Applied Thermodynamics

Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

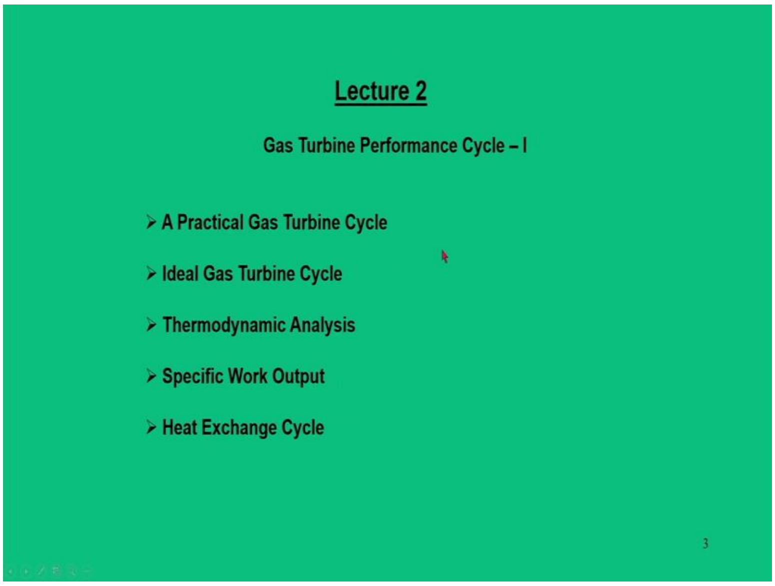
Module - 04
Gas Turbine Engines
Lecture - 32
Gas Turbine Performance Cycle-I

Dear learners, greetings from IIT Guwahati. We are now in the MOOCS course Applied Thermodynamics and we are in the Module 4 and title of this module is Gas Turbine Engines.

(Refer Slide Time: 01:07)

So, under these headings we have covered the first lecture that is gas turbine engines, basic Components, Thermal Circuit Arrangements. Today, we are going to start a new topic that is Gas Turbine Performance Cycle. So, it has 2 parts. So, we will cover the 1st part today and the in the next lecture we will cover its 2nd part.

(Refer Slide Time: 01:33)



So, in this particular lecture that is gas turbine performance cycle part 1. We are going to discuss about a practical gas turbine cycle, how it is supposed to operate? Second is ideal gas turbine cycle, because to have a practical gas turbine cycle we must have a theoretical estimates.

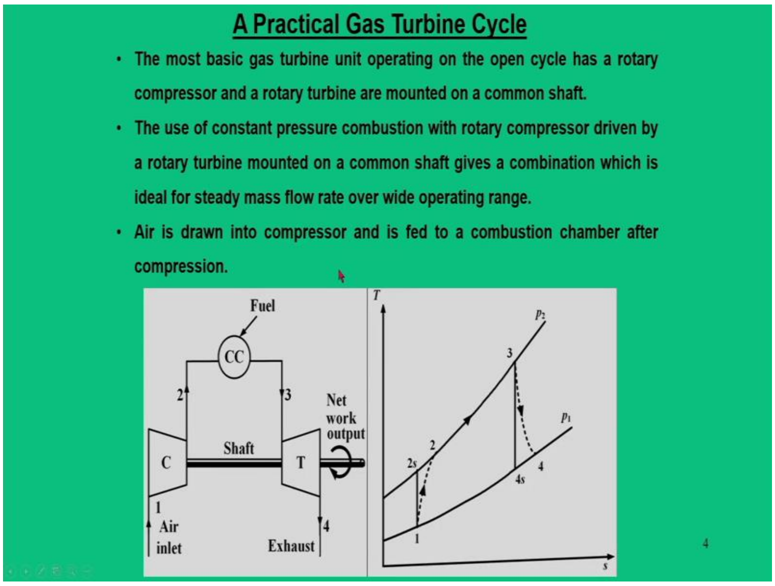
So, here we will talk about ideal gas turbine cycle and in fact, ideal gas turbine cycle is nothing but the Brayton cycle. This is similar to the concept that, when you deal with IC engines, Otto cycles are the ideal air standard cycle for SI engines and diesel cycle is ideal air standard cycle for diesel engines.

And, in similar sense the gas turbine cycle has air standard cycle which is known as Brayton cycle. And, for this Brayton cycle we are going to do the thermodynamic analysis, by doing this thermodynamic analysis we are going to calculate the thermal efficiency. Apart from that important aspect of a gas turbine cycle, because we want to think about how much work output or work ratio you are going to get?

So, for that also we are going to find out mathematically what is the specific work output? Now, apart from this ideal cycle, there is some limitations and for which we require the incorporation of an heat exchanger. So, such a cycle is known as heat exchange cycles.

So, in the gas turbine cycle performance, basically there are two variants which works on the Brayton cycle. The first variant is the simple gas turbine cycle and second one is heat exchange cycle. Now, in today's lecture we will summarize both these cycles.

(Refer Slide Time: 03:58)

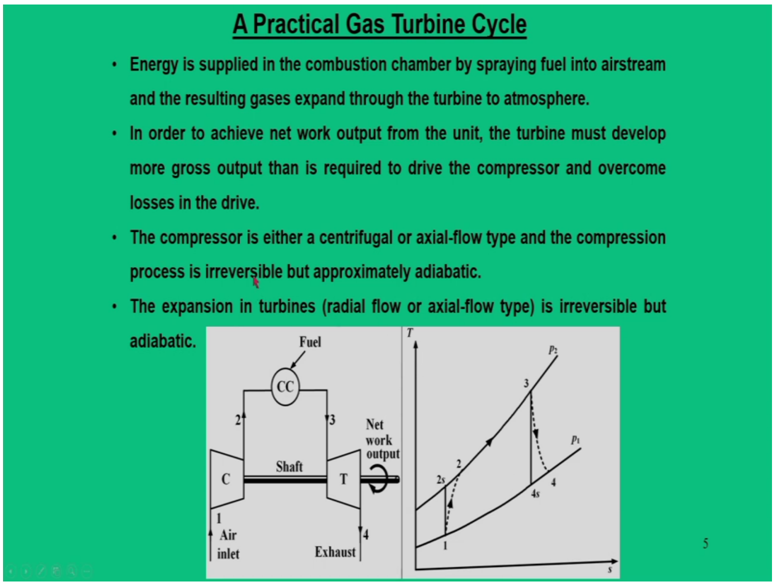


As I mentioned earlier or in the last class the most basic gas turbine unit has the two major components that is compressor and turbine. They are coupled with a shaft and apart from that we have a combustion chamber where fuel is added and in our analysis we will say that this fuel is equivalent to heat addition. And, finally, air comes from the atmosphere and enters to the compressor. And, finally, exhaust gas leaves at the state 4 at the end of the turbine.

And, the by virtue of the expansion process in the turbine, we get net shaft work output. And, typically we use a constant pressure combustion mode in this cycle. So, this was the summary which we got in our last class and another important aspect is that typically, we use rotary compressors driven by the rotary turbines, I mean they come under the category of turbo machines.

And, the thermodynamic circuit or thermodynamic cycle is represented in the T-s diagram process 1, 2, 3, 4 and if you look at this particular process 1, 2s, 3 and 4s, that is your ideal cycle and 1, 2, 3, 4 is your actual practical cycle. And, here the practical cycle involves component efficiency for the compressor and turbine. And, this part we will going to debate in the subsequent lecture. So, in your first thing we will consider the ideal cycle that is 1, 2s, 3 and 4s.

