Evolutionary Computation for Single and Multi-Objective Optimization
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Lecture - 25
Performance Assessment of Multi-Objective EC Techniques

Welcome to the session on Performance Assessment of EC techniques.

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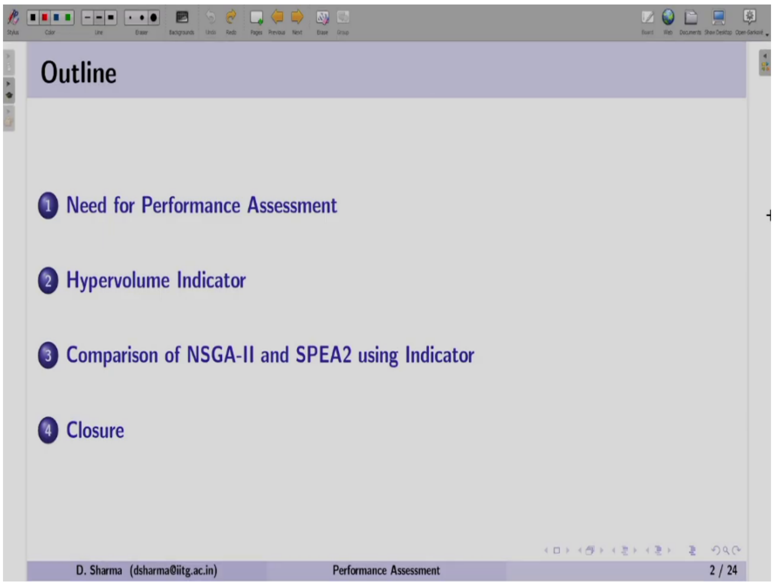

In this particular session, we will be looking for the need that why we need the performance assessment. Once we complete it, then we will be going through one of the indicators called hypervolume indicator. After performing some hand calculations, we will see the performance of NSGA-II and SPEA2 using this indicator. Thereafter, we will close this session.

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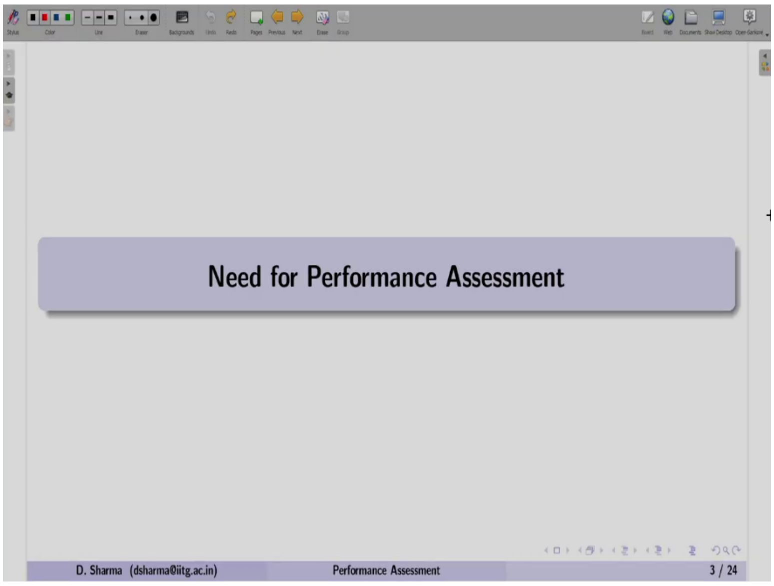

So, let us understand what is the need for the performance assessment.

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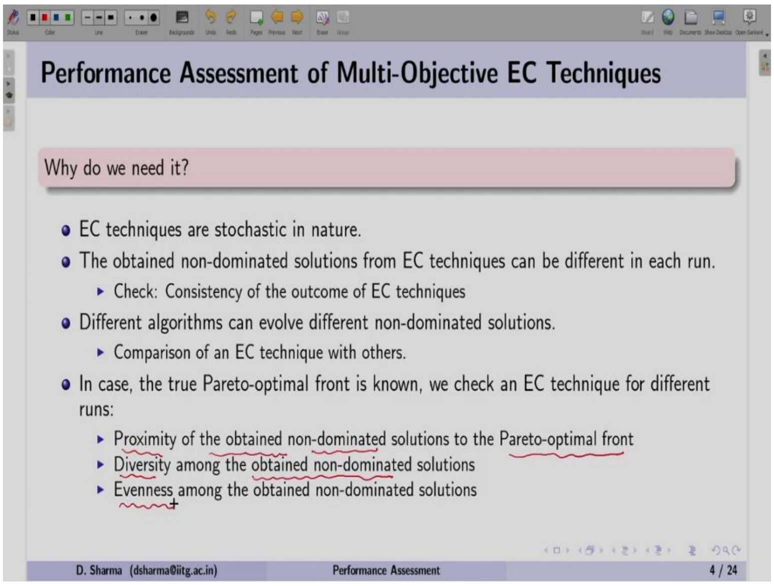

So, as we can see here that we have to understand the need of the performance assessment. So, we know that EC techniques, these are stochastic in nature. So, when we say these techniques are stochastic. So, we understand that we generate population randomly. The operators which we need for EC techniques, all of them are stochastic in nature.

So, when we are running EC techniques with the different random numbers, so these EC techniques are going to make or these EC techniques are going to generate different sets of solutions. In multi objective optimization, these EC techniques are going to generate different sets of non-dominated solutions. So, as we can understand the obtained non-dominated solutions, they can be different in each run. So, our objective is we want to see the consistency of the outcome of the EC techniques.

Why we are looking for the consistency? It is because that when we are running EC techniques for multiple times, sometimes it will give us a good set of non-dominated solutions and sometimes this EC technique can be prematurely converge to the local Pareto optimal front. So, in order to check their consistency that every time or most of the time, multi objective EC techniques are converging on the Pareto front or close to the Pareto front, we need performance assessment.

There are different EC techniques that are available in the literature. So, when these EC techniques are available and we and when we make a new technique, so that technique can be compared with the existing one. So, in order to make a comparison, so this kind of performance assessment is needed.

There are certain problems, where we know the Pareto optimal front. So, these problems are generally the mathematical multi objective optimization problems. So, in the previous sessions, we have gone through such problems such as ZDT problems, DTLZ problems and there are other problems as well.

Now, for such kind of a problems, when the Pareto front is known, so in that case, we can run our EC techniques and can look for various properties such as we can see the proximity of the obtained non-dominated solutions to the Pareto -optimal front, we can also see the diversity among the obtained non dominated solutions and we can also check the evenness. As per our earlier discussion, we know that there are two goals in multi objective optimization; first is the convergence, another is called diversity.

So, the proximity to the non-dominated or the known Pareto front means that how this non-dominated solution, how much they are closer to the known Pareto front. Second is what is the diversity among the solutions which are conversed to the Pareto front? Even though, there could be a good diversity, but if evenness is not there, then we may not get the actual picture of the Pareto optimal front.

