Evolutionary Computation for Single and Multi-Objective Optimization

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Lecture - 16
Simulations of Constraint Handling Techniques

The next problem which we have taken is g08; that is again from CEC 2006 competition.

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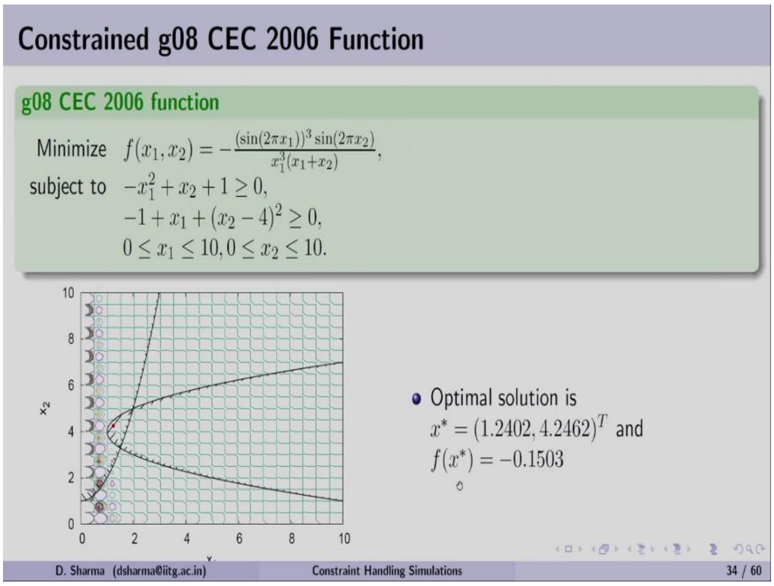

$$Minimize \quad f(x_1, x_2) = -\frac{(\sin(2\pi x_1))^3 \sin(2\pi x_2)}{x_1^3(x_1 + x_2)}$$

$$subject\ to \quad -x_1^2 + x_2 + 1 \geq 0,$$

$$-1 + x_1 + (x_2 - 4)^2 \geq 0,$$

$$0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10\,.$$

If we look at here, we want to minimize the two variable function. This function is written in terms of sin and there is a cube at the denominator. This particular problem is subjected to two constraints as given here; both the variables are lying in the range of 0 to 10. If we look its contour, now we can see that this particular problem is difficult to solve in a way that it involves sin function and cubic function in the denominator.

As we can see there are two constraints and because of these two constraint, the optimized line in between. So, as per this detail there is both the constraints are not at f at the optima. The optimal solution on the right hand side; up to four decimal places; the accuracy is shown here. So, the point is given as well as the function value is minus 0.153; so this value is quite small.

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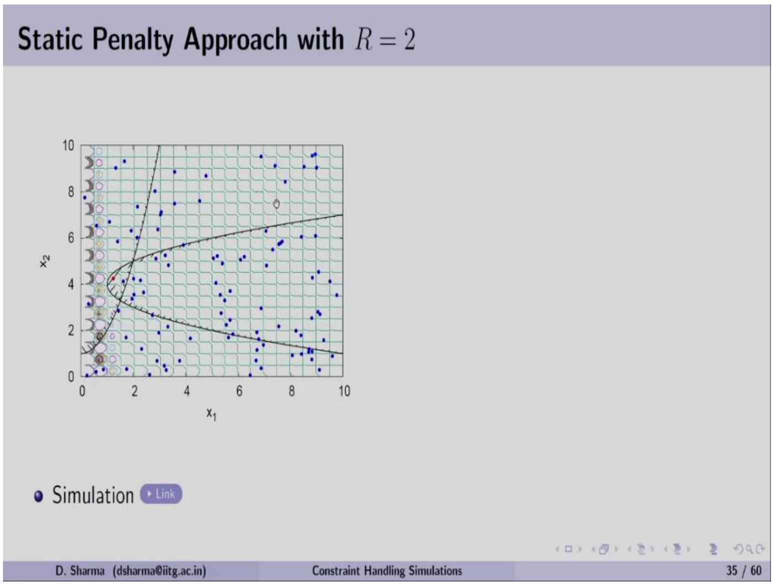

On this particular function, we have to see how these constraint handling techniques will work. We will start with static penalty approach having R equals to 2. So, we started with the small value of a R and we can see initially the solutions are distributed randomly in x 1 and x 2 plane. Let us see how these solutions will be moving towards the optimal.

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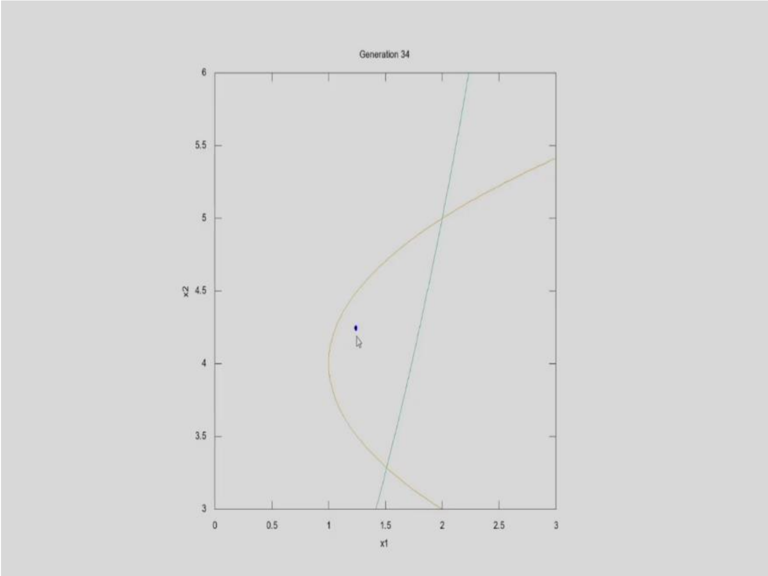

Now, as can be seen that the solutions have converged to the feasible region which is defined in this small area. And there after all the solutions are converged to the optimal point. Now, we have to see whether after few generation; the solutions are diverging to another solution that we have observed in the earlier examples. Since R equals to 2 is a small value; we have to find whether these solutions will converge to the another solution.

So, till 120 generation; all the solutions are converged to a single solutions and it seems that all the solutions have converged to the optima and the value of R which we have taken; R equals to 2 all of them have that is sufficient for solving the given problem. Still, we almost finish off 200 generation and we are at the optimum solution.

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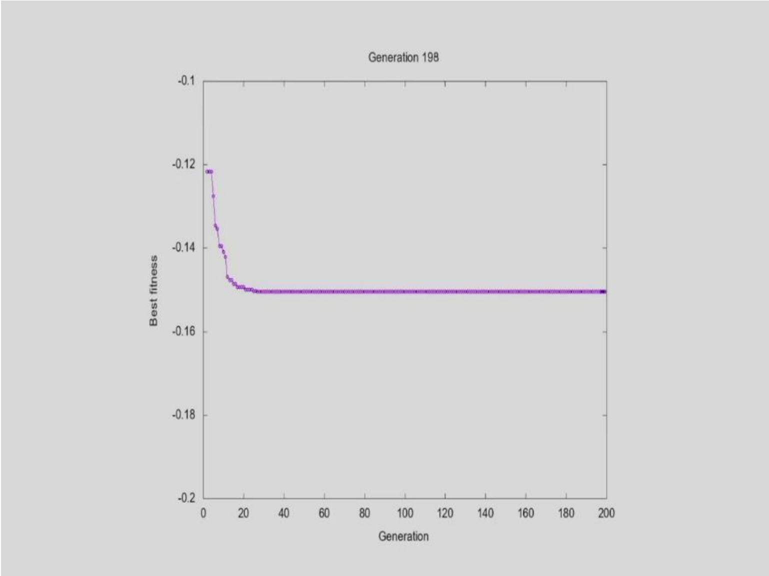

So, this says that for R equals to 2; the solutions have converged to the optimum point. If we see the progress, so number of iteration in x axis; the best fitness in y axis. It can be seen that almost close to 30 generation; there is no change in the fitness value because we have reached to the optimum point.

So, we will see the simulation of the same here. Now, in this case it is started with the small value and as we have understood; close to the 30 number of generation, the fitness remains the same; the best fitness in the population remains the same. Its only because all solutions are converged to the optimal point.

Similarly, as in our previous discussion we have taken two different values of R. So, R is 2 that was not sufficient for some problem; so, we have taken a large value of R. Now, we will again see whether R equal to 100 will make any change in the progress of a solution towards the optimal solution.

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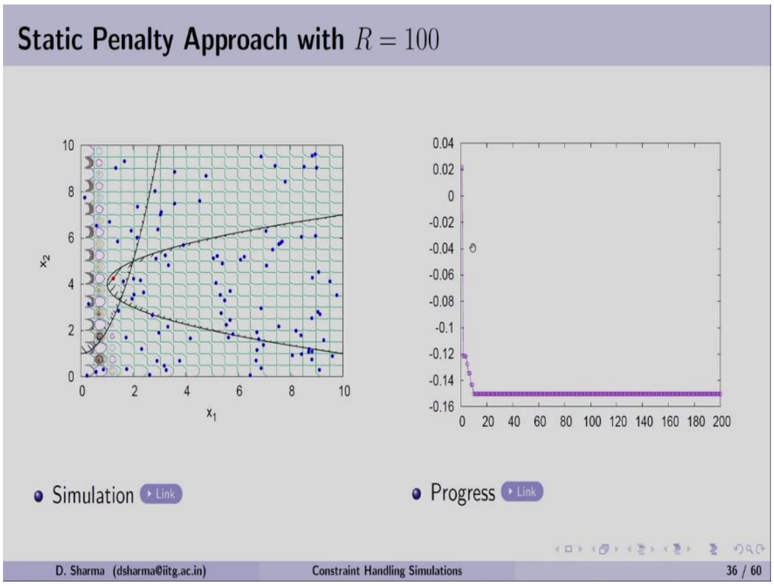

So, in this here; we have taken R equals to 100 for static penalty approach. Again the same set of solutions are taken in x 1 and x 2 plane and we will see how these solutions will be converging.

