Evolutionary Computation for Single and Multi-Objective Optimization
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Lecture - 18
Introduction to Multi-Objective Optimization

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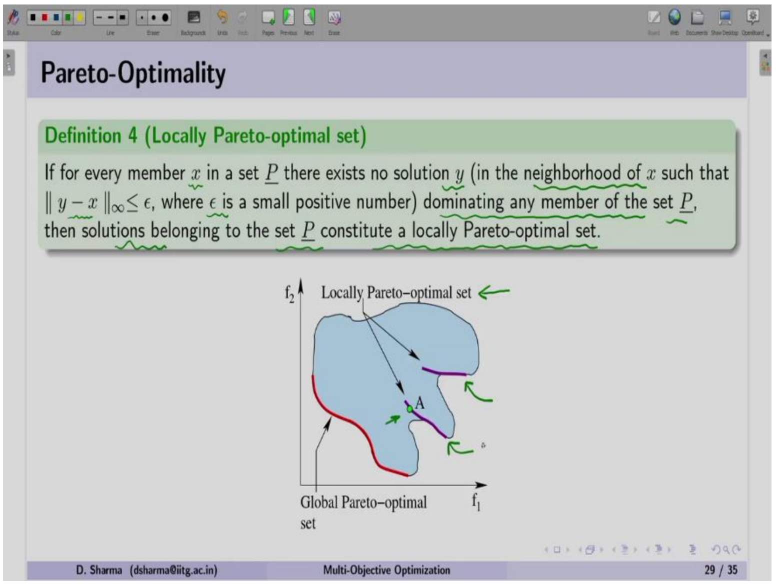

We have discussed the global Pareto set, now we will see the local Pareto set. So, as we can see here, the definition 4 says that if for any member x in a set P there exists no solution. So, this particular solution we are assuming in the neighborhood of the x such that the gap so, y minus x norm in finite is smaller than equal to epsilon value.

This epsilon a we are taking a small value small positive number. So, if there is no solution which is dominating any member of the set P, then solutions belong to the set P constitute a locally Pareto optimal set. If we look into the figure here as we can see we have two objective, we want to minimize both the objective. So, let us take the local Pareto optimal set.

So, if we take a solution A. So, if we are going to perturb this particular solution in the neighborhood so, then there is no solution which will be dominating the A. Similarly, the other solutions which will be lying on this particular set. Same thing can happen with the another local Pareto optimal set. Since we are comparing in the neighborhood only so, therefore, it is called as local Pareto optimal set.

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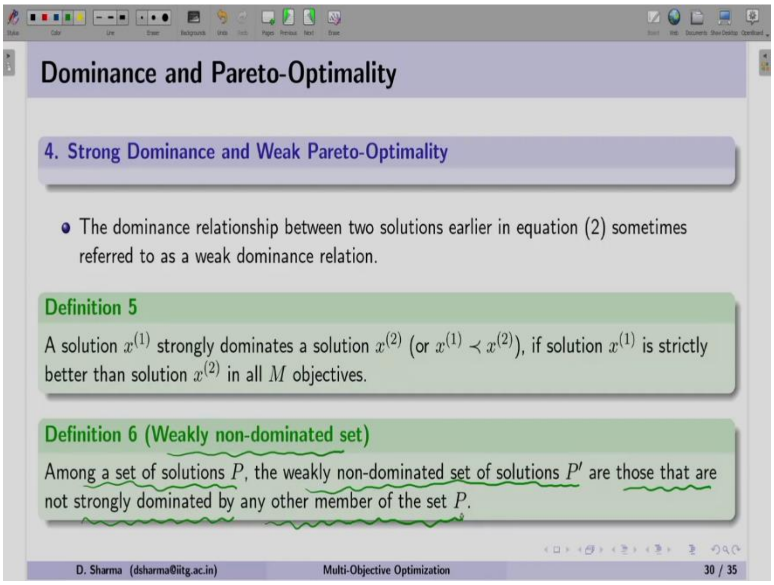

Now, we will discuss the strong dominance and weak Pareto optimality. It is only because the dominance relation which we have come across that is a that is still a weak why because, the notation which we have used that has something similar to smaller than or equal to.

So, if we look into the strong dominance relationship so, as we can see here, the dominance relationship between any solutions earlier that we have found in equation number 2. So, equation number 2 is the relationship for the concept where we use concept of dominance that we consider as a weak dominance. So, the definition 5 suggests that solution x 1 strongly dominates solution x 2 meaning that as you can see we are using this notation and earlier with that we have a equality as well.