So, in this case, when the Pareto optimal front is known for certain problems, we can use EC techniques for generating the solutions and using performance assessment with the help of indicators. We can look whether these solutions are close to the Pareto front; how diverse they are and what is the evenness among the solutions obtained by the EC techniques. So, now, we will be discussing the indicator which is called hypervolume indicator.

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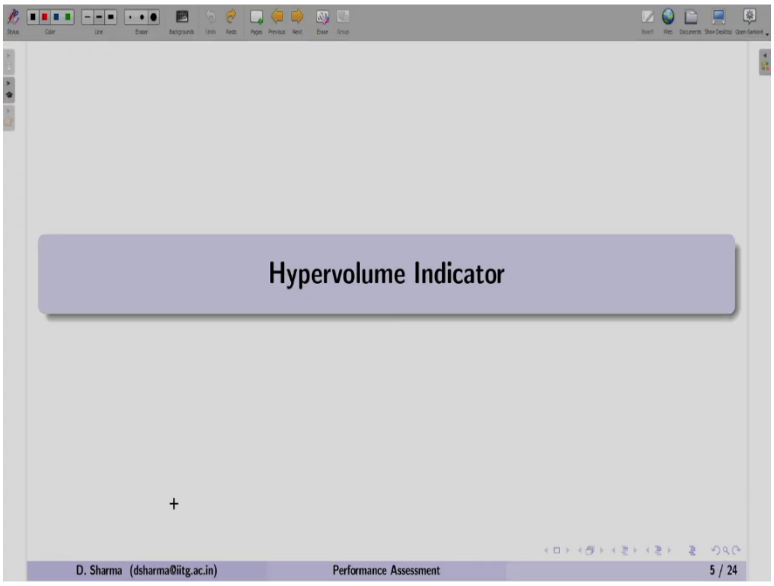

In the literature, there are various kinds of indicators available. Hypervolume is one of the indicators that has been used in various literature, while comparing the new algorithm with the existing algorithm. So, let us understand what is hypervolume indicator.

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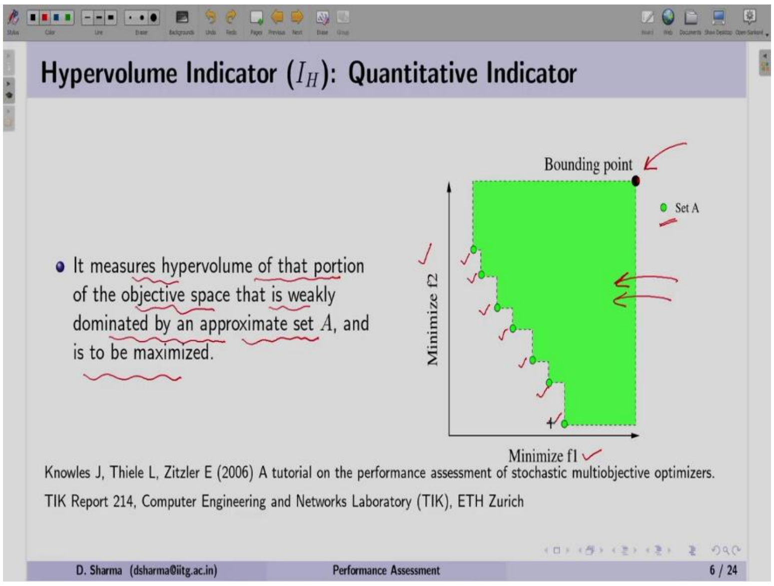

Now, this indicator measures volume of that portion of objective space that is weakly dominated by an approximate set A and is to be maximized. So, let us look at the figure on the right hand side. So, here these green dots, these are the set of non dominated solution say obtained by the EC technique.

So, in this case, what we are doing is we are considering one point called the bounding point and in this bounding point, we have chosen this bounding point, it is only because this point is dominated by all the points available in this set A. So, as the definition says that we will be finding that portion of the objective space that is weakly dominated by an approximate set.

o, this green space what you can see here, this is the space we are talking about and we want to maximize. So, as we know that this particular problem is the minimization for both the objective. When we are going to maximize this green region, this means that the solutions will be closer to the known Pareto -optimal front.

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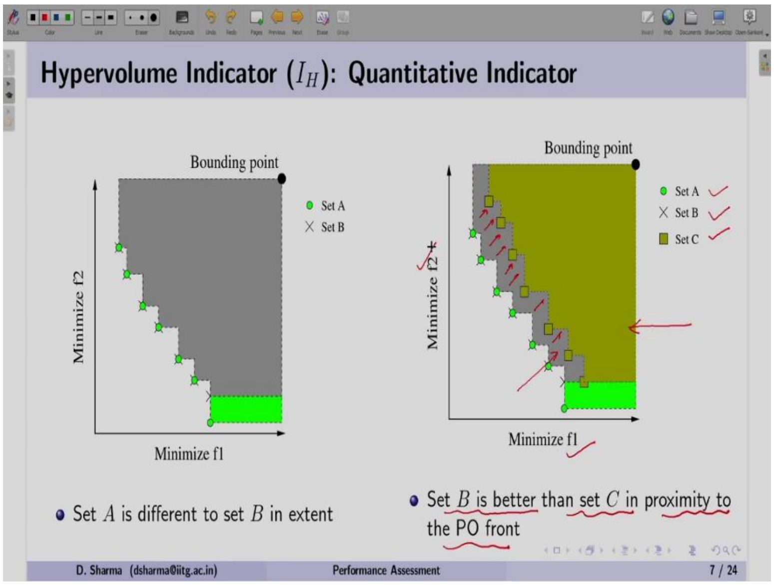

Let us compare the different sets of non dominated solution now. As we can see in the figure, we can see we have two different sets; set A and set B. Now, looking into the figure, we have the green dots that will be making the area green, what we have seen in the previous slide and the set B is shown by this gray area.

Now, if we want to compare the outcome in terms of set A and set B, what this hypervolume indicator suggest that set A is different to set B in extent. What we mean by extent? Now, you can see that certain portion of the green region is seen and because of this last solution, we can see that for the same bounding point, the green has or the set A has more better extent than set B.

Let us consider the another case. Now, in this case the set A and set B remains the remain the same and set C we have taken. Now, in this case, what we can see that the grey region is again by set B and this another region which is by the set C.

If we are going to compare set B and set C, we can see there is a gap and because of this gap, we can say that the set B is better than set C in the proximity to the Pareto front. It is only because the problem which we are solving is minimizing f1 and f2.

