Evolutionary Computation for Single and Multi-Objective Optimization
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Lecture - 17
Introduction to Multi-Objective Optimization

Welcome to the session on Introduction to Multi-Objective Optimization. This particular session, we will be introducing the multi objective optimization, thereafter we will discuss the approaches to multi objective optimization. Afterwards, we will be discussing the problem formulation and various other details about the same.

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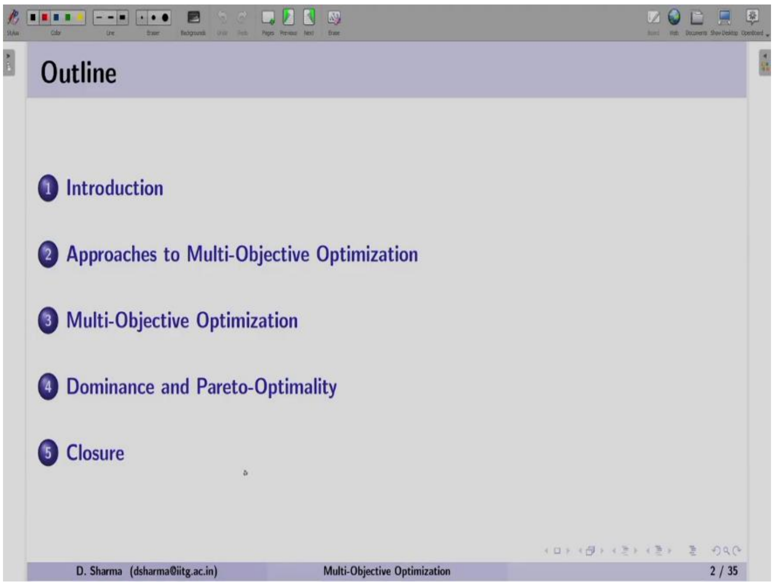

Then, we will discuss about the dominance and the Pareto optimality and then finally, we will conclude this session. So, let us start with introduction to multi-objective optimization.

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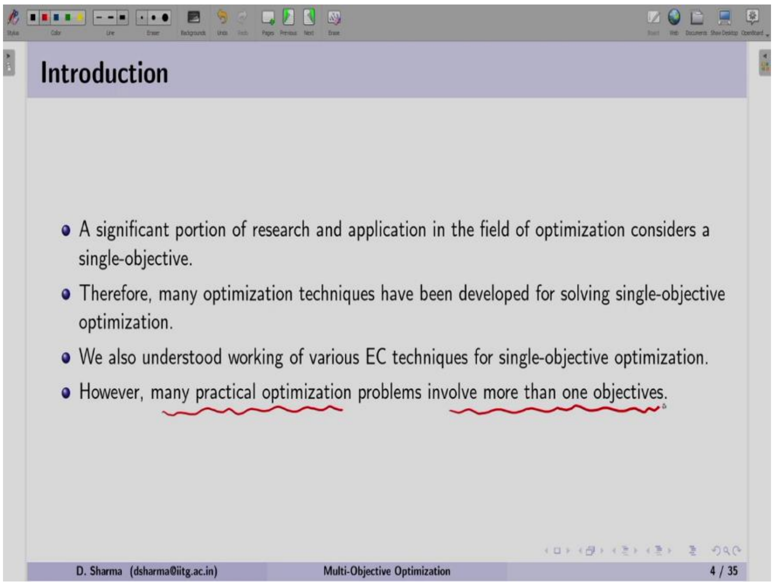

Now, as we can see in the literature as well as the other application, a significant portion of the research and application that are developed using single objective optimization. So, in those particular field, when we are constructing the optimization problem, we generally consider the single objective. Therefore, we have developed so many single objective optimization techniques to get an optimal solution.

We also understood our EC techniques for solving the single objective because in our previous sessions, all the EC techniques which we have discussed all the time we consider just one objective. However, if we see many practical optimization problems, those problems involve more than one objective.

So, when we talk about more than one objective that itself says that we are talking about multi-objective optimization. In this scenario, when we have many objectives so, these objectives essentially are conflicting in nature. So, let us see few of the examples of the multi objective optimization.

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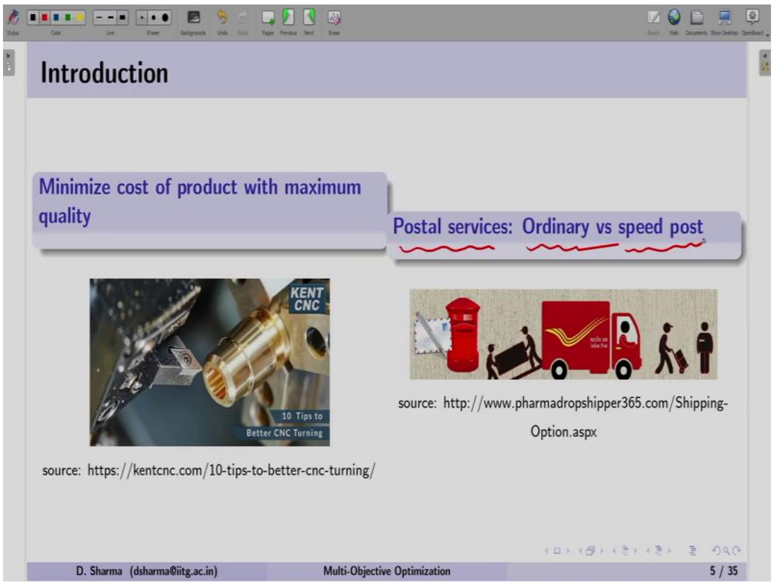

Let us begin with the very first example which is coming from the domain of manufacturing. In this case, suppose we want to minimize the cost and at the same time, we want to maximize the quality. So, as we understood, if we are going to minimize the cost of the product, we have to compromise with the quality.

Similarly, if we want a quality product, then most of the time the price of the product is also high. So, what we understood from this particular example that when we want to improve one objective, we have to compromise in another objective and that is multi-objective optimization. Let us look at the another example which is on the postal services.

So, let us compare ordinary versus speed post. Now, for a ordinary post when we have to send our say letter from our source to the destination so, we generally pay very nominal amount and our letter goes there, at that time, it takes little bit time, but if we send the same letter or a packet via speed post so, we have to pay little more and this particular letter will be reached in a shorter span of the time.

So, in this particular example, we can see that if we want to send our post a quickly, then we have to pay more. So, if we want to design a problem in which we want to minimize the cost of the postal versus says we want to minimize the time to reach the destination so, both the objectives are conflicting in nature.

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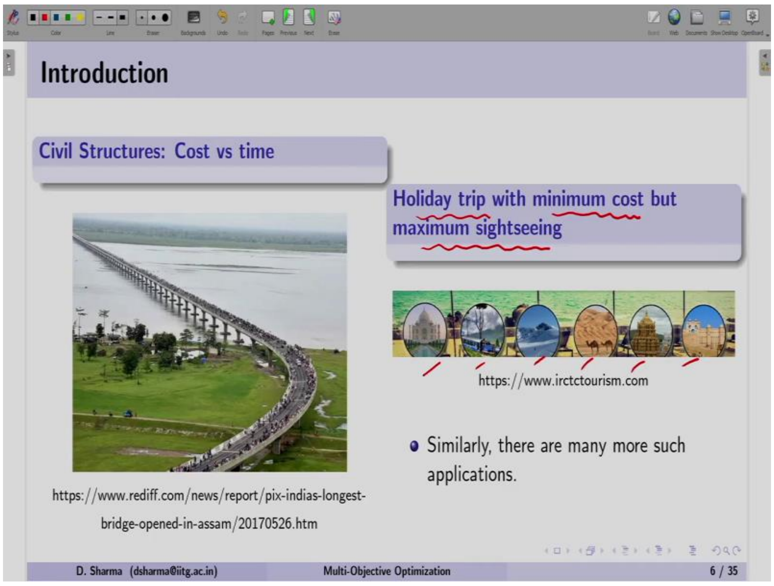

Let us take the another example now. So, this example we have taken from the civil structure where we want to minimize the cost and the time. Now, the figure which is shown here this is still date is the longest bridge on the in India that is on the river Brahmaputra.

