Evolutionary Computation for Single and Multi-Objective Optimization
Dr. Deepak Sharma
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 14
Constraint Handling with Evolutionary Computing Techniques

Welcome to the session on Constraint Handling with Evolutionary Computation. In this particular session, we will target two types of ways in which we can handle constrained in our Optimization problem.

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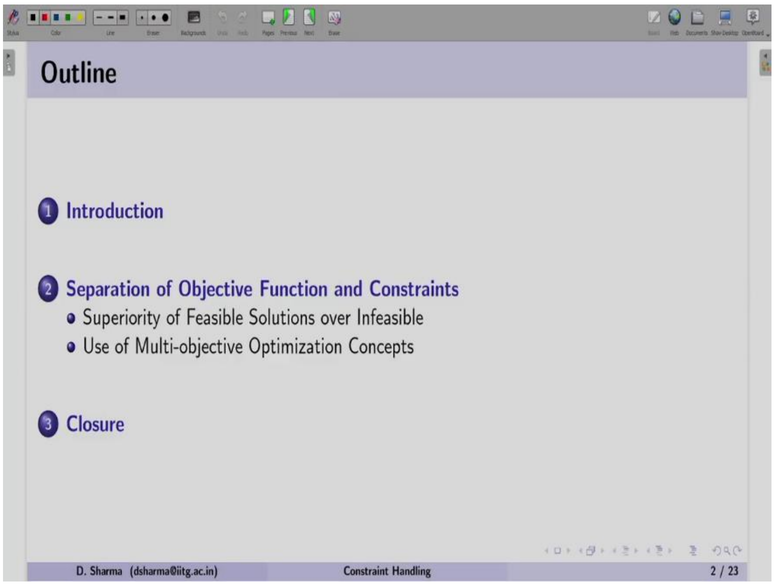

So, one of the way is to handle the constraint is to separate objective function and constraints and deal those constraint with them. The second method is using the concept of multi-objective optimization. So, in this session, we will start with these two approaches and then we will close this session.

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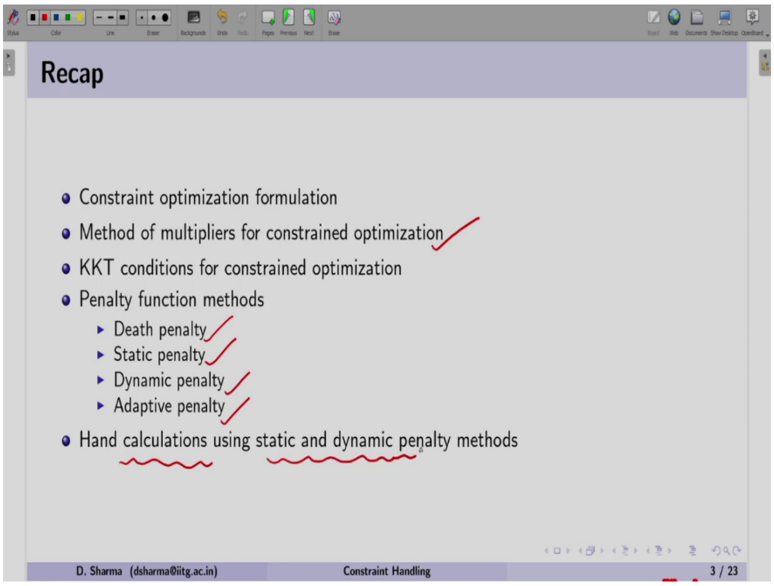

So, as a recap till now what we have covered in a constraint optimization. We started with a constraint optimization formulation which consist of an objective function subjected to inequality constraint, equality constraint, and the variable bounds. There after we discuss the methods of multipliers for constrained optimization.

In this particular method, we converted a constraint optimization problem in to an unconstrained optimization problem by using Lagrangian multiplier. So, for both equality and inequality constraints, we find the optimality condition for that unconstraint problem using Lagrangian multiplier.

There after we also discuss the Karush Kuhn Tucker condition in which is also known as KKT condition, that KKT condition we have written for an optimization or a constraint optimization problem having both type of constraint that is inequality and equality constraints. There after we discuss about the one of the famous method or in which that is called penalty function methods.

In this penalty function method. We have gone through various kinds of method such as death penalty static penalty dynamic penalty and adaptive penalty. So, we have gone through various kinds of penalty function methods; we perform the hand calculations using static and dynamic penalty, so that we can understand how these penalty function methods can be used with the constraint optimization with EC techniques.

Now, let us move to the another way to handle the constraint. So, before we begin let us have an introduction again.

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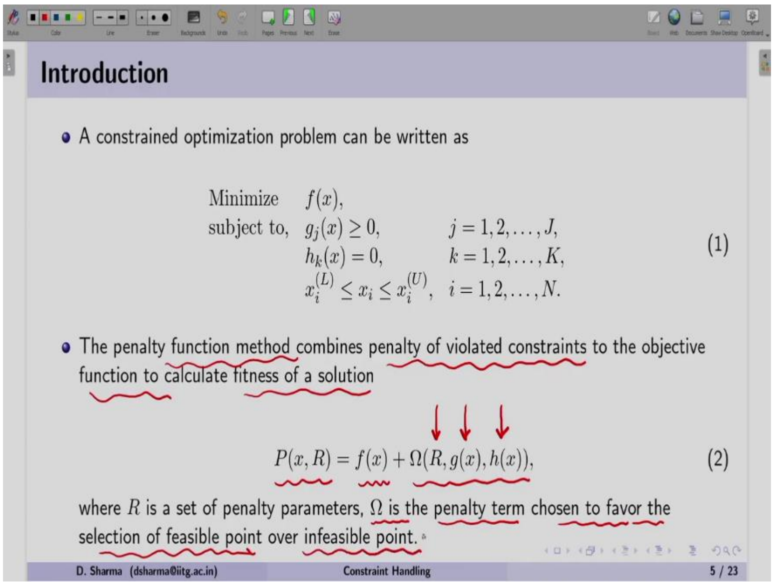

In this particular introduction, we will see that a constrained optimization problem can be written as we want to minimize an objective function and this objective function is subjected to various constraint. As can be seen in equation number 1, we can have inequality constraints, we can have equality constraints and the problem is having the variable bounds. So, this is typically a constrained optimization problem that we can write in a generalized form.

$$Minimize \quad f(x),$$

$$subject\ to, \quad g_j(x) \geq 0, \quad j = 1, 2, \ldots, J,$$

$$h_k(x) = 0, \quad k = 1, 2, \ldots, K,$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad i = 1, 2, \ldots, N.$$

$$P(x, R) = f(x) + \Omega\left(R, g(x), h(x)\right)$$

In the previous session, we have understood a penalty function method in which we combine penalty of violated constrain to the objective function to calculate the fitness of a solution. In this case, the penalty function method which is given in a in equation number 2 as P of x R equals to the objective function plus the penalty term.

In this particular penalty term, it is made of the penalty parameter called R, we can have inequality constraint as well as we can have equality constraint. So, in this particular case, we find this omega which is the penalty term, we choose this term in order to favor the selection of feasible point over infeasible point and that is the whole objective here, that whenever when we are working with EC techniques we have multiple solutions.