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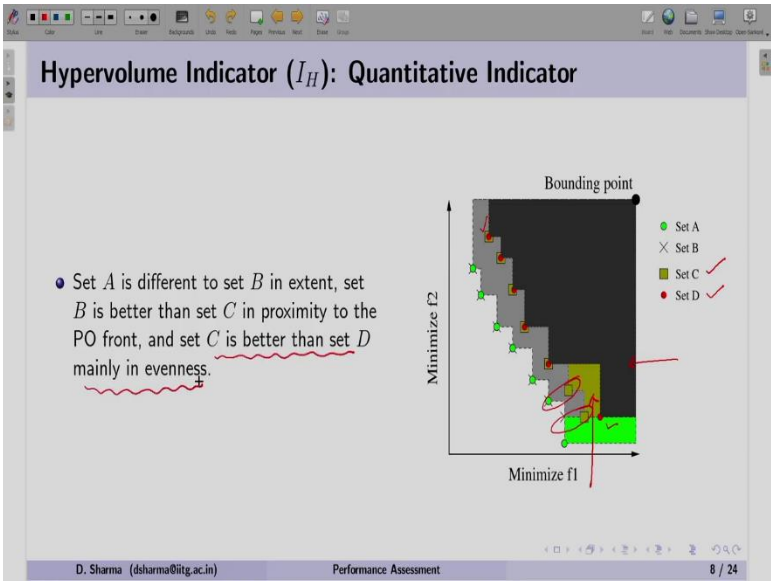

If we consider the another set called D and when we compare D with set C. So, here with the color coding, we can see that set C is represented by the color and the set D is represented by the black color. Now, if we look at the extremes of these two solutions that are same.

However, because of the two solutions of the set C, we can say the set C is better than set D mainly in the evenness. From this figure, what we can understand that when we are running multi objective EC techniques different times, so we may get different sets of non dominated solution.

So, set A, B, C and D all of them are different sets that we have obtained. Now, when we are going to compare them, some set is better than other set in some contest. It can be extent; it can be the closeness or we can say the proximity and someone something can be based on the evenness. Now, since these EC techniques can generate different sets, so we need to quantify it.

So, graphically, we can see that because of the different sets of non dominated solutions, we have to quantify the performance of the EC technique. Let us understand the hypervolume with the help of some hand calculation.

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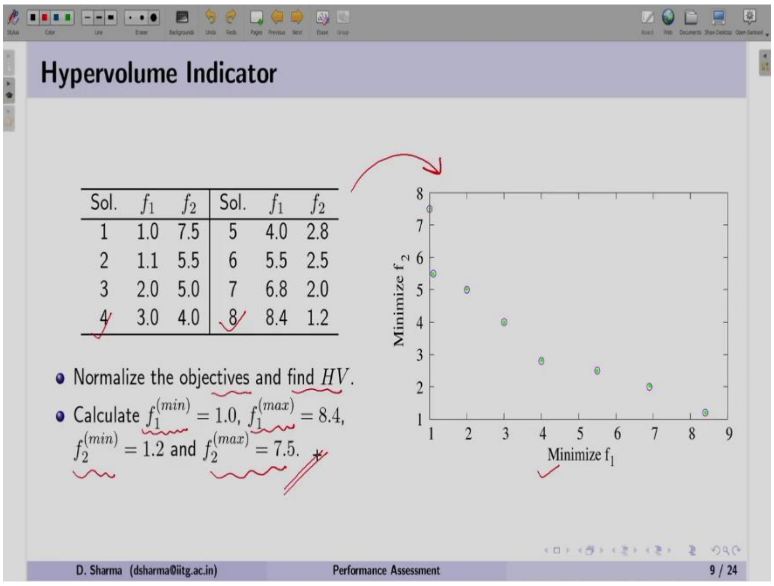

$$f_1^{(min)} = 1.0, \qquad f_1^{(max)} = 8.4, \qquad f_2^{(min)} = 1.2 \text{ and } f_2^{(max)} = 7.5$$

Now, in this case, we have taken a two objective case and both the objectives are of minimization type and we have taken 4 plus 4, 8 solutions. In order to calculate the hypervolume, we will first normalize the objective space and then, find the hypervolume. So, looking at these 8 solutions, we can find what is the minimum f 1, maximum f 1, minimum f 2 and maximum f 2.

So, on the right hand side, the figure shows the solutions or we can say the non dominated solutions for the given problem. Once we have identified this f minimum and f maximum in both the objectives, we can normalize these values.

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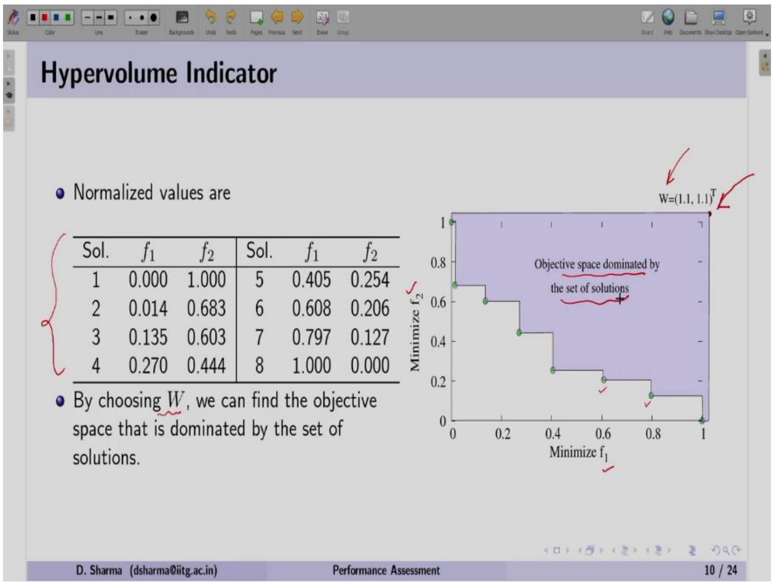

So, these solutions are the same; only after normalization, we are getting these values. Thereafter, we calculate the hypervolume. So, as per our discussion, we are looking for a bounding point say W that should be dominated by the given set. Since all the members are normalized between 0 to 1. So, what we can understand from the figure on the right hand side that we have chosen this W as 1.1 and 1.1.

It is only because all the solutions are lying between 0 to 1 in f 1 as well as in f 2. So, all the solution as shown in the green color. So, these solutions are dominating the point W. So, with respect to W, we will be calculating the objective space that is dominated by the given set of solutions.

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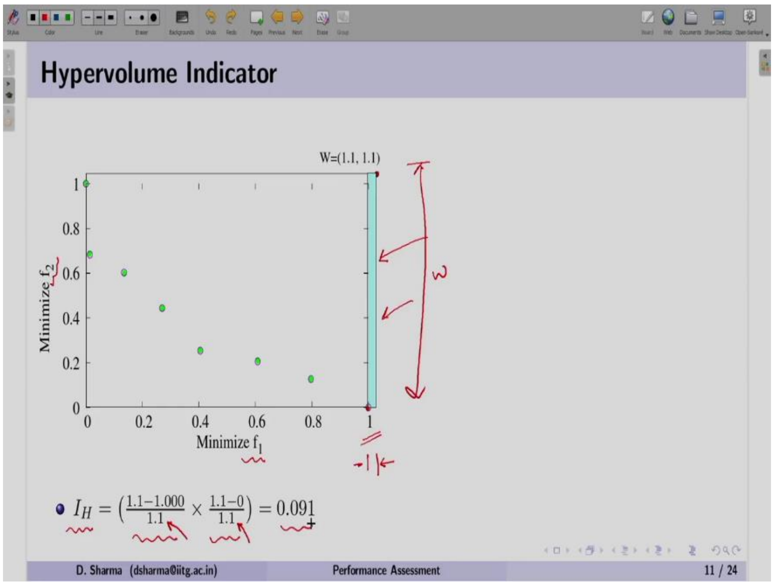

$$I_H = \left(\frac{(1.1 - 1.000)}{(1.1)} \times \frac{(1.1 - 0)}{(1.1)}\right) = 0.091$$

In order to calculate the hypervolume, the same figure we are drawing in term in terms of these rectangles. Now, we have to understand that since we are solving two objective problem, so it is easy to make the hyperspace in terms of the rectangle for a given problem. So, let us identify what is the area of this rectangle.