Now, suppose if we want to minimize the time of the construct the particular bridge, in that case we have to bring machineries, we have to bring lot of manpower's as well as the raw material so, in that case, we have to pay more or we have to put lot of money into it.

So, in this way, when we want to minimize the time to construct the bridge, we have to spend lot of money, but at the same time, if we want to minimize the cost of the construction, then we may be employing certain amount, we are not employing too much of say machineries or the labour to make the bridge.

So, in this case, the time will automatically increase to construct the bridge. So, this is another example where you can see that when we are gaining in one objective, we have to compromise in another objective. Let us take one more example here. This example is on the holiday trip where we want to minimize the cost, but at the same time, we want to maximize the sightseeing.

Suppose we want to travel all the places as you can see in the figure and as we know that if we are going to see different places, we have to pay accordingly. So, if we want to minimize the total travel time versus maximize our sightsee so, these two objectives are conflicting in nature and that will construct a multi objective scenario for us.

Similarly, there are various examples as well as practical applications where we have more than one objective. It can be two, it can be three or it can be many more and all these objectives are conflicting in nature.

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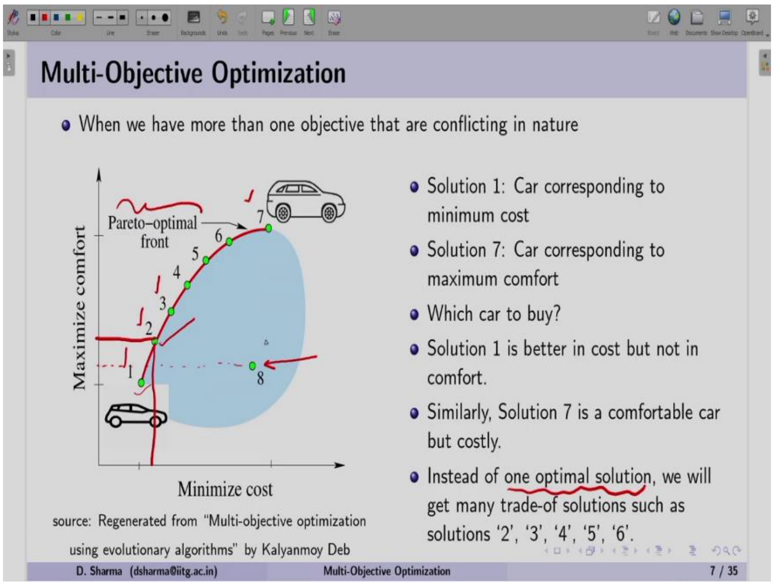

Now, since we have understood about the multi-objective scenario, let us discuss about the multi-objective optimization. So, as we have understood that when we have more than one objective and this objective are conflicting in nature, this particular problem can be referred as multi-objective optimization. We will see this particular scenario with another example as it is given here.

Now, in this case, we want to minimize the cost and maximize the comfort. So, let us assume that we want to buy a car, there could be different parameters on which we can buy the car and definitely our primer primary parameter is the cost. Along with that there could be other like fuel efficiency and one of the parameter as we have mentioned in this example could be the comfort.

So, here, we are taken two particular objectives to understand more about the multi-objective optimization. So, let us see the example. Now, in this particular example, you can see that we are showing a, we are showing the multiple solution which are referred as 1, 2, 3, 4, 5, 6, 7 and there is one more solution.

So, let us see what is this scenario. If we consider say car number 1 and car number 7. Now, we know that this car 1 or the solution 1 is corresponding to the minimum cost. So, as if I draw a line on the x-axis, this is going to be the minimum cost and if we look at the solution number 7, this is corresponding to the maximum comfort. So, the solution 7 is the car which is going to give us the maximum comfort.

Now, the question is which car to buy? Now, here as we have understood that this particular car number 1, this is minimum in cost, but the comfort is not as good. Similarly, if we look at the car number 7 or the solution 7, this is a comfortable car, but it is costly.

So, here we have a scenario that which car we have to buy because both of the car they are good in one objective and they are not so good in the another objective. So, what we can understood from this particular scenario is that when we are going to have a multi-objective optimization problem in this case, we can have multiple solutions that can be optimal.

So, here instead of say one solution so, till now, we have understood single objective optimization and as and when we solve that problem, we always give, we always get a one solution, but when we are going to solve a multi-objective optimization problem, we are going to get multiple solutions and there have those are multiple optimal solution.

So, as we have discussed in the figure, the solutions which are lying on the red line so, these this particular red line is referred to as a Pareto optimal front and the solutions which are lying say 1, 2, 3, till 7 all these solutions are the trade-off solutions and they are optimal solution for the given problem.

If we look at the solution number 8, now this particular solution if I compare with solution number 1 so, in this case, you can see that the solution one is better in cost, but the it is not as good as solution 8 in the comfort. However, if I compare solution 2 versus solution 8, we can see that the solution 2 is good in cost as well as it is better in comfort meaning that solution 8 or the solution 2 we say solution 2 is better in both the objective.

In that scenario, solution 8 is not going to be our optimal solution and that is why, if you consider a practical scenario where lot of cars are available in the market. There are some cars say for example, serving a one particular sector like a family car, hatchback cars, SUV cars or it may be a luxurious car, in those different different sectors there are some cars which are very famous.

But there are other cars which come into the market after certain time, those cars are gone from the market basically, they are not able to sell those car. So, if we refer those car as solution number 8 in this particular slide so, that car may not be serving or may not be as good as the existing car which will be satisfying or which will be giving the better value in both the objective.

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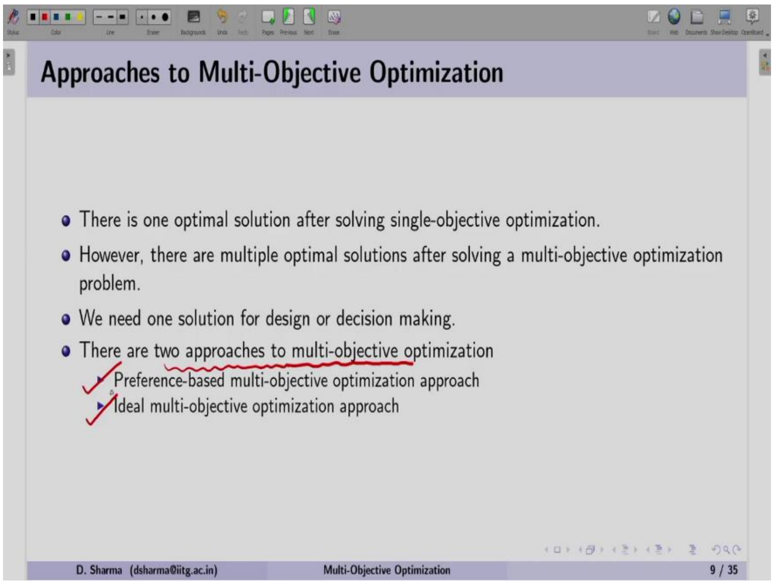

Now, let us move to the approaches to the multi-objective optimization. As we have understood, when we are going to solve a single objective optimization problem, we are always going to get a one optimal solution. At the same time, if we are going to solve a multi-objective problem, then there are multiple optimal solutions which will be optimal for the given problem.

Here, our main task is we need one solution either for a design or for a decision making. Even though we will get a multiple optimum solution in the case of multi-objective optimization, our task is always to find one optimum solution that is best for us. So, in order to.

In order to have one solution, there could be two approaches to multi-objective optimization so, the first approach is called preference-based approach and another approach we refer it as a ideal multi-objective optimization. So, let us discuss these approaches one by one. We will begin with the preference-based approach.

