fraction is

$$s \geq d \rightarrow \boxed{V_f = 78.54\%}$$

Similarly, in the case of a hexagonal array (as shown in Fig.), in the limit is that $d = s$ when the two adjacent fibers touch each other and the maximum volume fraction is

$$\frac{d}{s} = \sqrt{\frac{2\sqrt{3}V_f}{\pi}}$$
$$s \geq d \rightarrow \boxed{V_f = 90.69\%}$$

However, it is not possible to to realize this kind of volume fraction because the fibers touching each other means that that there is not sufficient wetting of the fibers by the matrix as the matrix actually binds the fibers. This is the theoretical maximum and not possible to use in structural composites.