So, we know that this particular member, this has a this has a coordinate 1 and a 0 and the W coordinates are given here. In order to calculate the hypervolume, which we can see at the bottom.

So, this is the length of the rectangle and this is the width of the rectangle. So, 1.1 minus 1 is going to give me the length and 1.1 minus 0, it is going to give us the width of this rectangle. Now, we are dividing with 1.1 and 1.1 because we are just normalizing these values. The outcome will become 0.091.

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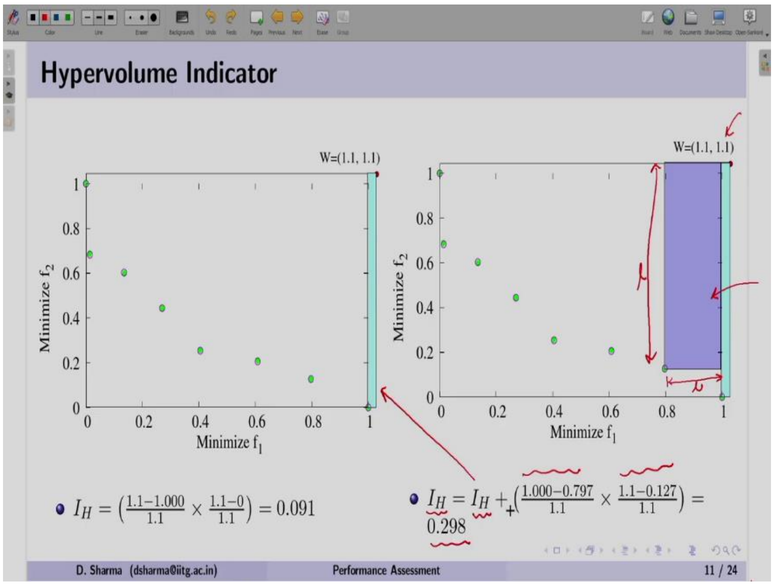

$$I_H = I_H + \left(\frac{(1.000 - 0.797)}{1.1} \times \frac{(1.1 - 0.127)}{(1.1)}\right) = 0.298$$

Similarly, we can have another rectangle which I have shown in the different color coding. In this case, we can see the length will become the difference between the f 1 value and the length of the rectangle will become difference in the f 2 value and 1.1.

So, here the hypervolume indicator is equals to the previous value which we have calculated earlier and this is the difference in the f 1 value and multiplied by 1.1 minus f 2 value of the given point. So, this will give me 0.298. So, this is the summation of those two rectangular areas.

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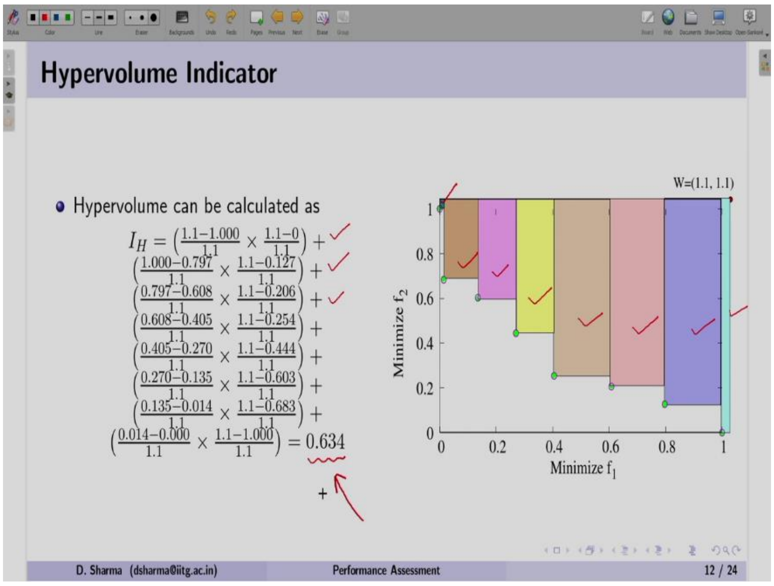

$$
\begin{aligned}
I_H = &\left(\frac{1.1-1.000}{1.1} \times \frac{1.1-0}{1.1}\right) + \left(\frac{1.000-0.797}{1.1} \times \frac{1.1-0.127}{1.1}\right) \\
&+ \left(\frac{0.797-0.608}{1.1} \times \frac{1.1-0.206}{1.1}\right) \\
&+ \left(\frac{0.608-0.405}{1.1} \times \frac{1.1-0.254}{1.1}\right) \\
&+ \left(\frac{0.405-0.270}{1.1} \times \frac{1.1-0.444}{1.1}\right) \\
&+ \left(\frac{0.270-0.135}{1.1} \times \frac{1.1-0.603}{1.1}\right) \\
&+ \left(\frac{0.135-0.014}{1.1} \times \frac{1.1-0.683}{1.1}\right) \\
&+ \left(\frac{0.014-0.000}{1.1} \times \frac{1.1-1.000}{1.1}\right) = 0.634
\end{aligned}
$$

Similarly, if we are going to make multiple rectangles based on the points which we get it after running the EC techniques. So, what we can do is we can sum them one by one and then finally, the total sum will become 0.634. So, we can say that for a given set of non-dominated solution, the hypervolume is 0.634.

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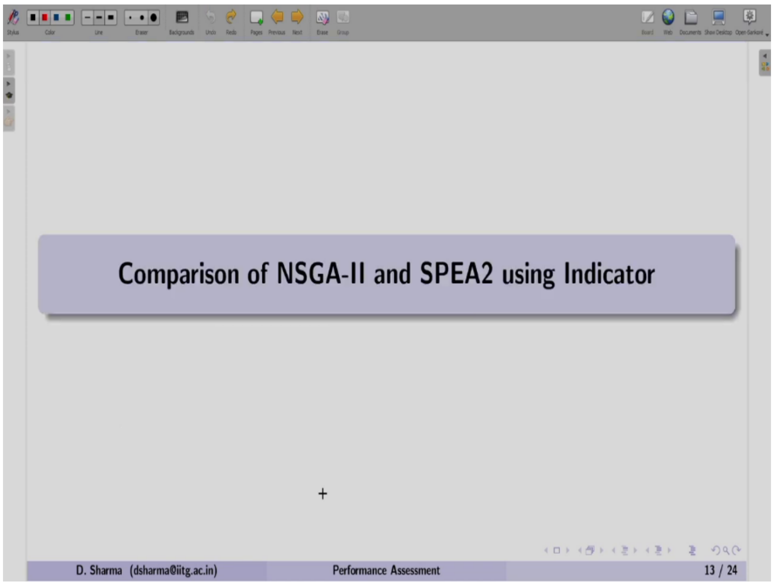

With this, basic understanding of hypervolume indicator, let us compare the two EC techniques which we have gone through in the previous sessions. Those techniques were NSGA-II and SPEA2. DTLZ problems are the mathematical multi objective optimization problem. These problems, for these problems we know where is the Pareto-front.