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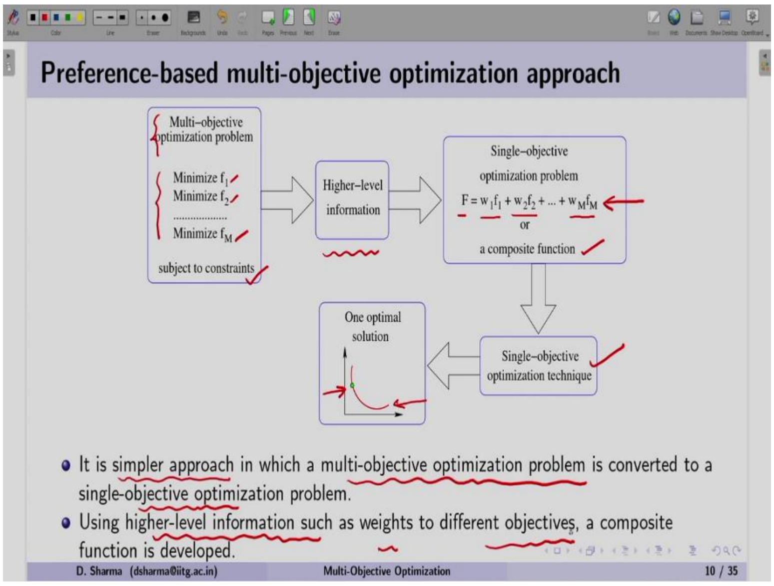

So, in preference-based approach, assume that we have a multi-objective problem, so we have M number of objectives as we can see we have f 1, f 2 and capital M so, f M number of objectives and the given problem has some constraints as well. While solving this problem, we if we use some kind of higher-level information. What we mean by higher level information?

So, although the problem is multi-objective, we already know that this particular objective will we will give more weightage say for example, objective number 3, we will be giving 50 percent, we will give 10 percent to objective number 1 and similarly, some other percentage to other objectives. So, here we are deciding, or we already know this higher-level information before solving a problem.

So, that higher level information we use it and then, we are converting the given problem into a single objective optimization. As you can see that we have the capital F which we can write as omega 1 f 1 plus omega 2 f 2 and till the summation of omega M f M or we can use some kind of a composite function that will be converting our multi-objective problem into a single objective optimization problem.

Since, we have just one objective now so, we know that we can use various techniques that we have already gone and those techniques, we can say that single objective optimization technique. After solving it, we are going to get a one solution as it you can see at the last box and this solution, we can use it.

Now, here the red line in this particular plot is define the Pareto optimal set. So, meaning that if we are going to use preference-based multi-objective optimization approach, at the end we are going to get a one optimal solution.

Now, this approach will be simpler in which a multi-objective optimization problem is converted into a single-objective optimization and as we understood, higher level information was used such as weights to different objectives and a composite function a was developed. So, that is the preference-based; preference-based multi-objective optimization approach.

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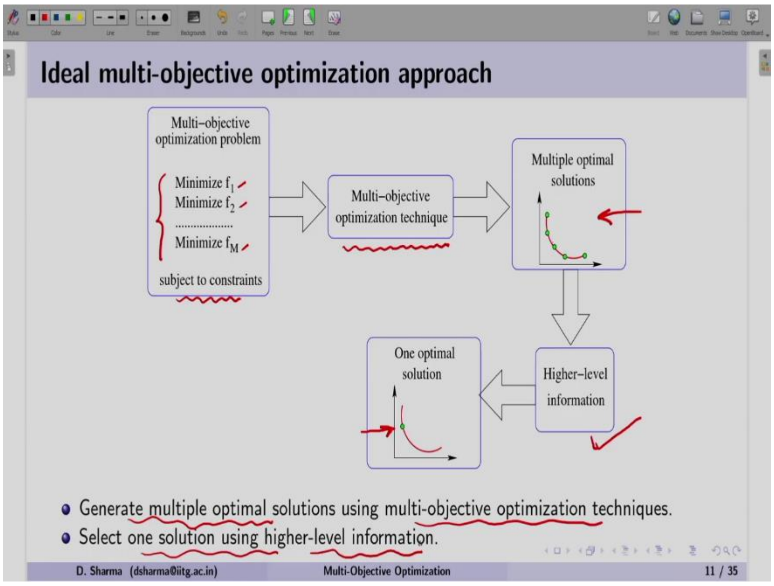

Now, let us see what could be the ideal multi-objective optimization approach. So, if we see here, we have the same problem and we have the same set of objective as f 1, f 2 and f M. This particular problem is subjected to the same set of a constraints. So, in. So, now, what we will do here is we are going to solve the problem in its original form using some multi-objective optimization technique.

And as we know, if we are going to solve this multi-objective problem, we are going to get multiple solutions as we can see in the third box. These three, these multiple solution will help us to understand what could be the Pareto front. Now, once we generated these multiple solution on the Pareto optimal set, now we can use the higher-level information as we have discussed like for example, we can decide the weights and then, accordingly and then at the last, we can choose the one solution which is the best for us according to our higher-level information.

In this particular case, we generate multiple solution using multi-objective optimization techniques and we select one solution using higher level of; high-level information. So, as we can see the difference here that in the previous case, we use the higher-level information at the beginning itself and then, we convert the multi-objective problem into single objective and solve it to get a one solution.

But here, when we use ideal multi-objective approach in this case, we are going to generate multiple optimal solution and then, use the higher level information to get the one solution that we are going to use say for design or decision making.

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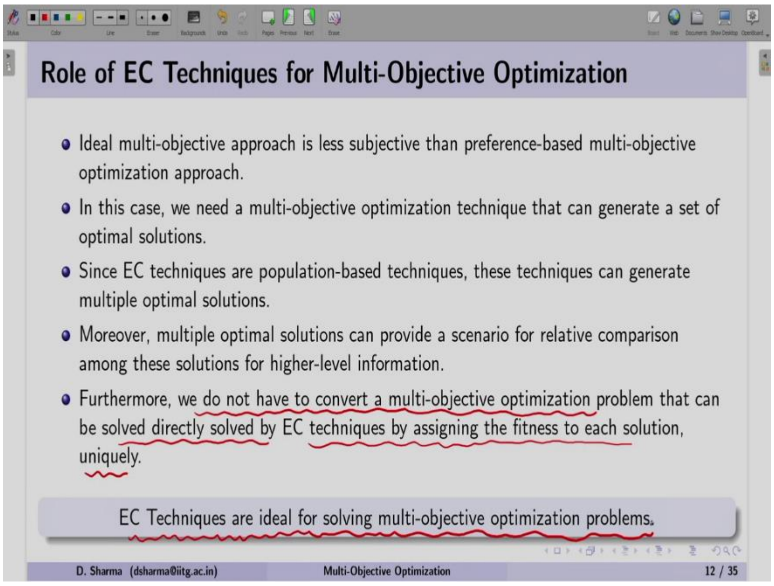

Now, since we have understood these two approaches, now the question is what is the role of EC techniques for multi-objective optimization? So, let us go one by one. In ideal multi-objective approach as per our explanation, we can understand that this ideal multi-objective approach is less subjective than the preference-based multi-objective optimization approach.

In this case, when we are dealing with the ideal multi-objective approach, the multi objective optimization technique, we need a multi-objective optimization technique that can generate a set of optimal solutions. Once this set is available, we can always do decision making.

Now, the point is we have understood that EC techniques are population-based techniques. So, these techniques if we use in a proper way, then these techniques can generate multiple optimal solution for the given problem. Moreover, these multiple optimal solution can provide a scenario for relative comparison among the solution for higher-level information.

So, here, as we have understood the two approaches so, what we can see that if we are going to solve a multi-objective optimization problem in its form say by using evolutionary computation technique, then these multiple solutions so, we have to change the fitness and accordingly, these solutions will be converged to the Pareto optimal front.

Now, when we apply our higher-level information so, we are going to choose one solution. Now, at that time, there could be a scenario that if suppose we are going to buy a car and if we see that as per our higher-level information, for example, we have decided that we are not going to spend money more than that much of amount, but when we solve with this problem, we understood that if we pay say more 10000 rupees, then the comfort of the car is increasing manifolds.