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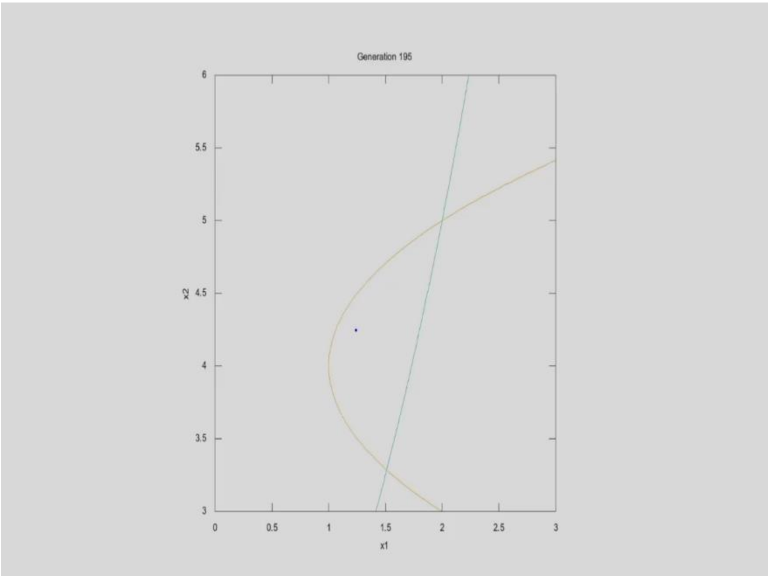

As expected since the value of R is large; within 8 generation or solutions are inside the feasible region and now all these population and the all members of the population, they are now converged to the optimum solution very quickly. We have to see in the previous simulation when we took R equals to 2; the solution were not converged.

So, we expect the same thing that once the solution are converged to the optimum solution, R equals to 100 will also not diverged and we are going to get the same optimum solution for R equals to 100. So, till 150 generation; we can see all the solutions have already converged here and there is no divergence meaning that members are not moving to the another solution based on the fitness here.

So, we have to see till 200 generation; yes; so using R equal to 100, we are able to get the solution. Now, we will see the progress; now again x axis is the number of generation, y is the best fitness. Since R value was large, this is the change we can see that even before 20 generation or close to 15 generation, the optima was find by the static penalty approach with RGE.

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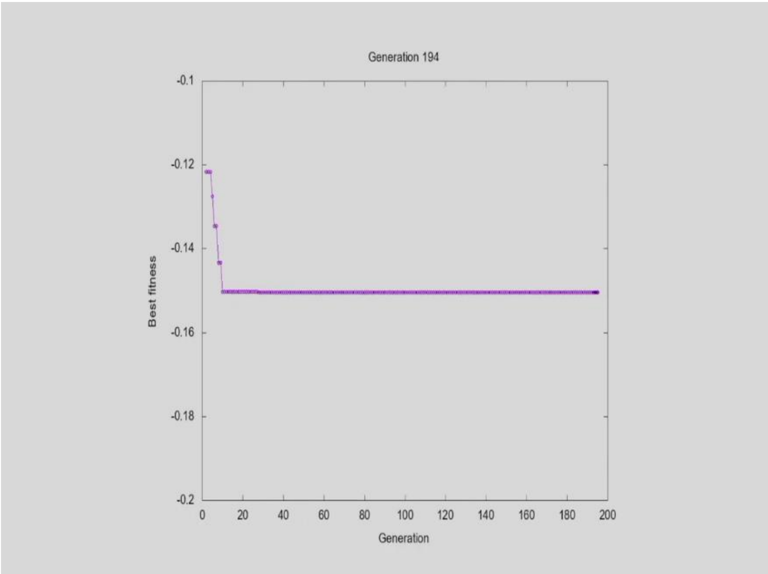

Now, as can be seen that close to 10 generations; the optima, optimum solution was find. It is only because the infeasible solutions were penalized heavily and therefore, the solutions what we get all of them are feasible and they search the optimum solution quickly. Now, we will see the simulation for dynamic penalty and as we have understood the penalty will be increasing with the iteration.

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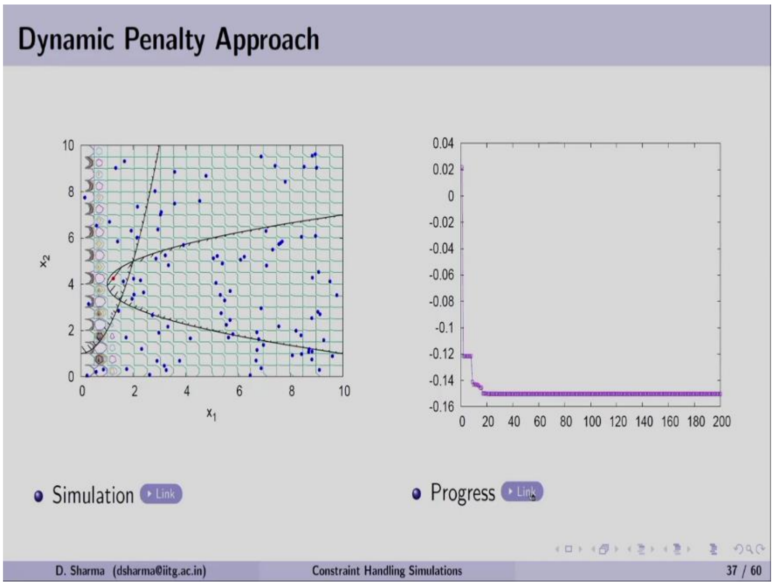

So, we will see the simulation now; here in x 1 and x 2 plane, we are starting with the same solution and let us see how these solutions will be converging to the optimum point.

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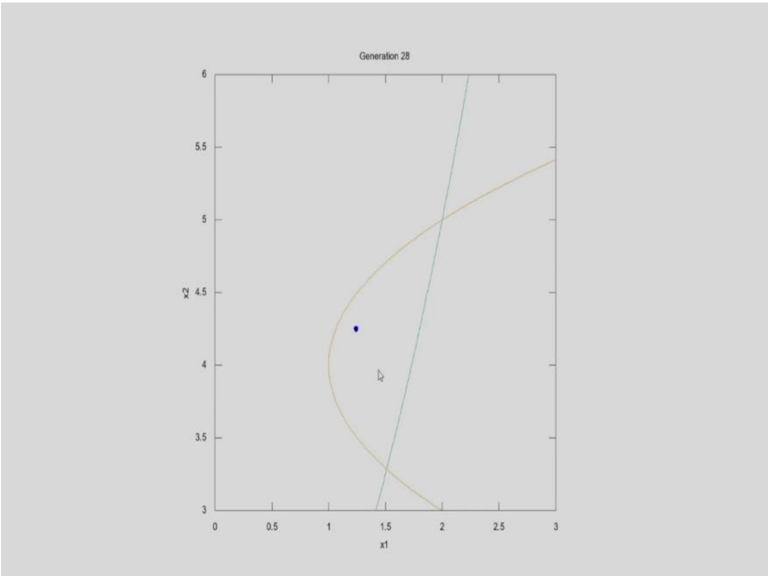

Now, as can be seen that solutions are now already converged in the feasible space and now they have converged to the optimum solution as well. In every generation, the penalty term is increasing; so if any solution which is generated infeasible that solution will get a very large value of a penalty using this dynamic penalty approach.

Now, in this case also the solutions have converged all the solutions of the population have converged to the optimum solution and they are not diverging. So, we have to see the behavior; it should be similar to the static penalty where the R equals to 2 and a 100 and till 150 generation, the points are still at the same point. So, all the members have converged to the optimum solution.

So, let us wait till 200 generations are over; just to make sure that there is no divergence of the solution from the optimum point; so yes we have seen that. Now, let us see the progress; so again x axis is the x axis is the number of iteration, y axis is the best fitness. Again close to 20 number of a generation or even less than that, the optimum point was found by the by this particular penalty approach.

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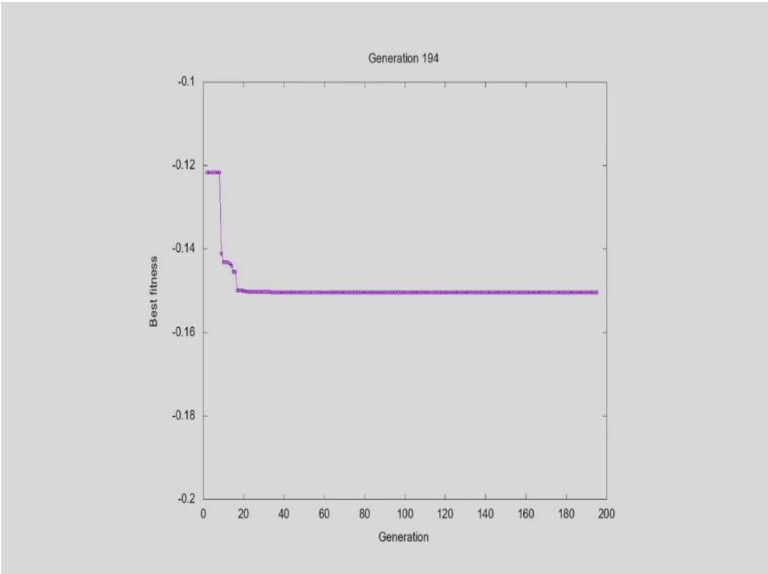

So, as can be seen that almost at the 16th generation the optima was found and the fitness; the best fitness remains constant, once the optimum solution was found. So, let us wait for 200 generation.

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Now, we have come to the last part of the approach for the g08 problem that is Deb's approach and as we understood; this particular approach is penalizing the infeasible solution more as compared to the feasible solution. We will see the simulation now; we are starting with the same set of solutions and let us see how these solutions are converging to the optimum point.

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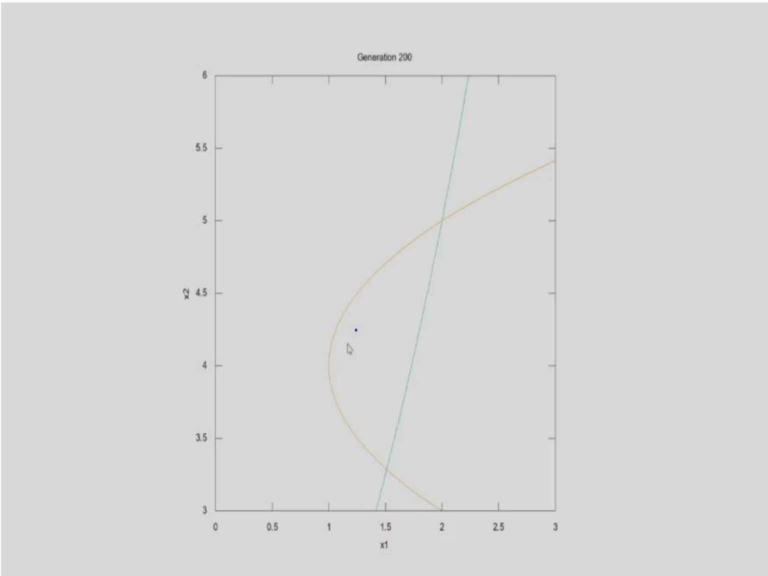

Now, here as you can see the solutions have converged in less than 10 generation. Now, all these solutions are again finding again converging to the local; again converging to the global optimum solution. Now, as we know that this particular approach will never allow to generate any infeasible solution; so this will not diverge. This we have seen with previous examples as well that all the solutions; the feasible solutions having a better fitness than the infeasible solution.

