**Evolutionary Computation for Single and Multi-Objective Optimization**
**Dr. Deepak Sharma**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Guwahati**

**Module - 01**
**Lecture – 01**
**Introduction to Optimization**

Welcome to the Module 1. This module 1 is divided into two sessions. Today I will be discussing on session 1 which is on Introduction to Optimization.

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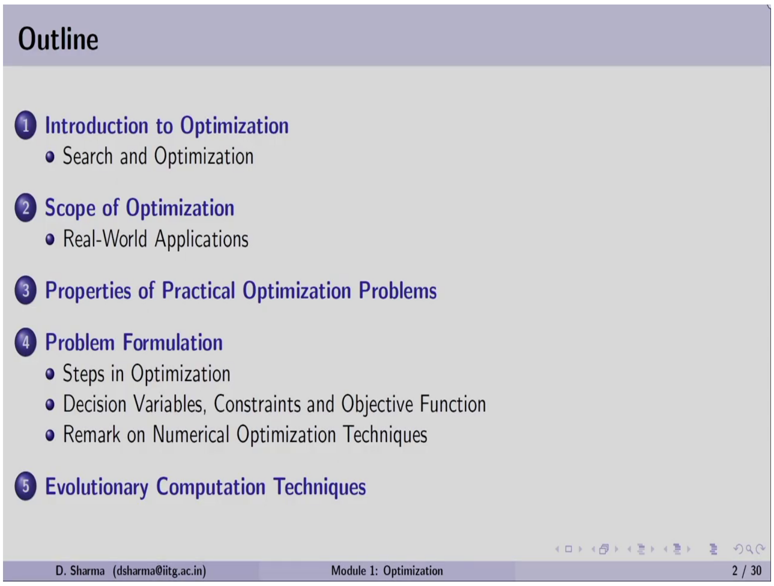


This session includes introduction to optimization, basically the definition and with the help of an example we will understand why we call them as a search and optimization. Thereafter, we will discuss some real world applications those have been solved using Evolutionary Computing techniques and with that we will be moving to the properties of practical optimization problem.

Since there are different characteristics or properties available, looking at those properties we generally decide what kind of algorithm we have to use it. Therefore, we will be going through all those properties one by one. Thereafter, we will discuss the problem formulation, generalized formulation, the mathematical equations, how to find the variables, constraints, the type of constraints, objective functions and variable bounds. We will discuss them one by one and thereafter we will give some remarks on numerical optimization techniques and finally, we will move forward to evolutionary computation techniques. Let us begin with introduction to optimization here.

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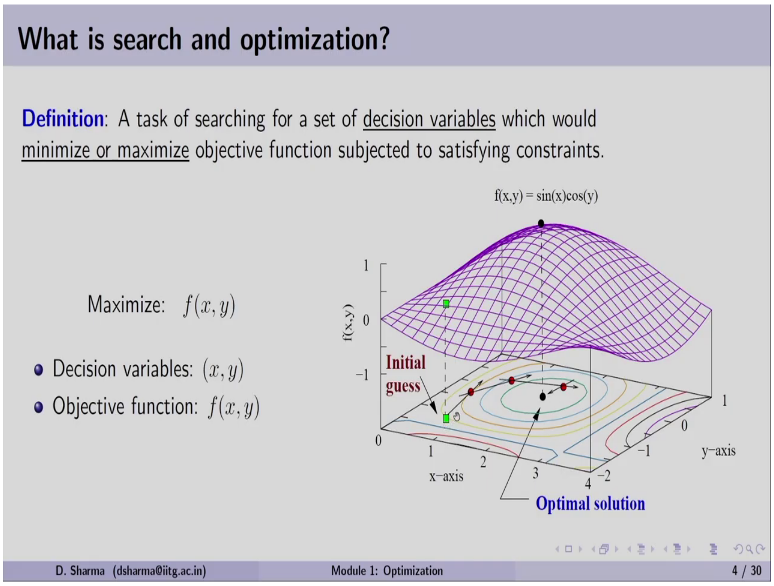


Now, before we start let us find what is the meaning of search and optimization. If we define it, it is a task of searching for a set of decision variable, which would minimize or maximize objective function subjected to satisfying constraint. Let us understand this definition with the help of an example. The example is given here. On the right hand side, you can see that we have a function called $f\ (x,\ y)$ which is $sin\ x$ and $cos\ y$. On the left hand side, if you see that we want to maximize this function. The problem is, we want to maximize the function $sin\ x$ and $cos\ y$. Now, this particular function is made of two variables x and y and using x and y we defined a function. It is called as objective function.