So, in this case, this will allow to have a scenario where I can compare. Similarly, I can also see that if I have to pay say 5000 less, then I am not compromising much with the comfort. So, accordingly, I will do the relative comparison and I can decide which car I can choose.

So, the higher-level information which we have decided earlier, even though we have decided, we can still do this relative comparison among the multiple optimal solutions that is generated by the multi-objective optimization technique. At the last, as we have understood, we do not have to convert a multi-objective optimization problem and this problem can be solved directly by EC techniques by assigning the fitness to each solution in a some way, in a uniquely unique way.

So, as we have understood, since EC techniques are population based techniques, if we work on assigning the fitness to the solutions, then EC can be the best choice for us to solve multi-objective optimization problem and therefore, at the last, we have mentioned EC techniques are ideal for solving multi-objective optimization problem.

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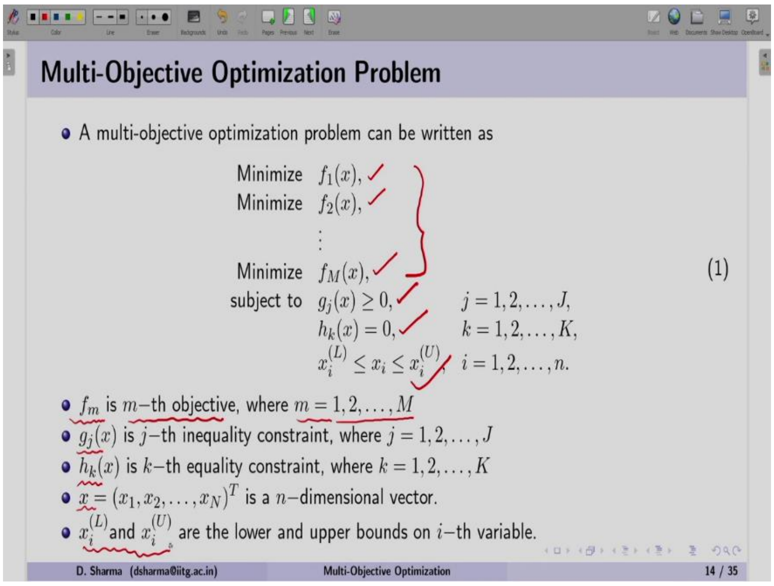

$$Minimize \;\; f_1(x) \;\; ,$$

$$Minimize \; f_2(x),$$

$$\vdots$$

$$Minimize \; f_M(x),$$

$$subject \; to \;\; g_j(x) \geq \; 0, \qquad j = 1,2,\ldots,J,$$

$$h_k(x) = \; 0, \qquad k = 1,2,\ldots,K$$

$$x_i^L \; \leq \; x_i \; \leq \; x_i^U, \qquad i \; = \; 1,2,\ldots,n.$$

So, with this introduction on multi-objective, now let us have more detail on the topic that is multi-objective optimization. Now, the problem, the problem formulation for multi-objective optimization, we can write as so, as we have understood we can have more than one objective. So, we want to say Minimize f 1, Minimize f 2 similarly, we have Minimized f M.

We want to minimize all these objectives and this particular problem is subjected to inequality constraint so, this format is known to us. Similarly, we have equality constraints, and we have variable bounds. So, what change we can see here that we have multiple objectives which we want to minimize, and the problem can be subjected to the similar kind of a constraints as we have understood.

If we are writing say f m, this means that we talk about the m th objective and this m can take a value from 1 to capital M. Rest of a our explanation is the same as g j is the inequality constraint, h k is the equality constraint, x is the vector so, is a n dimensional vector and x i L and x i U are the upper and the lower bound on the given problem.

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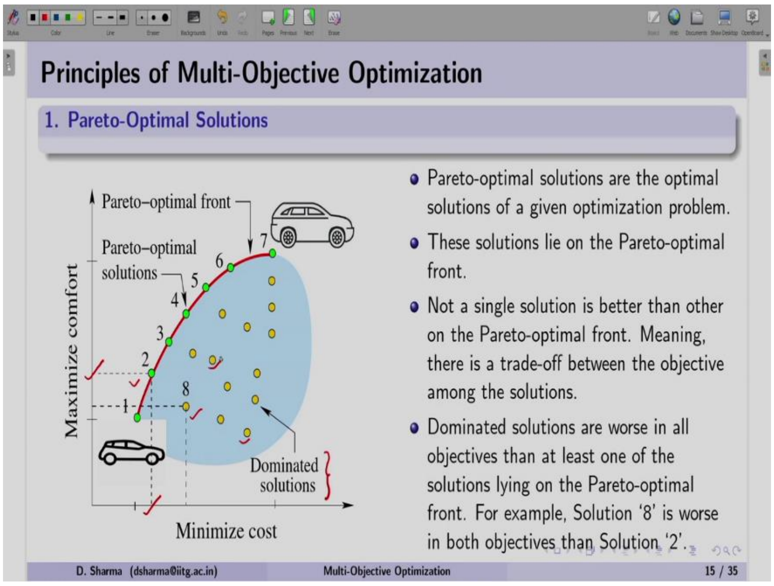

Now, we begin with principles of multi-objective optimization. We will start with Pareto optimal solution. So, in this particular case, we are going to take the same example of the car where we want to minimize the cost and maximize the comfort. Now, what is Pareto optimal solution? So, Pareto optimal solutions are the optimal solution of the given optimization problem. Now, these solutions lie on the Pareto optimal front. Now, let us look at the figure.

So, as we can see, the Pareto optimal front is represented by the red line. This particular line, in this particular line, the solutions which are lying for example, solution 1, 2, 3, 4, 5 and 6 all these solutions are lying on the Pareto front. So, these solutions are the Pareto optimal solution. If I am going to compare say solution versus; solution 1 versus solution 2, what we can see that one solution will be good in one objective, but it has to compromise in another objective.

So, in this case, there is not a single solution which is optimal for the given problem. So, there is a trade-off among the solution with from 1 to 8. Now, in this particular figure, we also see some solution which are referred as dominated solution. So, let us compare solution say 2 versus solution 8. Now, if we see solution 2 is lower in cost and it is giving better comfort than solution 8 meaning that solution 2 is better than solution 8.

So, we can say that solution 8 is worse in both the objective so, it is a dominated solution. If we look into the figure, the solutions which are filled with this yellow colour, we can say all these solutions, I they are dominated by at least one solution on the; one solution that is lying on the red line which is called Pareto optimal front.

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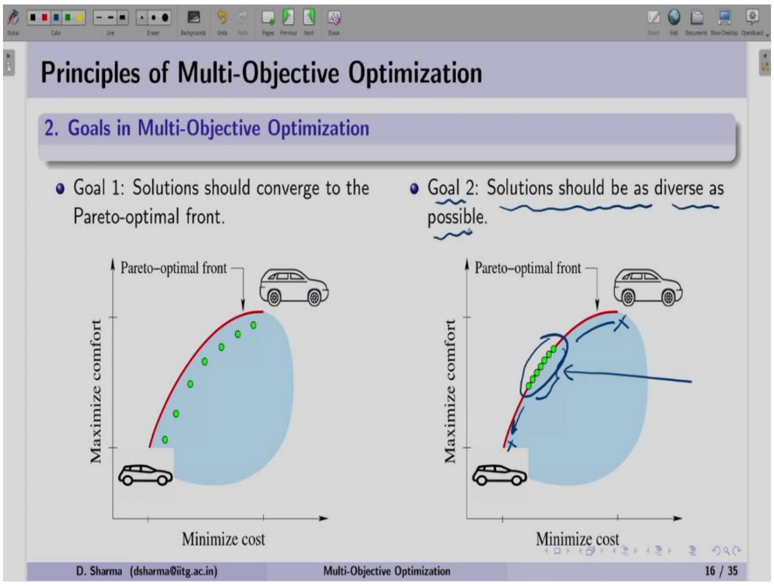

The second principle is the goals. So, the goal is in multi-objective optimization. So, let us understand that. Now, if we take the same example here. Now, let us we are solving this particular problem and when we solve it, we are going to get the solution as shown in the green dots. What we can observe from this figure that these solutions; these solutions are not converge; so, these solutions are not converge to the Pareto optimal front.