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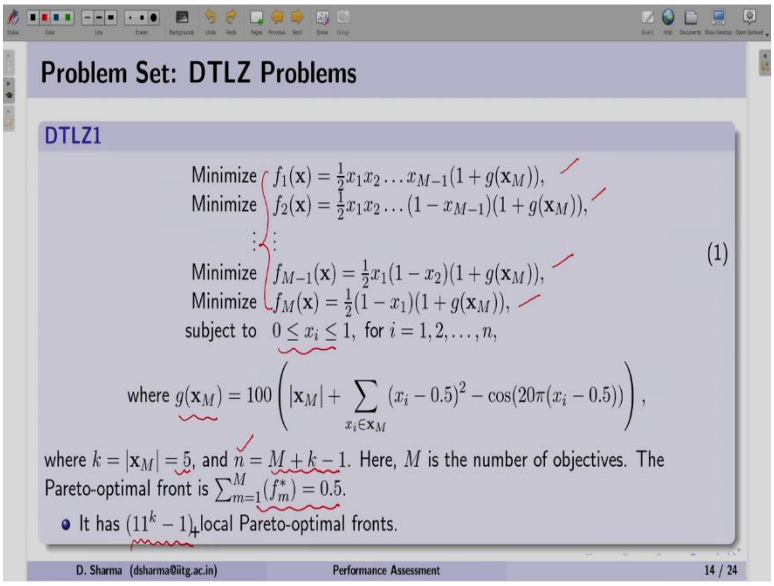

$$Minimize\ \ f_1(x) = \frac{1}{2}x_1x_2 \ldots x_{M-1}\big(1+g(x_M)\big),$$

$$Minimize \ \ f_2(x) = \frac{1}{2} x_1 x_2 \ldots \ (1 - x_{M-1})\big(1 + g(x_M)\big),$$

$$\vdots \ \vdots$$

$$Minimize \ \ f_{M-1}(x) = \frac{1}{2} x_1 (1 - x_2)\big(1 + g(x_M)\big),$$

$$Minimize \ \ f_M(x) = \frac{1}{2}(1 - x_1)\big(1 + g(x_M)\big),$$

$$subject\ to \quad 0 \leq \ x_i \leq \ 1, \qquad for \ \ i = 1,2, \ldots, n,$$

$$where\ g(x_M) \ = \ 100 \left(|x_M| \ + \sum_{x_i \in x_M} (x_i - 0.5)^2 \ - \ cos\big(20\pi \ (x_i \ - \ 0.5)\big) \right),$$

$$where\ k = |x_M| = 5,$$

$$n \ \ = M + k - 1. \ \text{Here,} \, M \text{ is the number of objectives. The Pareto}$$
$$- \text{optimal front is} \sum_{m=1}^{M} (f_m^*) \ = 0.5$$

These DTLZ problem, we remember that the DTLZ problems are scalable problems and their format are given here. The variable bound for DTLZ 1; the function g is given; value of a k is 5; number of variable is M plus k minus 1 and for this given problem, we know that the Pareto surface will be the plane intersecting the axis at 0.5. This particular problem has 11 to the power k minus 1 local Pareto-optimal front.

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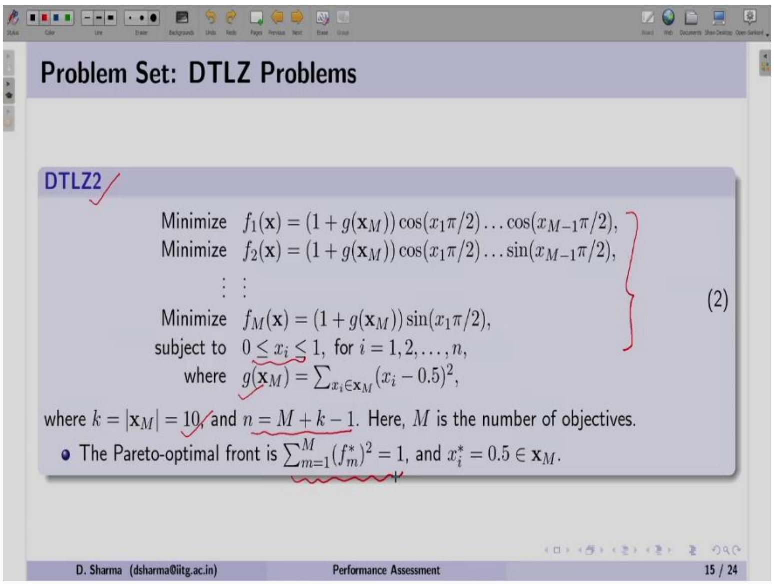

$$Minimize\ \ f_1(x) = \big(1+g(x_M)\big)\cos\left(x_1\frac{\pi}{2}\right)\ldots\ \cos\left(x_{M-1}\frac{\pi}{2}\right),$$

$$Minimize\ \ f_2(x) = \big(1+g(x_M)\big)\cos\left(x_1\frac{\pi}{2}\right)\ldots\ \sin\left(x_{M-1}\frac{\pi}{2}\right),$$

$$\vdots\ \vdots$$

$$Minimize\ \ f_M(x) = \big(1+g(x_M)\big)\sin\left(x_1\frac{\pi}{2}\right),$$

$$subject\ to\ \ 0 \le x_i \le 1,\ \ for\ i = 1,2,\ldots,n,$$

$$where\ \ g(x_M) = \sum_{x_i\in x_M}(x_i-0.5)^2\ ,$$

$$k = |x_M| = 10,\qquad n = M+k-1.\ Here, M\ is\ the\ number\ of\ objectives.$$

$$The\ Pareto\ optimal\ front\ is\ \sum_{m=1}^{M}(f_m^*)^2 = 1\,,\qquad x_i^* = 0.5 \in x_M$$

The another problem which we have considered for comparison is the DTLZ2 problem. Again, we can see these scalable objectives with the bound on x i, the function g, the value of a k is now 10, number of variable remains the same. In this case the Pareto front is the quarter of the sphere.

