Viscous Fluid Flow
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Module - 08
Laminar Boundary Layers - 11
Lecture - 02
The Correlation Method by Thwaites

Hello, everyone. So, in last class, we have used these carbon pollution and approximation and we describe the steps how to calculate the momentum thickness, displacement thickness, boundary layer thickness, then wall shear stress and the friction coefficient. Today, we will use the correlations proposed by Thwaites.

So, similar procedure we will follow, but scientist Thwaites actually used some experimental data and proposed some correlations to find the displacement thickness, boundary layer thickness, momentum thickness and the wall shear stress. Because our for design point of view we are more interested to know the wall shear stress.

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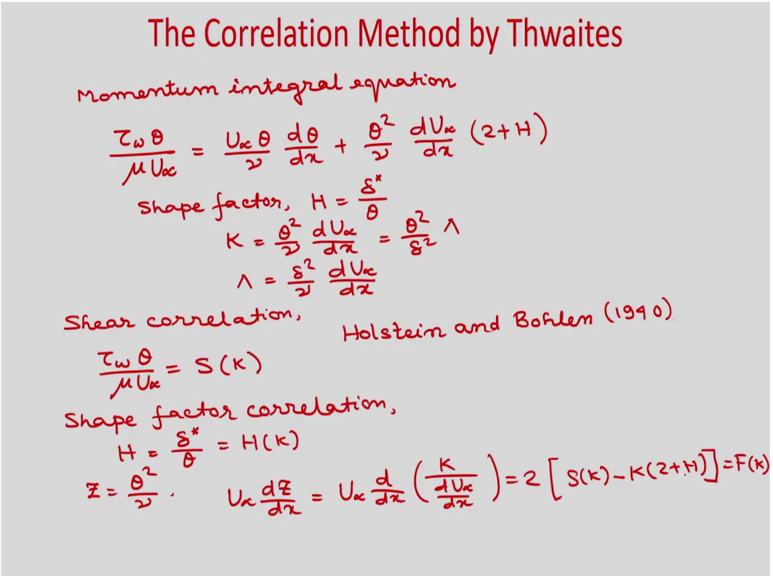

So, if you recall that in last class, we have already derived these momentum integral equation ok written by Holstein and Bohlen that is tau w theta by mu U infinity is equal to U infinity theta by nu d theta by dx plus theta square by nu d U infinity by dx 2 plus H ok. So, if you remember that H is the a shape factor.

So, H is delta star by theta displacement thickness by momentum thickness. This K is the pressure parameter that is theta square by nu dU infinity by dx which is theta square by delta square lambda and lambda is also the pressure parameter delta square by nu dU infinity by dx ok.

So, now, we can also write this shear correlation and shape factor correlation whatever we have derived in last class. So, shear correlation ok proposed by Holstein and Bohlen. So, that is tau w by theta mu U infinity is equal to S K. So, this is the shear correlation.

And, shape factor correlation so, H is equal to delta star by theta is equal to H K. So, these are proposed by Holstein and Bohlen. So, now, if we write that Z is equal to theta square by nu then we have written this from the momentum integral equation U infinity dZ by dx is equal to U infinity d of dx K by dU infinity by dx is equal to 2 S K minus K into 2 plus H ok and this we have represented as F K ok.

So, you can see that here S K is the shear correlation and H is the shape factor and these we have already derived in last class.

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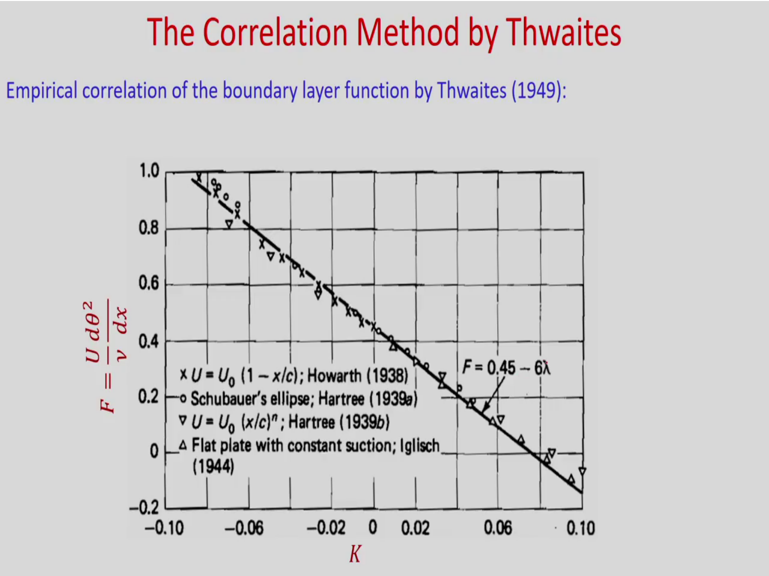