We will just see all these simulation that whether we are going to converge in another area or the members will remain the same. We will wait for 200 generation now; as can be seen that still all the members, they have converged to the same solution that is the optimum solution for the given problem.

Now, in the last generation and as can be seen that Deb's approach was found the same solution; if we look at the progress just before 20 generation, the optimum solution was found by this particular approach.

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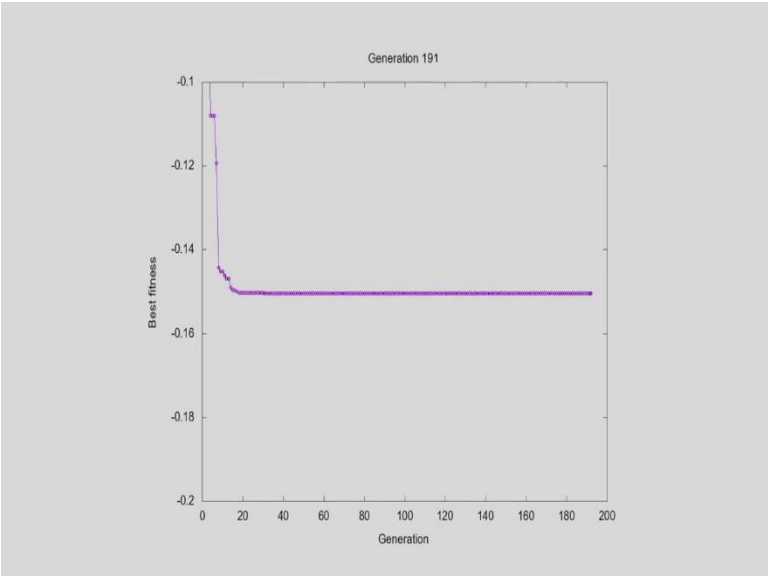

So, we will see the progress now; as we can see just at the 15th generation; we get the optimum solution and that is why the best fitness for the given problem will remain the same throughout the 200 generation.

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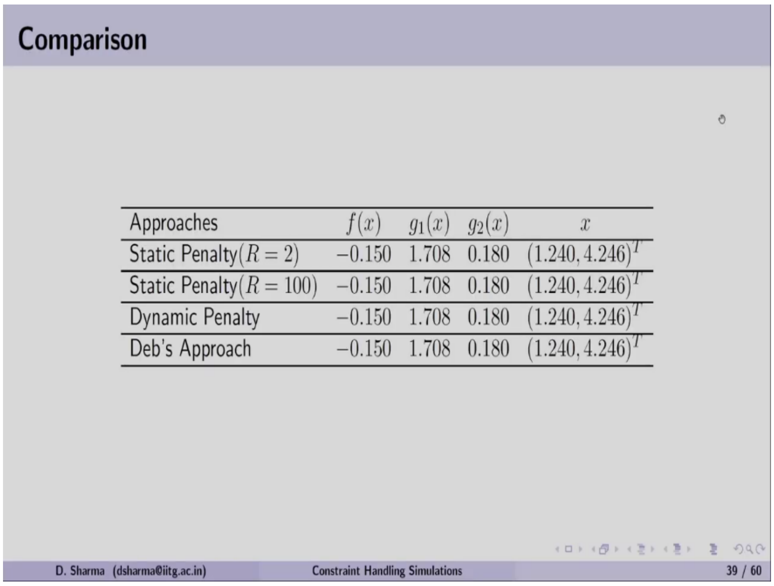

Approaches	$f(x)$	$g_1(x)$	$g_2(x)$	x
Static Penalty($R = 2$)	-0.150	1.708	0.180	$(1.240, 4.246)^T$
Static Penalty($R = 100$)	-0.150	1.708	0.180	$(1.240, 4.246)^T$
Dynamic Penalty	-0.150	1.708	0.180	$(1.240, 4.246)^T$
Deb's Approach	-0.150	1.708	0.180	$(1.240, 4.246)^T$

Now, as we have solved the this particular problem using different approaches, let us compare the solution. If we look into this particular table, here we can see that up to three decimal places; all the methods are giving us the same function value. Looking at the g 1 and a g 2 value; it can be seen that both the constraints are inactive constraints and up to three decimal places, these penalty these constraint handling techniques gave us the same solution.

As of now, we have solved five numerical constraint optimization problem for which optima was known to us. Now, we will be solving three constraint optimization problem that are practical problems; these three problems are taken from the literatures. So, in most of the papers these problems are solved and the performance of the algorithm was testis.

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Milling Process Parameter Optimization

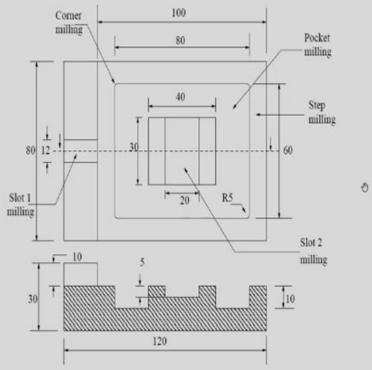

Figure: Final design of workpiece that is manufactured using five milling operations (all dimensions in mm).

So, we will start with the very first problem that is a milling process parameter optimization problem. This problem comes in the domain of mechanical engineering, but we will see this problem as how it is complex.

Let us understand that for making this particular product, we need five processes which are given as slot 1 milling, slot 2 milling, step milling, pocket milling and the corner milling. So, when we are using these five operations; then only we are going to get this final product.

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Milling Process Parameter Optimization

Problem formulation[1]

$$
\begin{aligned}
&\text{Minimize:} && C_u && \text{(Unit cost in \$)},\\
&\text{Subject to:} && && \\
& && C_5 V f^0.8 \leq 1 && \text{(Power constraint)},\\
& && C_6 f^2 \leq 1 && \text{(Surface finish constraint for end milling)},\\
& && C_7 f \leq 1 && \text{(Surface finish constraint for face milling)},\\
& && C_8 F_C \leq 1 && \text{(Cutting force constraint)}.
\end{aligned} \qquad (4)
$$

- Ten real continuous variables, four constraints.

[1] Deepak Sharma and Prem Soren, "Infeasibility Driven Approach for Bi-objective Evolutionary Optimization", IEEE Congress on Evolutionary Computation (CEC), June 20-23, 2013, Cancun, Mexico, pp. 868 - 875.

$$Minimize:\ C_u \qquad \{(Unit\ cost\ in\ \ )\}$$

Subject to:

$$C_5 V\, f^{0.8} \ \leq\ 1 \quad (Power\ constraint),$$

$$C_6 f^2 \leq\ 1 \qquad (Surface\ finish\ constraint\ for\ end\ milling)$$

$$C_7 f\ \ \leq\ 1\ (Surface\ finish\ constraint\ for\ face\ milling)$$

$$C_8 F_C \leq\ 1\ \ (Cutting\ force\ constraint)\ .$$

The problem formulation is defined as; we want to minimize the cost subject to there are four constraints that are on power, surface finish for a one process, surface finish for the another process and the cutting force. If we look it, its mathematical significance; we have basically the tan continuous variable and we have four constraints to solve the given particular problem.

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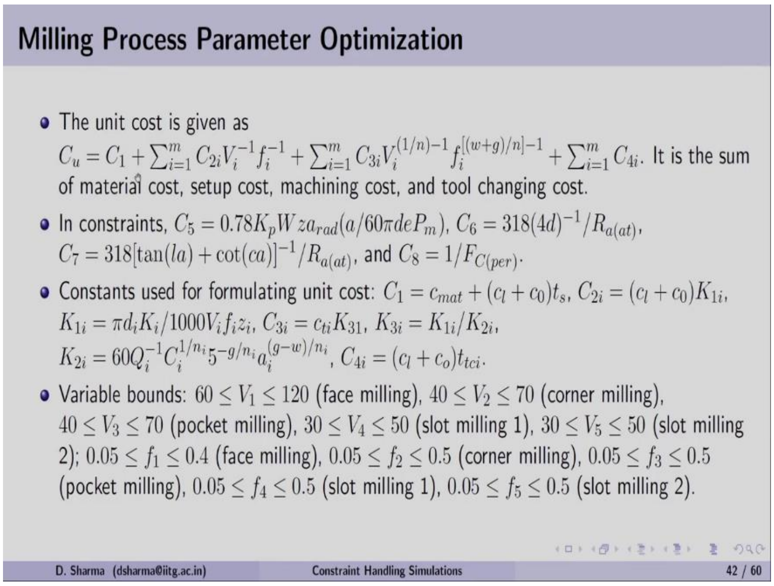

Now, for finding the cost; there are various costs which are associated. As can be seen that V i and f i are the variable for the given problem and looking at their form; we can see the cost function is a non-linear function. Similarly, there are certain constants like C 4 to C 8 are used for cost as well as in the constraints.

These constant for example, this C 1 constant to C 4 i; all these constants are used for the cost function. Now, we can see the tan variables for example, V 1, V 2, V 3, V 4; so this is the speed and f 1, f 2, f 3 and f 4 and f 5 these are the field. So, basically we have 10 variable and we can see the range of each and every variable at the bottom.

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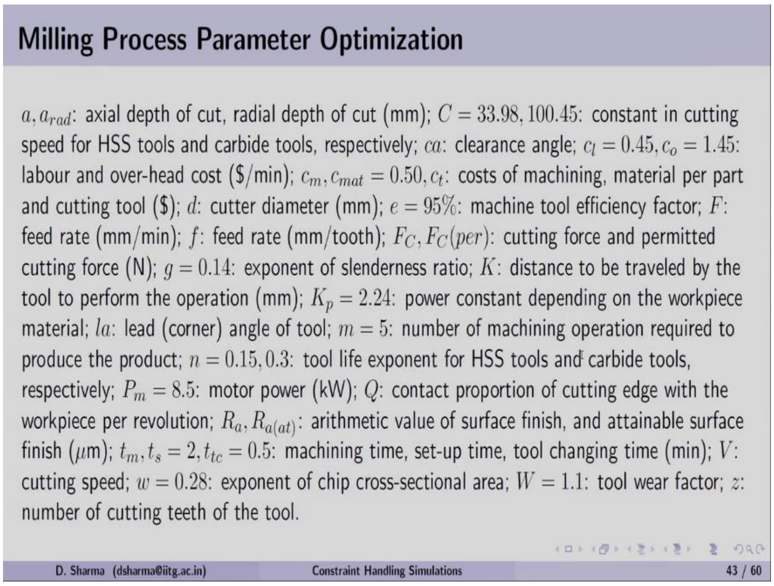

These are the some constant that are needed for calculating the cost; as well as the constraints.