At certain stage, we can have feasible solutions as well as infeasible solution. So, how we can differentiate them or how we can assign a fitness to them? So, in this case, when we are adding a penalty to infeasible solution, so that fitness always favor the feasible solution over infeasible solution as and when we perform selection or the survival stage. Now, let us move to the new way to handle the constraint handling that is separation of objective function and constraint.

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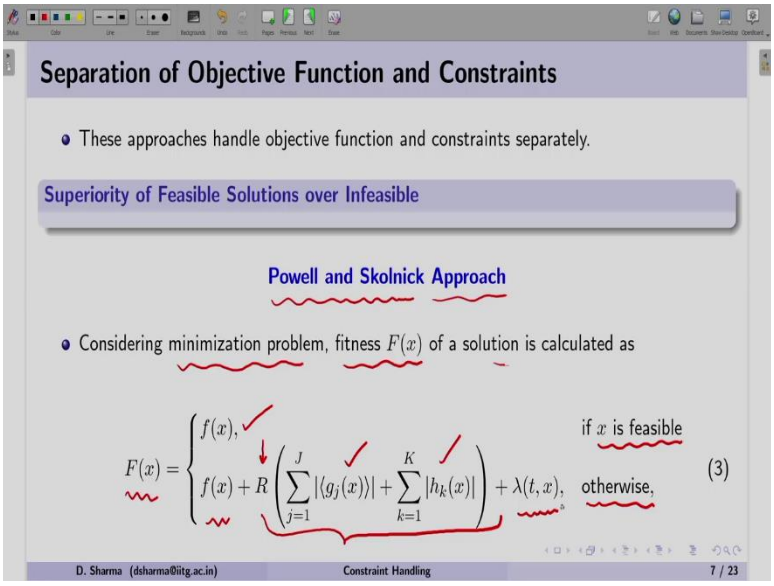

$$F(x) = \begin{cases} f(x), & \text{if x is feasible} \\ f(x) + R\left(\sum_{j=1}^{J} |\langle g_j(x)\rangle| + \sum_{k=1}^{K} \left|h_{k((x))}\right|\right) + \lambda\,(t, x), & \text{otherwise} \end{cases}$$

Now, this particular the approaches that comes that comes under this category, they handle the objective function and constraints separately. So, in this category, we have superiority of feasible solution over infeasible solution. So, we will start with one of the approach is called Powell and Skolnick approach.

In this case, let us assume we have a minimization problem. For a given problem, we can calculate the fitness of a solution as given in equation number 3. We can see that the fitness of a solution which is capital of F of x this is equals to the objective function when a solution is feasible. So, we are considering the fitness same as objective function when the solution is feasible, otherwise means the solution is infeasible.

In this case, we have the objective function then we can see this particular term as we have understood that we this R is the penalty parameters, the terms inside this big bracket, so these terms represents the constraint violation, and at the last we are adding lambda t, x.

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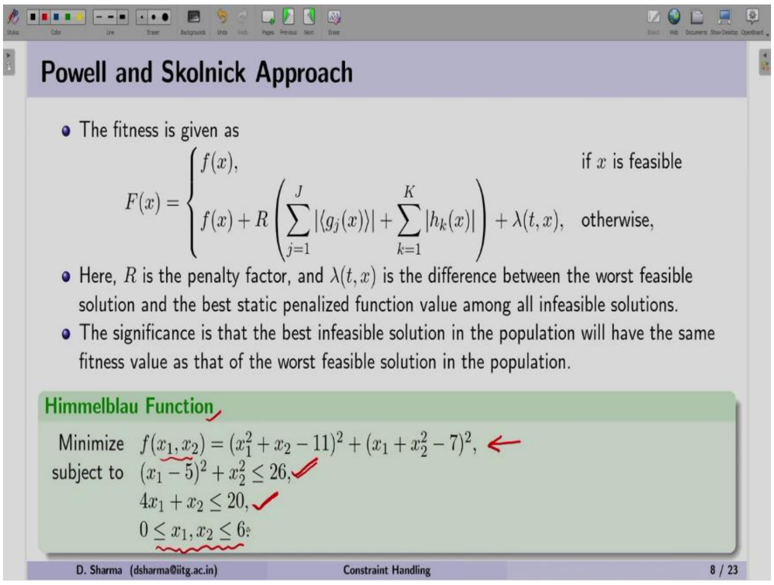

$$Minimize\ \ f(x_1,x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

$$subject\ to\ \ (x_1 - 5)^2 + x_2^2 \le 26$$

$$4x_1 + x_2 \le 20$$

$$0 \le x_1, x_2 \le 6$$

So, let us understand this equation in detail now. So, in this particular equation, R is the penalty factor, lambda t, x is the difference between the worst feasible solution and the best static penalized function value among all infeasible solutions. So, here what we can see that the value of a lambda we will be calculating with respect to the worst feasible solution in the given population and at the current iterations at t.

Similarly, in the same iteration what is the best infeasible solution. So, that we have to choose or we have to find carefully and that will represents the value of lambda here. Now, here the significance of such kind of a fitness is that this significance is that the best infeasible solution in the population will have the same fitness value as the worst feasible solution in the population.

So, the lambda value will be adjusted in a such a way that these two solution; that is worst feasible solution and the best infeasible solution both are going to have the same fitness value. So, let us understand this method using the Himmelblau function.

So, the Himmelblau function is an unconstrained problem in which we want to minimize the function as given. But this particular problem now subjected to two constraint and both of them are inequality constraint. Now, the first constraint is a quadratic form and the second constraint is a leaner form. For our simplicity let us assume that x 1 and x 2 will take a value between 0 to 6.

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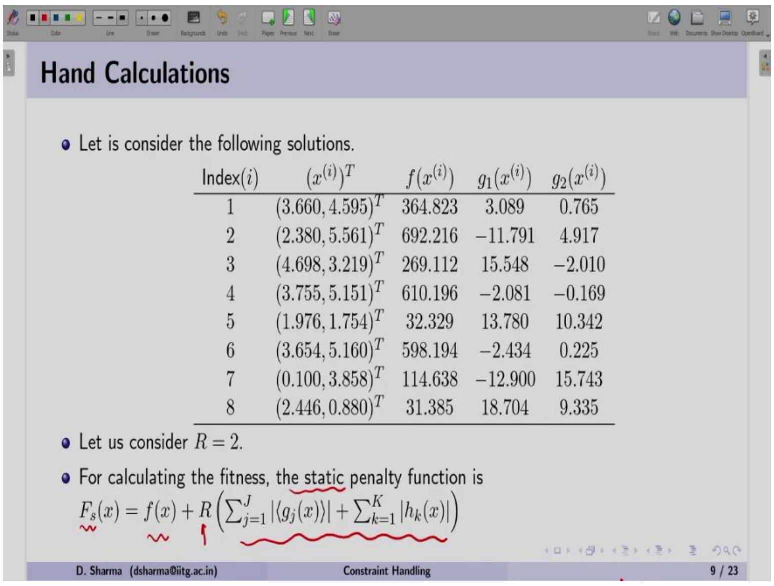

So, let us consider the solutions one by one. Here we choose the first solution as given in the table. And at this particular solution we can find the objective function by putting the value of x 1 and x 2 component. there after the same solution if we put in g 1 constraint the value is 3.089 and the same solution when we include in g 2 it is 0.765.

$$F_s(x) = f(x) + R\left(\sum_{j=1}^{J}|\langle g_{j(x)}\rangle| + \sum_{k=1}^{K}|h_{k(x)}|\right)$$

So, in this case what we are going to do here is, for a given example we are generating 8 solutions. Meaning that we are considering this x 1 and x 2 component of each solution and we created randomly between 0 minus x. And for every solution we are finding the fit function value. We are finding the constraint g 1 value as well as g 2 value.

