

Evolutionary Computation for Single and Multi-Objective Optimization
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Lecture - 26
Closure of EC for Single and Multi-Objective Optimization

We have come to the closure of this course that is Evolutionary Computation for Single and Multi-Objective Optimization. We started this course with the understanding that there are different problem characteristics of an optimization problem that we can encounter with any engineering optimization problem or any real world problem.

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Properties of Practical Optimization Problems

- Non-differentiable functions and constraints
- Discontinuous function and search space
- Discrete search space
- Mixed variables (discrete, continuous, permutation)
- Large dimensional problem (variables, constraints)
- Non-linear constraints
- Multi-modal function
- Multiple objectives
- Uncertainties in variables
- Computationally expensive problems
- Multi-disciplinary optimization

Need for an innovative and flexible optimization algorithm

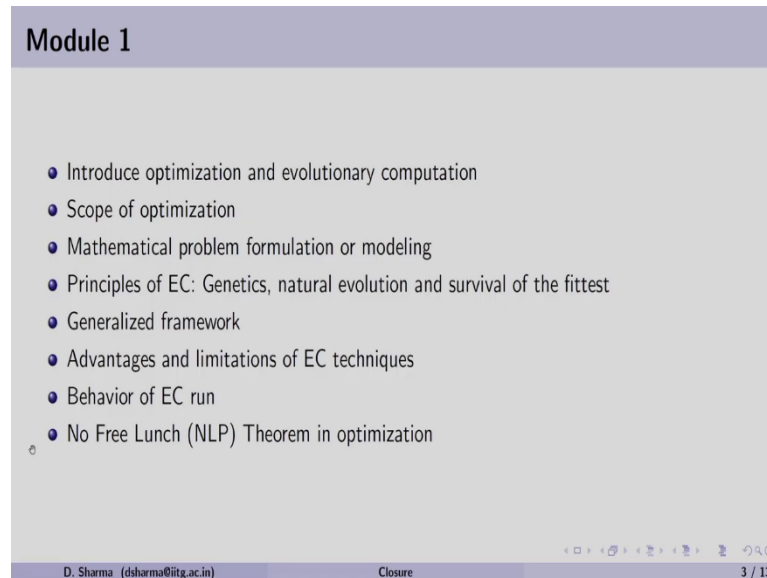
D. Sharma (dsharma@iitg.ac.in) Closure 2 / 13

Some of the problem characteristics are shown here such as this problem can be non-differentiable functions, they can have a discrete search space, discontinuous functions, they can have mixed variable, the function can be non-linear, it can be multi modal, multi-objective optimization, some problem can have uncertainties, some problem can be computationally expensive and some problem involves multiple discipline to get an optimal solution.

So, here, our understanding made that if we are looking for an innovative and flexible algorithm that can solve a large variety of problem. So, in this course, we decided the content based on the fact that these optimization problems can have different complexities.

So, what are the algorithms that are available, how they work? We can understand them by performing the hand calculation showing their performance using the simulation.

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Module 1

- Introduce optimization and evolutionary computation
- Scope of optimization
- Mathematical problem formulation or modeling
- Principles of EC: Genetics, natural evolution and survival of the fittest
- Generalized framework
- Advantages and limitations of EC techniques
- Behavior of EC run
- No Free Lunch (NLP) Theorem in optimization