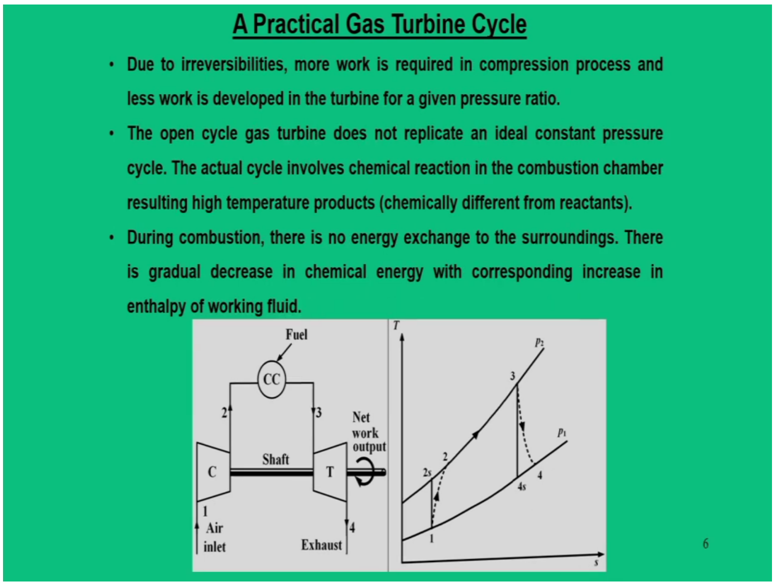
(Refer Slide Time: 05:47)



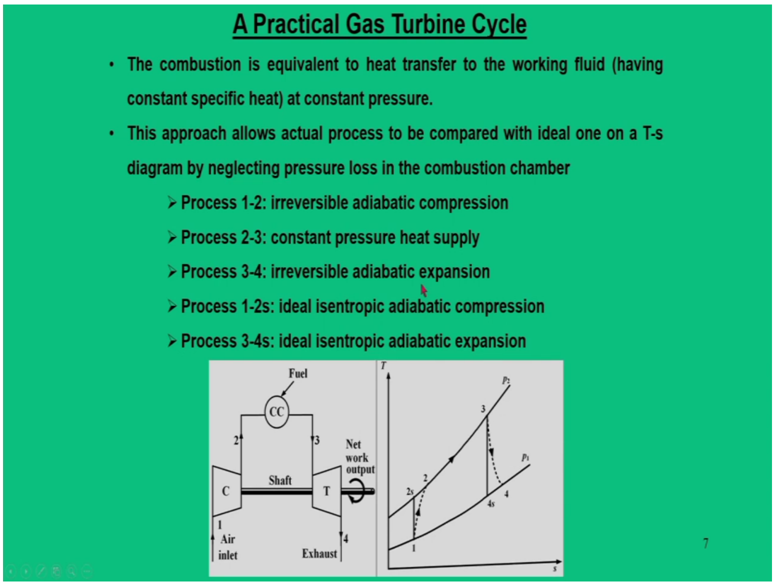
And, as I mentioned earlier the compressor is rotary in nature and under this rotary category, we can have a centrifugal or axial flow type compressor. And, the process is typically irreversible, but approximately adiabatic.

And, the expansion process is in the turbine and that is also irreversible, but adiabatic. But our first analysis when you deal with the ideal cycle, we will talk about the situation where the process is a reversible one. That is 1, 2s, 3 and 4s. .

(Refer Slide Time: 06:28)



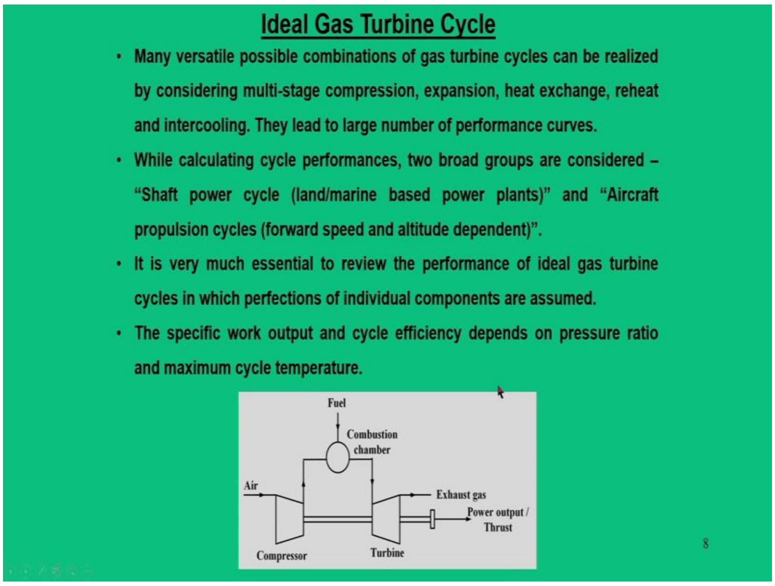
(Refer Slide Time: 06:40)



Now, with this summary we are going to discuss about a ideal gas turbine cycle and the typical processes that goes as process 1-2, that is irreversible adiabatic compression. Process 2-3 constant pressure heat supply, process 3-4 irreversible adiabatic expansion. So, here the actual cycle 1, 2, 3, 4 completes and cycle starts from again 1.

But to give a theoretical estimates we assume that if the process would have been an isentropic one, then you make an assumption it is an ideal isentropic compressions. So, the word adiabatic is redundant here. So, process 1-2 is known as an ideal isentropic compression and process 3 to 4s is ideal isentropic expansion.

(Refer Slide Time: 07:37)

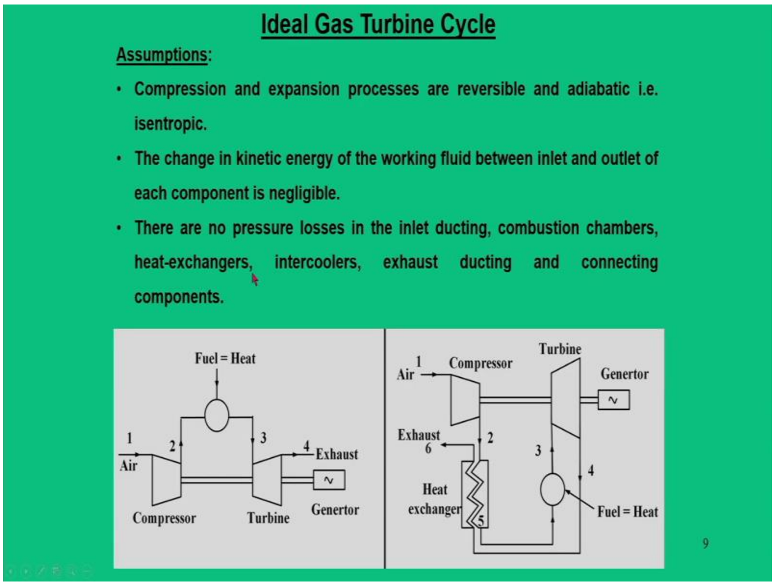


Now, let us see that why this ideal cycle is very important, because it gives a theoretical estimates. And, in the beginning I have emphasized the fact that when you use these gas turbine engines, they are used for two broad groups; one is considered as the "shaft power cycle" other is considered as "aircraft propulsion cycle".

Now, when you talk about shaft power cycle, here the power output is given as a emphasis, your main concern is that you are going to get work as a output. Whereas, in the aircraft propulsion system where thrust is given a prime importance and, that gives forward speed and this aircraft propulsion cycle depends on altitude, because higher and higher altitude we go, speed will become more and more, because density comes down when you go with altitude.

So, our main concern, we will talk about shaft power cycle and the second part is aircraft propulsion system. But, currently let us concentrate on shaft power cycle where work output and thermal efficiency are major concern.

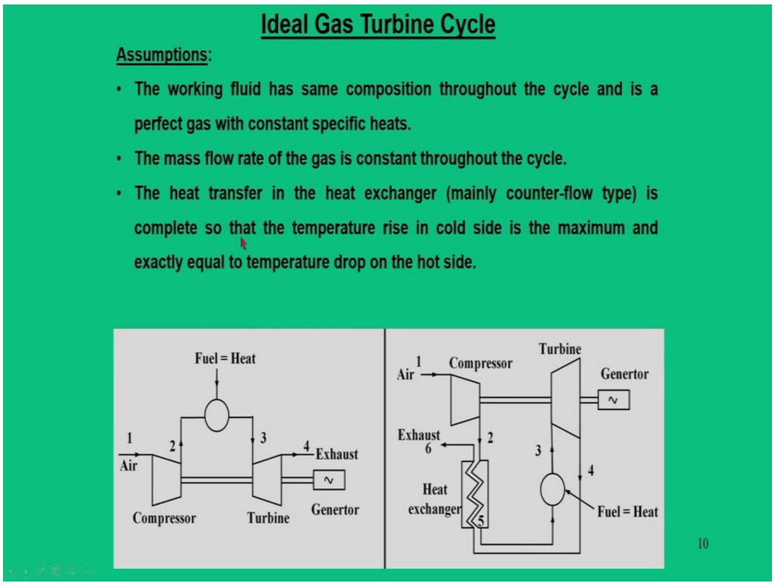
(Refer Slide Time: 09:17)



Now, when you deal with the ideal gas turbine cycle, assumptions are made and some of the important assumptions are compression and expansion processes are irreversible adiabatic or they are considered as an isentropic. Changes in the kinetic energy of working fluid between inlet and outlet is negligible, this is one of the important assumption for the shaft power cycle.