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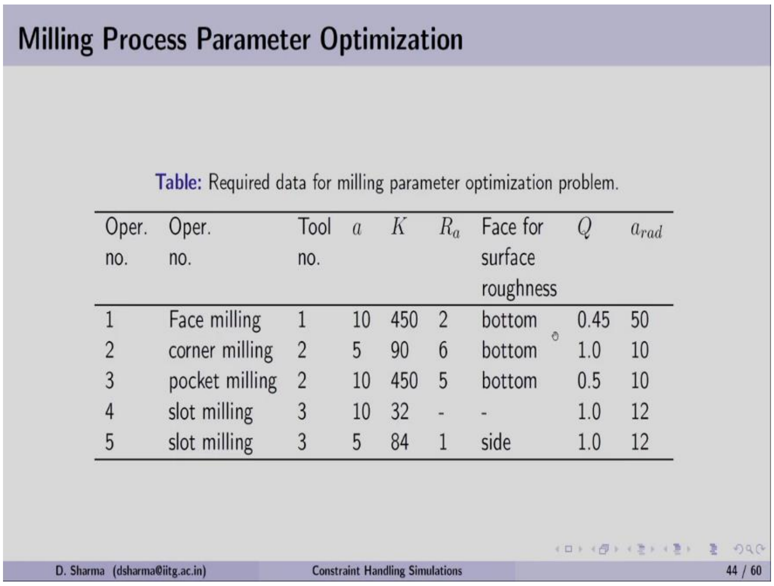

Some data is also required for the; for this particular process called milling and all the data related to the tool are given here.

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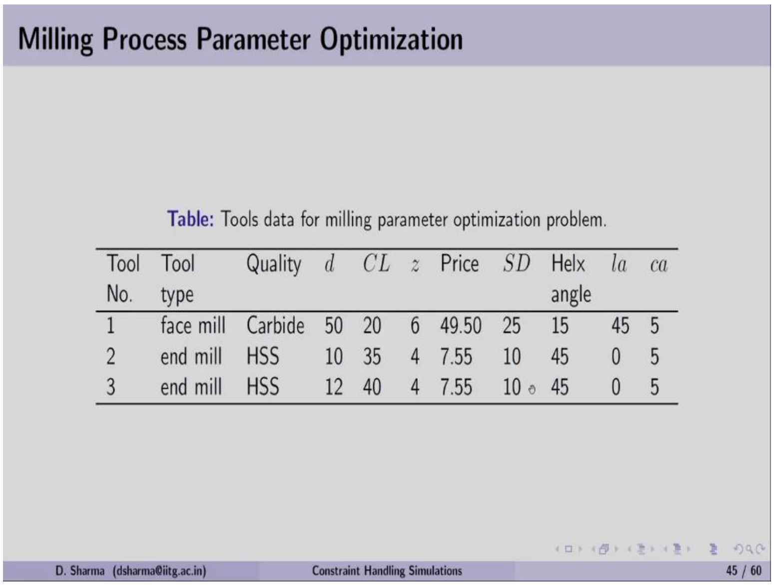

Tool No.	Tool type	Quality	d	CL	z	Price	SD	Helx angle	la	ca
1	face mill	Carbide	50	20	6	49.50	25	15	45	5
2	end mill	HSS	10	35	4	7.55	10	45	0	5
3	end mill	HSS	12	40	4	7.55	10	45	0	5

Similarly, when we have the tool like the material is given here; for them the other data, the price, the angle everything is given that will be needed in the formulation.

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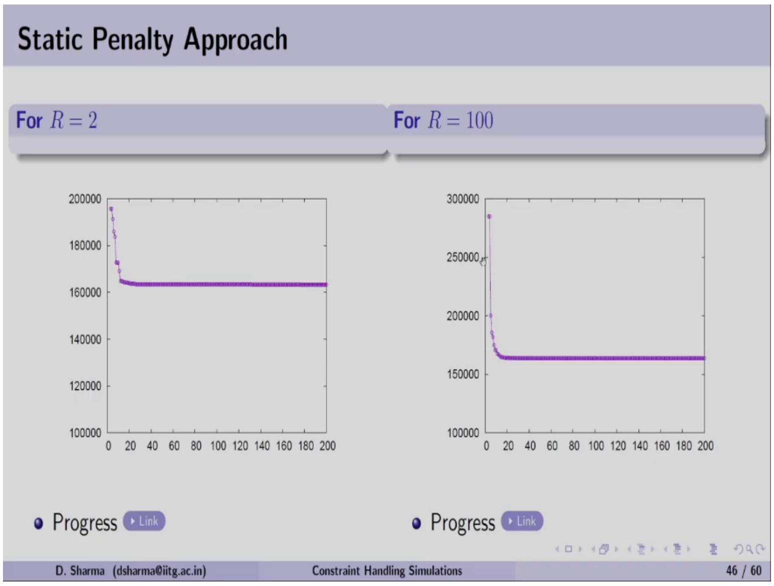

Let us see the solution using this static penalty with R equals to 2. Now, here as we know the problem is tan variable; so we cannot show the simulation; however, we will see the progress.

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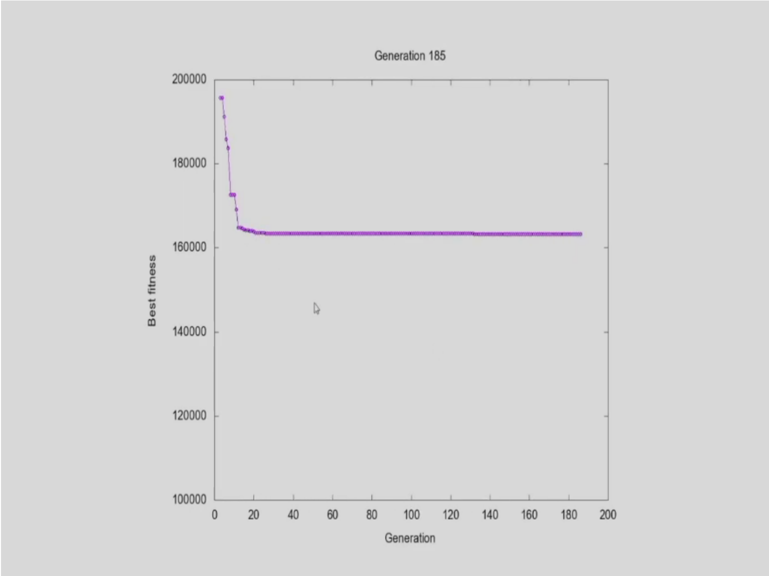

So, as we can see close to 20 or close to 30 generation the this particular approach has find the solution for the given problem. Now, since it is a practical problem; engineering optimization problem, so we do not know what is the best solution only what we have is the best known solution so far in the literature.

So, as can be seen close to the 30 generation; it has find there are some improvement at the later phase, but those that particular solution or solution corresponding to the best fitness was found. Now, in this case if we increase the value of a R which is now 100; we can see there is a change in the convergence, it is only because we are penalizing the infeasible solution more.

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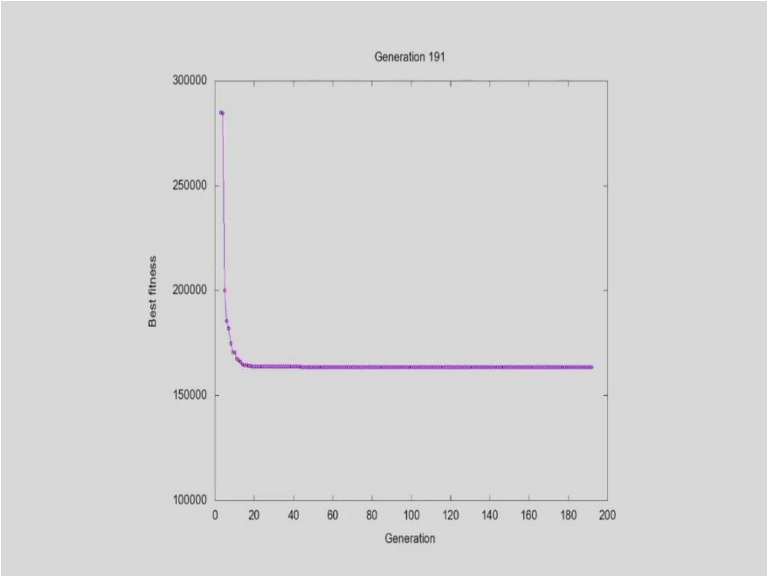

We will see the progress now; as can be seen that closed to 16 generation, we have we found a solution and at that particular solution; the fitness is not improving. So, according to R equal to 100; that is the solution which is best for the or the optimal for the milling process parameter optimization.

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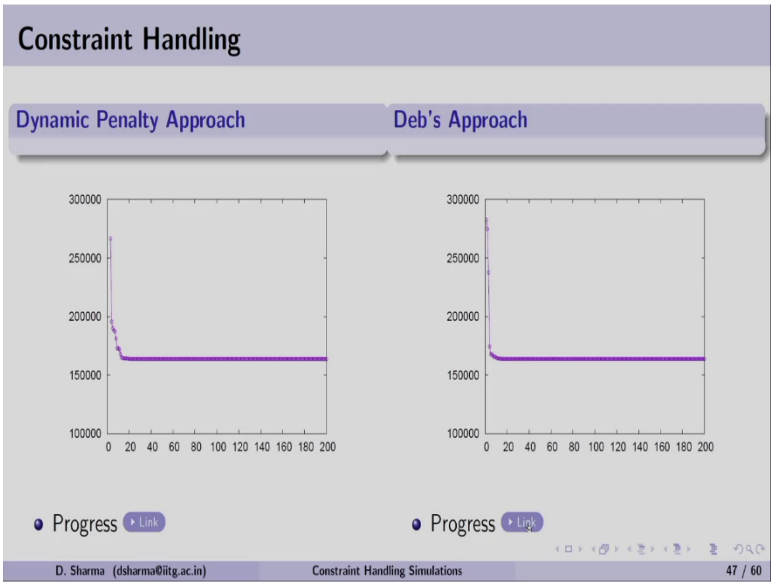

Now, we have a dynamic penalty approach; as we can see in the progress again close to 20 generation, it has converged to the solution found by the found by this particular approach.