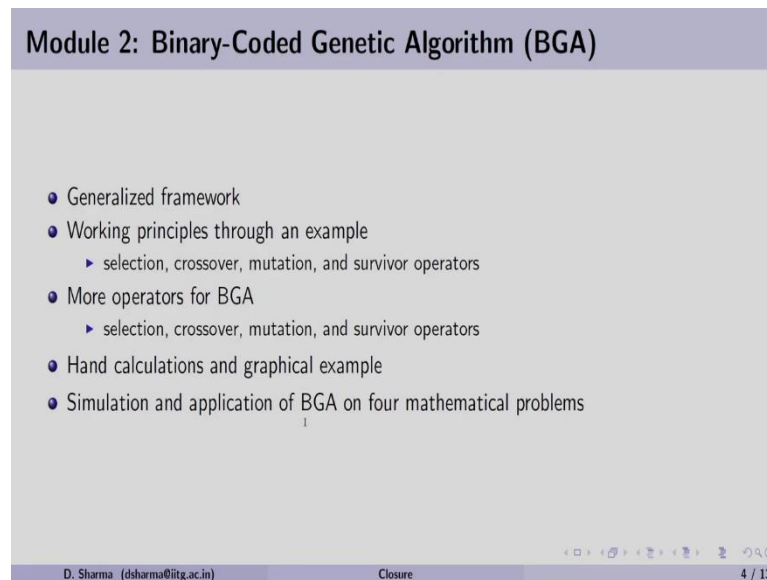
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In the module 1, we introduce the optimization and they and in the same, we should also introduce the evolutionary computation. Thereafter, we presented many real world problems to show the scope of an optimization. We discussed mathematical problem formulation in which constraint, decision variables, parameters, objective functions, bounds everything we discussed.

Thereafter, we focused our discussion on the principle of EC techniques that we understood with the features of genetics, natural evolution and the survival of the fittest. We started our discussion understanding the EC techniques using generalized framework and the same generalized framework; we have taken for different EC techniques in this course.

We also discuss certain advantages and limitations of these techniques, it is because we should have a fair idea in which area EC techniques can be good and what could be the limitations as well. The behavior of EC techniques we also have seen and then, we presented the no free lunch theorem in the parlance of optimization in module 1.

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Module 2: Binary-Coded Genetic Algorithm (BGA)

- Generalized framework
- Working principles through an example
 - ▶ selection, crossover, mutation, and survivor operators
- More operators for BGA
 - ▶ selection, crossover, mutation, and survivor operators
- Hand calculations and graphical example
- Simulation and application of BGA on four mathematical problems

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Based on that, the module 2 was based on binary coded GA. In this module, our focus was mainly on understanding the first algorithm that is BGA. So, we fit our binary coded genetic algorithm on the generalized framework. We understand the working principle through an example and this we have followed a, with each and every techniques which we have discussed in this particular course.

In this module, we understood the selection, crossover, mutation and survivor operators that are needed to make a binary coded GA. Followed by we also discuss more operators such as selection, crossover, mutation and survival. In this module, we showed the performance or the hand calculation of binary coded GA on one of the mathematical example and thereafter, we also showed the graphical illustration of the same. Thereafter, simulation of BGA was shown on four mathematical problems.

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Module 3: Real-Coded Genetic Algorithm (RGA)

- Limitations of BGA for solving problems with continuous search space
- Generalized framework for RGA
- Working principles of RGA through an example
 - ▶ Selection operator: No change in its functioning was observed with respect to BGA because it used the fitness values.
 - ▶ Crossover operator: Properties of single-point crossover operator, SBX crossover operator
 - ▶ Polynomial mutation operator
- More RGA operators
 - ▶ Crossover operators; Properties of crossover operators; Similarity among crossover operators on a flat landscape function
 - ▶ Mutation operators
- Hand calculations and graphical illustration
- Simulations and application of RGA
- Algorithmic implementation of BGA and RGA

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Once we have understood the binary coded GA, then we understood the limitation and those limitation tell us that we are looking for those evolutionary computing techniques that can be used for real parameters.

So, in module 3, we discussed about real coded genetic algorithm in short, we call it as a RGA. We discussed the RGA on the same generalized for framework as we discussed in the in module 1 following the same structure of understanding the RGA through an example.

So, under this example we have gone through various operators. While going through RGA, we understood that the selection operator remains the same for binary coded as well as real coded, it is only because the selection operator depends on the fitness, so that is why we did not change anything.