But, this assumption is not true when you deal with aircraft cycle, because the kinetic energy between inlet and outlet of the components are significance. So, the emphasis is given in a different aspect on a different context, in those situations. The third important aspects is that there is no pressure losses in the inlet ducting, combustion chamber, heat exchangers, intercoolers, exhaust ducting and connecting components.

(Refer Slide Time: 10:19)

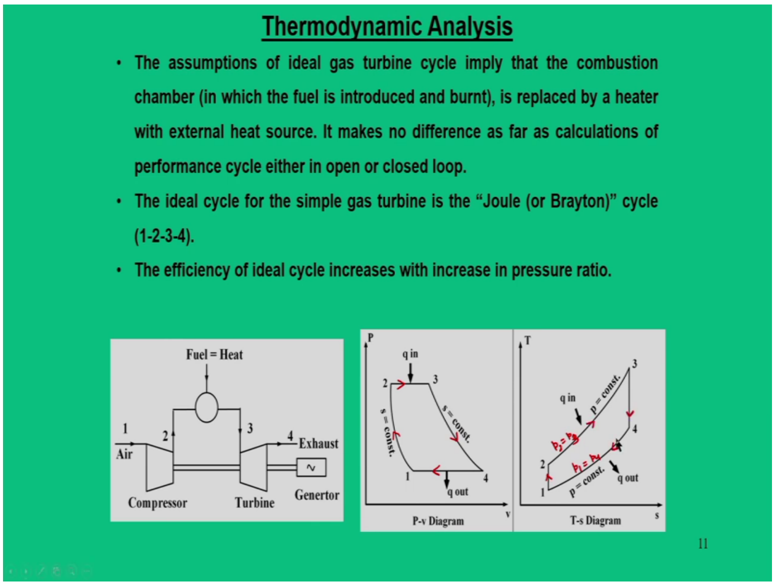


Apart from this the working fluid is same composition throughout and it is a perfect gas with constant specific heats, mass flow rate of the gas is constant. The heat transfer in the heat exchanger mainly counter flow type is complete so that temperature rise in the cold side is maximum and exactly equal to the temperature drop in the hot side.

So, in the ideal gas turbine cycles we are going to see two variants; in first variant there is no heat exchanger and the second variant there is a heat exchanger. Why we think about a heat exchanger? Because, when you take a first variant that is exhaust from the turbine, that state is at thermodynamic state 4, the gases are at very high temperatures.

So, in order to tap those heat, we try to use an heat exchanger that takes the heat from the exhaust and preheats the compressed air. So, this is the main motive that we are going to increase the efficiency of the cycle by incorporating a heat exchangers. So, these two things are known as ideal gas turbine cycle one with simple cycle other is with heat exchange cycle.

(Refer Slide Time: 11:42)



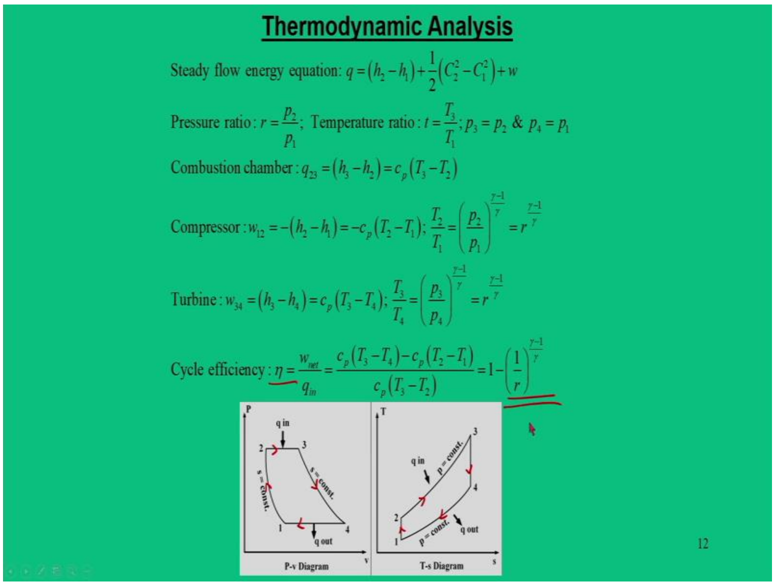
Now, let us see the thermodynamic analysis of the first case, which is a simple gas turbine cycle. So, just to give the explanation that in the compressor, air is drawn at state 1 it is compressed to 2 through this compressions, the temperature and pressure also goes up. So, it is now in state 2. Now, it is goes to a combustion chamber where fuel is added and in our terminology when fuel is added, it is equivalent to a heat.

So, that is basically we are adding heat to the air in our simple analysis of Brayton cycle. And, finally, the heated air comes out from this combustion chamber or which is at a state 3 and it expands in the turbine. When it expands in the turbine, the state goes from 3 to 4 and throughout the process the power output is developed. And, out of this power output, it is partly used to drive the compressor and rest of the work output go in the form of work or finally, in the form of electricity.

Now, this particular process can be represented thermodynamically in the pressure volume diagram and temperature entropy diagram. So, what you see is that, when the cycle goes from state 1 to 2 it is an compression process. So, it is a isentropic. Process 2-3 is considered as heat addition, process 3 to 4 process is expansion in the turbine and 4 to 1 is the exhaust process.

So, here it is written as a cyclic, because more or less the point 1 coincides with the last point although the cycle is open in nature, but the way we represent, it resembles the close cycle. Now, similarly in the temperature entropy diagram we have 2 constant pressure lines that is p_1 is equal to p_4 and p_2 is equal to p_3.

(Refer Slide Time: 14:09)



Just to give I will represent the direction of this thermodynamic cycle 1, 2, 3, 4 and this is nothing but the Brayton cycle. So, you start the thermodynamic analysis with our understanding from the steady flow energy equations, which is $q=(h_2-h_1)+\frac{1}{2}(C_2^2-C_1^2)+w$; that means, when the system goes from state 1 to state 2, the first law equations can be used for an open flow systems and here we say that there is no change in the potential energy. So, it is only change in the kinetic energy. So, change in the enthalpy takes care about the internal energy change as well as the flow work. And, q is your heat transfer, w is the work transfer.

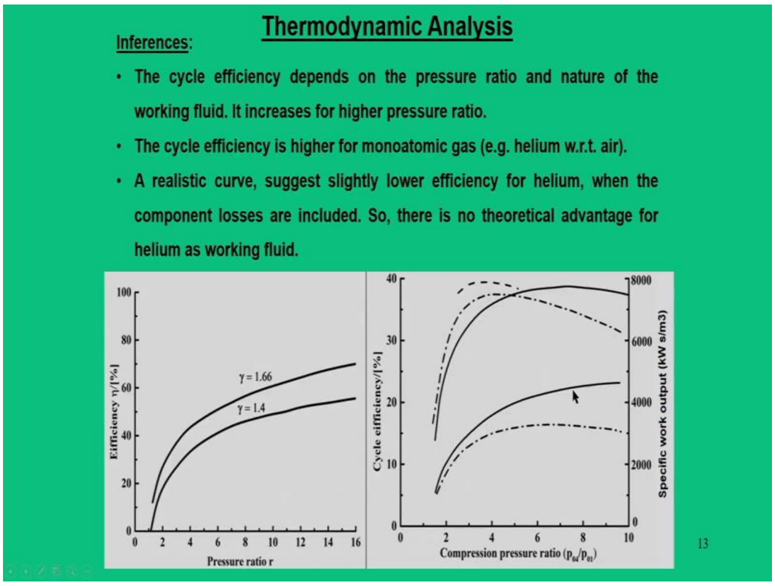
Now, this particular equation we are going to use for our gas turbine cycle, which is ideal in nature. But, before you do that we defined some important parameters that is $\text{Pressure ratio}: r=\frac{p_2}{p_1};\ \text{Temperature ratio}: t=\frac{T_3}{T_1}; p_3=p_2\ \&\ p_4=p_1$. So, from this, in the combustion chamber the heat addition process can be represented as a $q_{23}=(h_3-h_2)$.