Now, let us look at the green point. Generally, when we solve a problem, we do not know where is the optimum. Optimization algorithms start with a random point that we are referring it as an initial guess. Now the green point in the $xy$ plane is the initial guess for us. Once we find the initial guess we calculate the objective function value.

If you look at this line and if you find this another green point on the surface that signifies what is the objective function value. Once we find it out then at this current point in the $xy$ plane we try to find out in which direction we should move so that our objective function should improve. Now, here if we say should improve; it means that the objective function should increase. Since is a it is a maximization problem objective function should keep on increasing in that direction.

With the local idea or some rule we find say direction $s1$ as you can see here, when we are moving along that direction we get a first point say $p_1$. This $p_1$ is the point which is better than the green point and this point have if it is better it means that it has a larger objective function value than the green point. Once we find this point $p_1$ again looking at the local information or the search direction we move in another direction say $s_2$.

Once we are moving on $s_2$ we will get another point say $p_2$. This $p_2$ we can calculate the objective function and when we compare the function value at point $p_2$ with $p_1$, the $p_2$ is better in terms of objective function it means that our objective function is improving.

Thereafter again we will move in the direction say $s_3$ and we will get a point say $p_3$. And finally, going in another direction we will get an optimum solution. The overall process if you understand it you can realize that we what we are doing is we are searching for a set of decision variable by moving from say green point to the point 1 then point 2 then point 3 and finally, reaching to the optimal solution. This whole process is defined as a search and optimization, where we look for an optimal value of $x$ and $y$ that will be maximizing the function, which you can see as a black dot. Now, let us move towards the scope of an optimization.

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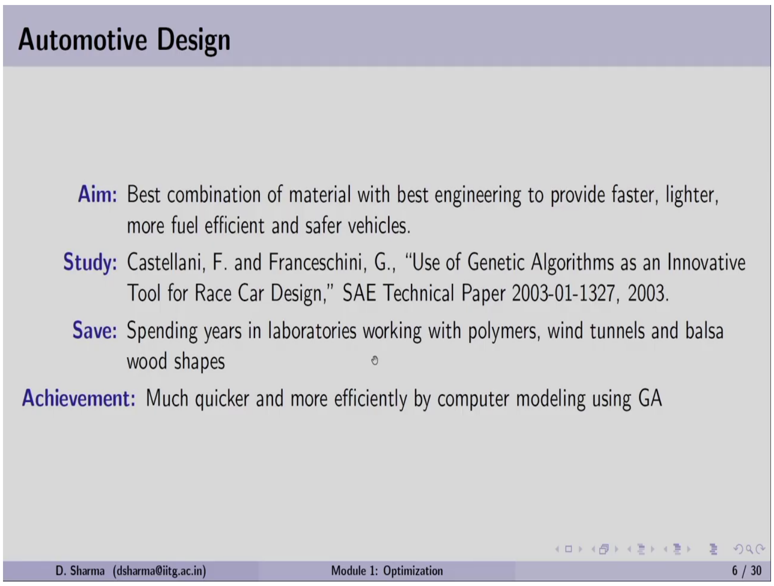

Let us begin with the automotive design. Now, in this particular application the aim was to find the best combination of a material with best engineering to provide faster, lighter, more fuel efficient and safer vehicle. Having this particular aim, the scientist and the researchers they use genetic algorithm with the models the available models and they try to optimize their car design. What they found is that if they are going to use any optimization or evolutionary computation techniques, they can actually save lot of years in laboratory working with polymers, wind tunnels and other equipment to find what could be the optimal design for a racing car. Their achievement is that they can find the solution the optimal design of the car much quicker and more efficiently by computer modeling using genetic algorithm. Here, you realize that when we are performing lot of experiments, it requires lot of time as well as funds. Both of them will be required to come up with the new racing design. However, if we couple our computer models with genetic algorithm or any evolutionary algorithm this will give us a solution on which the designers will work on it and can further improve the design.