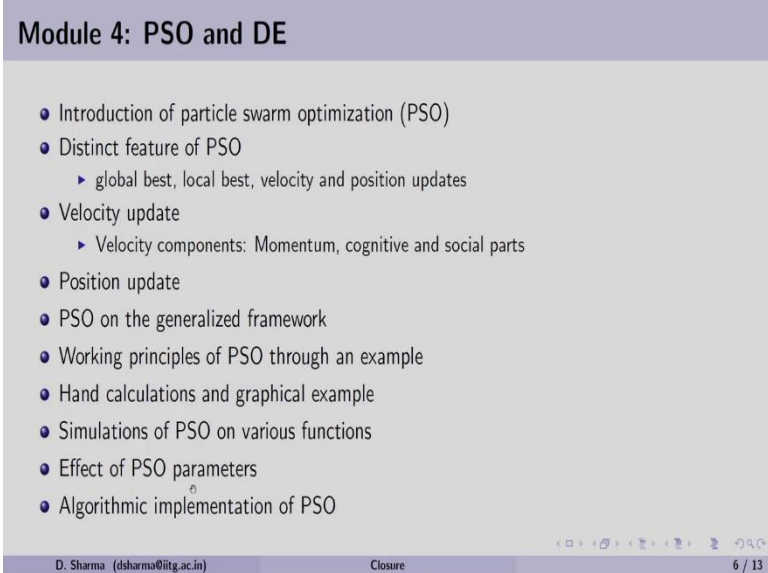
Thereafter, we discussed the crossover operator. In that session, we discussed SBX crossover operator which was developed using the properties of single-point crossover operator. We again discussed the polynomial mutation. So, both the crossover operator and polynomial mutation, both of them are made using the non-linear probability distribution function.

In the another session under module 3, we discussed various other crossover and mutation operators, we discussed the properties of crossover operator, similarity among the

crossover operator. The real coded GA, we understood using hand calculation and the graphical illustration. So, these hand calculations can help us to understand how this algorithm work.

Finally, we showed the simulations of RGA on the mathematical problems. Since these BGA and RGA we have to develop the source code, so, we discussed the algorithmic implementation. So, that algorithmic implementation was made in such a way that it can be used for any for developing BGA and RGA on any of the programming platform such as C, Python, Java etcetera.

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Module 4: PSO and DE

- Introduction of particle swarm optimization (PSO)
- Distinct feature of PSO
 - ▶ global best, local best, velocity and position updates
- Velocity update
 - ▶ Velocity components: Momentum, cognitive and social parts
- Position update
- PSO on the generalized framework
- Working principles of PSO through an example
- Hand calculations and graphical example
- Simulations of PSO on various functions
- Effect of PSO parameters
- Algorithmic implementation of PSO

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Module 4 was based on the two algorithm that were on particle swarm optimization and differential evolution. So, we started our discussion on introducing the particle swarm optimization.

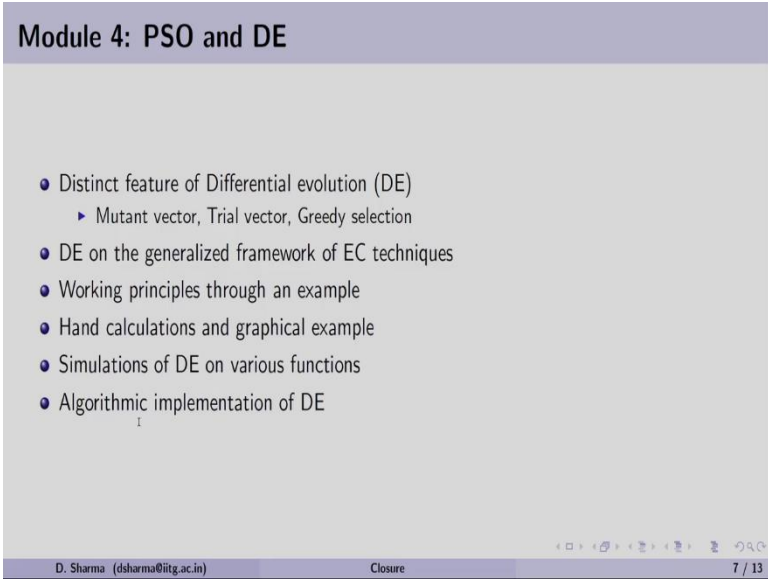
We discussed the distinct feature of PSO. Under this, we have discussed global best, local best, velocity and position updates. The velocity update it has a three component; momentum, cognitive and a social part we understand this using the graphical example and finally, we updated the position.