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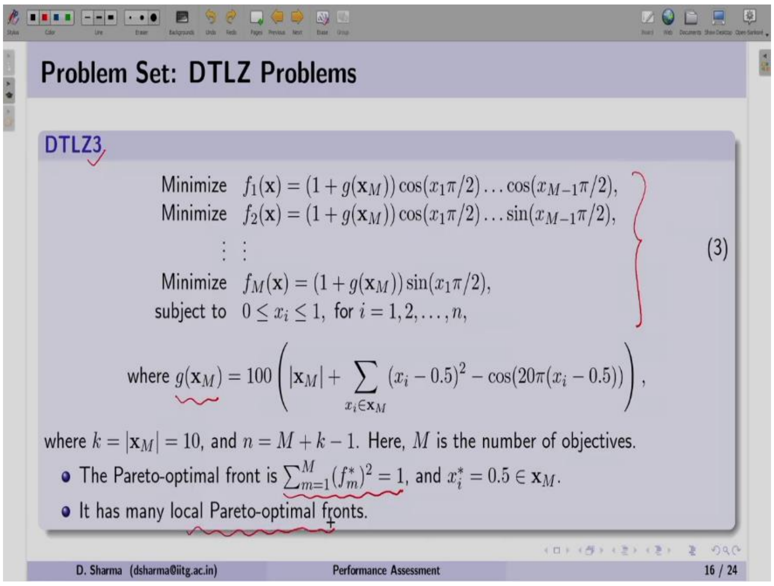

$$Minimize \;\; f_1(x) = \left(1 + g(x_M)\right)cos\left(x_1\frac{\pi}{2}\right) \ldots \; cos\left(x_{M-1}\frac{\pi}{2}\right),$$

$$Minimize \;\; f_2(x) \; = \; (1 + g(x)_M)) \; cos(x_1\frac{\pi}{2}) \;\; \ldots \; sin(x_{M-1}\frac{\pi}{2}),$$

$$\vdots \, \vdots$$

$$Minimize \;\; f_M(x) = \left(1 + g(x_M)\right)sin\left(x_1\frac{\pi}{2}\right),$$

$$subject\ to \quad 0 \le \; x_i \le \; 1, \qquad for\ i = 1,2, \ldots, n,$$

$$where \;\; g(x_M) \; = \; 100 \left(|x_M| \; + \sum_{x_i \in \, x_M} (x_i - 0.5)^2 \; - \; cos\left(20\pi \; (x_i - \; 0.5)\right) \right),$$

$$k = |x_M| = 10, \qquad n = M + k - 1. \; Here, M\ is\ the\ number\ of\ objectives.$$

$$The\ Pareto\ optimal\ front\ is \sum_{m=1}^{M} (f_m^*)^2 = 1 \quad and\ x_i^* \; = \; 0.5 \in x_M.$$

The third problem which we have considered is the DTLZ 3 problem. The formulation is given in equation number 3. The function g is given to us. This particular problem has the same Pareto front as DTLZ 2 which is quarter of the sphere that is in the first quadrant and this particular problem has many local Pareto fronts.

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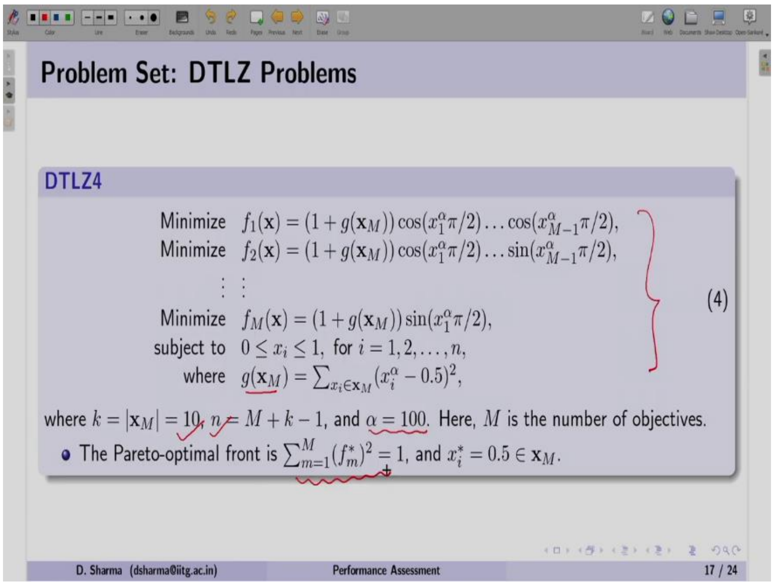

$$Minimize\ \ f_1(x) \ = \ \left(1 + g(x_M)\right) cos\left(x_1^\alpha \frac{\pi}{2}\right) \ldots \ cos\left(x_{M-1}^\alpha \frac{\pi}{2}\right),$$

$$Minimize\ \ f_2(x) = \ (1 + g(x_M))cos\left(\frac{x_1^\alpha \pi}{2}\right) \ldots \ sin\left(x_{M-1}^\alpha \frac{\pi}{2}\right),$$

$$\vdots \ \vdots$$

$$Minimize\ \ f_M(x) = \ (1 + g(x_M))sin\left(x_1^\alpha \frac{\pi}{2}\right),$$

$$subject\ to \ \ 0 \le \ x_i \le \ 1, \qquad for\ i = 1,2, \ldots, n,$$

$$where \ \ \ g(x_M) \ = \sum_{x_i \in x_M} (x_i^\alpha - 0.5)^2$$

$k = |x_M| = 10,$

$$n = M + k - 1, and\ \alpha = 100.\ \ Here, M\ is\ the\ number\ of\ objectives.$$

$$The\ Pareto\ optimal\ front\ is \sum_{m=1}^{M} (f_m^*)^2 = 1 \ ,and\ x_i^* \ = \ 0.5 \in \ x_M.$$

The last problem on which we will be comparing our techniques is the DTLZ 4 problem. We can see the mathematical equations in equation number 4. The function g is given to us; k is 10; n is the number of variable. Here alpha is taken which is a big value 100. The Pareto front for the given DTLZ 4 problem is again the quarter of the sphere in the first quadrant.

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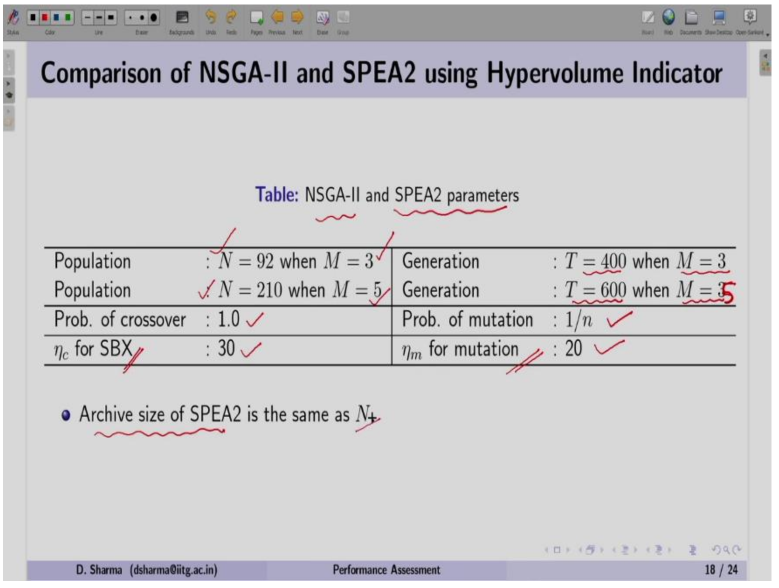

In order to compare NSGA-II and SPEA2, we have set the common parameters. So, this population size is 92 for 3 objective problem and population size is 210 for 5 objective problem. Similarly, the number of generation is decided that T is equal to 400, when the objective is number of objectives is 3; T equals to 600, when the number of objective is 5. So, this is 5.