So, using different experimental results Thwaites actually tried to find this F K. You can see in this figure that for different experimental values in the in this range of K. These are the results of Howarth, then other scientist it is reported at different symbols you can see. So, this is F versus K and you can see that again this can be represented by a straight line and Thwaites proposed that F can be represented as 0.45 minus 6 lambda.

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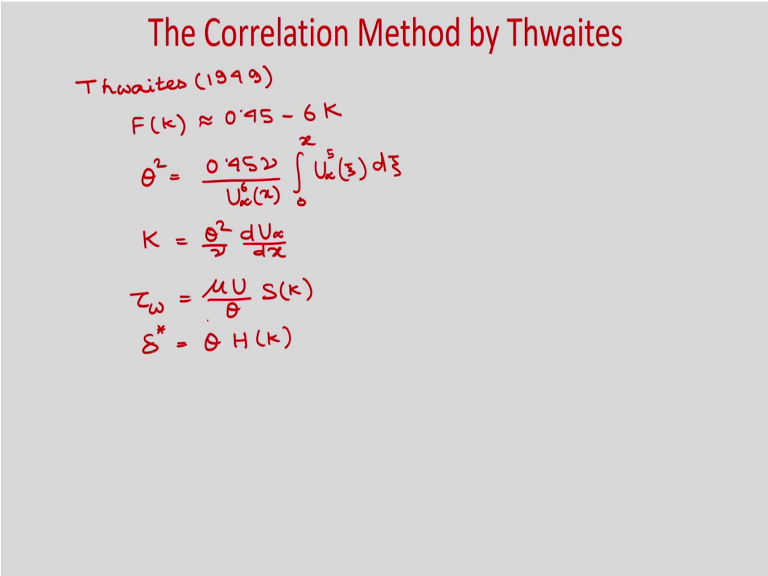

So, from the empirical correlations Thwaites proposed that proposed this expression F K as 0.45 minus 6 K ok. So, from here you can now find the theta square. So, similar way whatever we have expressed in carbon pollution method. So, similarly we can write theta square here as 0.45 nu by U infinity to the power 6 integral 0 to x U infinity to the power 5 zeta d zeta ok.

So, this is the theta square. So, similarly once theta square is known we can find the K from this expression theta square by nu dU infinity by dx, ok. And from there we can calculate the other parameters like skin friction coefficient and the displacement thickness.

So, we can write tau w is equal to mu U by theta the shear correlation S K and the displacement thickness delta star is equal to theta into the shape factor H. So, you can see from here actually Thwaite suggested correlation for S K and H K and these are tabulated here.

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The Correlation Method by Thwaites

Shear and Shape Functions Correlation by Thwaites (1949):

K	$H(K)$	$S(K)$	K	$H(K)$	$S(K)$
+ 0.25	2.00	0.500	−0.056	2.94	0.122
0.20	2.07	0.463	−0.060	2.99	0.113
0.14	2.18	0.404	−0.064	3.04	0.104
0.12	2.23	0.382	−0.068	3.09	0.095
0.10	2.28	0.359	−0.072	3.15	0.085
+0.080	2.34	0.333	−0.076	3.22	0.072
0.064	2.39	0.313	−0.080	3.30	0.056
0.048	2.44	0.291	−0.084	3.39	0.038
0.032	2.49	0.268	−0.086	3.44	0.027
0.016	2.55	0.244	−0.088	3.49	0.015
0.0	2.61	0.220	−0.090	3.55	0.000
				(Separation)	
−0.016	2.67	0.195			
−0.032	2.75	0.168			
−0.040	2.81	0.153			
−0.048	2.87	0.138			
−0.052	2.90	0.130			

So, you can see these are the shear and shape function correlation by Thwaites. So, this is the value of K, this is shape factor H and the shear correlation S. So, for different values of K the H and S are tabulated, ok.