We discussed PSO on generalized framework that we are following from module 1. The working principle of PSO is again understood through an example. We perform hand

calculation, show the graphical illustration and finally, simulation of PSO was shown on various mathematical functions.

Apart from that, we also discuss the effect of PSO parameter. So, since PSO involve more parameters, so, we change those parameter and look the performance. We also presented the algorithmic implementation of PSO so that we can develop the source code of a PSO.

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Module 4: PSO and DE

- Distinct feature of Differential evolution (DE)
 - ▶ Mutant vector, Trial vector, Greedy selection
- DE on the generalized framework of EC techniques
- Working principles through an example
- Hand calculations and graphical example
- Simulations of DE on various functions
- Algorithmic implementation of DE

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Thereafter, we discuss the differential evolution; this has a distinct feature based on mutant vector, trial vector and a greedy selection. After understanding DE, we fit that DE on the generalized framework of EC techniques. We discuss the principles, working principle through an example. We showed hand calculation for DE, graphical illustration. Simulations of DE on various functions are also shown. Algorithmic implementation of a DE was shown in module 4.

After this module 4, we focused on constraint optimization. It is only because most of the real world problems have constraints. So, in module 1, 2 and a 3, 4 in these modules, the problems which we have solved all of them have variable bounds, but there was no constraint. To understand that, the module 5 was developed on the same.

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Module 5: Constraint Handling With EC Techniques

- Constraint optimization formulation
- Optimality conditions for unconstrained problems
- Method of multipliers for constrained optimization
- KKT conditions for constrained optimization
- Penalty function methods
- Hand calculations using static and dynamic penalty methods
- Constraint handling via separation of objective function and constraints
- Hand calculations for Powell and Skolnick's approach and Deb's approach
- Constraint handling via multi-objective optimization concepts
- Simulation of real-coded genetic algorithm (RGA) on five mathematical and three practical constrained optimization problems

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In this particular module, we discuss constraint optimization formulation, the optimality condition for unconstrained optimization, method of multiplier for constrained optimization and KKT conditions for constrained optimization. So, these are the basics which we have covered under the constrained optimization.

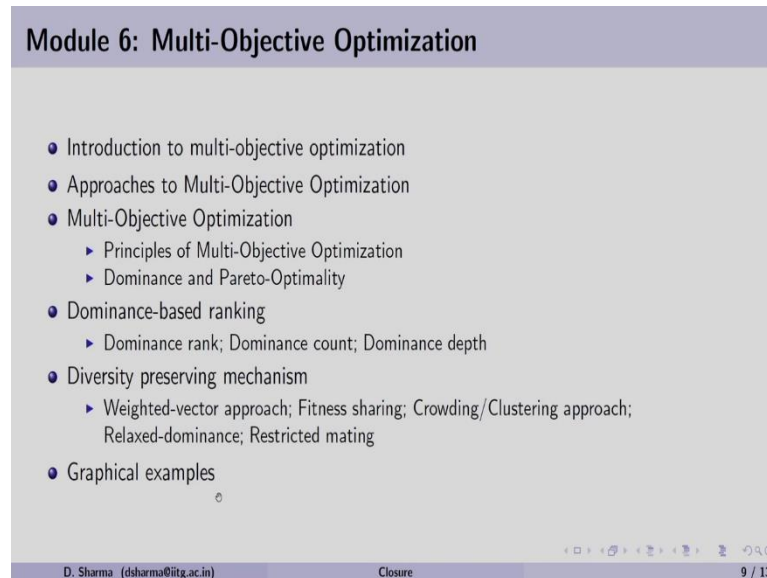
Since when we are solving any of the optimization problem, we should understand the optimality condition so, those mathematical optimality condition, necessary and sufficient condition where we discussed for unconstrained optimization and we discussed KKT condition for constrained optimization.