And, since air is used as an ideal gas. So, it can be represented as $c_p\left(T_3 - T_2\right)$. Similarly, compression process; it is a work consuming device that is $-\left(h_2 - h_1\right) = -c_p\left(T_2 - T_1\right)$. And, apart from that the compression process is isentropic.

So, the relation between pressure and temperature relations can be represented in this manner $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}$. And, moving to the turbine this is similar to compressor analysis, but with a fact the turbine is a power producing device. So, there is no negative sign here. $w_{34} = \left(h_3 - h_4\right) = c_p\left(T_3 - T_4\right)$

And, finally, $\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}$. And, by definition we can define the cycle efficiency $\eta = \frac{w_{net}}{q_{in}} = \frac{c_p\left(T_3 - T_4\right) - c_p\left(T_2 - T_1\right)}{c_p\left(T_3 - T_2\right)}$. And, finally, the cycle efficiency for an ideal Brayton cycle is written as $1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}$.

(Refer Slide Time: 17:17)



Now, from this equation one can find out that efficiency is function of two parameters; one is the pressure ratio, other is the nature of working fluid. By nature of working fluid I mean this specific heat ratio γ; that means, efficiency depends on r and γ. And, of course, it increases with increase in the pressure ratio.

For various pressure ratios efficiencies can be plotted. One for gamma is equal to 1.4 and gamma is equal to 1.66 that is for monoatomic gas. So; that means, instead of air if you use helium, cycle efficiency will go up.

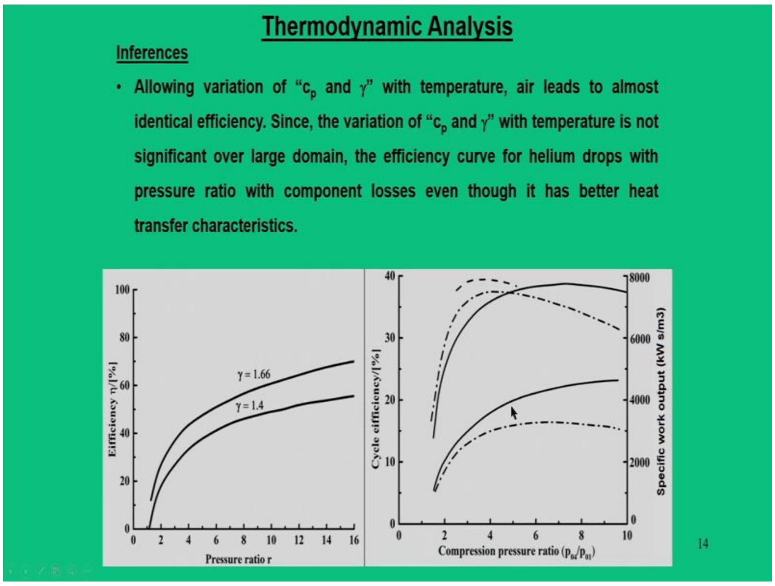
Any value gamma comes down below 1.4, their efficiency will come will reduce. So, that is the reason Brayton cycle does not recommend any gas other than air. But, apart from that we have seen that if you have gamma is higher, the efficiency also goes out.

But one interesting thing is that there are many limitations when you deal with the helium. Because helium is a lighter gas and it requires large volume of helium to run the cycle.

And, what will happen is that, at one particular point although efficiency increases in the beginning at lower compression pressure ratio, but at higher compression pressure ratio, efficiency falls.

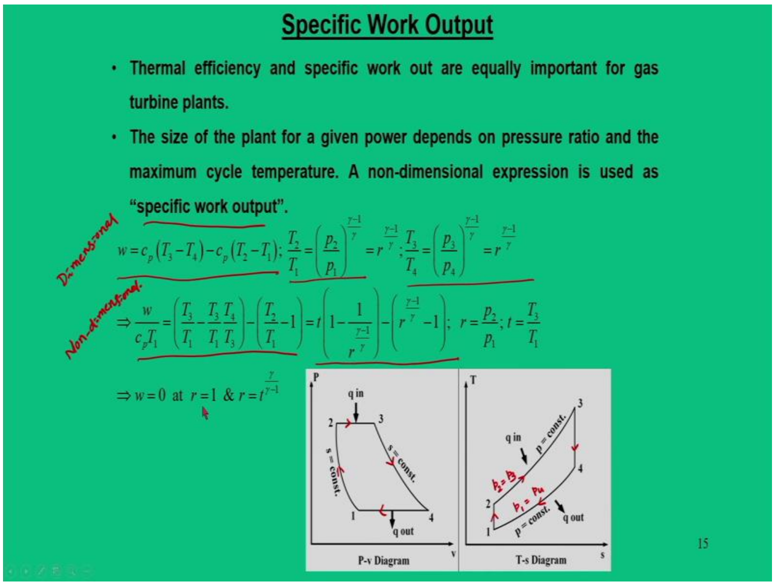
Side by side specific work output kW-s/m^3 then that also falls with the pressure ratio, but at some point of time air takes over because air is continuously higher. That means, a realistic curve when you use air at a wide range of pressure ratio gives a theoretical advantage than the helium. So, because of this reason air is the only choice as the working fluid for a gas turbine cycle.

(Refer Slide Time: 19:59)



And, there are other uncertainties, that is involved that we can think of variation of c_p and gamma with temperature, it leads to almost identical efficiency, when you think about helium and air. So; that means, when you take the variation of c_p and gamma at higher temperature the helium and air will have similar identical efficiency. So, over a large domain of pressure ratio air is the best choice.

(Refer Slide Time: 20:29)



Moving further apart from the efficiency another important aspect of a gas turbine engine is the work output and here we will analyze a very important factor specific work output.

So, if you look at this cycle in the P v and T s diagram the cycles are represented as 1, 2, 3, 4. And, the directions of the arrows are given in this fashion. So, here $p_2 = p_3$ and $p_1 = p_4$. And, here this particular analysis we will focus on the calculation of specific work output.

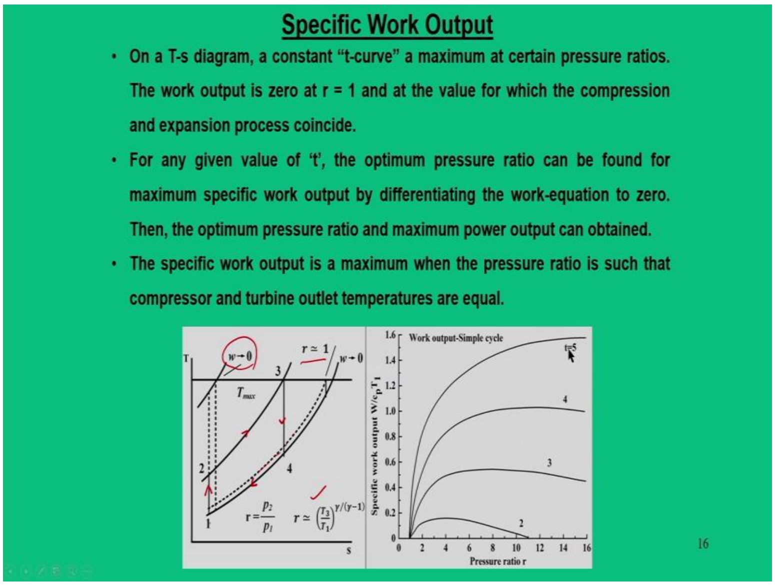
One ideal way of looking at the work output is the difference between the turbine work and compression work. So, the specific work per kg of air, the work output can be represented as $w = c_p(T_3 - T_4) - c_p(T_2 - T_1)$, but the other important aspect that we want to use it in a non dimensional form.