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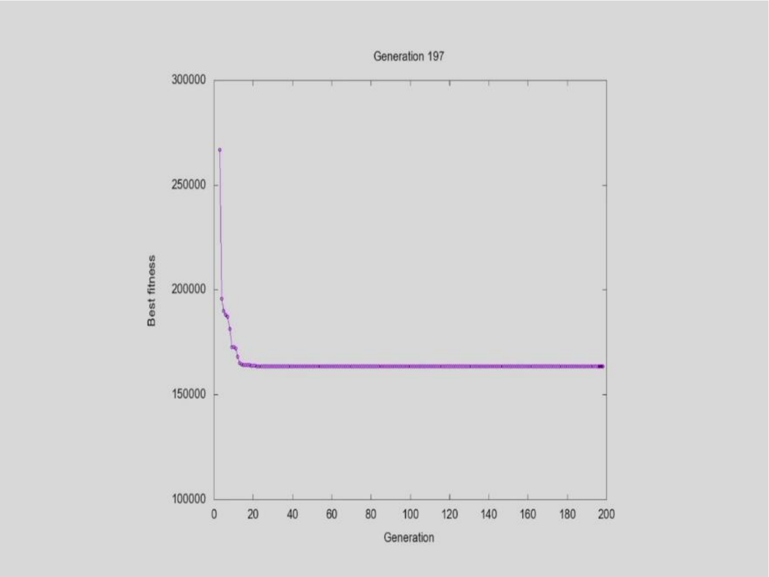

Now, here we can see that close to 15 and 16 generation; we find the solution by this particular approach when we have R equals to 100 which is a large value and it is a same fitness throughout the 200 generation. Now, we have come to the Deb's approach; now in Deb's approach also just close to 200 generation, a solution was found.

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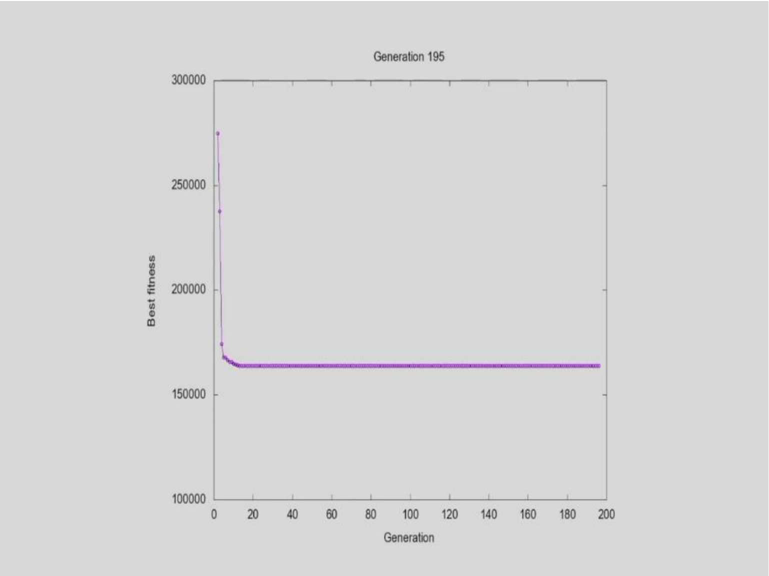

Now, it can be seen here that almost close to 10th generation; the solution was found by Deb's approach and the fitness remains the same throughout 200 generation.

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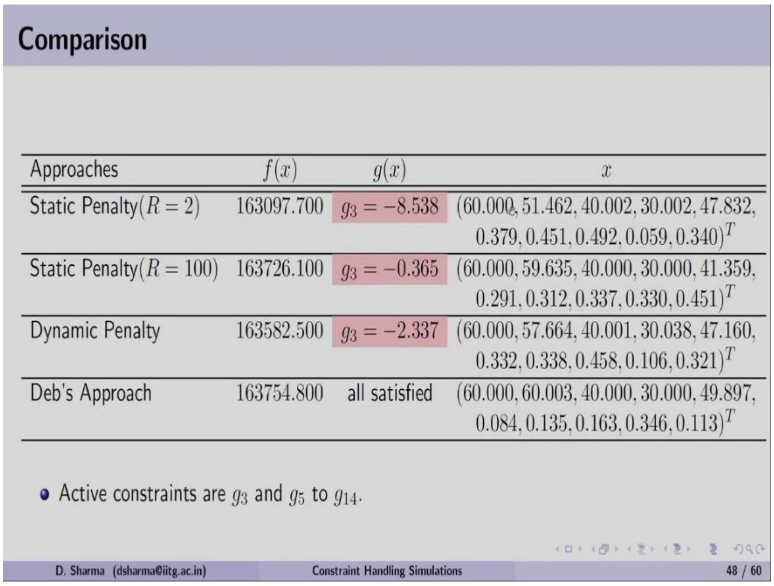

So, this constant there is a the fitness remains the constant till 200 generation now. Now, if we compare their numerical values here; so as we have done the simulation for static value R equals to 2; 100, dynamic penalty and Deb's approach; since there our many constraints, so here we are showing only the constraint which is not satisfied. For static penalty as well as dynamic penalty, the constraint g3 was not satisfied.

The corresponding x vector is given on the last column, but when we use the Deb's approach; all the constraints were satisfied and that is why this function value what we found is the best known so far in the literature. Similarly, if we look at the other constraint; we found that g 3 and from g 5 to g 14; all these constraints were active when solving the problem using Deb's approach.

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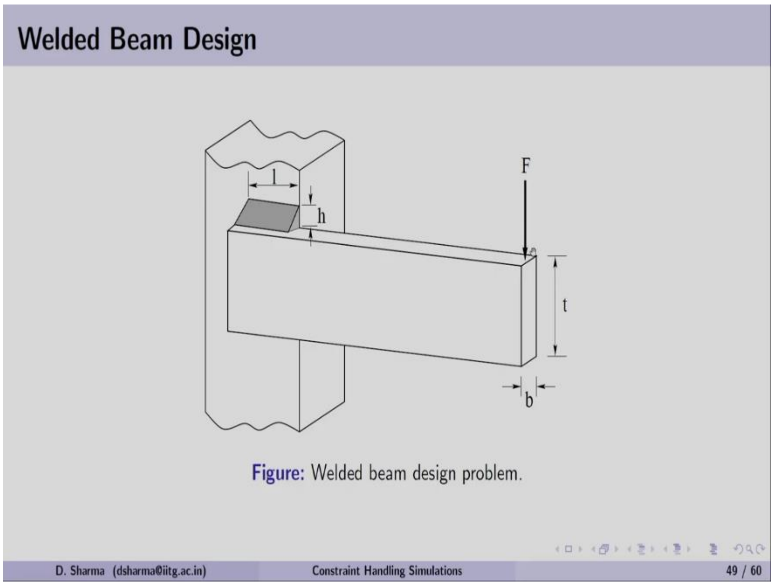

Now, we have come to the second problem; that is the second engineering optimization problem which is also solved in the literature very frequently. This problem is called as welded beam design problem; here we can see there is a vertical component and a horizontal component; these two component are joined together by the weld. Now, this grip area is the weld; it has certain length, it has height. The horizontal component there are some forces applied and it has certain dimension.

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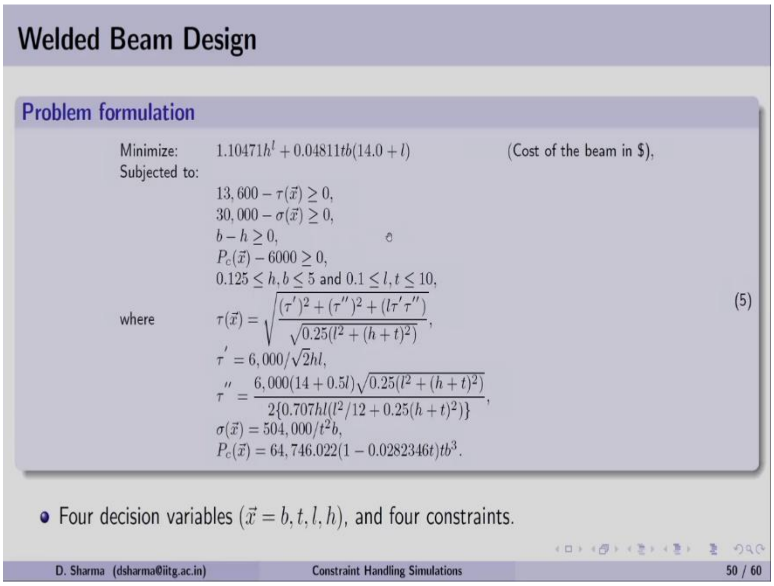

$$Minimize: \quad 1.10471h^{l} + 0.04811tb(14.0 + l) \quad (Cost\ of\ the\ beam\ in\ ),$$

Subjected to:

$$13{,}600 - \tau(\overrightarrow{(x)}) \geq 0,$$

$$30{,}000 - \sigma(\overrightarrow{(x)}) \geq 0,$$

$$b - h \geq 0,$$

$$P_c(\overrightarrow{(x)}) - 6000 \geq 0,$$

$$0.125 \leq h, b \leq 5 \quad and \quad 0.1 \leq l, t \leq 10$$

$$where \quad \tau(\overrightarrow{(x)}) = \sqrt{\frac{\left((\tau')^2 + (\tau'')^2 + (l\tau'\tau'')\right)}{\left(\sqrt{0.25(l^2 + (h+t)^2)}\right)}}$$

$$\tau' = \frac{6{,}000}{\sqrt{2}hl}$$

$$\tau'' = \frac{6{,}000(14 + 0.5l)\sqrt{0.25(l^2 + (h+t)^2)}}{2\{0.707hl\left(\frac{l^2}{12} + 0.25\,(h+t)^2\right)\}}$$

$$\sigma(\overrightarrow{(x)}) = \frac{504{,}000}{t^2 b}$$

$$P_c(\overrightarrow{(x)}) = 64{,}746.022(1 - 0.0282346t)tb^3$$

So, the problem is defined as; we want to minimize the cost of the weld which is given as the objective function. Similarly, when we are designing it; the problem is subjected to various constraint. So, the two constraint is based on the stress meaning that the load which is applied should withstand by the weld; it should not break.