So, you can see that S is the shear correlation. So, that means, it represents the shear stress. So, obviously if it becomes 0, that means, you will get the separation at that point. So, S K for value of K as minus 0.09 this S K is becoming 0. So, this is the point where flow will separate ok.

So, the limit of case would be minus 0.09 for flow separation because once the flow separates, then the boundary layer theory is not valid. So, from the table you can see that the for different values of K these shape factor H and the shear correlation S are represented and Thwaites actually plotted this H versus K ok.

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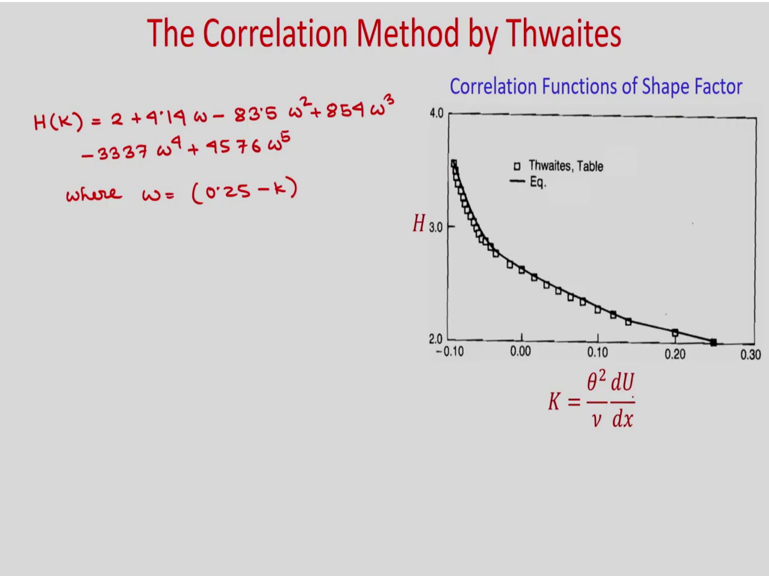

So, you can see in this plot this square symbol these are the correlation function of shape factor. So, H versus K so, this is the variation ok. So, here Thwaites proposed this simple function H K from this correlation as H K is equal to 2 plus 4.14 w minus 83.5 w square plus

854 w cube minus 3337 w to the power 4 plus 4576 w to the power 5, where w is 0.25 minus K.

So, you can see that this feet whatever we are represented here this is the straight line that is the equation ok. So, this equation, so from this Thwaites stable whatever it was actually plotted. So, from this equation this is the feet and you can see that from here you can represent this H relation with K.

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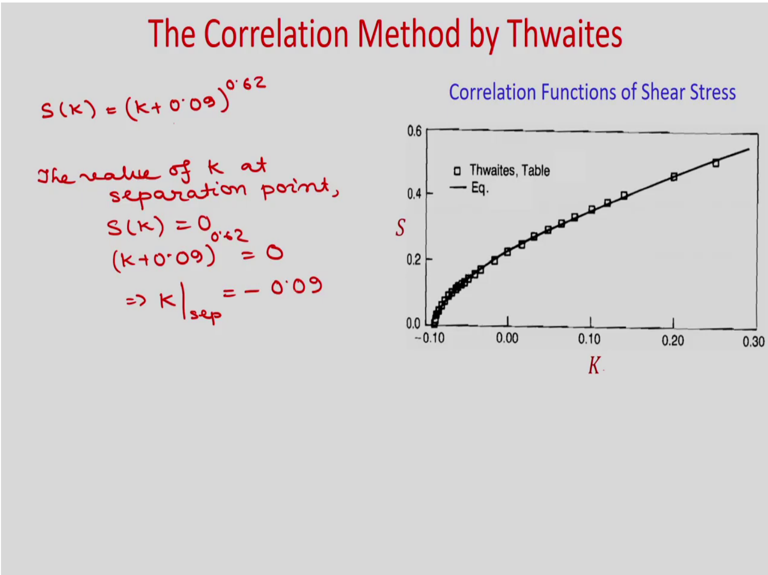

Similarly, from that table S versus K are plotted and you can see these are the square symbol this is the plot and the feet is given in this equation S K is equal to K plus 0.09 to the power 0.62. So, you can see that it is a simple, but accurate curve feet for these Thwaites table whatever it was plotted. So, this is the shear correlations given in terms of K.