So, here it is a strong domination. So, x 1 is strongly dominating x 2 if solution x 1 is strictly better than solution x 2 in all objective. So, that suggests that if I am comparing two solutions x 1 and x 2 and at the time when we are comparing the f 1 objective and f 2 objective. So, in this case the f 1 value as well as f 2 values; both the objectives for x 1 solution are better than the x 2 solution. In this case we can see the that x 1 is strongly dominating x 2.

Now, we have weakly non dominated set. Suppose among a set of solution P, the weakly non dominated set solutions P bar P prime are those that are not strongly dominated by any member of the set P. So, meaning that when we are using the concept of a dominance and when we are trying to compare each and every solution; in this case since the relationship the dominance relationship which we have understood earlier.

If we are using it, then there are some set of a solutions that when we are following the definition of the concept of a dominance, they are not weakly dominated by the other members of the P.

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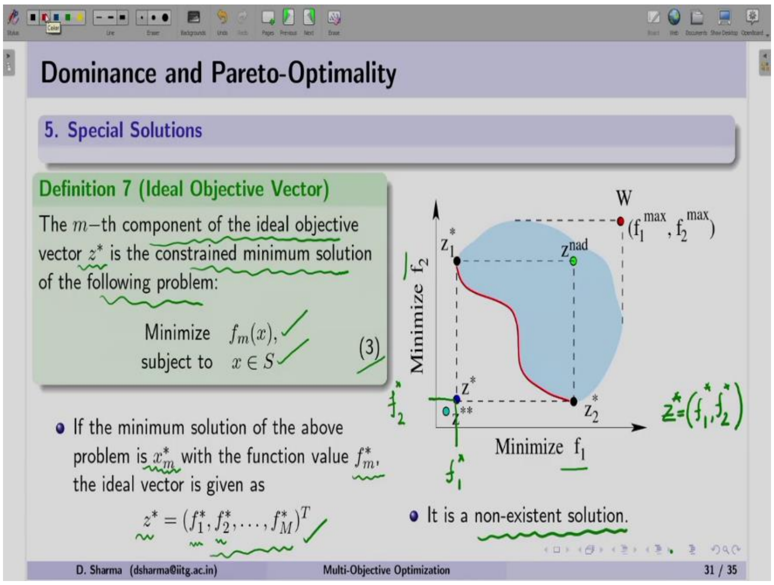

$$Minimize\ f_{m(x)},$$

$$subject\ to\quad x \in\ \ S$$

$$z^*\ =\ (f_1^*, f_2^*, \ldots, f_M^*)^T$$

Under the dominance and Pareto optimality, now let us discuss these spatial solutions. We will start our discussion with ideal objective vector. So, let us see what is the definition. The m-th component of the ideal objective vector. So, we are saying we are representing it as z star is the constraint minimization minimum solution of the following problem.

Now, here suppose we are minimizing one of the objectives subjected to the various constraints so, the solution which we are going to get it that is that will construct the ideal vector. So, suppose when we are minimizing the above problem, the solution is x m star and its function value is f m star.

So, what is ideal vector? So, z star is represented as. So, f 1 star means this is the minimum function value when we are minimizing equation number 3 with respect to f 1. Similarly, f 2 when we are minimizing again the equation 3 with respect to f 2 and so on.

So, the minimum objective function value when we are independently minimizing f 1, f 2 then up to f M, the vector which we are going to get in terms of their objective function, it is called the ideal vector. Let us look at the figure on the right hand side. Here we are minimizing f 1 and minimizing f 2.

In this case if suppose we are going to minimize say f 1, if we minimize it so, this is the point which we are going to get it. So, let us take this is as a f 1 star. Similarly when we are minimizing f 2 independently, we are going to get the another point that is f 2 star as per our definition.

So, in this case z star which is the ideal point vector will become f 1 star and f 2 star. So, that is this ideal objective vector for the given problem. Here it is important to note that this particular vector is non existence; why because as we have understood when we discussed about the multi objective optimization properties so, one of the properties that the objective should be conflicting in nature.

Now, since when we are minimizing say f 1 independently and then when we are minimizing f 2 independently, we are going to get a 1 point. Since f 1 and f 2 are conflicting in nature what we can understand is if we are going to minimize f 1, then at the same time the value of f 2 will be more.

So, we have to compromise in the f 2 value and vice versa when we are minimizing f 2, then we have to compromise with the f 1 and that result in a set of Pareto optimal solution. If we minimize both the objectives and these objectives are not conflicting in nature so, as we remember the solutions will converge to a single solution.

Here we have taken an example of conflicting objective as we have seen in the figure. So, this means that if I am independently minimizing f 1 and f 2, the resultant vector which I get in terms of f 1 star and f 2 star. So, that particular value we cannot achieve. So, that is why it is called nonexistent vector.