Having understanding on constrained optimization, we focused on the penalty methods. In these penalty methods, we have take we discuss various kinds of methods and afterwards, we showed hand calculations using static and dynamic penalty methods. Since there are multiple ways, we can understand or we can handle the constraints so, we discussed about separation of objective function and constraints.

Under this category, we discussed Powell and Skolnick's approach and Deb's approach, we perform hand calculation for both these approaches and finally, we also shown that constraint handling can be done using multi-objective optimization concept. At the end, we showed simulation of real coded GA on various mathematical, so, five mathematical and three practical optimization problem.

In that case, we take penalty function method and Deb's approach to show the performance of RGA on constrained optimization problem.

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Till this module, we were focusing on single objective and constrained handling. From module 6, we focused on multi-objective optimization. Since multi-objective optimization is a different kind of a problem so, we started this with the introduction to multi-objective optimization, they approaches to multi-objective optimization and under multi-objective optimization, we discussed such as principle of principles and the dominance and Pareto optimality.

We also discuss dominance based ranking methods such as dominance rank, dominance count and dominance depth. Since solving a multi-objective optimization problem, we need convergence and diversity. So, dominance based rank methods will help us in the convergence and in order to maintain the diversity, we discussed various methods and approaches such as weighted vector approach, fitness sharing approach, crowding and clustering approach, relaxed dominance and restricted mating.

All these approaches the dominance ranking and the diversity, we understood through graphical examples.

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The slide is titled "Module 7: Classical Multi-Objective Optimization Methods". It contains a bulleted list of topics to be discussed in the module. The list includes: "These methods converted multi-objective optimization problem into single-objective optimization problem." followed by a sub-list of "Weighted-Sum method", " ϵ -constraint method", "Weighted metric methods", and "Benson's method". Below this, there are two more main bullet points: "Graphical illustration" and "Hand calculations". The slide footer shows the presenter's name "D. Sharma (dsharma@iitg.ac.in)", the word "Closure", and the slide number "10 / 13".

- These methods converted multi-objective optimization problem into single-objective optimization problem.
 - ▶ Weighted-Sum method
 - ▶ ϵ -constraint method
 - ▶ Weighted metric methods
 - ▶ Benson's method
- Graphical illustration
- Hand calculations

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In module 6, we focused on the introducing the topic of multi-objective optimization. In module 7, we discussed more of classical multi-objective optimization method. Now, since we know there are two approaches such as preference based and ideal multi-objective optimization approaches so, these classical multi-objective optimization methods, they follow the preference-based meaning that we convert our multi-objective problem into single objective.

So, there are various methods available, we discuss these methods one by one. In this module, we covered weighted-sum method, epsilon constraint method, weighted metric methods and Benson's method. For all of these methods, we discuss their advantages and limitations. We also showed graphical illustration on a two objective problem and we also perform hand calculations for weighted-sum and epsilon constraint method.

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Module 8: Multi-Objective EC Techniques

Elitist non-dominated sorting genetic algorithm (NSGA-II)

- NSGA-II on generalized framework
- Working principles of NSGA-II through an example
 - ▶ Crowded tournament selection operator
 - ▶ Non-dominated sorting and crowding distance
 - ▶ Survivor
- Simulations of NSGA-II and SPEA2 on ZDT and DTLZ problems
- Hypervolume indicator
- Comparison of NSGA-II and SPEA2 on DTLZ problems

Strength Pareto Evolutionary Algorithm (SPEA2)

- SPEA2 on generalized framework
- Working principles of SPEA2 through an example
 - ▶ Fine-grained fitness through strength and raw fitness
 - ▶ k -th nearest neighbor approach for diversity
 - ▶ Archive truncation method
 - ▶ Survivor
- Simulations of NSGA-II and SPEA2 on ZDT and DTLZ problems

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Afterwards, we came to the last module of this particular course that is multi-objective EC techniques. In this particular session, we discussed two main benchmark algorithm. We started our discussion with NSGA-II which is referred as non-dominated sorting genetic algorithm and two stands for part II.