So, a non dimensional expression would be $\frac{w}{c_p T_1}$. So, it is a non dimensional form of representation. So when you do this, you have to do some mathematical jugglery. So, side by side we have to use this conditions for an isentropic process, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}; \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}$

So, ultimately we can get a non dimensional form of power, which is represented as $\frac{w}{c_p T_1} = \left(\frac{T_3}{T_1} - \frac{T_3}{T_1}\frac{T_4}{T_3}\right) - \left(\frac{T_2}{T_1} - 1\right) = t\left(1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right) - \left(r^{\frac{\gamma-1}{\gamma}} - 1\right)$ which is a function of 2 important parameters, which is r that is pressure ratio and t is nothing but the temperature ratio.

So, here the most important things that $\frac{w}{c_p T_1}$ is function of temperature ratio as well as the pressure ratio. And, an interesting fact that we can find out from this equation that, w is equal to 0 at 2 locations when we put r is equal to 1, and $r = t^{\frac{\gamma}{\gamma-1}}$.

(Refer Slide Time: 23:41)



So, let us see what is this physical significance? Now to do that we recall our temperature entropy diagram and try to see the actual cycle that is 1, 2, 3, 4. So, this is the actual cycle we have been studying and analyzing.

Now, let us see when w is equal to 0; at 2 locations that is when r is equal to 1. So, when r is equal to 1 means, p_2 is equal to p_1. So, that means, if you can see this dotted line, that the expansion process in the turbine as well as the compressor, they are very close to each other.

So; obviously, there will be no work output. So, this becomes a unrealistic situation; that means, why we should go for r is equal to 1? So, there is no meaning as far as the practical utility is concerned, but the very fact that when you put $r = t^{\frac{\gamma}{\gamma-1}}$ again the w value goes to 0.

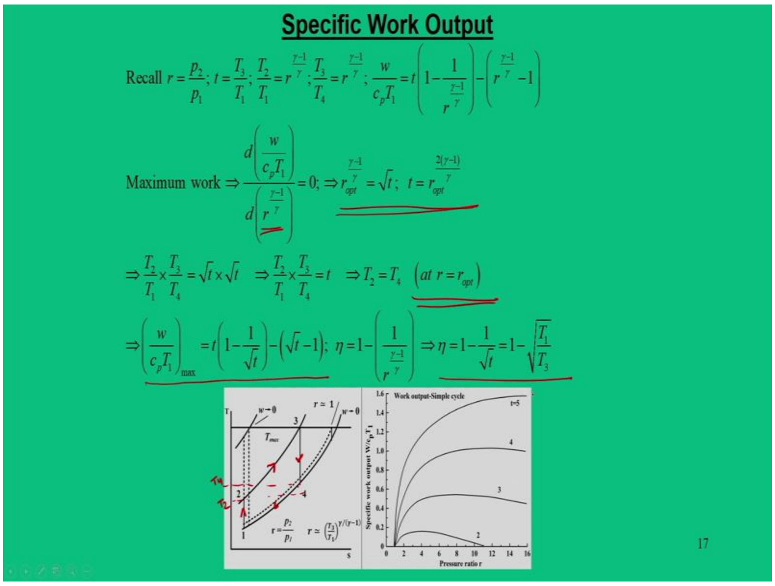
So, this represent a situation that, we should not operate the r value, the pressure ratio at this number, which means it is close to the other extreme. That means, process goes in this dotted line instead of 2 it will be like 1, then you will land after compression will have this 3, 2 to 3 and 3 to 4. And, in this case ideally if you look at both the lines are coinciding each other; that means, compressor and turbine work are equal.

So, this has also no meaning. So, then what is the better choice; that means, so, between these two value that is 1 and these, there should be a optimum pressure ratio. So, this is what the mathematical analysis or treatment tells to us. So, for any given value of ‘t’, the optimum pressure ratio can be found out for maximum specific power output by differentiating the work equation to the zero.

And, when you do this we can find out optimum pressure ratio and maximum power output. And, we will see that the specific work output will be maximum, when pressure ratios are such that the compressor and turbine outlet temperatures are equal. So, this will tell us that when compressor outlet and turbine outlet temperatures are equal then work output is maximum.

Now, by doing so, here another plot we can represent that is $\frac{w}{c_p T_1}$ for the way we have calculated that this w by c p times T is a function of pressure ratio as well as temperature ratio. So, this plot is given for different values of t.

(Refer Slide Time: 26:54)



So, let us recall that we were going to find out the condition at which maximum work output can be obtained. So, first thing from our previous analysis, we can see that we have pressure ratio r, temperature ratio t and isentropic relations $\frac{T_2}{T_1}=r^{\frac{\gamma-1}{\gamma}}; \frac{T_3}{T_4}=r^{\frac{\gamma-1}{\gamma}}$.

And, also we have this non dimensional form of specific work.

So, to have maximum work we have to differentiate this particular $\frac{w}{c_p T_1}$ with respect to this particular parameter, that is $r^{\frac{\gamma-1}{\gamma}}$. So, when you differentiate with respect to this then we will find a relation between the temperature ratio and optimum value of pressure ratio. $\frac{d\left(\frac{w}{c_p T_1}\right)}{d\left(r^{\frac{\gamma-1}{\gamma}}\right)} = 0; \Rightarrow r_{opt}^{\frac{\gamma-1}{\gamma}} = \sqrt{t}; \;\; t = r_{opt}^{\frac{2(\gamma-1)}{\gamma}}$

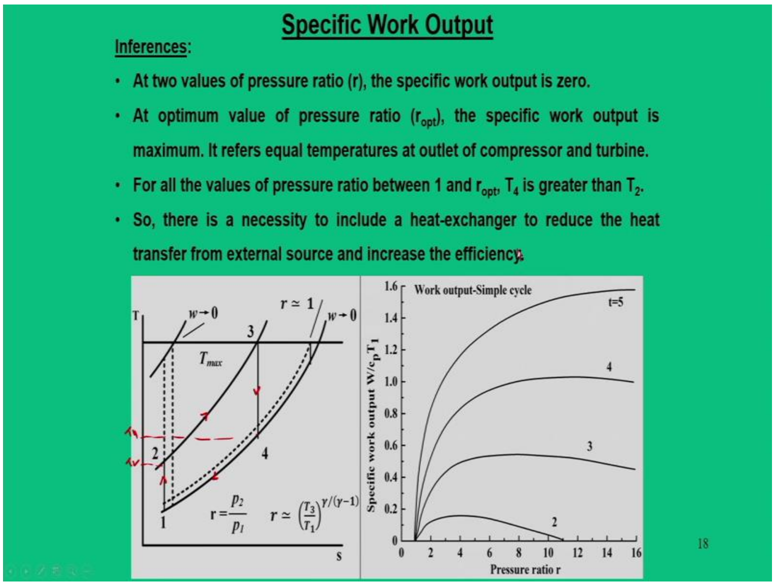
$$\frac{T_2}{T_1} \times \frac{T_3}{T_4} = \sqrt{t} \times \sqrt{t} \quad \Rightarrow \frac{T_2}{T_1} \times \frac{T_3}{T_4} = t \quad \Rightarrow T_2 = T_4 \quad \left(at\ r = r_{opt}\right)$$

And, when you put this particular things then we will note that this is nothing but situation at which T_2 is equal to T_4; the temperature at the exit of the compressor and turbine is equal, that conditions will have optimum pressure ratio.

But, that again is a very hypothetical situation due to variety of reasons, drop in pressures or pressure losses, we will not land off in that manner. So, T_4 is always higher than T_2. So, we will come back to that conclusion later, but here when you put this r optimum conditions, we can find out another expression what is the conditions for maximum work output?

That means, $\left(\frac{w}{c_p T_1}\right)_{\max} = t\left(1 - \frac{1}{\sqrt{t}}\right) - \left(\sqrt{t} - 1\right)$. So, here you can see the maximum work output is only a function of temperature ratio. And, subsequently for efficiency at this maximum work conditions would be $\eta = 1 - \left(\frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right) \Rightarrow \eta = 1 - \frac{1}{\sqrt{t}} = 1 - \sqrt{\frac{T_1}{T_3}}$. So, this is another important consequence that efficiency conditions at the maximum power output.