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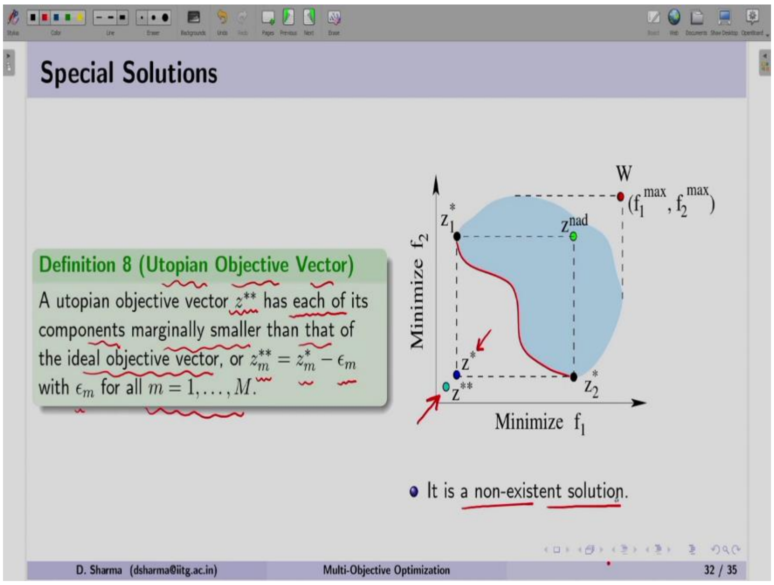

Now, let us move to the another solution that is called Utopian. So, as we can see in definition number 8 utopian objective vector this particular vector is represented as a z star star. And this vector has each of its component marginally smaller than that of ideal objective vector that is z m star star equals to z m star minus epsilon m with epsilon for all the objective.

When we are presenting this particular definition, we are assuming that we are minimizing all the objectives. If we look into the figure on the right hand side so, here this figure so, this z star we have already identified. Now, when we are going to subtract epsilon in f 1 and epsilon in f 2, we are going to get a point z star star that is an utopian objective vector. Again this particular vector is a nonexistent solution for a multi objective optimization.

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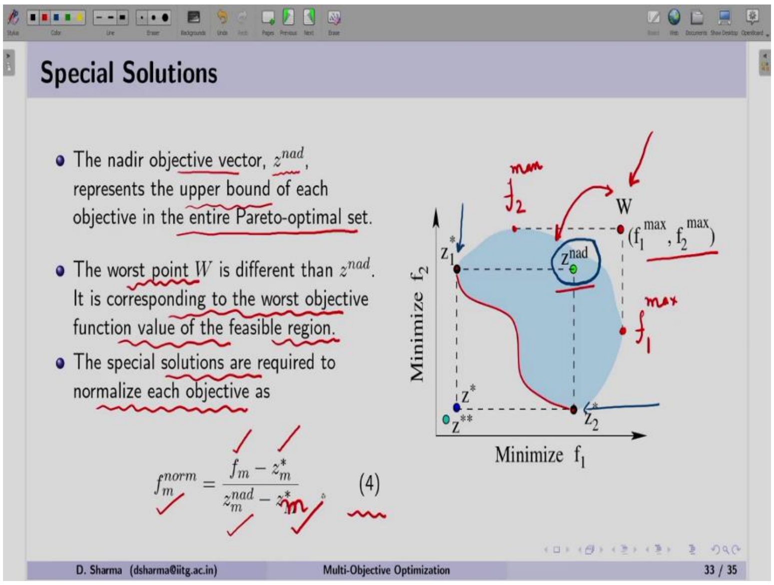

$$f_m^{norm} = \frac{(f_m - z_m^*)}{(z_m^{nad} - z_m^*)}$$

The third kind of a solution which we know is called nadir point. So, let us see that. So, the nadir objective vector as we can see it is represented as z nad that represents the upper bound of each objective in the entire Pareto optimal set. So, this is an important line where we are saying it is the entire Pareto optimal set. So, let us look into the figure on the right hand side.

So, this particular red line is this particular red line is the Pareto optimal set and if we look at the corner so, this is one of the corner and this is another corner of the Pareto optimal set. So, when these two points are constructed and using these two point, we are going to find the z nadir. So, this is the nadir point here.

So, this particular point should not be confused with the worst point which is represented by W here. How we are getting W? Suppose I want to maximize f 1 so, if I maximize f 1 I am going to get this particular point which is f 1 max. Similarly, if I independently maximizing f 2, I am going to get this particular point which is represented as f 2 max.