As we can see NSGA-II we discussed based on the generalized framework. We understand the working principle of NSGA-II through an example. So, since the other operators we have discussed earlier so, our main focus was on the different features of NSGA-II such as crowded tournament selection operator, non-dominated sorting and crowding distance and the survivor.

Similarly, we discussed the another benchmark multi-objective evolutionary computation technique that is called SPEA2. In this SPEA2, we again fit SPEA2 our generalized framework. We understand SPEA2 through; through an example where the spatial feature such as fine-grained fitness through strength and raw fitness, k th nearest neighbor approach for diversity, archive truncation method and the survival stage of the SPEA2 were discussed.

Thereafter, we perform simulation of NSGA-II and SPA; SPEA2 on ZDT and DTLZ problems. So, these two problems are mathematical problems, and we know where is the Pareto optimal front for these problems. Thereafter, we discussed hypervolume indicator. This hypervolume indicator we used for performance assessment of multi-objective EC

techniques. We understood its definition; we perform some hand calculation, so that we can calculate hyper volume for a given set of non-dominated solutions.

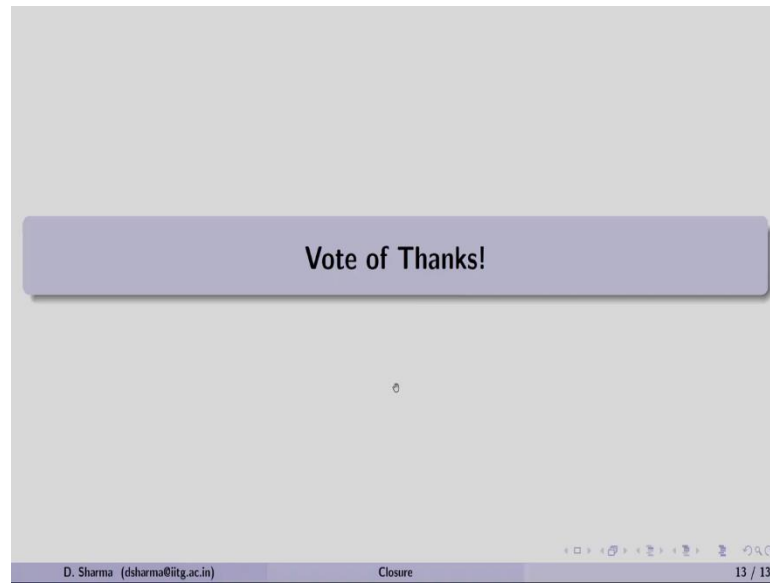
Further, we have shown the performance of NSGA-II and SPEA2 using this hyper volume indicator on the various sets of DTLZ problem from DTLZ 1 to DTLZ 4. Throughout this course, if we try to maintain the consistency in understanding each and every algorithm that we have covered for single and multi-objective optimization.

We used our generalized framework to understand every algorithm followed by we understand each and every algorithm by performing the hand calculation and the graphical illustration. Since we have to show how these algorithm are behaving or solving certain class of problems, we showed simulations. So, the same common framework, we have used to understand each and every algorithm in the course.

So, the outcome of this course what we can expect is when we are going through all of these EC techniques for single and multi-objective optimization, we would be trained to understand it these techniques, we can use these techniques for our purpose, we can even modify them, or we can come up with some new EC techniques based on the principles, theory and concept that we have covered.

Moreover, since we now get the fair idea of EC techniques, we can even use for our research purpose, we can use in our industries to solve various kinds of optimization problem.

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I would like to conclude this course with a vote of a thank to this initiative of the Government of India, the hosting institute, all the people involved in making the course, IIT Guwahati, the team behind organizing this course, hosting the course material and video editing, video uploading, assignment everything. I would also like to thank all the participants who have taken this course and constantly involved in understanding the courses, various lectures, solving the assignment.

With this note, I conclude this course on evolutionary computation for single and multi-objective optimization.

Thank you very much.