Now, in order to find the fitness let us consider we have R equals to 2. So, as you can see in the previous equation that the R will remain the same. Now, the value, so the fitness using the static penalty function is; so, we are using this term called F under F s, s is in the subscript, so F of x is equals to the objective function we have one penalty parameter which is currently 2 and we have a constraint violation. So, let us identify or calculate the static penalty function value using this formula.

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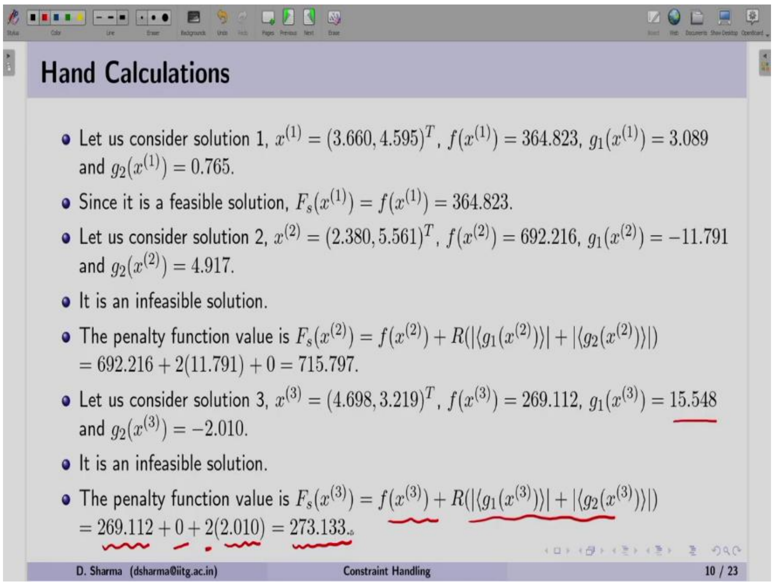

We will start with solution number 1. In this particular solution, as we have already calculated the objective function the g 1 value and the g 2 value. Now, here looking at the value of g 1 and a g 2 we can see that both the constraints are satisfied for a solution number 1. So, we can say the particular solution is feasible solution and therefore, the fitness of the solution is equals to the objective function which is 364.823.

Now, let us consider the solution number 2. Now, here the solution number 2 we already calculated the objective function value, similarly the g 1 and the g 2 value. What is the observation here is the g 1 value is negative meaning that this particular constraint is not satisfied for x 2, therefore the solution is infeasible.

Since, the solution is infeasible let us calculate the penalty function value here the penalty function value as we have we know it is made of objective function plus the constraint violation multiplied by R. Now, looking at the value here we have the objective function value as 692.216 plus.

Now, if we look inside the bracket, now we know that g 1 is not satisfied, but g 2 is satisfied. Since g 1 is not satisfied we are putting the value here, and g 2 is not satisfied and this bracket operator as we remember that f the value of a g 2 is 0 or positive it is going to be 0. By considering those things, we can have two times of g 1 value and since g 2 is already satisfied, so we are taking a 0 value. So, the penalty function value for the solution 2 is given here as 715.797.

Similar exercise we will do for solution number 3 here. Now, as you can see the solution number 3 is given, we already calculated the objective function the constraint g 1 and the g 2. Now, the observation here is this g 2 is negative meaning that the solution is infeasible.

So, we are going to use the static penalty function here using the same formula as given here we will do first we will consider the objective function plus now the 0 is written for a g 1. Why? Because it is already satisfied plus 2 times of now as you can see that constraint g 2 is not satisfied, so we are taking a positive value. In this case, the fitness of the solution number 3 is given as 273.133.

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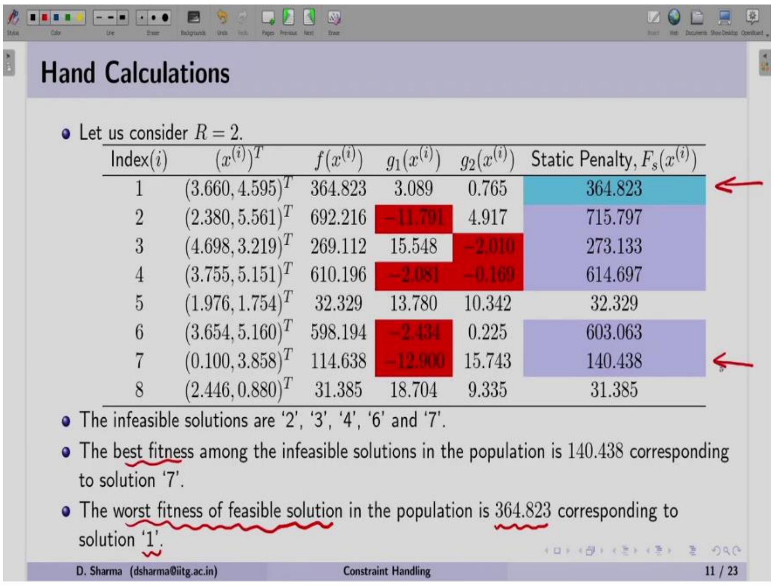

Now, since we have done calculation for 3 solutions let us move ahead. So, in this case, we are considering the R equals to 2, we have performed our calculation for solution 1 and as we understood that since this particular solution is feasible, so the objective function value will become the penalty function value as you can see in the column number 3 and column number 6.

Look at the solution number 2 here. Now, as you we can see here g 1 is g 1 is not satisfied, g 2 is satisfied, so it is an infeasible solution and using this static penalty we found that this is going to be 715.797. Similar calculation we did it for solution number 3 and in this particular solution we can see that g 2 is not satisfied, so the solution is infeasible. By using the formula, we calculated these static penalty function value.

Now, looking at equation looking at solution number 4, we can see that the objective function is given here the constraint g 1 as well as constraint g 2 for solution number 4, both of them are not satisfied. So, in this case the both the constraint violations are included and multiply by 2 and added into the objective function value. Same procedure we follow and we can calculate the static function value for all the solution.

Now, for solution number 5 since it is a feasible solution. So, the objective function value is the same or the fitness is the same as the objective function value. For solution number 6 and 7 as we can see that g 1 constraint is not satisfied. So, these two solutions are infeasible and accordingly we calculated the fitness value for both of them. And the solution number 8 it is feasible, so we can see the fitness is the same as the objective function value.