Now, as you can see the component of a W is represented by the f 1 max and f 2 max. So, therefore, we should not be we should be clear about what is the difference between the worst point and the nadir point. So, as again this nadir point we constructed with the help of the extreme solutions of the Pareto optimal set; however, the worst point W it is different; it is corresponding to the worst objective function value of the feasible region.

So, the particular figure which we have shown here that is help us to understand that if we are going to maximize say f 1 and then independently maximizing f 2, it will give me the worst point rather than the nadir point. Now, the question comes that these spatial solutions starting from the say ideal point, utopian point and nadir point and we have one more one more point called worst point. So, what are these what is the significance of this point?

As we can see in here in equation number 4, these special solutions are required to normalize the each objective. How we can do it? So, suppose f m norm. So, I am talking about the m th objective, this we can normalize as the objective function value f m minus the m th component of the ideal point divided by the m th component of the nadir point minus and this should be small m. So, this should be the m th component of the ideal point.

So, what you can see here is when we are going to use for example, idel point and nadir point. So, the objective function value will be lying between 0 to 1 for example. In this case suppose we are solving a problem in which f 1 objective value is given in terms of 10 to the power 6 and if we look at the value of f 2, it is lying in terms of say 0.1, 0.2, 0.3.

So, there is a huge difference between the values of f 1 and f 2. If we are going to solve this particular problem, then our EC techniques can prefer f 1 over f 2. In order to have a unbiased search, we normalize both the objective.

Whenever we normalizing it, f 1 will be lying between 0 to 1 and f 2 will be lying also 0 to 1 and in that case the selection criteria for our EC techniques will make emphasis to both the objectives and that is why we normalize our objective function when we when we solve a multi objective optimization problem.

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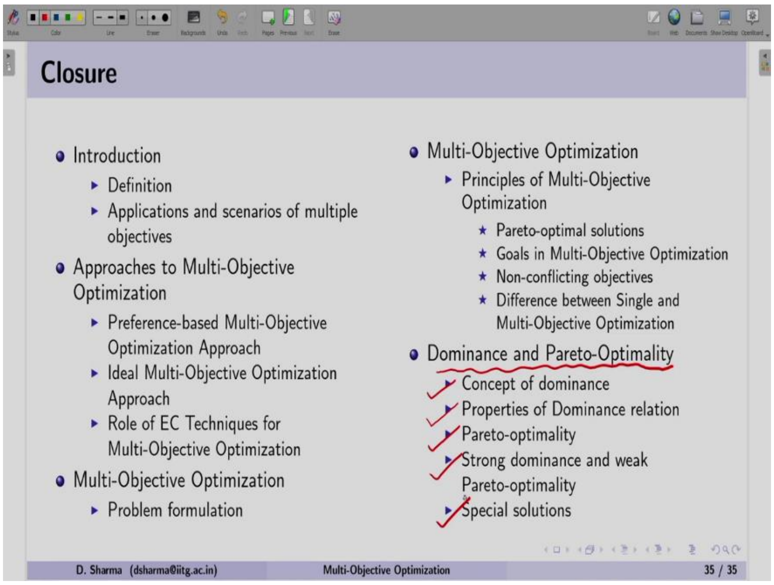

So, in this particular session we have come to the closer. Here we started with the introduction where we understood the definition of multi objective optimization. There after a very few practical examples we have gone through to understand that these multi objective is so common in nature that the problem which we solve or we handle that are also sometimes multi objective in nature.

After that we discussed about the approaches to multi objective optimization where we understand about the preference based multi objective optimization in which we convert the multi objective optimization problem into single objective optimization. Then we discussed about the ideal multi objective optimization approach in which we can generate multiple optimal solutions and then finally, we are going to select one solution using some higher information.

Thereafter we also discuss the role of EC for solving the multi objective optimization problem. Thereafter we discussed about the problem formulation in multi objective optimization. The principles, under the principle we talk about Pareto optimality, the two goals that we have found in multi objective optimization, then the non conflicting objective which we discussed and we also understood what will be the difference between single and multi objective optimization.

And finally, we discuss about dominance and Pareto optimality, there we started our discussion with the concept of a dominance where we have shown the relationship then there are certain properties which we have discussed symmetric, asymmetric and etcetera other properties we discussed. We also discussed about the Pareto optimality the global Pareto optimal, set the local Pareto optimal set that we have undergone through it.

We also discussed about the strong dominance and weak Pareto optimality to in order in order to find the solution is strongly dominating the another solution. And finally, we discuss about the special solutions, these special solutions such as ideal vector, utopian vector, nadir point and the worst point. These points we use it to normalize the objective functions. With this introduction, I conclude this session.

Thank you very much.