The third constraint is based on the dimension and the fourth constraint is based on the force apply. So, the fourth constraint is applied based on the force applied. Here we can see h and b are lying between the range of 0.125 to 5 and l and a t having other range; looking at the value of a tau; which is calculated with respect to tau prime, double prime and tau prime and double prime; so it has numerator and denominator and everything is under the root.

So, these equations signify that we have the objective function which is non-linear, as h to the power l we can see one term here. Similarly, the constraints are also are also non-linear. Looking at the problem, we have four design variables which are given as b, t, l and h and there are four constraints as well.

So, meaning that the problem has to be designed in a such a way that we want to minimize the cost; it should withstand the force F. So, what should be l, h, t and a b combination? That will be minimizing the overall cost of the weld.

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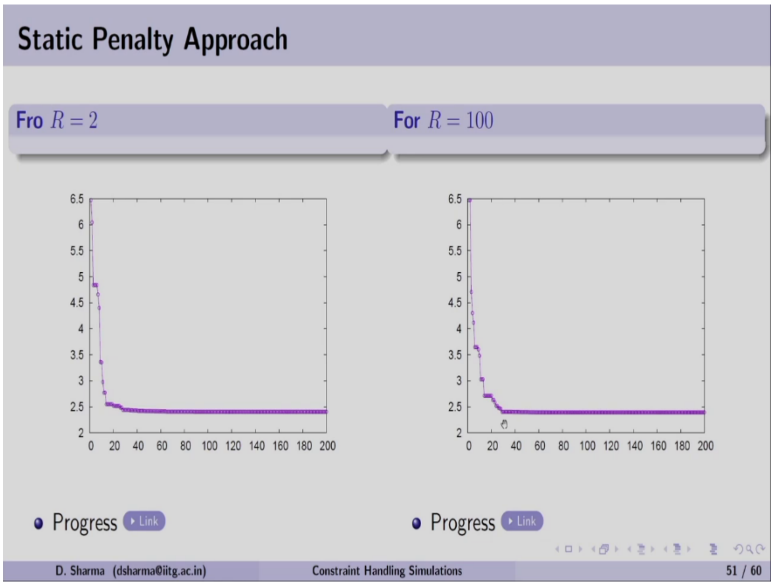

Let us see the simulation for the static penalty with R equals to 100.

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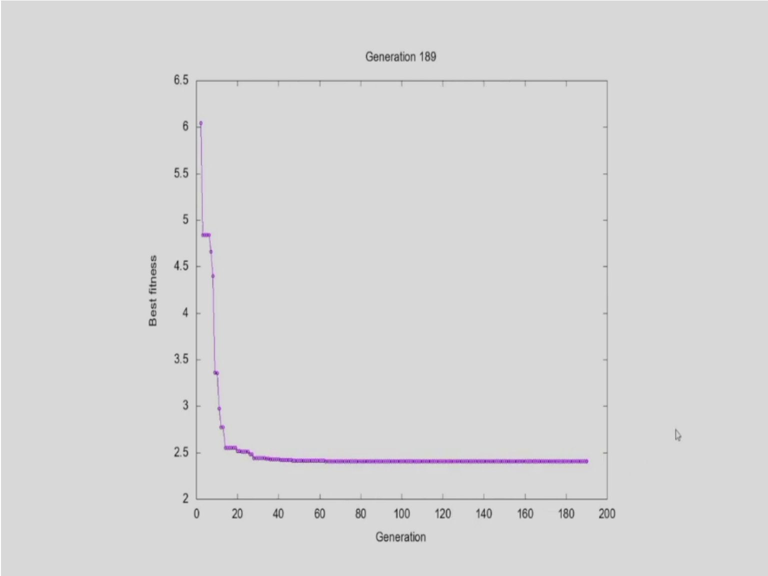

Now, as it can be seen that close to 20 generation; now it is 29 generation. We found a solution which is showing the best fitness and that solution remains the same till 200 generation. Now, if we increase the value of R; now, that we have to see that it is starting with large value and then close to again 30 gen number of generation; we find the best fitness.

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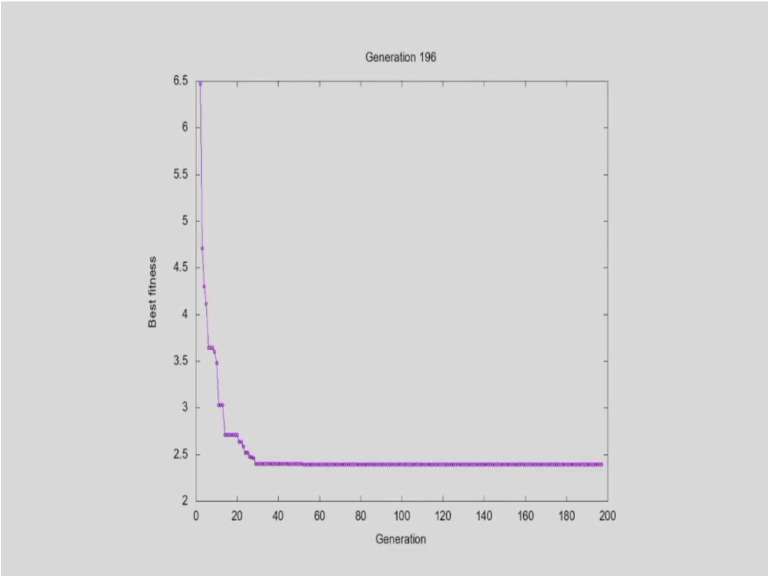

Now, here we can see that the fitness is improving and close to 30 gen number of generations; the we find a solution which is having the best fitness for the given problem and that fitness will remain the same till 200 generation.

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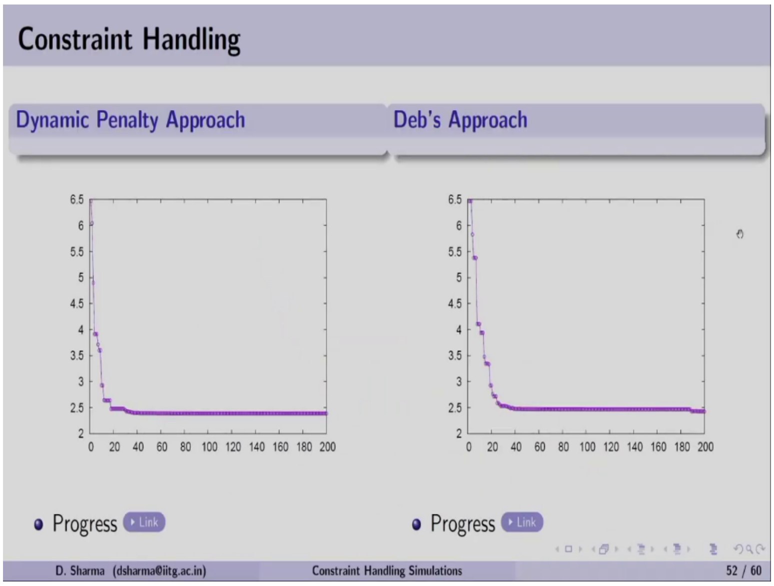

We are going to solve the same problem using dynamic penalty approach. In this particular approach, as we remember the penalty will be increasing with the number of generation.

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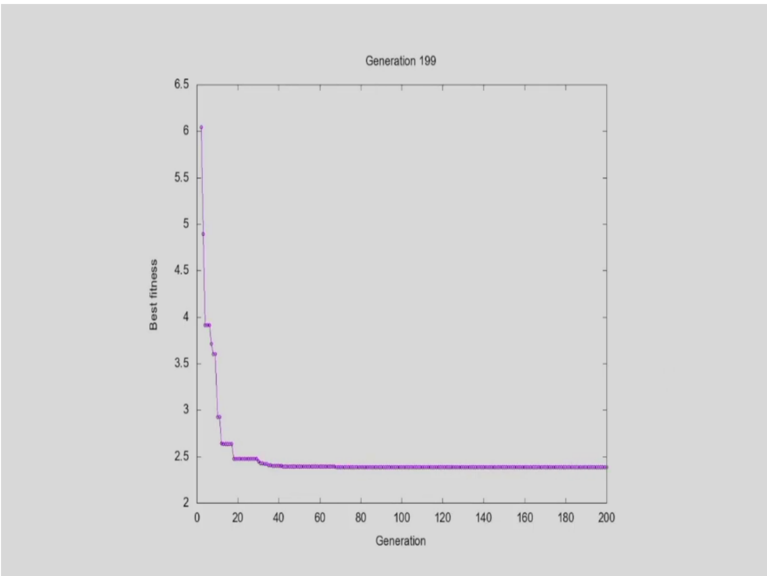

So, here we will see that the fitness is improving and close to 31 and 32 generation, the solution was find which was showing the best fitness and still the fitness remains the same till 200 number of generations. At last, we are going to solve the same problem using Deb's approach.

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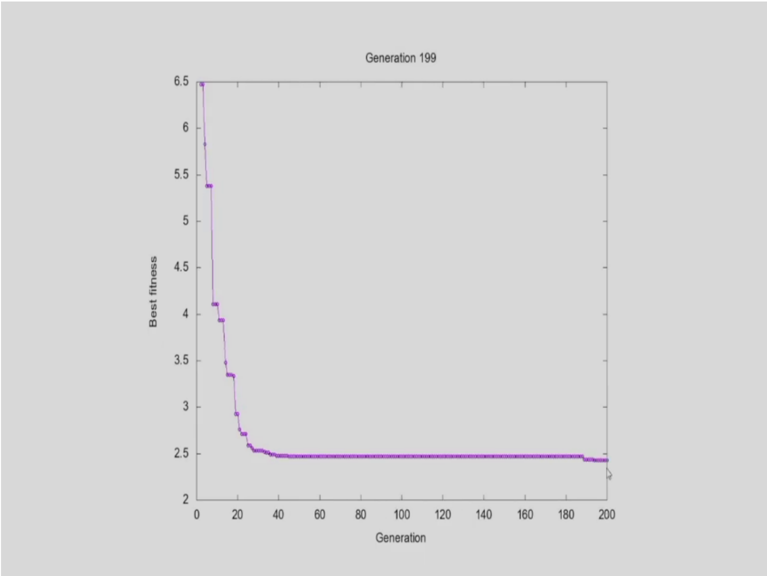

Now, here the fitness is improving and close to 35 generation; the fitness this particular approach found a solution for which the fitness is not improving. And we have to see at the last there is a little improvement that we can see finally to get solution which is the best having the best fitness. And the same thing we can find in the progress chart at the last; there it was a little improvement in the fitness.