(Refer Slide Time: 29:50)

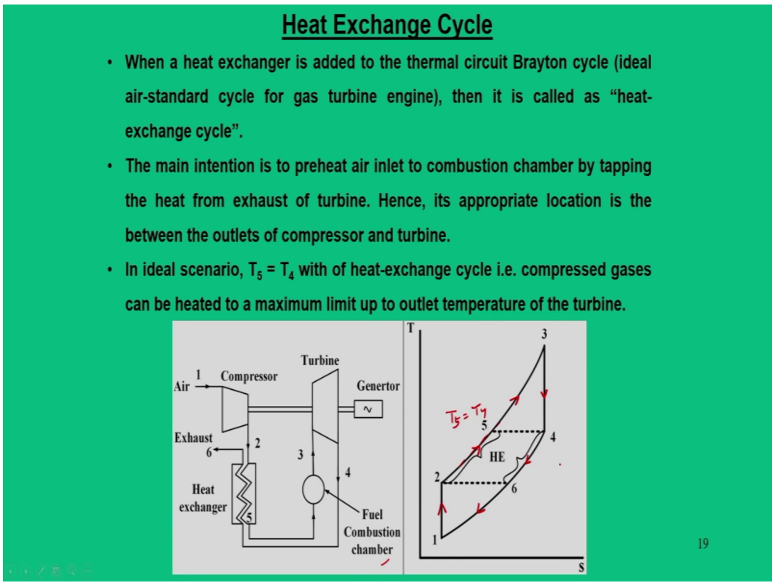


So, this particular study gives some important inferences. That is at two values of pressure ratio, the specific work output is zero. The optimum value of pressure ratio the specific work output is maximum.

And, it refers to a equal temperature at the outlet of compressor and turbine. For all values of pressure ratio between 1 and r_{opt} T_4 is always greater than T_2. Hence, there is a necessity to include a heat-exchanger to reduce the heat transfer from the external source and increase the efficiency.

So, the simple Brayton cycle analysis tells us that the condition of optimum power is a situation, where compressor exit temperature and turbine exit temperature are equal. So, these two points must coincide for optimum work output, but that normally does not happen.

(Refer Slide Time: 31:07)



Because of this reason, the new cycle comes into picture; the simple cycle with inclusion of a heat exchanger. So, what the heat exchanger does then, the heat from the exit of the turbine from the exhaust is tapped by preheating the compressed air that comes out from the compressor at state 2.

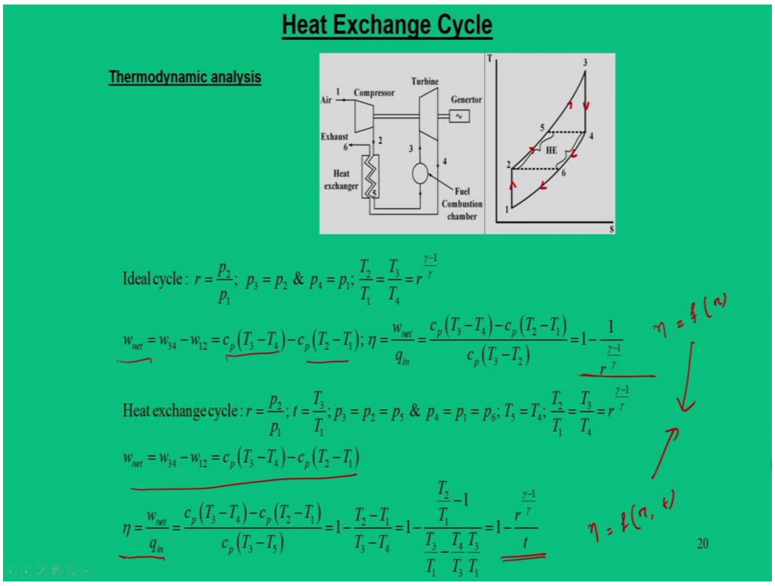
So, we have another process goes from 4 to 6 the other process goes from 2 to 5. So, in this process the changes that happens in the T-s diagram goes this way. So, you have 1-2 is the compression process, then 2 to 5 is a heat exchange process, then 5 to 3 that is a heat addition process.

So, heat addition does in two parts; one is preheating part, other is the through the fuel combustion chamber. And, then of course, they are in a different pressure line, but ideally we are maintaining at same pressure for our cycle analysis. Then, 3 to 4 is the expansion in the turbine, then 4 to 6 is the heat rejection from the turbine in the heat exchanger and finally, from 6 to 1 it goes to the exhaust.

So, we call this as a heat exchange cycle. So, the air standard cycle is now referred as heat exchange cycles. With the main intention to pre heat the air inlet to the combustion chamber by tapping heat from the exhaust of the turbine.

So, hence the appropriate location is between the outlet of turbine and compressors. And, in ideal scenario T_5 is equal to T_4 with a heat exchange cycle that is compressed gases can be heated to a maximum limit to the outlet of turbine temperatures. That means from 2 to 3 we cannot go because the heat exchanger has a limiting situation that, as T_5 is equal to T_4 and that is that much heat we can tap.

(Refer Slide Time: 33:41)



So, now we will go back to this thermodynamic analysis that is 1 - 2 - 5 -3 - 4 - 6 - 1. And one thing that we want to emphasize is the fact that when you calculate the net work it remain same. That means, the difference between the turbine work and compressor work remains same.

Now, here we are comparing two aspects. One is the ideal cycle; other is heat exchange cycle, when you say ideal cycle this is nothing, but a simple Brayton cycle. So, net work output and this efficiency as a function of pressure ratio. Now, if you recall our analysis with respect to heat exchange cycle, we can find out that the net work output does not change for the heat exchange cycle.

And, that is quite obvious because there is no extra components where work output is derived. But, what interesting fact is that heat transfer process. And, in the process what we did is that we reduced to the value of q_{in} so; that means, instead of T_3 - T_2 which was supposed to be given by this combustion chamber. Now, it is becomes T_3 - T_5.

So; in actual process when it goes from 2 to 3 and the first part that is from 2 to 5, we are getting it from the exhaust gas of the turbine. So, that particular component gets reduced in heat transfer process.

And, finally, when you calculate the cycle efficiency, it is now represented by 2 terms that is the cycle efficiency is now function of r and t. Whereas, in a simple cycle the cycle efficiency is a function of only pressure ratio. So, these two things are different with respect to heat exchange cycle.

$$\text{Ideal cycle}: \; r = \frac{p_2}{p_1}; \;\; p_3 = p_2 \;\&\; p_4 = p_1; \; \frac{T_2}{T_1} = \frac{T_3}{T_4} = r^{\frac{\gamma-1}{\gamma}}$$

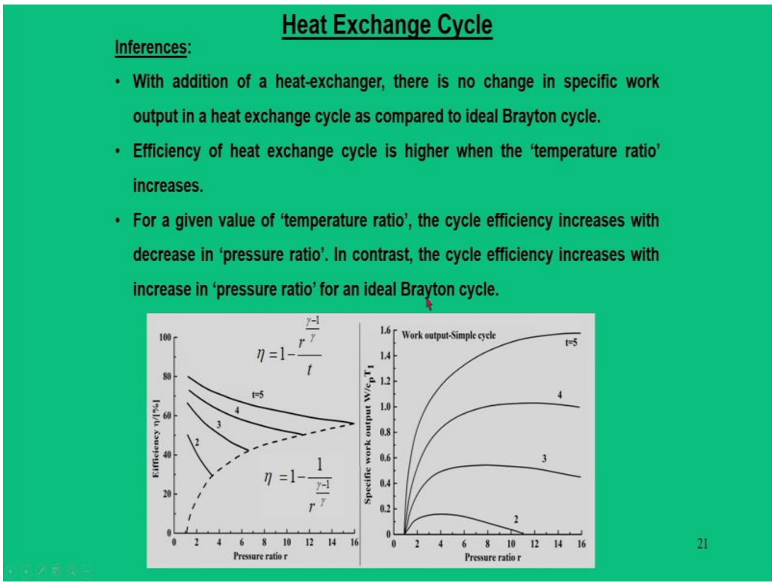
$$w_{net} = w_{34} - w_{12} = c_p\left(T_3 - T_4\right) - c_p\left(T_2 - T_1\right); \; \eta = \frac{w_{net}}{q_{in}} = \frac{c_p\left(T_3 - T_4\right) - c_p\left(T_2 - T_1\right)}{c_p\left(T_3 - T_2\right)} = 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}$$