So, our goal is so, the first goal is the solutions should convert to the Pareto optimal front. So, that is the first objective and we can say the primary objective that when we are solving a multi-objective optimization problem, our technique should give us the solution that are converge to the Pareto optimal front.

What could be the second objective? As we can see, suppose we are solving and we get one algorithm that has converged to the Pareto front, but if you look at these solutions, these are converged into the very small region of the Pareto front. These solutions are not going to give us any idea what is the Pareto front of the given problem.

Therefore, the second goal of the problem is solution should be as diverse as possible. So, what is the best situation? The best situation if we look into this particular figure that the solution as we can see here, the solution 1 to 7, they are converged to the Pareto front as well as these solutions have a good diversity. So, these are the two goals.

So, here what we can understand that solving a multi-objective problem itself will multi-objective in nature because we have to achieve two goals that is the convergence as well as the diversity among the solution on the Pareto optimal front.

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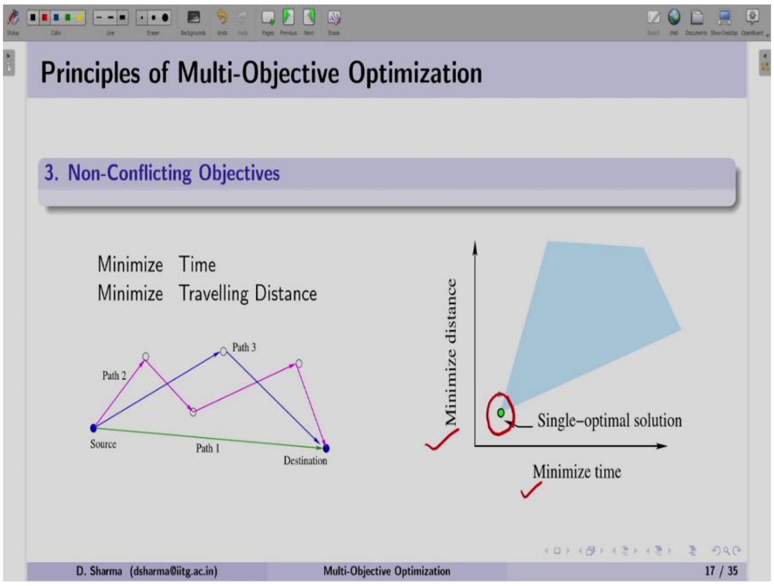

The 3rd principle is non-conflicting objectives. So, since the beginning of this session, we always emphasize on that as when as and when we have more than one objective, those objectives should be conflicting in nature. So, let us take the given example here. We want to minimize the time and we want to minimize that travelling distance.

Suppose the source is given to us, our destination is known, and we have three paths; path 1, 2 and 3. From this particular this simple figure, if we want to minimize the time, we are going to choose path 1. Similarly, if we want to minimize the travelling time that will also choose the path 1. So, in this case, either I minimize the first objective or the second objective, all of the cases I am going to choose the same solution or I am going to get a one solution.

So, what will happen in this case? We can see the figure on the right-hand side that even if we want to take both objectives as our multi-objective problem, we will end up into a one solution that is called single-optimal solution. So, therefore, our objective should be conflicting in nature otherwise, we are end up in getting just one solution rather than multiple optimal solution for the given problem.

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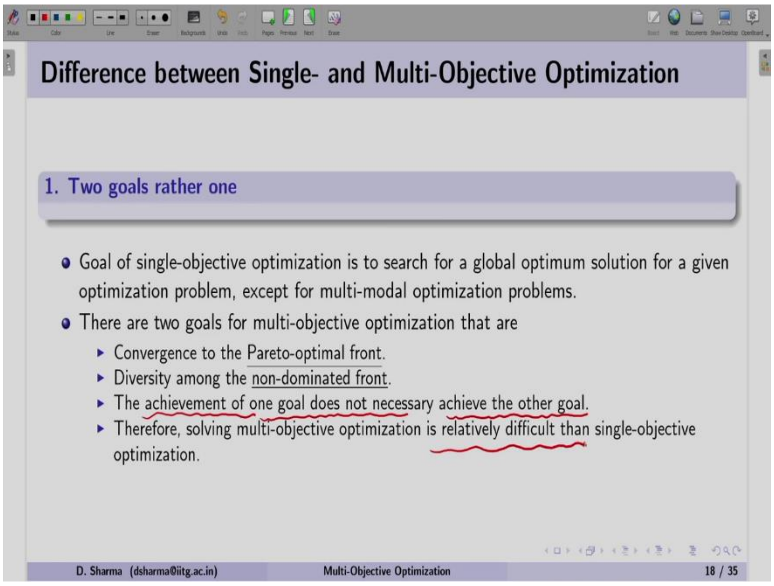

Now, with having understanding on the principles of multi-objective optimization, now let us discuss the differences between single and multi-objective optimization. We will begin with say two goals rather than one. So, we remember that when we solve single-objective optimization, we have just one objective and that says suppose we want to minimize it.

So, the goal of single-objective optimization is to get a global optimum solution for a given optimization problem, if we do not consider the multi-modal optimization problem. So, multi-modal we discussed in the very first session of this course. So, if you remove that multi-modal optimization, every time we are looking to generate a global optimum solution for the given problem.

However, if we talk about multi-objective optimization, there are two goals so, as we discussed in the previous slides. So, there are two goals for a multi-objective optimization. First goal is the convergence to the Pareto optimal front and there should be a diversity among the solution.

Now, here the achievement so, as we have to understood that if suppose we achieve one goal in multi-objective optimization, it is not necessary that we are going to get the another objective. So, both the objectives are different so, if we are formulating our algorithm just to cater the need of say one of the objective say either convergence or the diversity, then in that case it is not necessary that the algorithm will be good in the another objective.

So, looking at this two goal scenario, what we can understand is solving a multi-objective optimization problem is relatively difficult as compared to the single-objective optimization and the same thing we have mentioned here that this problem is relatively difficult.

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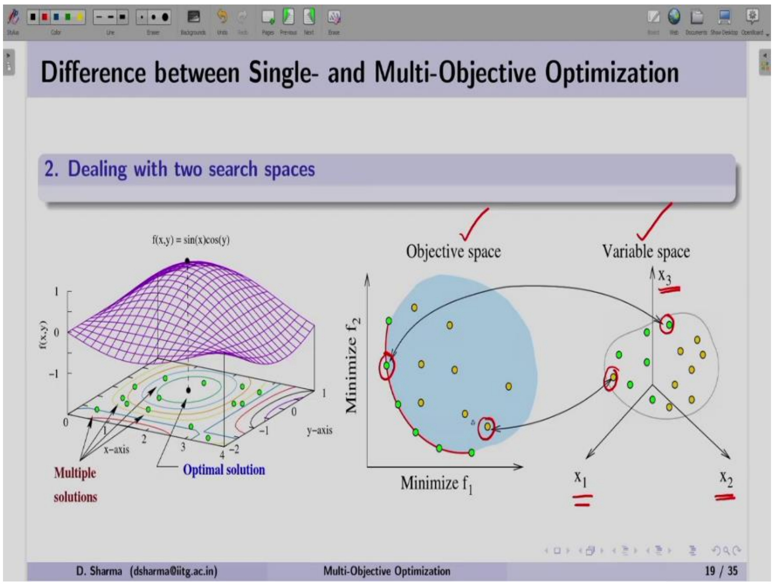

Another difference between the single and multi-objective optimization is the search space. In single-objective optimization we deal with just one space called variable space. So, let us look into the example here. So, in single-objective, this figure is familiar to us. In this case, suppose we want to maximize our objective function and we have two variable say x and a y. So, we always look in x and a y and we try to find out our optimal solution.