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Comparison

Approaches	$f(x)$	$g(x)$	x
Static Penalty($R = 2$)	2.942	0.128, 3.012, 0.003, 12773.410	$(0.374, 0.377, 4.533, 6.677)^T$
Static Penalty($R = 100$)	2.783	0.147, 10.666, 0.008, 8034.875	$(0.329, 0.337, 5.034, 7.059)^T$
Dynamic Penalty	3.939	0.162, 22.813, 0.044, 63432.780	$(0.579, 0.623, 3.346, 5.194)^T$
Deb's Approach	2.426	16.499, 3552.643, 0.005, 0.099	$(0.236, 0.240, 5.987, 8.902)^T$

- The active constraint is g_3 and g_4.

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Let us compare their values; now here we can see all the approaches gave us the feasible solution. The best solution is given by Deb's approach which is having the function value as 2.426. Now, looking at the constraints; all four constraints are written here for our reference; as we can see none of them is negative, all of them are positive; so the all constraints are satisfy. So, the solution which we get on the last column is a feasible solution.

Since Deb's approach gave us the optimal solution for the given problem; so we found that constraint 3 and constraint 4 are the active constraint for the given problem. Now, we have come to the last engineering optimization problem that come from the domain of civil and mechanical engineering, this problem is known as two bar truss problem.

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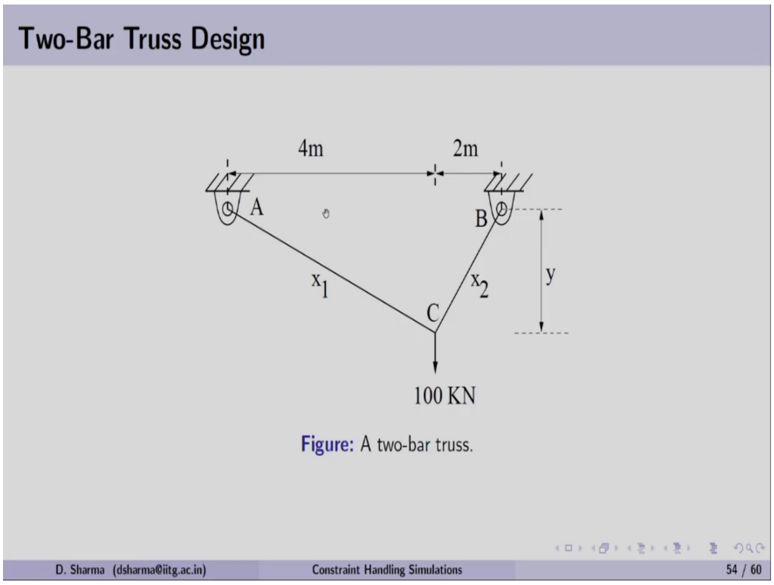

As we can see in this particular figure, there are two bars x 1 and x 2 length and there is a y length as well and the load of 100 kilo Newton is applied. Dimension between or distance between A and a B are given as 4 meter and 2 meter.

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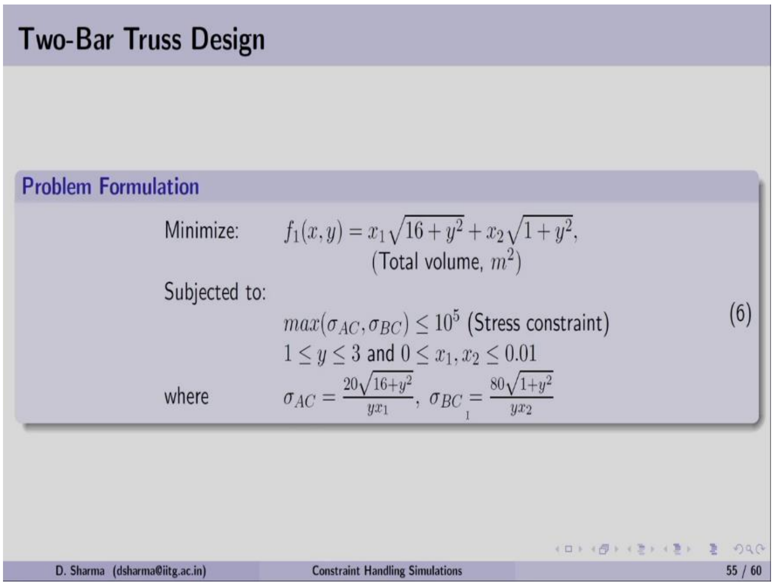

$$Minimize: \quad f_1(x,y) = x_1\sqrt{16+y^2} + x_2\sqrt{1+y^2}$$

$$(Total\ volume, \quad m^2)$$

$$Subjected\ to:$$

$$max(\sigma_{AC}, \sigma_{BC}) \leq 10^5\ (Stress\ constraint)$$

$$1 \leq y \leq 3\ and\ 0 \leq x_1, x_2 \leq 0.01$$

$$where\ \sigma_{AC} = \frac{20\sqrt{16+y^2}}{yx_1}, \qquad \sigma_{BC} = \frac{80\sqrt{1+y^2}}{yx_2}$$

It is a simple problem; we have to see how these constraint handling methods will work. Here we want to minimize the function which is assisted forward equation in x; x 1, x 2 and y. And here we are making sure that; so we have a one constraint that make sure that the particular two bar truss should have a sufficient strength so that it can withstand; it should be within the allowable stress. The range of y, the range of x 1 and x 2 are given to us; here the sigma AC and sigma BC; these values we are calculating using these equations.

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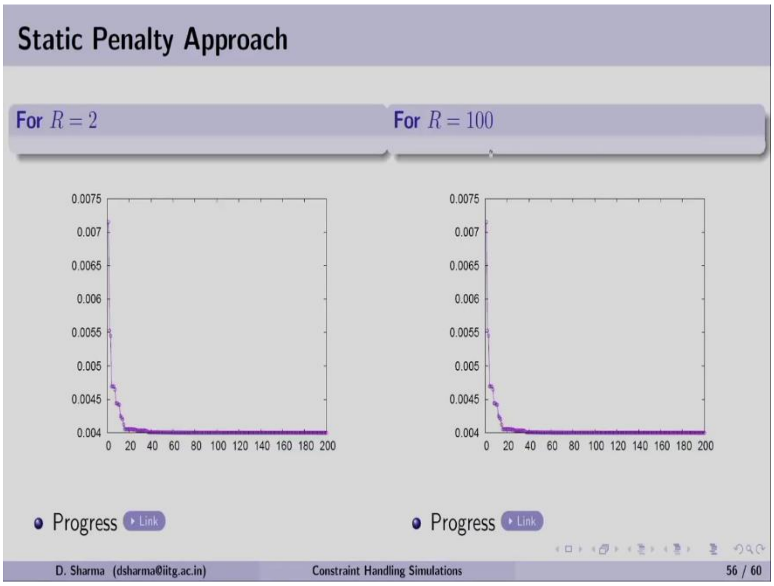

Now, the problem is a three variable problem; we have to see the progress. As we can see which is already starting with the very small value and just before 40 generation, R equal to 2; the constraint handling method found the optimal solution; so we have to see that.

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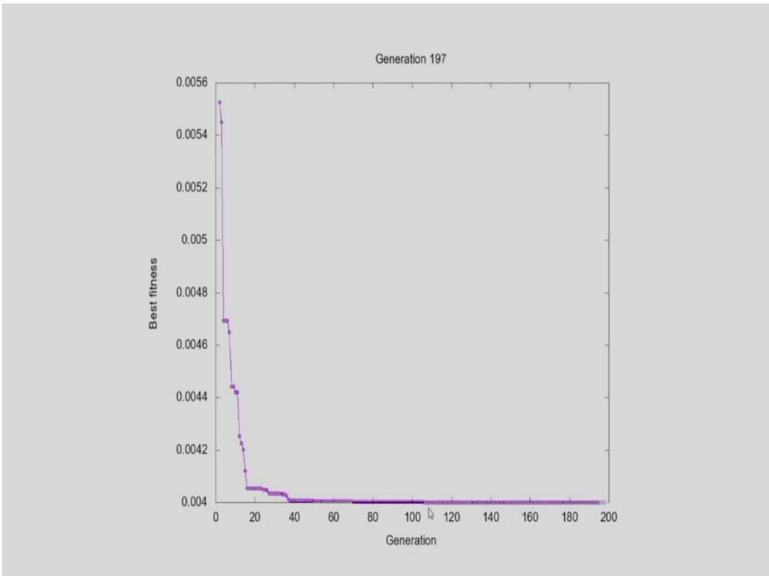

So, as we can see; the fitness is keep on improving and almost at the 30th generation; now there is a little improvement at 17th 70th generation and afterwards there improvement was seen close to 110 generation. So, that small improvement can be seen in the fitness and the solution which was found at 110th generation that was remain the same. We will see the performance when we increase the value of R to 100.

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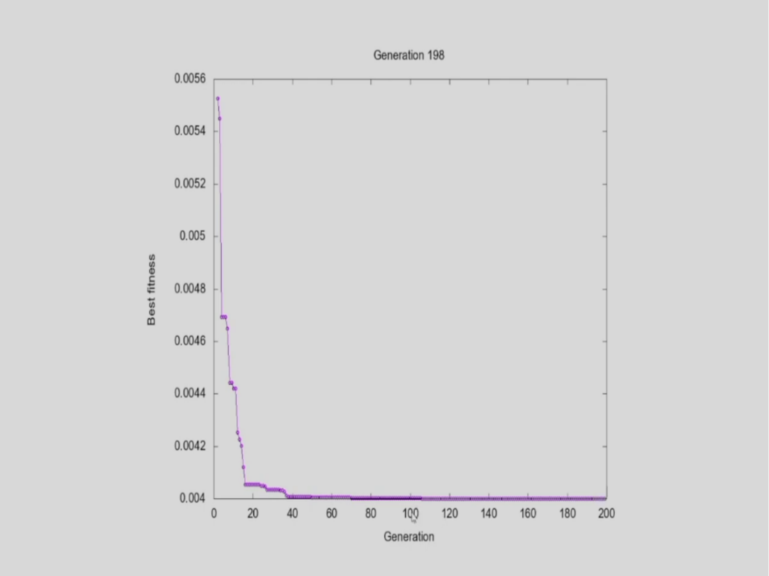

In this case, the fitness is reducing; we have to see how much the fitness is reducing. So, there was around 38, there was a small change. Then close to 70 generation, then 110; meaning that for the value of R equal to 2 and R equal to 100; the performance of both the static penalty was found to be the same.