Now, probability of crossover is 1. Probability of mutation depends on the number of variables; eta c is 30 and eta m is 20. So, these are the SBX is the crossover operator and polynomial mutation is used. Since we use archive in SPEA2, the size of archive is the same as the population size N.

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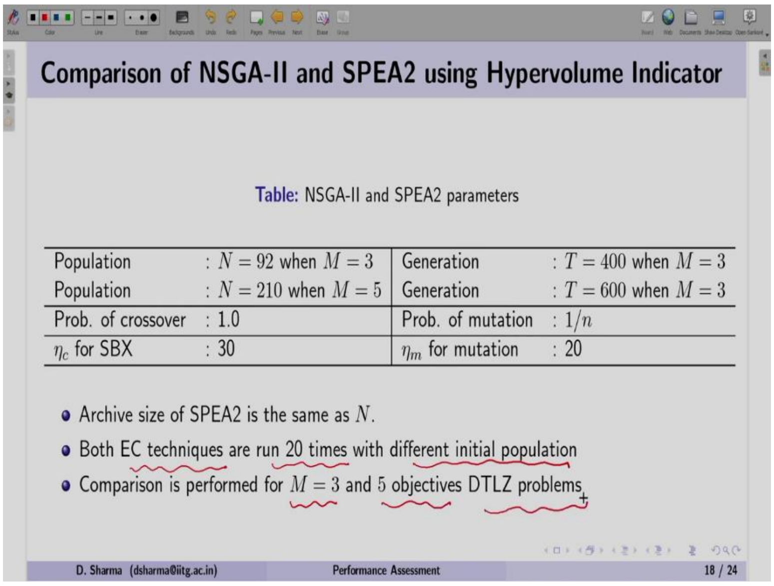

Both the EC techniques are run 20 times with different initial population and the comparison as we have mentioned is performed for 3 and 5 objective DTLZ problems. Here, it is important to note that we are running our EC techniques multiple times that is 20. It is only because since these are stochastic in nature.

When we are starting with different initial population, 20 times when we are running, we should get different set of non dominated solution obtained by these EC techniques. Let us see the performance of these techniques on DTLZ problem one by one. We will take DTLZ 1 first.

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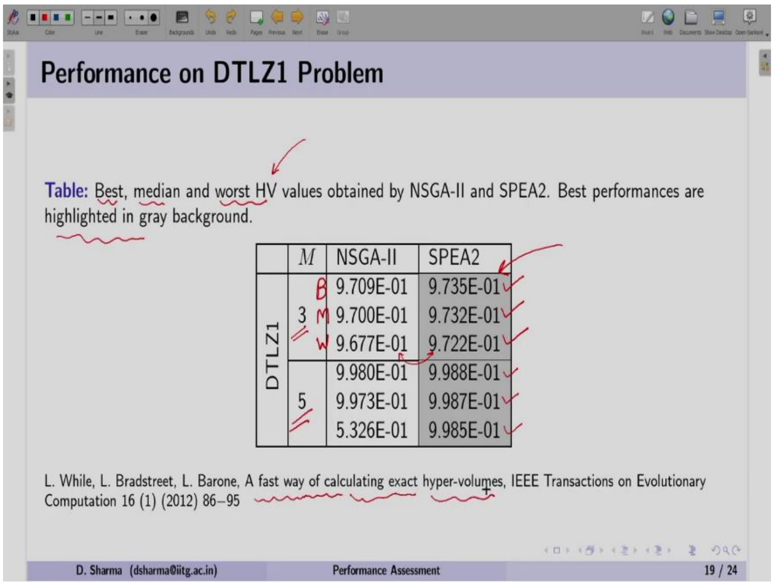

Here in this particular table, we can see the best median and worst HV; HV stands for Hyper-Volume obtained by NSGA-II SPEA2 and best performance are highlighted in a grey background. So, when we are saying the first value. So, this is the best value; this is the median and this is the worst value. So, what we mean by that? When we are running any of the EC techniques, then we get a one non dominated set.

For the given non dominated set, we can calculate what is the hypervolume. Similarly, we will be running another time. So, since we are running 20 times, so we can get 20 hypervolume values. So, among these 20 values of hypervolume, we find which one is the best, which one is the worst or which one is the median. These statistical values, we are showing in this particular table.

So, let us look into the table now. In this table, we can see that the performance of SPEA2 is better than the performance of NSGA-II for 3 objective as well as for the 5 objectives. However, if we are going to compare them, there is not much difference; but since this value is more, so SPEA2 is showing better value. Now, what we can understand that since as per the definition of the hypervolume indicator, we want to maximize that particular area or the hyper volume.

So, the value which is going to be close to the one is the better value. So, that we can understand from this particular table. So, here these hypervolume values, we have calculated by following the paper that is a fast way of calculating the exact hyper volume.

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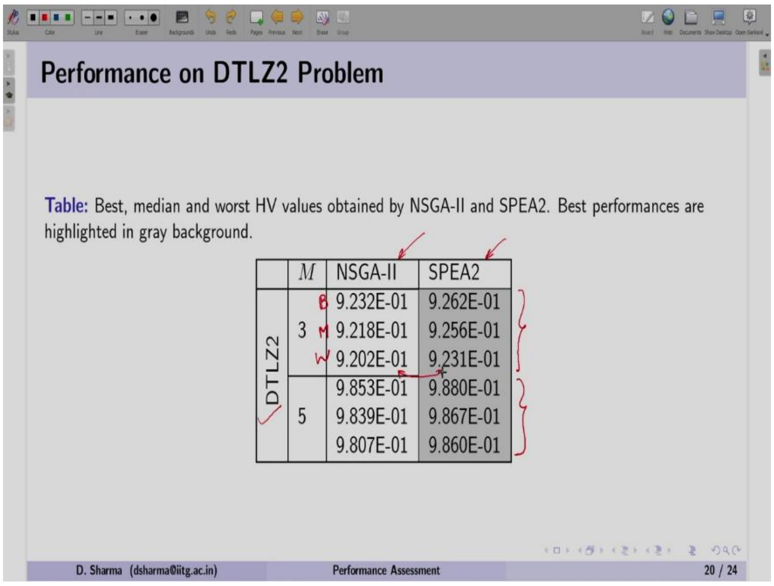

The second problem which we have considered is the DTLZ 2 problem. Again, the best value, the median value and the worst value are given. Now, looking at the performance of NSGA-II and SPEA2, the shaded region suggest that in both the objectives, again SPEA2 is generating better set of non dominated solution as compared to the NSGA-II. However, if we look at what is the difference. So, the difference you can find only after the third decimal places.