So, single-objective optimization only deals with just one space, but if we look at multi-objective optimization, it deals with two spaces; one is called objective space, second is called variable space. So, as the variable space name suggests so, the value of our variable or the vector say x so, the value of x 1, x 2, x 3.

So, those values we will be searching in the variable space and then, we have a mapping. So, as you can see this particular mapping, for this solution, we have this mapping into the objective space. This mapping we get based on the objective function and here, you can see we are showing the two mapping.

So, what you can see here is we always search in the variable space in multi-objective optimization as well as single-objective optimization, but when we have to decide a solution, we always do our selection in the objective space where we deal with multiple objectives and therefore, we deal with two spaces in multi-objective optimization rather than just one space in single-objective optimization.

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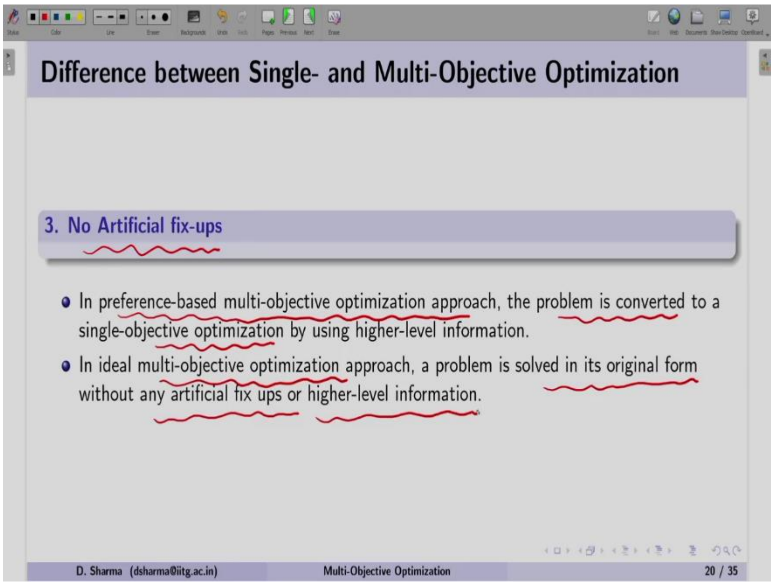

The third difference what we can understand is the artificial fix up or the higher-level information. So, let us look into it. So, we do not need artificial fix up while solving the multi-objective optimization problem. So, in preference-based multi objective optimization approach as we have understood, we convert our multi-objective problem into single-objective optimization problem. However, in ideal multi-objective approach, the problem is solved in its original form without any artificial fix ups or the higher-level information.

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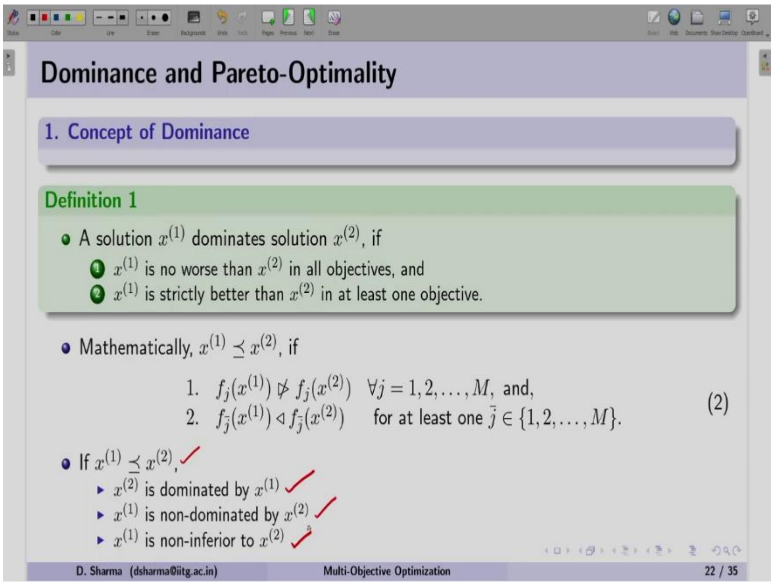

Now, we have come to discuss the dominance and the Pareto optimality. This is needed why because we are dealing with multiple solutions, multiple solutions can be optimal, there are already multiple objectives so, how we can compare them? So, this dominance and the Pareto optimality will help us in differentiating the solution. Along with that, we will also discuss few more details about the Pareto optimal solutions and other solutions.

Let us begin with the first is that is called concept of dominance. The concept of dominance as we can see in definition number 1, suppose we have a solution 1 which is represented by x 1, we have solution 2 which is represented by x 2. Let us take let a solution x 1 dominates x 2, if so, there are two conditions to satisfy that x 1 is no worse than x 2 in all objective. At the same time, x 1 is strictly better than x 2 in at least one objective.

So, as we have understood these two condition meaning that x 1 is not going to be worse than x 2 in all of the objective. So, it means that it can be same, or it can be better, but it cannot be worse. The second condition says that it should be better at least in one objective. If we write the concept of dominance in a mathematical form, we will see the equation number two now.

Mathematically, if we have to say x 1 dominates x 2, then the objective function should not be worse than the objective function of a two for all the objectives as well as the solution 1 at least one objective should be better than the solution 2. So, when we are saying that x 1 is dominating x 2, this means that x 2 is dominated by x 1 or we can say x 1 is not dominated by x 2 or we can say x 1 is non-inferior to x 2. So, there are different ways we can when the when we are comparing these two solution, we can refer these two solution in a different ways.

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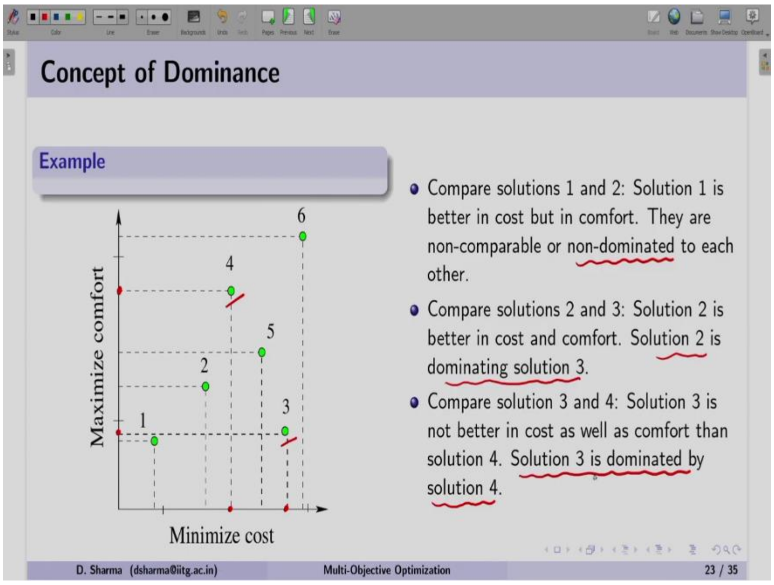

Now, let us understand the concept of a dominance within an example. So, we are taking this example as suppose minimizing the cost and we want to maximize the comfort. In this example, we have taken 6 solution. Now, suppose let us compare the solution 1 versus solution 2. Now, as we can see solution 1 is having less cost so, it is good in cost, but at the same time if we compare the comfort the solution 2 is better than solution number 1.

In this scenario, not both the solution, one solution is good in one objective and it has to compromise in another objective. So, in that case scenario, we call these solutions a solution 1 and a 2, they are non-dominated to each other. So, as we have mentioned that these solutions are non-comparable or non-dominated to each other.

Now, let us compare solution number 2 versus 3. So, we are following the concept of dominance here. Solution 2, we can see it is better in cost than solution 3. If we look at the comfort, solution 2 is again better than solution 3 in comfort meaning that in both the objective, solution 2 is better. So, we can say that solution 2 is dominating solution 3.

Let us take the another scenario here. We are going to compare solution 3 versus solution 4. Now, here when I am comparing 3 versus 4, 3 has the more cost so, means 4 is better and when we are comparing the comfort, 3 is having worse comfort as compared to the solution 4. So, we can say that solution 3 is dominated by solution 4.