$$\text{Heat exchange cycle}: r = \frac{p_2}{p_1}; \; t = \frac{T_3}{T_1}; \; p_3 = p_2 = p_5 \;\&\; p_4 = p_1 = p_6; \; T_5 = T_4; \; \frac{T_2}{T_1} = \frac{T_3}{T_4} = r^{\frac{\gamma-1}{\gamma}}$$

$$w_{net} = w_{34} - w_{12} = c_p\left(T_3 - T_4\right) - c_p\left(T_2 - T_1\right)$$

$$\eta = \frac{w_{net}}{q_{in}} = \frac{c_p\left(T_3 - T_4\right) - c_p\left(T_2 - T_1\right)}{c_p\left(T_3 - T_5\right)} = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{\frac{T_2}{T_1} - 1}{\frac{T_3}{T_1} - \frac{T_4}{T_3}\frac{T_3}{T_1}} = 1 - \frac{r^{\frac{\gamma-1}{\gamma}}}{t}$$

(Refer Slide Time: 36:13)



Now, when you plot efficiency versus pressure ratio by considering for an ideal simple Brayton cycle; efficiency is equal to $1-\frac{1}{r^{\frac{\gamma-1}{\gamma}}}$, we see this is the dotted line. So, this is the typical nature of the plot.

Now, on this we are superimposing the efficiency term for heat exchange cycle. So, when you put the efficiency of heat exchange cycle. So, it is a function of r as well as t, that is $1-\frac{r^{\frac{\gamma-1}{\gamma}}}{t}$. So, for different values of t, we get the curves like this. So, this curve shows for t is equal to 5, then 4, then 3, and then 2.

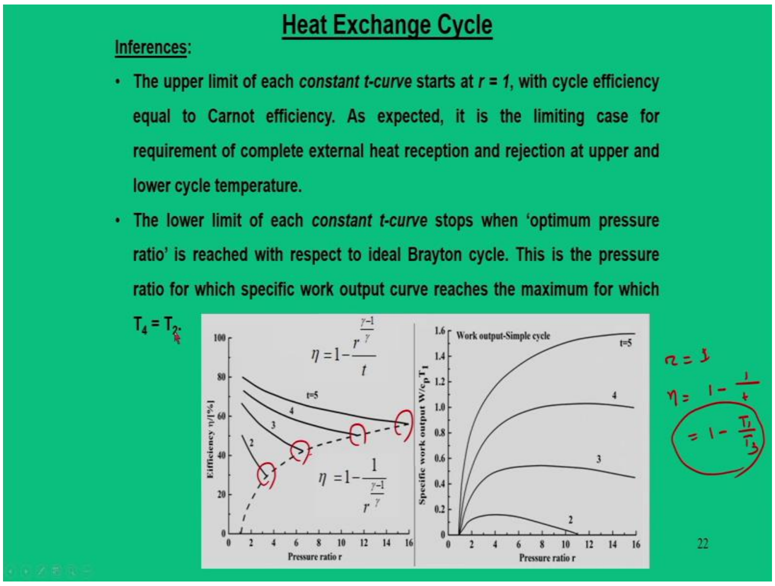
But, the interesting fact is that at some point when you see here, that for an simple cycle with increase in the pressure ratio, the efficiency goes up. But, for a heat exchange cycle with increase in the pressure ratio at a given temperature ratio, the efficiency fall this is the quite interesting factor.

So, these are the two contradicting factor, but one point of time somewhere at some locations they intersect each other. And, in fact, this is the optimum locations that we have to use; that means, we have to use this particular temperature, and for that this is the optimum pressure.

So, whatever I explained if you can make some inferences, we can write like this that with an addition of heat exchanger, there is no change in the specific work output in a heat exchange cycle as compared to Brayton cycle.

So, efficiency of heat exchange cycle is always higher when the temperature ratio increases. For a given value of temperature ratio; the cycle efficiency increases with decrease in the pressure ratio. For a given value of 'temperature ratio', the cycle efficiency decreases with increase in the 'pressure ratio' or we can say in a reverse way. But in contrast the cycle efficiency increases with increase in the 'pressure ratio' for an ideal Brayton cycle.

(Refer Slide Time: 38:46)

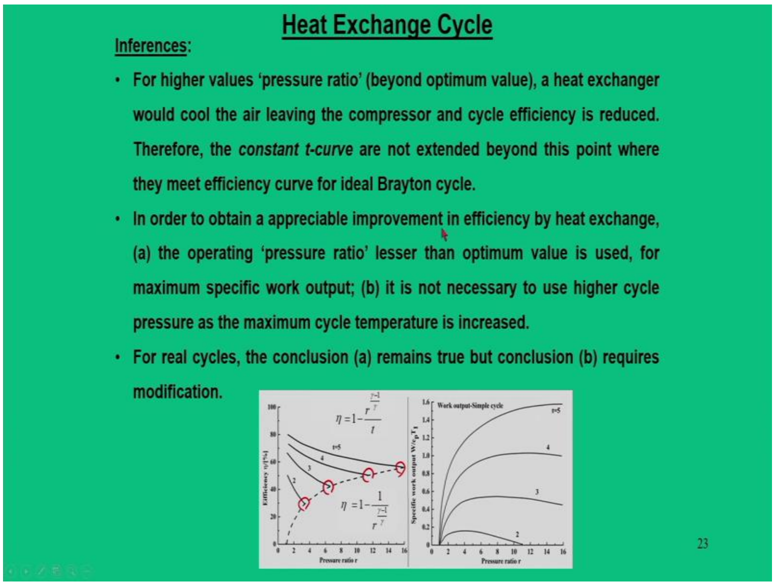


Another point is that the upper limit of constant t-curve starts with r is equal to 1. So, now, if you look at this particular plot, when you put r is equal to 1, it gives a situation that it is nothing, but efficiency becomes 1 by t. And, that is nothing, but $1 - (T_1 / T_3)$. So, this is nothing, but your Carnot efficiency.

So, as expected for the limiting case, the cycle efficiency is nothing but the Carnot efficiency. So, it is the limiting case for the requirement of complete external heat reception and rejection at upper and lower cycle temperatures. The lower limit of the constant t-curve stops at the 'optimum pressure ratio' is used.

So, these are the locations where this constant t-curve and simple cycle intersect. So, this particular intersecting points, we call as a optimum pressure ratio. And, the optimum pressure ratio refers to a situation, where the turbine outlet temperature and compressor outlet temperatures are equal.

(Refer Slide Time: 40:11)



For higher values of pressure ratio that is beyond optimum value, the heat exchanger would cool the air leaving the compressor and cycle efficiency is reduced. That means, beyond these locations, normally these curves are not extended, because it will have a different interpretation that heat exchanger will not play its role; that means, the air leaving the compressor would be cooled instead of getting heated.

So, finally, in order to obtain appreciable improvement in the efficiency by heat exchange, there are two possible situations; one is operating pressure ratio is lesser than the optimum value is used for maximum specific work output. Other option is that always it is not necessary to use high higher cycle pressures.