The infeasible from the table we can see that we have solution 2, 3, 4, 6, and 7 are the infeasible solution. Now, as we remember that in this particular approach, we have to find the value of a lambda as well. Now, the lambda will be calculated with respect to the worst feasible solution and the best infeasible solution.

So, we have to identify both of them. So, let us see a let us see the table again here. In this particular table, as we know the solution number 1, solution number 6, and solution number 8 are feasible solution. Among these 3 solution number 1 is the worst feasible solution looking at their fitness value. Similarly, if we look at the infeasible solution, we have to find which is the best solution.

Now, the among the infeasible solution as we have used these colours, we can see that the solution number 7 is the best infeasible solution which has the fitness value 140.438. So, this particular solution we are considering because it is the best infeasible solution similarly.

Similarly, we have a worst feasible solution as we have discussed earlier. Looking at the fitness value the solution 1 is going to be the worst feasible solution. So, we have selected solution number 1 and solution number 7 to calculate the value of lambda.

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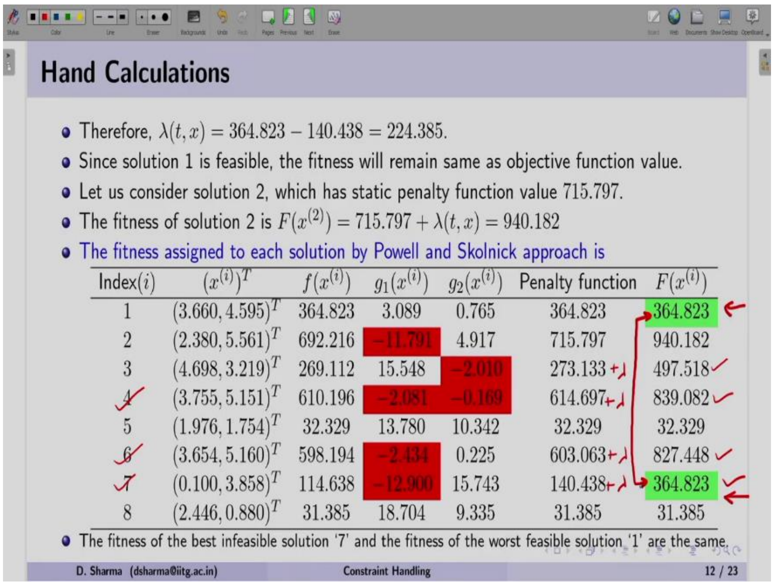

So, let us move forward and find the value of a lambda here. The lambda value is the worst feasible solution minus the best infeasible solution. If we take a difference this is what we are going to get a value of lambda.

$$\lambda(t,x) \ = \ 364.823 - 140.438 \ = \ 224.385.$$

$$F\left(x^{(2)}\right) \ = \ 715.797 \ + \lambda\left(t,(x)\right) \ = \ 940.182$$

Now, let us use the value of a lambda in order to calculate the fitness using this approach. Now, as we know solution 1 is the feasible solution, so the fitness will remain same as the objective function value. Now, consider the solution number 2, the static penalty function value as we calculated earlier it is given as 715.797.

So, the fitness of solution number 2 is the static penalty function value plus the lambda value that is giving me 940.182. So, the final fitness of solution 2 we are going to calculate with respect to lambda because a solution 2 was a infeasible solution. So, by following the same procedure we can assign fitness to each solution using Powell and Skolnick approach here.

Now, solution 1 as we know this is the feasible solution, so that is why the fitness value as you can see in the last column is the same as the objective function value. The solution number 2 as we know that in the previous calculation, we have already calculated the penalty function value. Now, we are adding a lambda, so in the last column we can see the fitness of a solution 2 as 940.182.

If we follow the same step for solution number 3, as we know solution number 3 is infeasible, so we are going to add this we have this penalty function value we are going to add a lambda here and the final fitness of this particular solution is 497.518. If we follow the same procedure, now you can see that solution number 4, 6, and 7 they are infeasible solution.

So, in this case we are going to add lambda with the penalty function and then we will get the final fitness as given in the last column. And for the other solution since they are feasible solutions, so we know that the fitness of the solution is the same as the objective function value.

So, from this table what is what is our observation? As it is mentioned earlier, the solution number 7 was the was the best infeasible solution, and solution 1 was the worst feasible solution. Looking at their fitness value we can see that both of them have the same fitness value and that is what this method does, that fitness of the worst feasible solution is the same as the fitness of the best infeasible solution.

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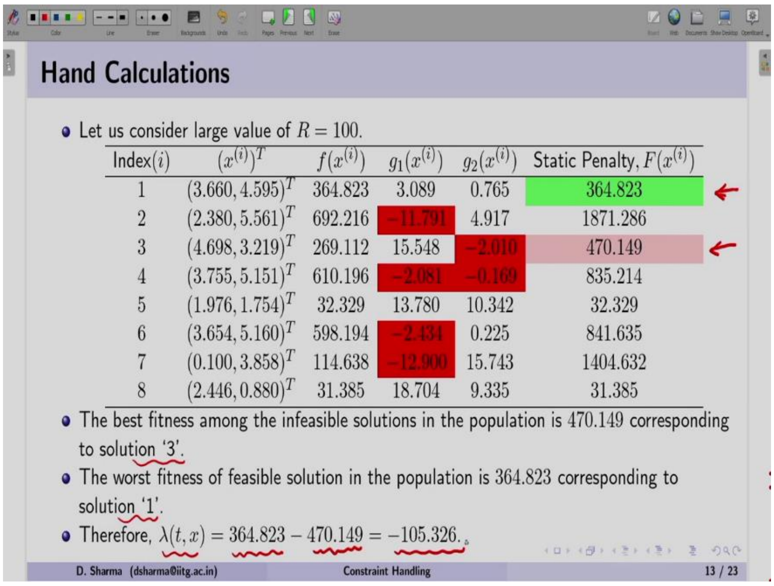

Now, here we perform the previous analysis with a small value of a R which is 2. If we consider R equals to 100, then let us observe what are the changes in the fitness value since we are going to take R very large value. So, we know that the static penalty function value will be large. So, let us observe the changes with a same solution, but considering R equals to 100.

$$\lambda(t, x) = 364.823 - 470.149 = -105.326$$

Now, looking at the table now here, so we are considering R is equals to 100 now. Solution 1 so it is a same solution we are considering, since this g 1 and g 2 are satisfied, so we know that it is going to be feasible and that is why the static penalty function value is the same as the objective function value.

Now, coming to the solution number 2 here. Now, solution 2 is infeasible as we can see here g 1 is not satisfied. Since g 1 is not satisfied and by using the formula of a static penalty the fitness value or the static penalty function value is given in the last column. So, the observation here is as and when we are going to increase the value of R, the static penalty function value increases according to the R which is currently a large value.

Similarly, we can calculate the fitness static fitness value for solution number 3 which is infeasible right now, 4 is also infeasible, 6 and 7 are also infeasible. Now, according to that by taking R equals to 100, we are going to get the values as mentioned here, in the last column of the table.

Now, since we have calculated the static penalty function value let us calculate what should be the value of a lambda. Now, as we know the definition of a lambda, we have to search two solutions. So, let us identify the worst feasible solution and the best infeasible solution from the table.