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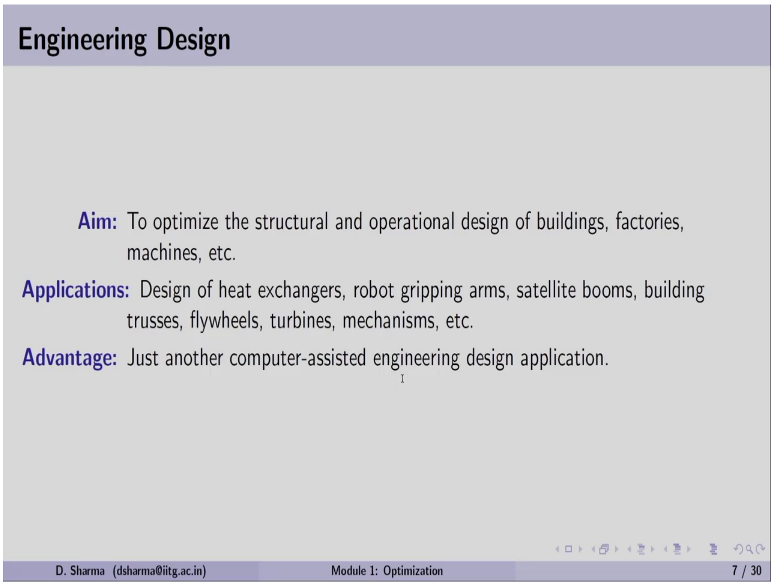


The second application is the engineering design where the objective is to optimize the structural and operational design of buildings, factories, machine etcetera. Now, this application is widely used in various application whether it is mechanical engineering or civil engineering, electrical engineering or chemical engineering we always try to find out the optimal engineering design. The application is varying. It can start from heat exchanger to the robot gripping, satellite booms, building trusses, flywheels, turbines, mechanism and etcetera.

Since there is a wide application, with the help of computer and an optimization algorithm we can find an optimal solution or the design for our engineering problems.

Another interesting example you can find is in the robotics. Here, aim is the intelligence and learning in robotics. This learning in the robotics can be done with the help of optimization. There is a lot of scope here.

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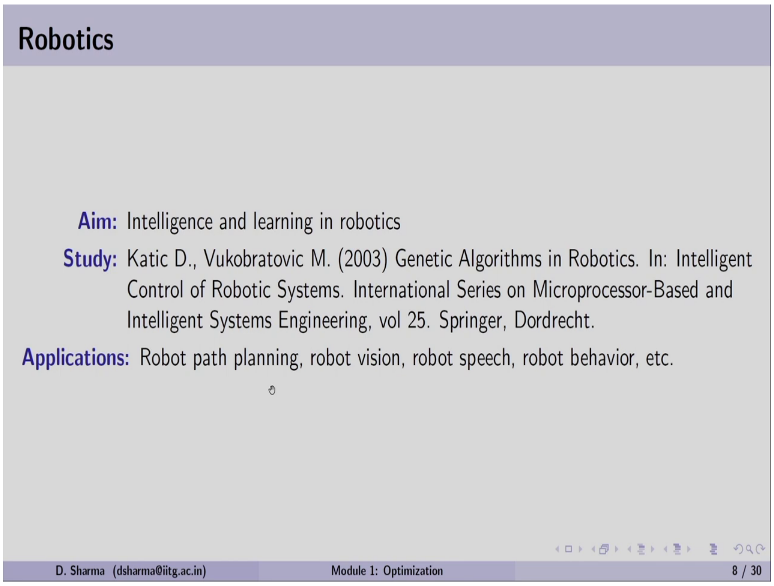


These two scientists long back in 2003 use Genetic Algorithms for Intelligent Control of Robotic System. With this they can find the application in robot path planning, vision, speech and behavior. It is a very interesting example where GA will guide a robot to learn and apply intelligence to do the tasks for which the robots are assigned.