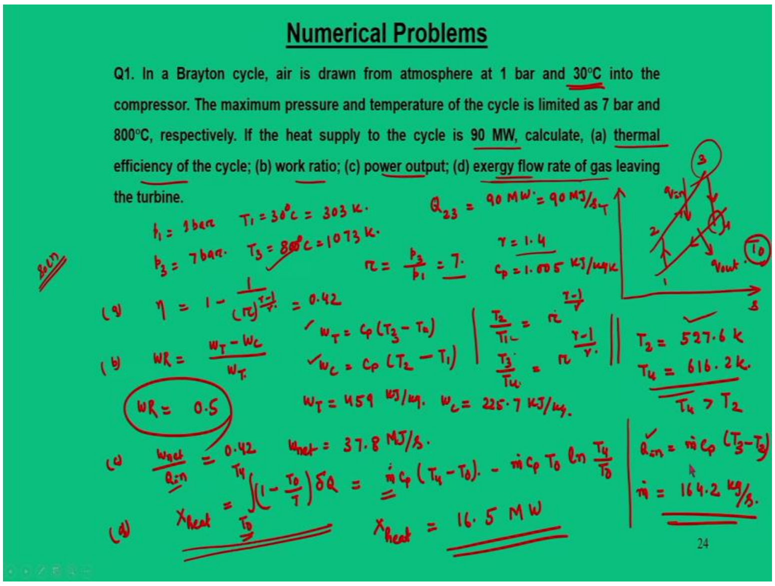
So, one way to increase the efficiency is that either you go for higher temperature ratio or you go for higher pressure ratio. But, what will happen is that higher pressure ratio will have a lot of other complications, because you require many kind of rotary devices through the compression mode.

So, the better option would be to go for a heat exchange cycle with higher temperature ratio. Through this; obviously, we land off in getting higher pressure ratio, at some location. For example, if you take t = 4, we land up a pressure ratio which is optimum and maybe at r is equal to 11.

But that means, we can achieve this pressure ratio in a heat exchange cycle with t = 4, for which we need not have a very big compressors, which can give you the pressure ratio of similar number. So, eventually it is the fact that, the use of heat exchange cycle is a very significant aspects in a inclusion of heat exchange or heat exchange cycle is a very significant aspect for gas turbine engines.

And, finally, we will say that, if you want to operate a higher pressure ratio lesser than optimum value, then you require certain modifications. So, what modification we will do, we will see in the next class. So, from the basic variant of simple gas turbine cycle and a heat exchange cycle, we will have few modifications that can be used for it is practical utility. To account for the fact that we have to get appreciable improvement in the efficiency by heat exchange.

(Refer Slide Time: 43:12)



So, after this we are now going to solve some numerical problems. So, in this numerical problems, we have basic aspect that we have covered here is the simple Brayton cycle. So, for simple Brayton cycle, first thing we are going to draw is the temperature entropy. And, we have to represent the cycle diagram 1 2 3 4; 1-2 compression 2-3 heat addition 3-4 turbine and 4-1 exhaust.

So, now, let us see the question that in a Brayton cycle air is drawn at atmospheric pressure 1 bar and 30°C into the compressor. So, state 1 is known the maximum pressure on temperature of the cycle is limited to 7 bar and 800°C. So, we have p_1 as 1 bar T_1 as 303 K, then p_3 that is 7 bar and T_3 that is 1073 K.

And, you have heat supply is q_{2-3} as 90MW or MJ/s. So, it is total heat because mass flow rate is involved. So, you are going to find out thermal efficiency, work ratio, power output, exergy. $\eta = 1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}; r = \frac{p_3}{p_1} = 7 \Rightarrow \eta = 0.42$.

Second problem is work ratio.

$$\text{WR} = \frac{w_T - w_C}{w_T}; w_T = c_p\left(T_3 - T_4\right), w_C = c_p\left(T_2 - T_1\right); \frac{T_2}{T_1} = r^{\frac{\gamma-1}{\gamma}}, \frac{T_3}{T_4} = r^{\frac{\gamma-1}{\gamma}}$$
$$\Rightarrow T_2 = 527.6K, T_4 = 616.2K; w_T = 459\text{kJ/kg}, w_C = 225.7\text{kJ/kg}; \text{WR=0.5}$$

So, we got this work ratio, third point is power output.

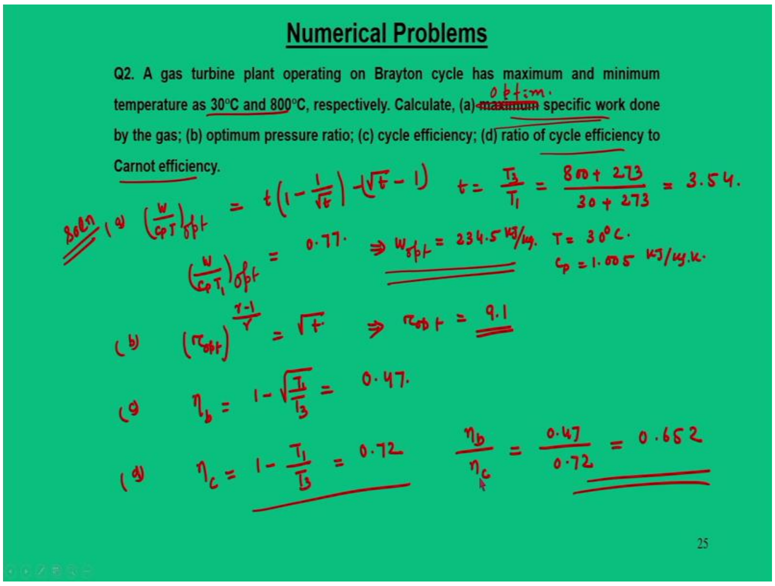
$$\eta = \frac{w_{net}}{q_{in}} = 0.42 \Rightarrow w_{net} = 0.42 \times 90 = 37.8\text{kW}$$

So, last part is exergy in the exhaust gas leaving from a gas turbine unit. So, here the exergy is in the form of heat. So, this is the expression for exergy which comes out of heat.

$$X_{exit} = \int_{T_0}^{T_4}\left(1 - \frac{T_0}{T}\right)\delta q = \dot{m}c_p\left(T_4 - T_0\right) - \dot{m}c_p \ln\frac{T_4}{T_0}$$
$$q_{in} = \dot{m}c_p\left(T_3 - T_2\right) \Rightarrow \dot{m} = 164.2\text{kg/s} \Rightarrow X_{exit} = 16.5\text{MW}$$

So, it means that, this exhaust heat has a tremendous value of energy that is about 16.5, which could have been trapped. And, because of this reason we say that effectiveness of heat exchanger is realized.

(Refer Slide Time: 52:34)



Now, we will go to the next problem, next problem is also a similar nature, but here we will try to concentrate on some important aspect that is about maximum work done and how you are going to compare with a Carnot efficiency?

So, with same realistic number and the previous example where the operating temperature limits are 30°C and 800°C, we have to find out the maximum specific work done and optimum pressure ratio cycle efficiency and ratio of cycle efficiency to the Carnot efficiency.

So, here the problem is simple with the fact that we have to refer a situation for maximum specific work output condition.

So,
$$\left(\frac{w}{c_p T_1}\right)_{max} = t\left(1-\frac{1}{\sqrt{t}}\right)-\left(\sqrt{t}-1\right); t = \frac{T_3}{T_1} = \frac{800+273}{30+273} = 3.54$$
$$\left(\frac{w}{c_p T_1}\right)_{max} = 0.77 \Rightarrow w_{max} = 234.5\text{kJ/kg}$$

So, you can get this much of maximum work from these situations. And, instead of maximum work I will correct it as optimum, because this is as optimum condition.

And second part would be r optimum, $t = r_{opt}^{\frac{2(\gamma-1)}{\gamma}} \Rightarrow r_{opt} = 9.1$

So, optimum pressure ratio is 9.1. Third is for this optimum pressure ratio, what is the cycle efficiency? So, Brayton cycle efficiency at optimum pressure ratio is $\eta = 1 - \frac{1}{\sqrt{t}} = 1 - \sqrt{\frac{T_1}{T_3}}$. So, this number would be 0.47. Now, last part we have asked to find out Carnot efficiency. So, recall that if you take the Carnot efficiency between these two temperature limits.

So, it will be $1 - \frac{T_1}{T_3}$. And, this number is 0.72. And, finally, the ratio would be 0.47 / 0.72 = 0.652. So, ratio of a Brayton cycle efficiency at optimum power is about 65 percent of the Carnot efficiency. And, this is how it says that it is the optimum air standard efficiency of a Brayton cycle when you compared with the Carnot efficiency. So, with this let us conclude for today.

Thank you for your attention.