So, you can see obviously, the value of K at separation point you will get as. So, S K should be 0, right, at separation point? So, if it is 0 so from this feet you can see that K plus 0.09 to the power 0.62 should be 0 and from here you can see that K at separation point should be minus 0.09 and it is tabulated by Thwaites ok. So, K is equal to minus 0.09.

So, now you can use the similar procedure which was discussed in last class the and you can find the displacement thickness, momentum thickness, boundary layer thickness and the shear stress and the skin friction coefficient. So, now let us apply this Thwaites a approximation to the Howarth's decelerating flow.

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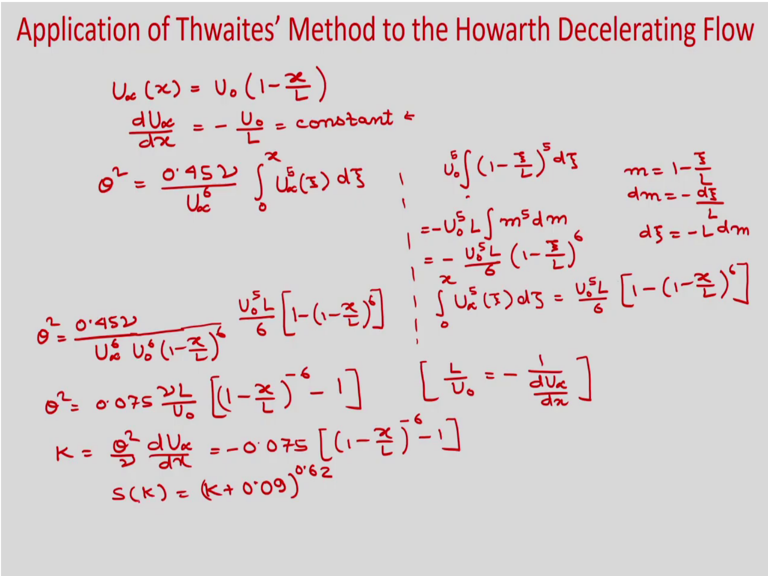

So, the linearly retarded flow of Howarth deals with a simple decelerating non similar pastime distribution. So, U infinity is given for this Howarth decelerating flow as U naught into 1 minus x by L ok. So, now from here you can find dU infinity by dx. So, from here you can see it will be minus U naught by L and U naught is constant. So, this will be constant ok.

So, once you know the U infinity so obviously, we can calculate the momentum thickness, right. So, we have the expression theta square is equal to 0.45 nu divided by U infinity to the power 6 integral 0 to x U infinity to the power 5 zeta d zeta.

So, from here you can see that if you evaluate this integral so, obviously, you can see here integral. So, if you write these so, it will be U naught to the power 5 that is constant you can take it outside 1 minus zeta by L to the power 5 d zeta right. So, now let us represent m is equal to 1 minus zeta by L. So, dm will be minus d zeta by L. So, you can see that you can write d zeta is equal to minus L into dm.

So, from here you can see that you can write U naught to the power 5 L and with a minus sign integral z m to the power 5 dm. So, you can see that now you can write it that u naught to the power 5 L by 6. So, m to the power 6 by 6 right, so m to the power 5 means 1 minus zeta by L to the power 6 ok.

So, you can write 0 to x U infinity to the power 5 zeta d zeta is equal to. So, if you put the limits ok then you will get U to the power U naught to the power 5 L by 6 1 minus 1 minus x by L to the power 6, ok. So, now if you put this value here then you will get theta square is equal to 0.45 nu by U infinity to the power 6 and here U infinity to the power 6 we have.

So, we will get U naught to the power 6 1 minus x by L to the power 6 and integral we have evaluated U naught to the power 5 L by 6 1 minus 1 minus x by L to the power 6 ok. So, now if you simplify it we are going to get theta square is equal to 0.075 nu L by U naught 1 minus x by L to the power minus 6 minus 1.

And, you can see from this expression you can write actually L by U naught is equal to minus 1 by dU infinity by dx ok. So, now we can write dU infinity by dx we know. So, now we can find the K. So, K will be just theta square by nu d U infinity by dx. So, dU infinity by dx we can write minus U naught by L and from here you can theta square is known.