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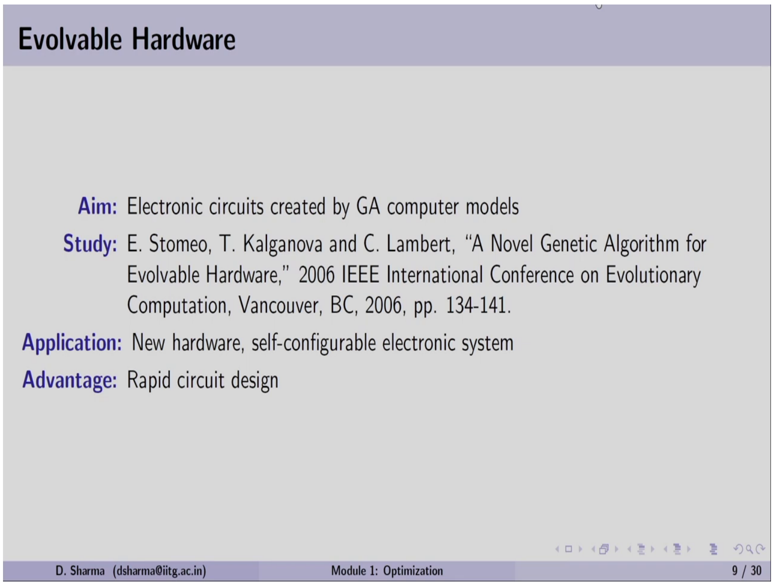


This is an another interesting application on evolvable hardware the aim is we have to create an electronic circuits created by GA model. These scientists they work on this problem they use GA for evolving some of the hardware's that is also long back in 2006. Now, the application of such kind of approach is they can come up with the new hardware, self-configurable electronic system.

You may realize that evolutionary computation may give you some kind of a design which may look simple, but it is not still available with us. Some new designs simple designs can be evolved by the evolutionary computation. what are the advantage? Since we are working on computer models, we are using GA to optimize it, we can have a rapid circuit design.

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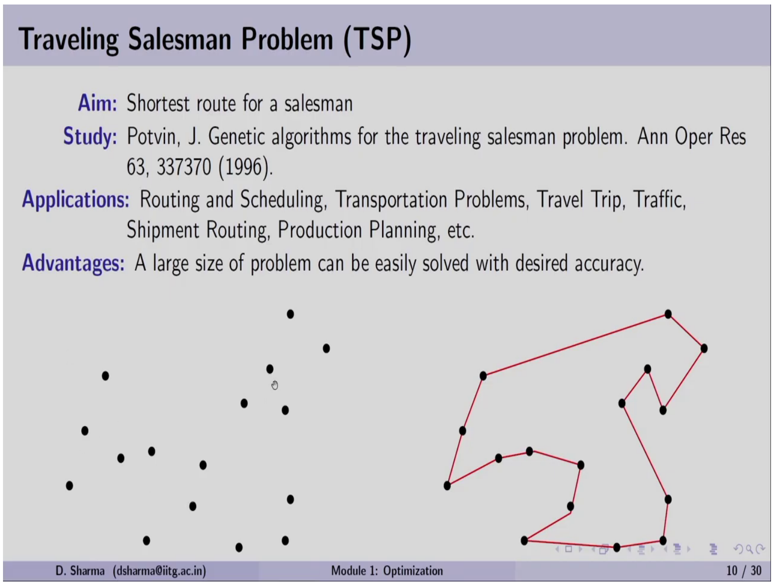


The another application which is a very well known application is the traveling salesmen problem. This problem is known to many people where the aim is to find the shortest route for the salesman. Let us look at into it. Now, here you can see that there are many dots are given. The problem is defined as the salesman has to start at a one particular node. He has to visit all the nodes just one and finally, it should come back to the original node. In these conditions we have to find what could be the shortest route for the salesman.

Long back in 1996 this author has used genetic algorithm for solving the TSP problems. Now, application of solving the TSP we can find in routing and scheduling, we can find the application in transportation problems, travel trip, traffic shipment routing, production planning and many other applications.

Now, what you can see here is the given example has some number of nodes. Suppose when we are solving a real word problem, when we have more number of a nodes in that case GA will help you to give you a desirable solution for the shortest route for a salesman.

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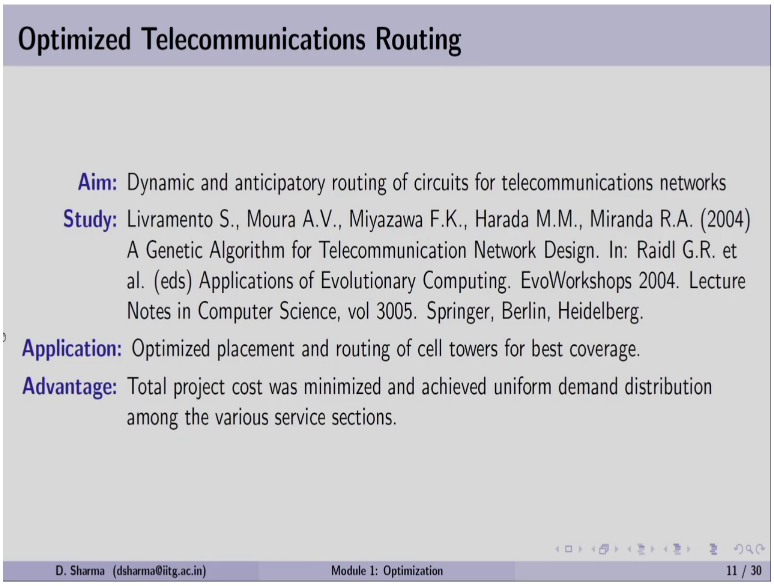