Now, looking at this particular table we can see that looking at the all the infeasible solution, the worst infeasible solution is corresponding to the solution number 3 having a fitness of 470.149. Similarly. if we look at the best feasible, so we have used in the last column two colour coding.

So, the best feasible solution the best infeasible solution is corresponding to solution number 3 and the worst feasible solution is corresponding to solution number 1. So, when we have identified solution 1 and solution 3, we can calculate lambda. So, as we know the worst feasible solution fitness minus, the best infeasible solution and we get a value of a lambda as minus 105.326.

So, what is the observation here? With respect to the previous example where we have taken R equals to 2 there the value of a lambda was a positive. So, basically, we are penalizing those constraint with the small value of R. But as and when, we take a large value of a R then the fitness or the static function value is large and in the current case we have lambda as a negative value.

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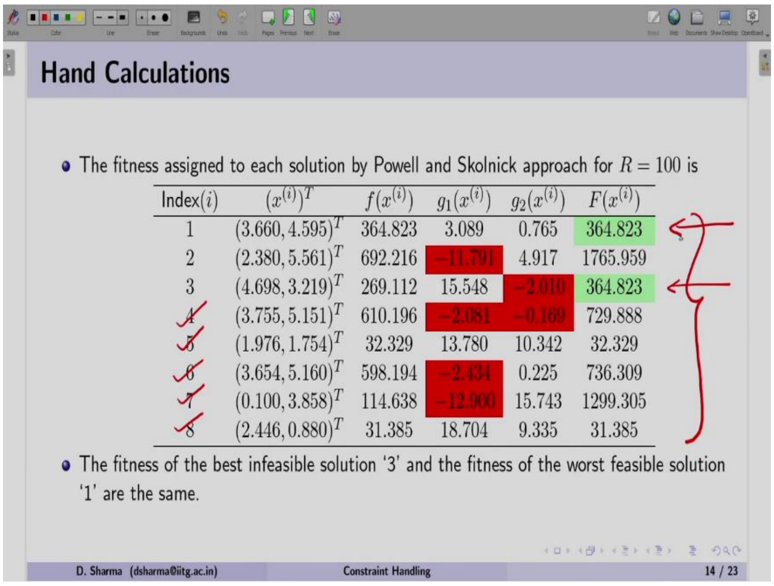

Now, let us calculate the fitness of each solution when R is equals to 100. Starting with the solution number 1 we know the it is a feasible solution, so the fitness and the objective function value are the same. Solution number 2 is a infeasible solution as we can see g 1 is not satisfied, so we are going to add the value of a lambda which is which was a negative value into the penalty function value.

Similarly, for a solution number 3, which is infeasible as g 2 is not satisfied, so we have we have this lambda value which we added into the penalty function value. Similarly, we have we perform the same calculation for solution number 4, 5, 6, 6, 7, and 8 here, and by using the negative value of a lambda we find the fitness of every solution in the last column.

Now, as we remember for the given case of R equals to 100, solution 3 was the best infeasible solution and solution 1 was the worst feasible solution. Now, looking at their fitness value both of them are the same and that is what this method does that both the fitness should be the same in this particular case.

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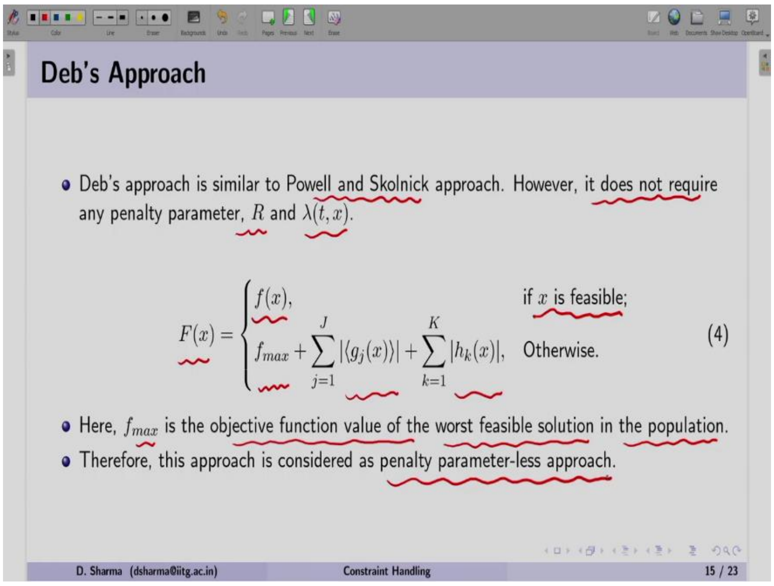

Now, in this particular case as you can observe that we are handling the objective function and constraint values separately. In this case, as and when the objective function is feasible we always consider the function value as a fitness value. And when the solution is infeasible, so that constraint violation we are going to add into the objective function value. Since we are dealing the constraint and objective function separately, so this is one of the efficient methods that can handle constraint with the EC technique.

$$F(x) = \begin{cases} f(x), & \text{if x is feasible} \\ f_{max} + \sum_{j=1}^{J} |\langle g_j(x) \rangle| + \sum_{k=1}^{K} |h_k(x)|, & \text{otherwise} \end{cases}$$

Following the similar concept, we have another method called Deb's approach. So, let us see what is that. In this Deb's approach, this is you will find it is similar to Powell and Skolnick approach; however, in this particular approach it does not require an in any penalty parameter called R or lambda.

In this Deb's approach, what they have done, looking at the equation number 4 the fitness of the function is equals to the objective function value when the solution is feasible. So, this is similar to the previous approach; however, if the solution is infeasible, so there is a term called f max, and then we are adding the constraint violation in to this f max. So, in this case what is f max? So, f max is the objective function value of the worst feasible solution in the population.

Now, you remember that the lambda was calculated with respect to the two solutions, in this particular approach the infeasible solution is not considered at all. Only, the solutions which are feasible among those feasible solution we identify which is the worst solution and we take the objective function value of the worst feasible solution as f max. As and when we get f max value we add the constraint violation into it and that constraint violation without a value of R.

So, in this particular formulation you can see that we have not used any penalty parameter called R, so therefore, this approach is also known as as we can see the penalty parameter less approach. This is also one of the famous constraint handling technique for evolutionary computation techniques.