So, you can write 0.075 with a minus sign 1 minus x by L to the power minus 6 minus 1. So, you can see that we have found the momentum thickness and the case. So, now from there we will be able to calculate the shear correlation S K, then you can calculate the shear stress and other parameters like displacement thickness and boundary layer thickness.

So, once K is known so, you can calculate the from the carpet the shear here correlation is S K which is K plus 0.09 to the power 0.62 ok. So, once S K is known then you can calculate what is the shear stress. Now, for this flow if we want to find that at which x the flow separation will take place, then obviously we have to make that S K must be 0.

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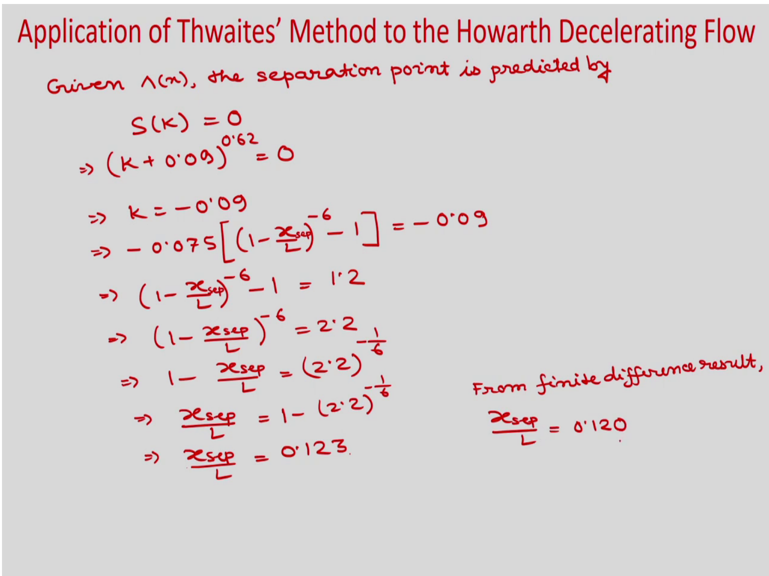

So, you can see that so for given lambda the separation point is predicted by so, S K is equal to 0; S K is equal to 0, that means, K plus 0.09 to the power 0.62 is equal to 0. That means, K should be minus 0.09 and we have the expression of K. So, that is minus 0.075 1 minus x by L to the power minus 6 minus 1 should be minus 0.09 ok.

So, from here you can see you can write 1 minus x by L to the power minus 6 minus 1 and if you divide it in the righthand side so, we will get 1.2. So, that means, we will get 1 minus and these x we can write at separation point right because S K is equal to 0 at the separation point. So, we will write sep separation.

So, 1 minus x separation by L to the power minus 6 will be 2.2 and from here we can write 1 minus x separation by L should be 2.2 to the power minus 1 by 6. So, from here you can calculate x separation by L should be 1 minus 2.2 to the power minus 1 by 6. So, we will get x separation by L is equal to 0.123 ok.

So, if you see that this problem if you solve using finite difference method, then H actually it is x separation by L is 0.120 ok. So, from finite difference result ok. So, you can compare ok. So, this is within 3 percent ok variation. So, this prediction by the Thwaites method of these separation point ok is just 3 percent a within the 3 percent of the exact solution using finite difference method.

So, in today's class, we started with the correlation which we derived in the carbon pollution method in last class and with that we discussed about the Thwaites approximation. So, what Thwaites did? Actually, he found the experimental values of F K versus K and from there he proposed the feet of F K.

And, using that he found the he actually tabulated the values K, the shape factor H and the shear correlation factor S. and he plotted this S K versus K as well as the H versus K and he proposed a feet. And, with that, he actually calculated the shear correlation S K and from there, he found the momentum thickness theta.

And, once theta is known then we can calculate the displacement thickness, the boundary layer thickness and the shear stress and from there you can calculate the friction coefficient. And, later this Thwaites method was applied to Howarth decelerating flow, where U infinity is given. And, from there we calculated the momentum thickness and the shear correlation factor.

And, from there we a try to find that at what x the flow will separate and putting S K is equal to 0. And we found that whatever results we get those are almost 2 to 5 percent with the exact results.

Thank you.