So, in this particular example, we have discussed three scenario that two solutions can be non-dominated to each other or a one solution can dominate other solution and the third scenario is one solution is dominated by the other solution. So, these are the three scenarios which we can get it from the concept of dominance.

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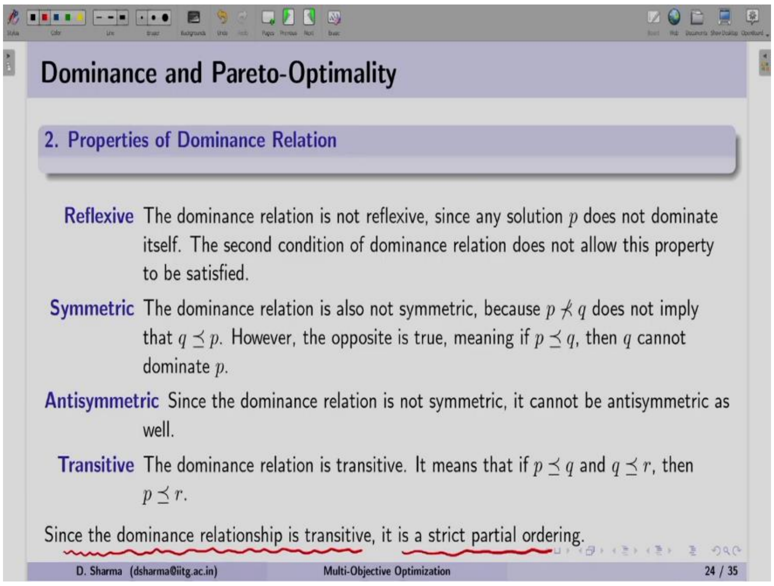

Let us move to the second section which is on the properties of the dominance relationship. Let us begin with the property called reflexive property. The dominance relation what we have discussed in the previous slide that is the concept of dominance equation number 2, this is not reflexive meaning that, any solution p does not dominate itself.

Now, let us remember the concept of dominance that a solution 1 should not be worse than solution 2 in all objective. So, this condition is satisfied when I am comparing solution p versus solution p, the same solution. The second solution; the second condition says that our solution 1 should be better at least in one objective, but this condition will not be satisfied if I compare solution p versus p.

So, therefore, as we have mentioned, second condition of dominance relation does not allow this property to satisfy. So, we understood that this the dominance relation is not reflexive because of the second condition of dominance relation. Second property is the symmetric.

Now, the dominance relation is also not symmetric why because let us understand that. If we see that the solution p is not dominating q so, this representation says that p is not dominating q. It does not mean that that q is dominating p. So, if p is not dominating q, there could be there two say, there could be two possible ways either q is dominating p or p and a q both are non-dominated.

And therefore, we can understand that the dominance relation which we have just understood it is not symmetric because when p is not dominating q, it does not mean that q is dominating p. So, when we have to use the concept of dominance, we compare solution say 1 to 2, 3, 4, 5 and when we take solution 2, then 2 should will again compare with 1 than 3, 4, 5.

So, that comparison so, this means when we take a solution p, it has to compare with all the solutions again. Now, there is an interesting part here is that the opposite is true, meaning that if p is dominating q, meaning that q cannot dominate p that is for sure so, that is a very interesting property. So, the dominance relationship is not a symmetric one.

Now, the third property is the anti-symmetric. As we have understood that the dominance relation is not symmetric so, it cannot be anti-symmetric as well. Now, come to the last property which is called transitive property. The dominance relation is transitive. So, this is interesting how?

Let us assume that we the solution p is dominating q and solution q is dominating r. So, we can directly say that the p is dominating r. So, this is a transitive rule which is followed by the dominance relationship. So, here as we have mentioned, since the dominance relationship is transitive so, it is a strict partial ordering.

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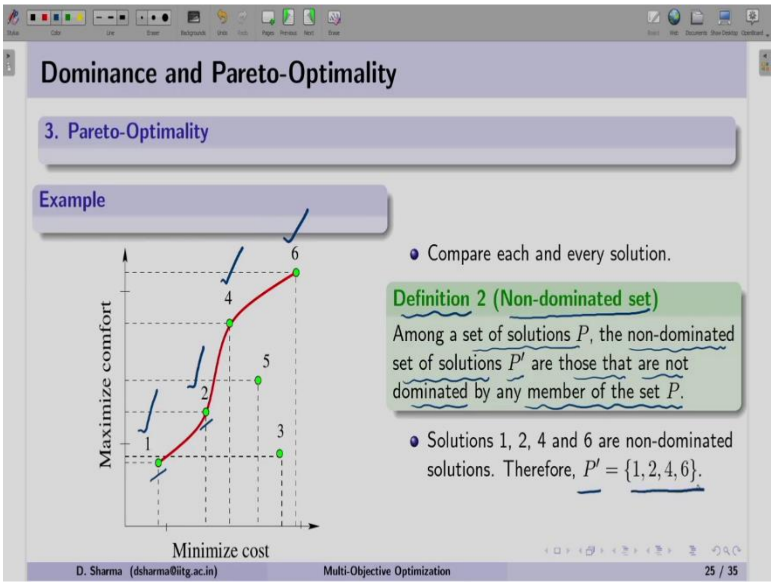

Now, we have come to discuss the Pareto optimality. So, we will be taking an example here. So, let us take this particular example where we have the 6 solution in which we want to minimize the cost and we want to maximize the comfort. So, as we have understood, we have to compare each and every solution. So, before we study, we before we go further, let us understand the definition number 2 that is given based on the non-dominated set.

What is that? Among a set of solution P, the non-dominated set of solutions say P prime are those that are not dominated by any member of set P. Now, let us look at the figure here so, we have 6 solutions. So, when I am comparing solution 1 with all, then I am looking whether solution 1 is dominated or a not. Similarly, when I am comparing 2 with a all including 1, then whether 2 is dominated by 1 or a not.

So, in this case, when I am comparing each and every solution, what we can see that solution number 1, 2, 4 and 6, they are non-dominated set among the 6 solutions which is currently in the P. So, P constitute of 6 solution and P prime is made of 1, 2, 4, 6 which are non-dominated solutions.

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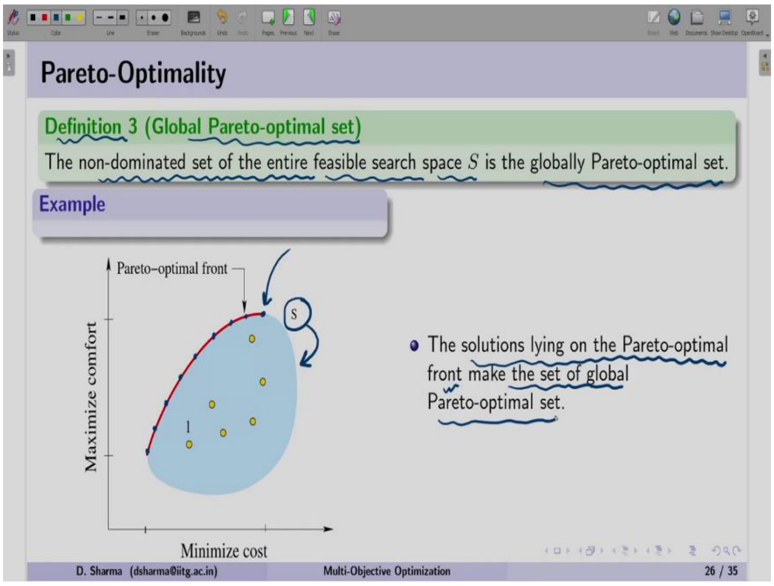

Going further, we can use the definition of non-dominated set. Now, the definition 3 suggests that about the global Pareto optimal set. What is that? The non-dominated set of the entire feasible search space say S is a global Pareto optimal set. Let us look at the figure now. Now, let us assume that the feasible search space S is given by the shaded region.