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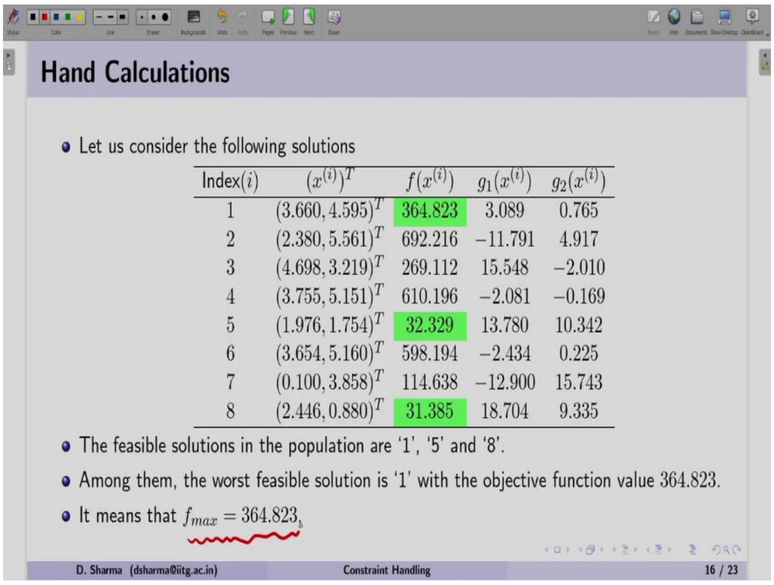

Hand Calculations

- Let us consider the following solutions

Index(i)	$(x^{(i)})^T$	$f(x^{(i)})$	$g_1(x^{(i)})$	$g_2(x^{(i)})$
1	$(3.660, 4.595)^T$	364.823	3.089	0.765
2	$(2.380, 5.561)^T$	692.216	−11.791	4.917
3	$(4.698, 3.219)^T$	269.112	15.548	−2.010
4	$(3.755, 5.151)^T$	610.196	−2.081	−0.169
5	$(1.976, 1.754)^T$	32.329	13.780	10.342
6	$(3.654, 5.160)^T$	598.194	−2.434	0.225
7	$(0.100, 3.858)^T$	114.638	−12.900	15.743
8	$(2.446, 0.880)^T$	31.385	18.704	9.335

- The feasible solutions in the population are '1', '5' and '8'.
- Among them, the worst feasible solution is '1' with the objective function value 364.823.
- It means that $f_{max} = 364.823$

D. Sharma (dsharma@iitg.ac.in) Constraint Handling 16 / 23

So, let us see how this method work. In this case, we are going to take the same set of a solutions which we have considered in our previous case. So, in this case, let us see the solution. Now, in the given table all 8 solutions are given their x value, similarly their function value, g 1 and a g 2 values are given.

Now, as we remember we have to just consider the feasible solution. So, in this in this case, we have solution number 1, solution number 5, and solution number 8 as our feasible solution. In order to calculate the f max value, we have to find what is the worst feasible solution.

Looking at the feasible solution, now we are looking at the green part here. Now, if we compare the objective function values of these three solution, we can find that the solution number 1, is the worst feasible solution and its objective function value is 364.823. So, this is going to be our f max in our formulation. So, let us see what will be the fitness here.

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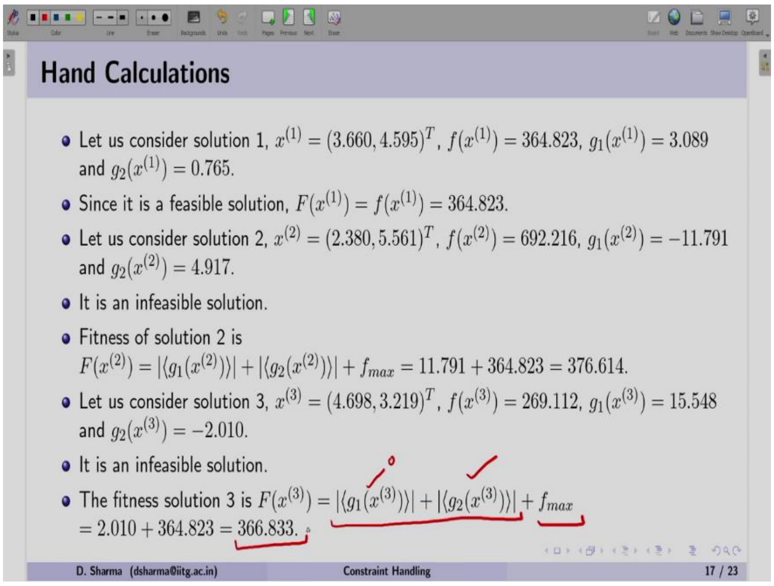

So, let us consider solution one by one here. Solution number 1, as we have calculated earlier g 1 and g 2 as satisfied, so it is a feasible solution, and since it is a feasible solution the fitness of the solution is the same as the objective function value. Let us consider now solution number 2, now for solution number 2 we know that the constraint g 1 is not satisfied. Meaning that the solution 2 is infeasible.

$$x^{(1)} = (3.660,4.595)^T, \quad f(x^{(1)}) = 364.823,$$
$$g_1(x^{(1)}) = 3.089 \; and \; g_2(x^{(1)}) = 0.765$$

$$F(x^{(1)}) = f(x^{(1)}) = 364.823$$

$$x^{(2)} = (2.380,5.561)^T, \quad f(x^{(2)}) = 692.216,$$
$$g_1(x^{(2)}) = -11.791 \quad and \; g_2(x^{(2)}) = 4.917$$

$$F(x^{(2)}) = |\langle g_1(x^{(2)}) \rangle| + |\langle g_2(x^{(2)}) \rangle| + f_{max} = 11.791 + 364.823$$
$$= 376.614$$

$$x^{(3)} = (4.698,3.219)^T, \qquad f(x^{(3)}) = 269.112,$$
$$g_1(x^{(3)}) = 15.548 \quad and\ g_2(x^{(3)}) = -2.010$$

$$F(x^{(3)}) = |\langle g_1(x^{(3)}) \rangle| + |\langle g_2(x^{(3)}) \rangle| + f_{max} = 2.010 + 364.823 = 366.833$$

So, by using a formula given by Deb's approach, so we have the constraint violation and then we are adding a f max value. In this case, since only one constraint is infeasible, one constraint is not satisfied, so the positive value we have taken and the worst f max value, so we get the fitness here. So, what is the observation here is we are not considering the objective function value of an infeasible solution. We are finding the fitness with respect to f max plus the constraint violation.

Now, let us look at the solution number 3 now. Now, solution number 3 as we know the constraint g 2 is not satisfied. Since, it is not satisfied this particular solution is a infeasible solution. So, in this case, we are adding the constraint violation plus f of f max. Now, since g 1 is satisfied it is going to be 0 and positive value of a g 2 we will consider, so we can find the fitness of a solution 3 as f max plus the constraint violation.

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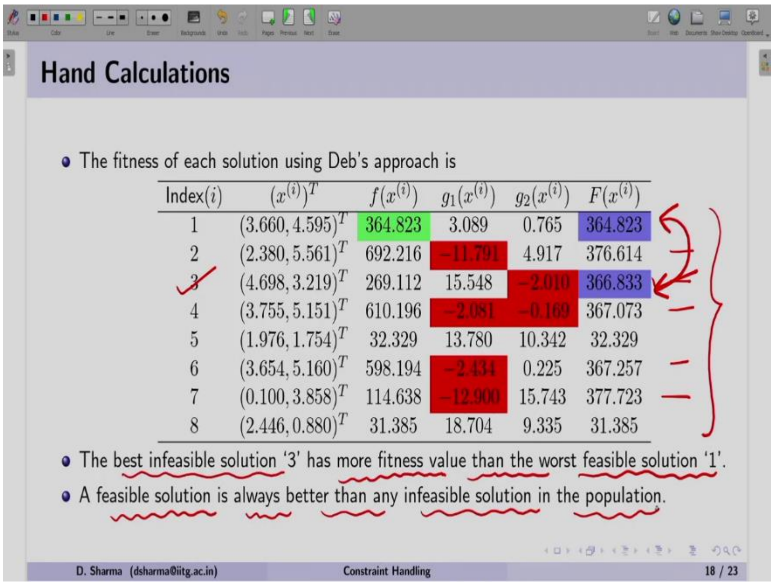

If we are going to follow the same procedure for all of the solution, then we can find the fitness. So, let us see one by one in a table. So, here the fitness of each solution using Deb's approach is we have a solution 1, which is a feasible solution. So, therefore, the objective function value is and the fitness value are the same.