Coming to a very interesting application which is optimized telecommunication and routing. Now, here the aim is dynamic and anticipatory routing of circuits for telecommunication network. This study has been taken by these researchers and applied in one of the cities.

What they found is when they include any evolutionary computation technique in their formulation they can achieve the optimize placement and routing of cell tower for the best coverage. For example, in a city with a lot of population then how we are going to place those towers in the city, so that everyone should get the desired signal.

Now, you may be realizing that in some portion of the city which may be highly dense population and in other part of the city they are maybe the discuss population. In that case how we are going to put those towers? Since it is applied in practical what these author found that the overall advantage is they can reduce the overall cost of a project to put those towers in the city as well as they achieved a uniform demand distribution among the various service sections.

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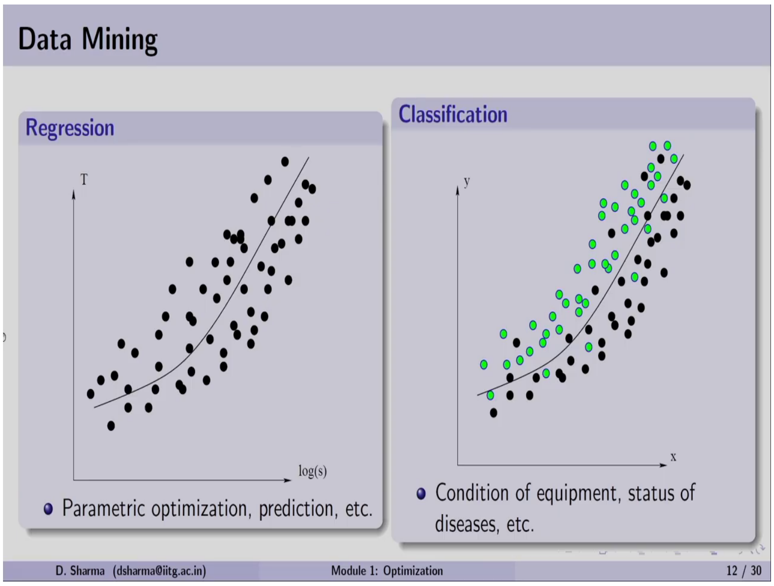


Coming to the data mining; as you know that, data mining is a very important field right now. It is because we have lot of data that is generated either from the machines or it is coming from some applications etcetera.

Now, the very first application in the data mining is the regression. Regression means that we have a set of a data and we try to find out the best fit curve. Now, here the optimization will help you to find out the user defined parameters that will be controlling the best fit curve for the given data. As you can see in the figure this particular best fit curve can be used for parametric optimization or it can be used for prediction or it can be used for prognosis as well.

Now, here it is important that when we are performing some parametric optimization the overall aim is if we can predict something that is for unknown value of the input variable then we do not have to perform so many experiments. We can save those experiments, we can save some fund so that this regression can help you to predict what could be the values for this set of input parameters without performing the experiments.

Another application you can find in classification. Now, classification as an example shown here, it is a binary classification. When we are dividing these dots in a grain and a black dots, our aim here is to demark them so that we can use this demarcation for finding what should be the what is the condition of our machine or what is the status of our disease. So, this classification also need an optimization. We have to solve it and that will give us a good accuracy in classification.

As you know when this kind of a binary classification can help us to tell whether the machine is working fine or a not or the person has this disease or a not.