The only region reason could be that since the fitness value, the best fitness value that is in very small range which is close to 0. So, this R equal 2 and 100 are sufficient to penalize the to penalize the infeasible solution and therefore, the solutions that are going from one generation to the another generation, all of them are feasible solution.

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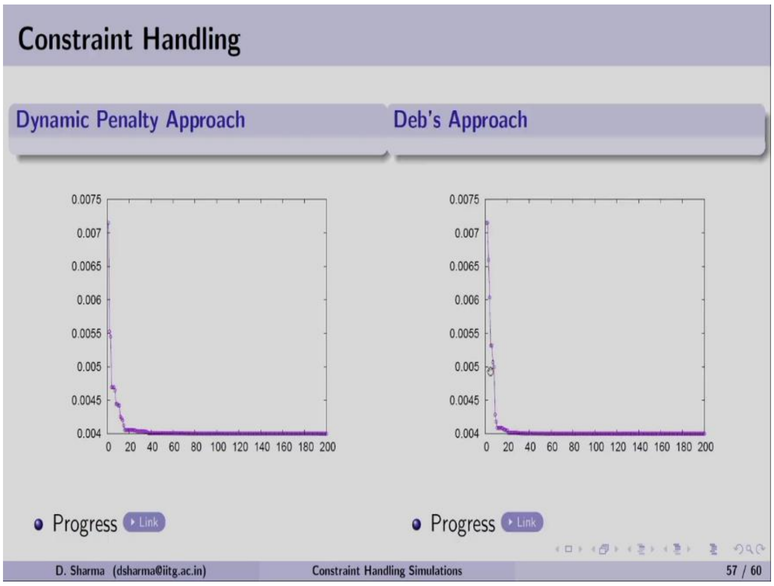

Now, we will test the same problem on dynamic penalty approach here.

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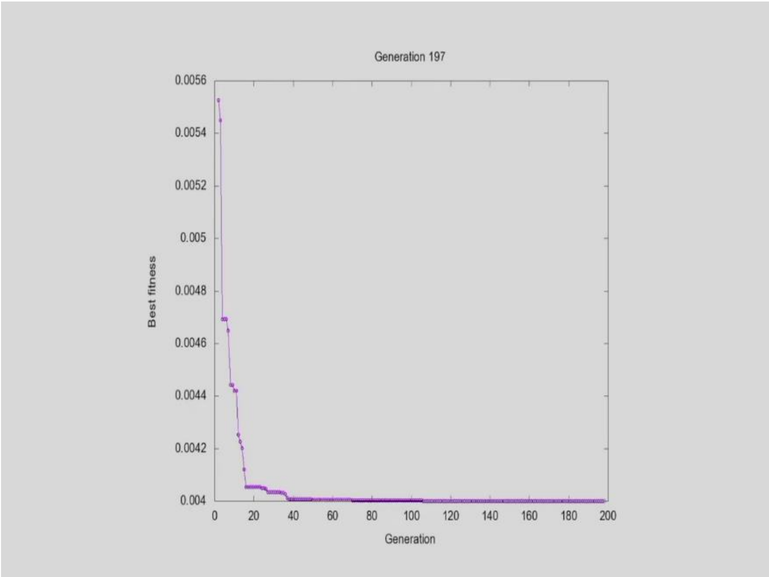

This particular penalty approach; we will see the fitness is improving and close to 38; again it has improved, now close 70; it has improved and now close to 110. So, the same simulation or the behavior we found with the dynamic approach and the reason remains the same that since the objective function value is very small; so therefore, these small penalties at the beginning is sufficient to penalize the infeasible solution.

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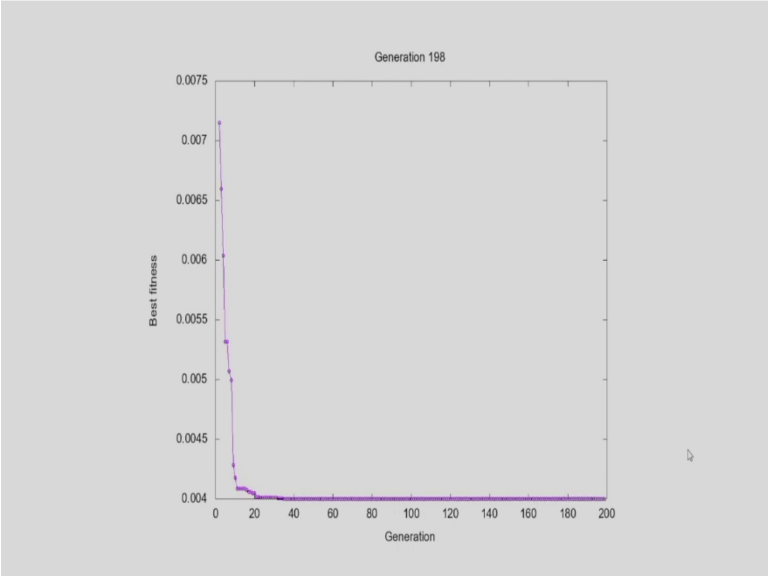

Now, we will look at the Deb's approach at the last. Here we can see that almost close to 33 35 generation; all the solution the this particular approach has found the best fitness solution for the given problem. So, that small change we can see with the Deb's approach; it is only because it give more emphasis to the feasible solution than the infeasible solution.

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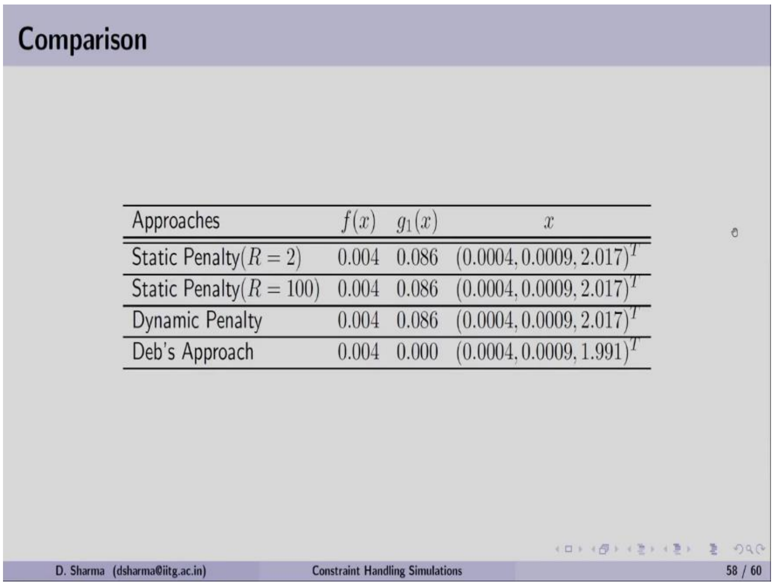

Comparison

Approaches	$f(x)$	$g_1(x)$	x
Static Penalty($R = 2$)	0.004	0.086	$(0.0004, 0.0009, 2.017)^T$
Static Penalty($R = 100$)	0.004	0.086	$(0.0004, 0.0009, 2.017)^T$
Dynamic Penalty	0.004	0.086	$(0.0004, 0.0009, 2.017)^T$
Deb's Approach	0.004	0.000	$(0.0004, 0.0009, 1.991)^T$

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Now, let us compare their values; numerical values here. So, here we can see up to three decimal places, all these approaches found the same solution. Similarly, if we look at the constraint; as can be seen that the constraint is active for us. Looking at the value of x 1, x

2 and a y; as we can see the value is quite small for x 1 and x 2; y value there is a certain value we can see.

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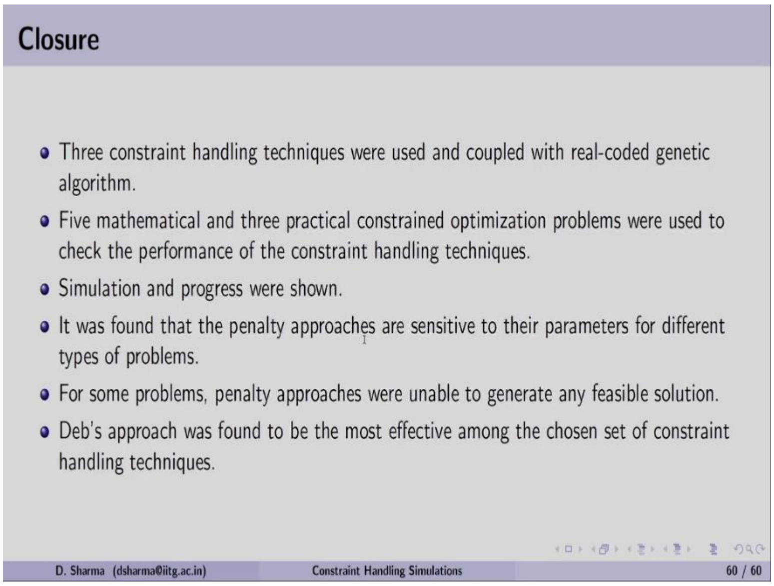

Now, we have come to the closure of this particular session on constraint handling techniques. In this particular lecture, we have gone through three constraint handling techniques that were used and coupled with real coded genetic algorithm. We have solved five mathematical and three practical optimization problem so that we can see the performance of static, dynamic and Deb's approach on solving such kind of problems.

Simulations and progress were shown. Now, it was found from our analysis that the penalty function methods or penalty approach is sensitive to the values of R or value of a c. It is because it is because these values sometimes may not be able to penalize the infeasible solution sufficiently so that, we can differentiate the objective function.

Also, in some of the problems; these approaches were find the infeasible solution, it is because the infeasible solution was having better fitness than the feasible. So, therefore, unless and until we are not able to penalize the infeasible solution sufficiently, our penalty approach or constraint handling techniques will always be sensitive towards the problem.

And at the last we have found that Deb's approach was the best among the chosen set and in every case for all the 8 problem; Deb's approach found the optimal or close to optimum solution for the given problem. From the theoretical problems, we can say that Deb’s approach found the optimum solution in all of the cases.

But for the engineering optimization problem, we can say that the Deb’s approach was the best among the chosen set of constraint handling technique. With this simulation and understanding the behavior of various constraint handling techniques; I conclude this session.

Thank you.