For solution number 2 we as we have seen g 1 is not satisfied, so it is a infeasible solution, so this is going to be f max plus constraint violation. Similarly, we follow we are getting say g 3, g 4, 5, 6, 7, and 8. For all these solutions wherever these are infeasible solution we are considering f max plus constraint violation and when a solution is feasible, for example, solution number 5, 8, the fitness is the same as the objective function value.

Now, there are certain observation from the table. Now, the best infeasible solution if we find from the table so let us look at the last column of the table. Now, as we know the solution number 2, 3, 4, 6, and 7, these are the solution which are infeasible. Among them the best infeasible solution is corresponding to solution number 3.

Now, here this particular solution. So, let us compare the fitness of solution 3 verses solution 1. Why 1? Because 1 is the worst feasible solution, and solution 3 is the best infeasible solution. So, the observation here is that the best infeasible solution number 3, has more fitness value than the worst feasible solution number 1, which suggest that a feasible solution is always better than any infeasible solution in the population.

So, there are certain differences which we can find between the two approaches which we have gone through. So, in Deb's approach first of all we are considering the f max, there is no penalty parameter called R, and we do not consider the objective function value of infeasible solution.

In this case, when we use Deb's approach, the infeasible solution will get more fitness value than the if than the if any of the feasible solution. Since we are solving a minimization problem, so we know that if the fitness is less, we are going to select that solutions favourably. However, in the previous approach we have this lambda term and that lambda term is making the fitness the same value of a fitness for both the solutions.

Now, as of now we have used the first approach as separating the objective function and a constraint in which we assign the fitness by giving superiority of feasible solution over infeasible solution. In both the approaches, we find that we are calculating the objective function and the constraint violation separately and that why these two approaches comes into that category.

Now, we are moving to another category where we again consider objective function and constraint separately. This particular approach is motivated from multi-objective concept. So, let us begin with that.

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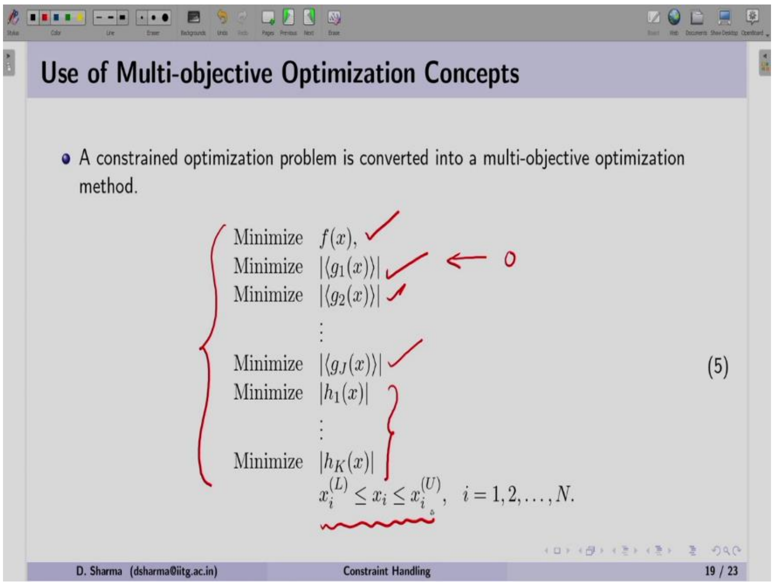

$$Minimize \quad f(x),$$

$$Minimize \quad |\langle\, g_1(x)\rangle|$$

$$Minimize \quad |\langle\, g_2(x)\rangle|$$

$$\vdots$$

$$Minimize \quad |\langle\, g_J(x)\rangle|$$

$$Minimize \;\&\; |h_1(x)|$$

$$\vdots$$

$$Minimize \quad |h_K(x)|$$

$$x_i^{(L)} \leq\; x_i \leq\; x_i^{(U)}, \&\; i = 1, 2, \ldots, N.$$

So, using the multi-objective concept we are going convert an constrained optimization problem. So, this particular problem is converted; as you can see in equation number 5, we want to minimize our function that is original objective function value, along with that we can minimize the mod of the bracket inside the bracket g 1.

So, we know that the minimum value for this particular function will be 0. So, as and when it is become 0, so the constraint is satisfied. Similarly, we can consider g 2, similarly all inequality constraint, and for equality constraint also we are considering this. So, we are considering all these constraints as a separate objective function.

Now, looking at this particular formulation that when we have more than one objective this is called multi-objective. Since we are considering constraint as our objective function, so we our formulation which was constrained optimization problem that has been converted into a multi-objective problem.

Now, here we do not have any constraint. What we have is only the variable bound. So, looking at the equation number 5, we are having multiple objective functions and we have the variable bound here. So, this is one of the way, we can convert a constraint optimization problem using multi-objective approach.

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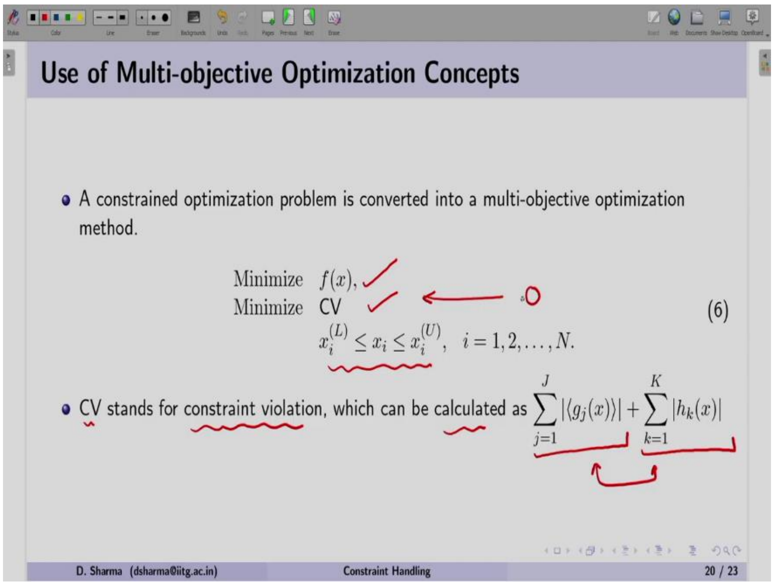

$$Minimize \quad f(x),$$

$$Minimize \quad CV$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)}, \& \; i = 1, 2, \ldots, N.$$

$$\sum_{j=1}^{J} |\langle g_j(x) \rangle| + \sum_{k=1}^{K} |h_k(x)|$$

What is the another approach? So, instead of having each and every constraint as an objective function what we can consider as you can see in equation number 6, we want to minimize our original objective function and we want to minimize CV and this particular problem is subjected to the variable bounds.