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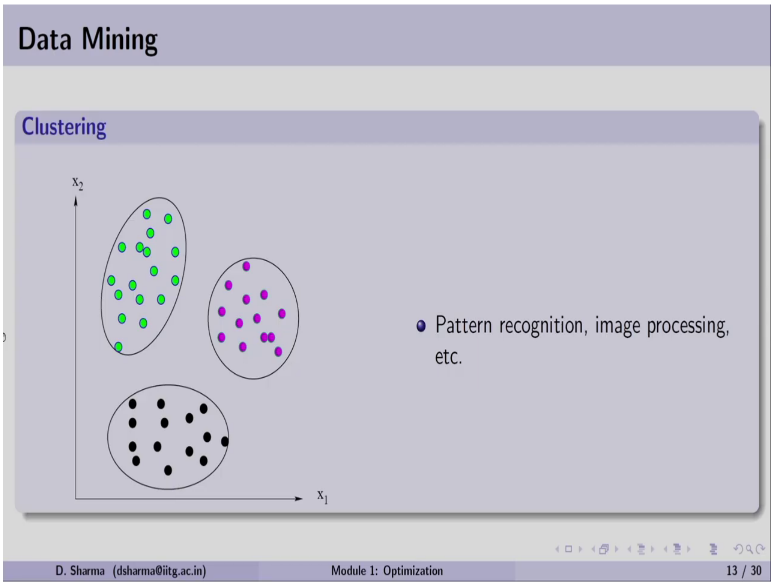


Another good application of optimization you can find in clustering; we are different varieties of the data's are may are put into different clusters. The application of the clustering you can find in pattern recognition as well as in image processing. Data mining need optimization and for solving that we need efficient optimization algorithms.

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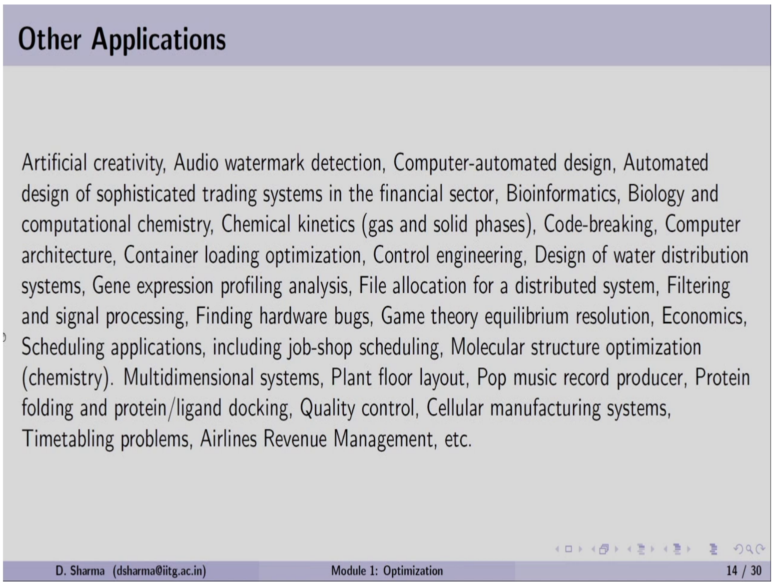


Now, in this slide you can find that there are various applications which are which has the scope of an optimization. Since these application are complex and large in nature, we need some algorithm that can solve these problems. Evolutionary computation can be helpful or maybe other algorithm which is flexible enough to solve such kind of a problems which are large and complex, although this particular list looks like a big list, but it is still not complete.

When you search you will find there are so many applications apart from what we discussed right now. There is a lot of application where there is a scope of an optimization.

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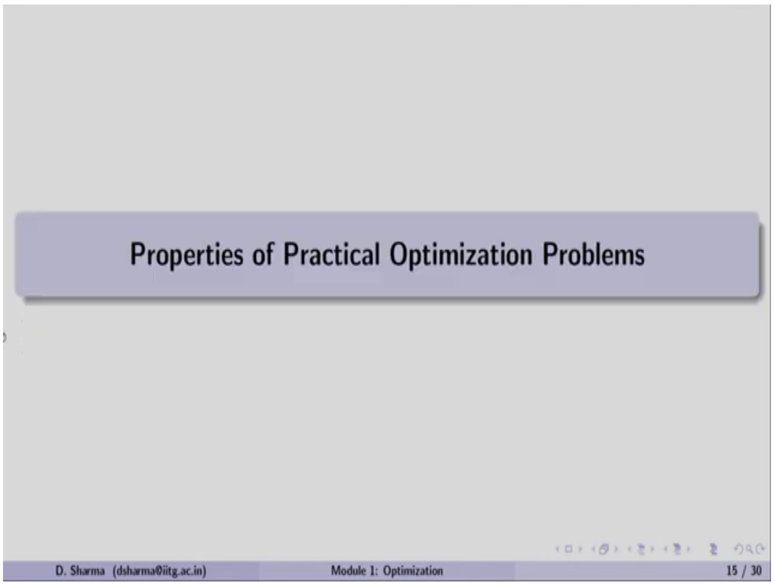

With this introduction on the application, we are moving towards the properties of practical optimization problem. When we say a real world problem is large and complex, then what are the properties they there exist with the practical optimization problems?