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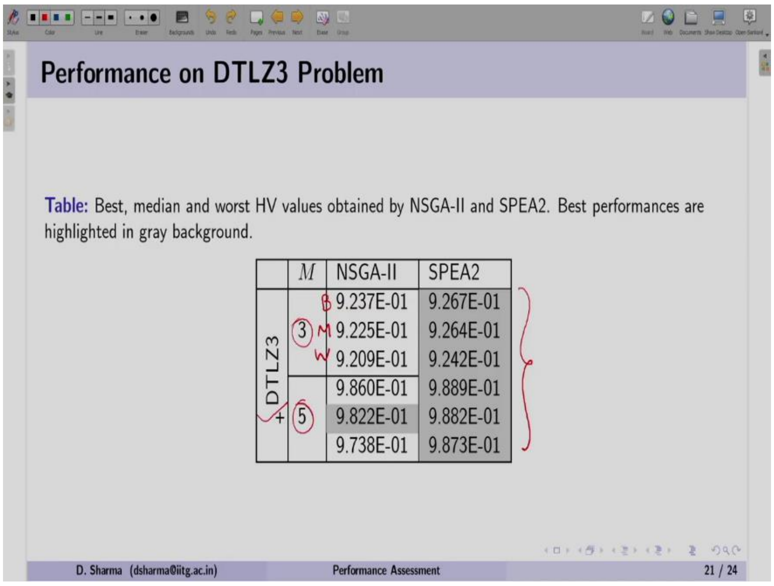

The next problem which we have considered is DTLZ 3. In this case the best mediam median and worst values are given. So, here we can see that again SPEA 2 is showing better hypervolume indicator value for t3hree as well as 5 number of objectives.

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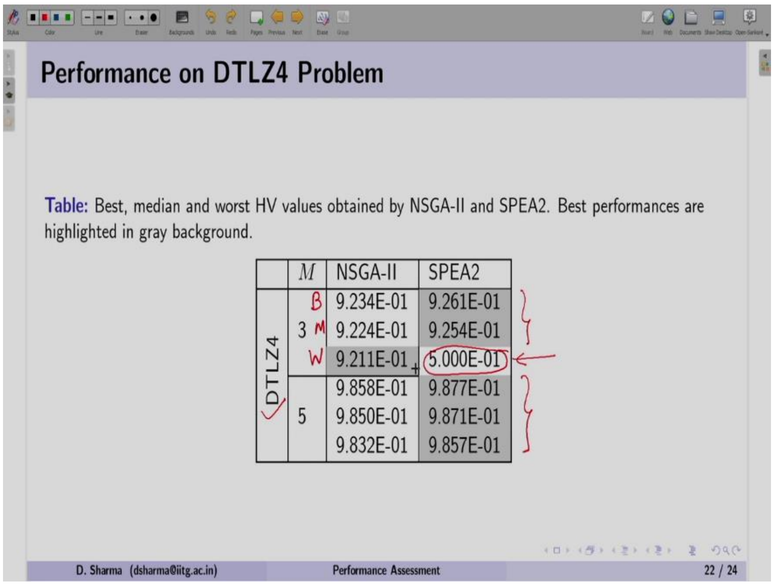

The last table on DTLZ 4, we can see that SPEA2 is showing better results over NSGA-II, except just one worst value. So, we remember that this is best mediam median and worst. So, just the worst value of SPEA2 is poor as compared to NSGA-II; otherwise SPEA 2 generated the better set of non dominated solution for the given problem.

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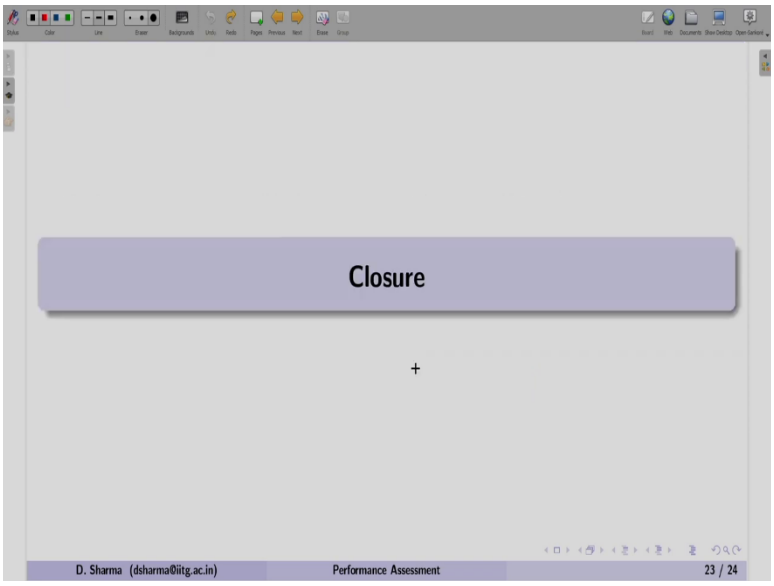

So, with this understanding on performance indicator, we have come to the closer of this particular session.

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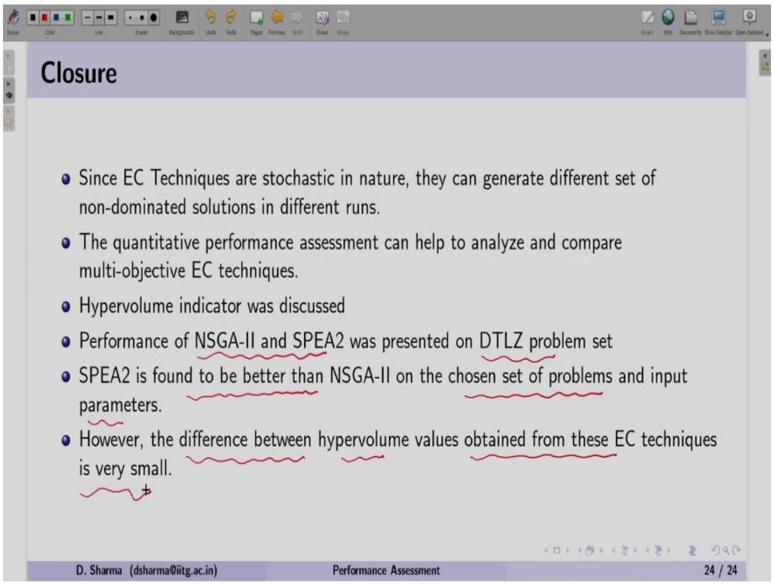

In this session, we understood that since EC techniques are stochastic in nature, we have to assess the performance be using some indicator. For the same, what we are looking is the quantitative comparison or the performance assessment that we can achieve using various indicators.

In this session, we focus on hypervolume indicator that can give some value and based on that, we can compare or assess the performance of EC techniques. On this hypervolume indicator, we see the comparison on NSGA-II and SPEA2 on the DTLZ problem set.

What we found that SPEA2 is better than NSGA-II on the chosen set of problem and the input parameters. However, the difference between the hypervolume values obtained from these EC techniques is very small and that we have understood that the difference can be seen at the third decimal place. So, with this understanding on hypervolume indicator, I conclude this session.

Thank you.