So, this is we have two objective functions and the bounds on the variable. So, what is CV here? CV is stands for constraint violation that we can calculated as looking at this we this is the constraint violation coming from inequality constraint, and the second term says that the constraint violation from the equality constraint.

So, when we are adding these two terms that will become the constraint violation. Now, when we are going to minimize looking at equation number 6, if you are going to minimize, so the minimum value will become 0, this means the solution which we are going to get is the feasible solution for us.

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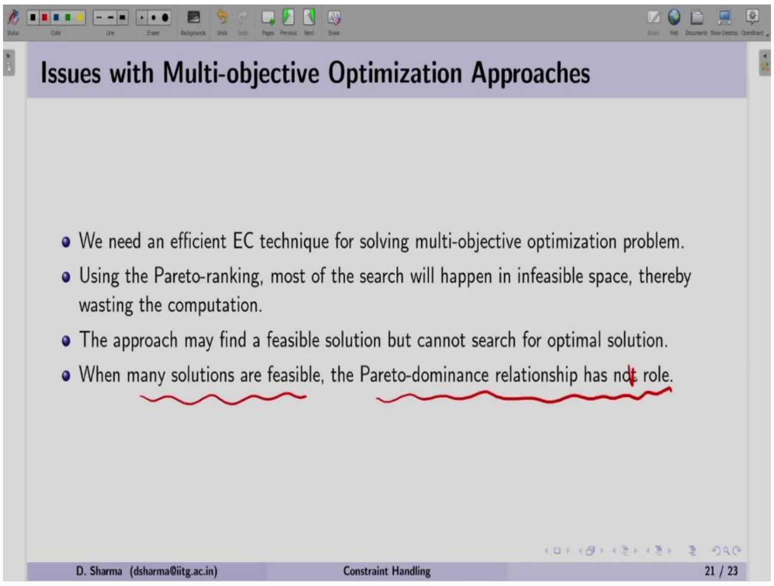

So, there are certain issues, although it looks simple that we can always covert our constraint problem into an unconstraint problem using multi-approach objective, but there are certain issues. So, let us understand one by one. So, first of all we should have an efficient EC technique for solving multi-objective optimization problem.

So, let us consider the first approach where we want to minimize our objective function as well as we want to minimize the constraint violation. If we consider we have 10 constraint, so overall the problem will have 10 plus 1, 11 objective functions. Why? Because very constraint is converted into an objective function.

So, in this case we will, we should need an efficient multi-objective optimization algorithm that can handle 11 constraints and as we know there are certain practical optimization problem which can have multiple constraints, mean meaning that the number is very large. So, we do not have or we should need such kind of efficient multi-objective optimization algorithm.

Second is, if in multi-objective techniques, generally we consider this Pareto ranking. In this Pareto ranking, what will happen that most of the search will happen in the infeasible space thereby wasting the computation. Why? Because when we perform Pareto computing or Pareto ranking, we have to do some computation because we compare each and every solution in each objective and then we know which solution is non-dominated which solution is dominated.

Now, when we are performing as we know that if the solution is infeasible, so the objective function will have certain value. So, every time when we are performing this Pareto ranking, we are actually performing for infeasible solution. And therefore, a lot of computation will be wasted when we perform this extra computing for single objective optimization.

Third is, the approach may find a feasible solution, but cannot search for the optimum solution. Now, remember that we have two ways to handle the constraint optimization, either we can have all constraint as independent objective functions or we can take the second objective as a constraint violation.

Now, from these, from both the ways what we can find is that as and when we get a feasible solution, the constraint violation is 0, and we can get a some objective function value. In this case, we will not be searching, we may not be able to search for an optimum solution because the algorithm has find an optimum solution using the multi-objective concept. So, therefore, sometimes we may not able to find the optimum solution, but may we may get the feasible solution using this approach.

Now, coming the next issue is when many solutions are feasible, then the Pareto dominance relation has no role. So, this is so in this case suppose we have 10 solutions all the solutions are feasible. So, this means that the constraint violation is 0. Only the objective function value we have it. So, that is minimization of f of x.

Now, since all the objective function values are 0, that is corresponding to the constraint violation, we have only change in the first objective which is minimization of f of x. When we perform the Pareto dominance relationship, it has no role because the second objective is always 0 for all the solution. So, therefore, it is not helping us to take the solution towards the optimum solution.

So, in this particular section, what we have gone through is another way to handle the constraints. In this, in the first category we separated the objective function and a constraint and we assigned a fitness when a solution is feasible and infeasible. When it is feasible as we understood we are giving the fitness is the same as objective function, when solution is infeasible we are adding the constrained violation into it.

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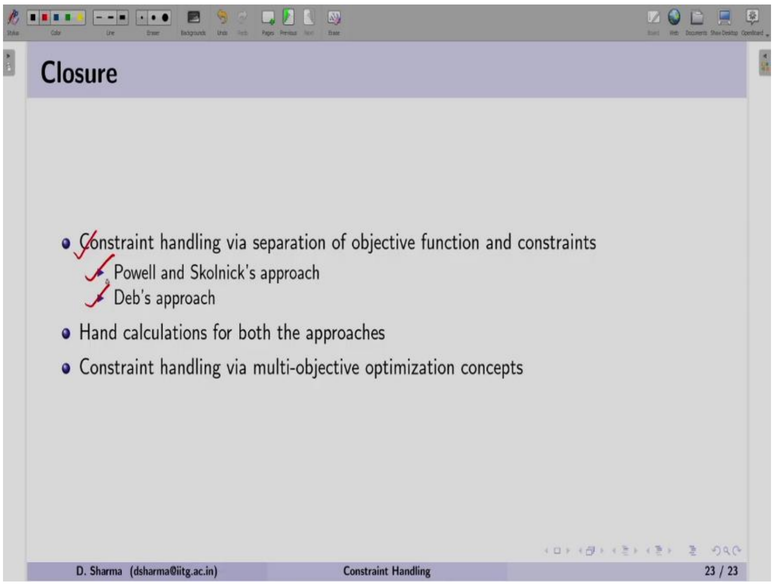

So, in this particular session, we have gone through these two methods called Powell and Skolnick's approach, and Deb's approach. Both of them handle the constraints via separating the objective function and the constraints we also perform the hand calculation for both the approaches and what we have certain observation for both the methods.

In the first approach, we need value of a R that is the penalty parameter. We also have to calculate the value of a lambda using the two solutions, that are worst feasible solution and best infeasible solution. But in the second approach that is Deb's approach, we only consider the worst feasible solution we do not need R, and therefore, this approach is used with many of EC techniques for handling the constraints.

Along with that we have also gone through the multi-objective concept, that multi-objective concept we have discussed very briefly just to understand how this constraint problem can be converted into an unconstraint problem by considering all the constraint as an objective. So, there are certain issues with a multi-objective approach; however, this is one of the ways to handle the constraint optimization method.

With the detail on these constraint handling techniques, I conclude this session.

Thank you very much.