Now, as we have understood, when we have say six solutions? When we are comparing so, we are going to get non-dominated solutions, but if I get the non-dominated solutions with respect to the entire search space, then this these are called Pareto optimal set. So, the solutions that are lying on the red line, those are become the global Pareto optimal set. So, we can say that these solutions which are lying on the Pareto front, these are the non-dominated solutions with respect to the feasible search space. So, these are the global optimum solution.

So, just to understand one more time, when we have a set of population and when we are comparing each and every solution and by using the concept of dominance, we can find out what are the non-dominated solutions. If I do the same exercise for the all the solutions which are lying in the feasible search space, the solutions which are non-dominated with respect to the entire feasible space will become the global Pareto optimal set.

So, therefore, as we have mentioned, the solutions lying on the Pareto optimal front make the set of global Pareto set. So, that is that difference we will be remembering and as and when we have to tell what is Pareto optimal set, we can define in terms of non-dominated solution with respect to the entire feasible search space.

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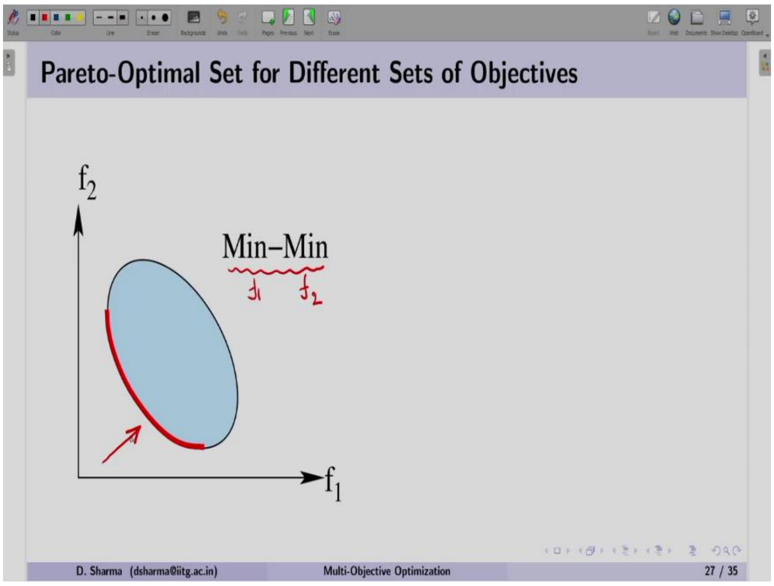

There are different sets of objectives. So, this is very interesting now. Suppose we have the common objectives, and we have a common feasible search space. Now, if we change the nature of objective how this Pareto front or the optimal Pareto front will change. So, let us look one by one.

So, in the present example, we can see that when we are having min-min, this means I want to minimize f 1 and also, I want to minimize f 2. In this case, for the given feasible search space as we can see that this red thick line on the boundary of this surface is the Pareto optimal set.

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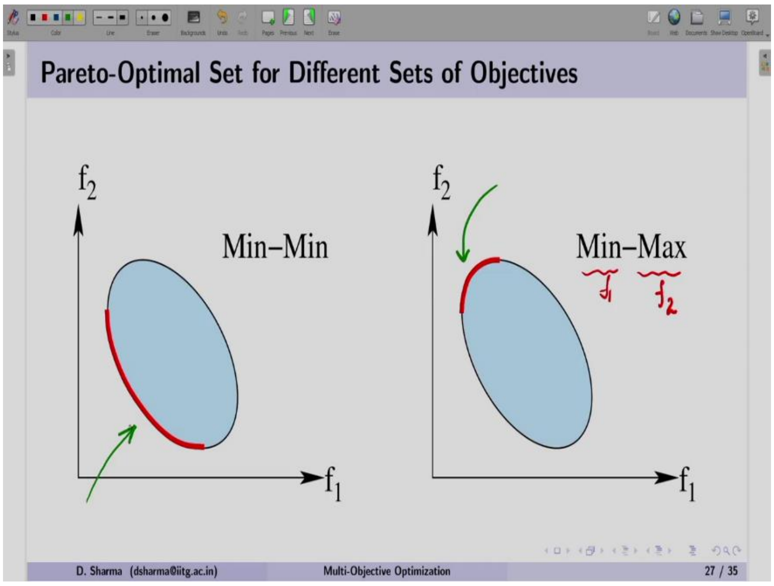

Suppose we change now, we want to minimize f 1 and we want to maximize f 2 so, when we are doing it, you can see the same search space, but now the Pareto optimal set is now changed. As per the objective or the nature of objective, we can see how this Pareto optimal set on the left-hand side figure and the right-hand side figure can change.

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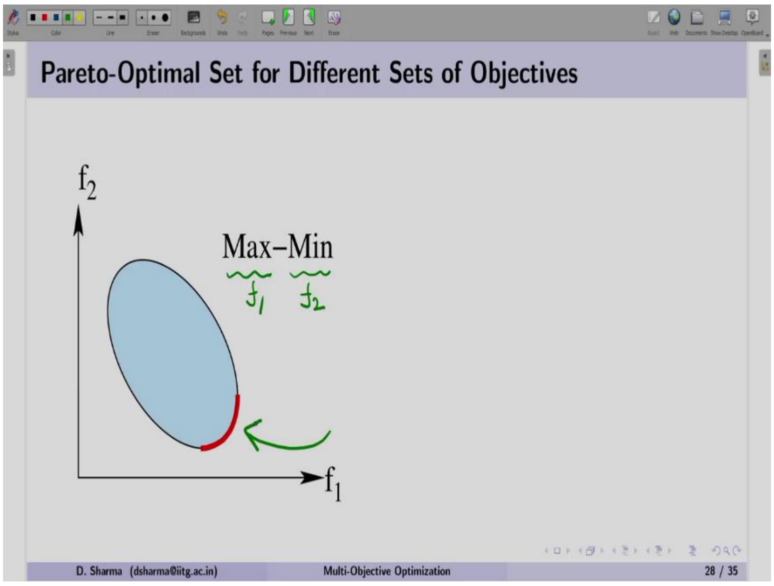

Let us take two more examples. Suppose we want to maximize f 1 and minimize f 2, when we will be doing here for the same feasible search space, the Pareto set will shift it to this particular corner.

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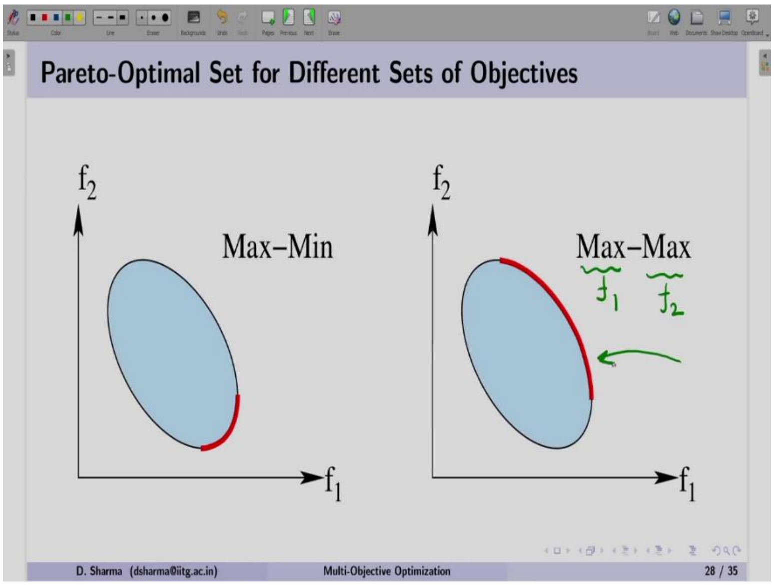

And at the last, if suppose we want to maximize f 1 and maximize f 2, we can see the Pareto front is moved to this particular space or the area of the feasible search space. Since the feasible search space is defined by say the constraints or the variable bounds so, in this case, the Pareto surface or the Pareto optimal front is lying on the boundary and the motive of showing these four example to understand that when the nature of the objective will change, the Pareto set or the Pareto optimal front, the location of the Pareto optimal front will also change.