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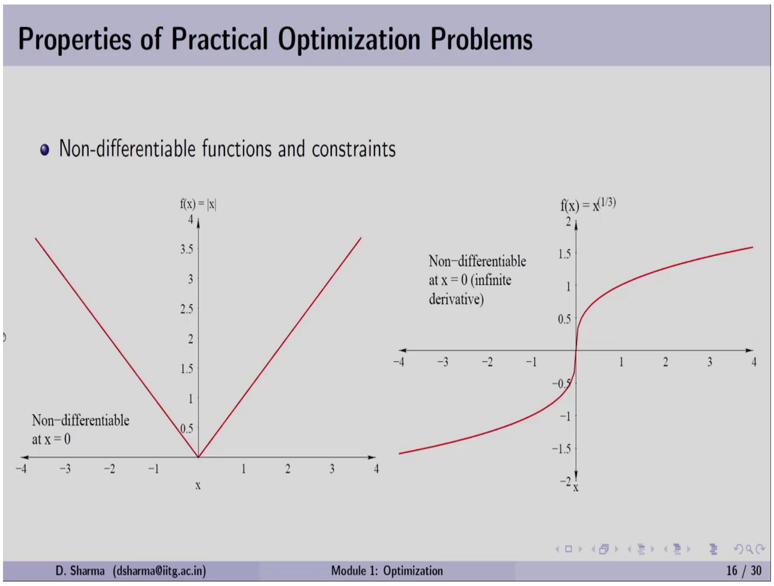


Let us begin with non-differentiable functions. Now, non-differential function as you can see from both the figure. So, the first figure on the left hand side says that if we plot y is equal to mod of x, so, at x = 0 function is non differentiable. Similarly, if we look the right hand side figure we have a function which is cube root of x which is also non differentiable at x = 0.

The issue with non-differentiable function is that if we look for the optimization method, that need gradient information we cannot use it.

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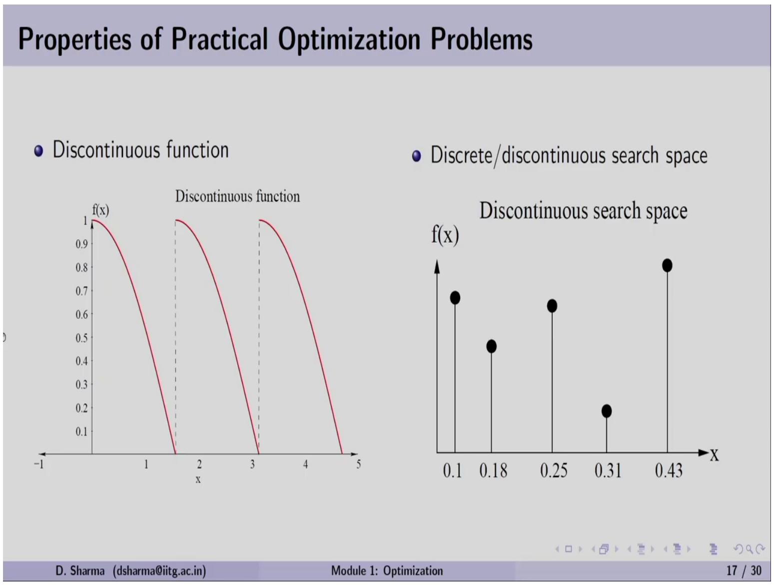


Coming to the second property it is a discontinuous function. Looking at the figure of the discontinuous function you can see that since the functions are discontinuous optimization algorithm may stuck in one of the portion and may not reach to the global optimum and that is a big challenge for an optimization algorithm to solve such kind of discontinuous function.

The another application which you can find is the discontinuous search space. Now, the discontinuous search space as you can see in the figure there are different values of x are given, it means that for a given problem these x values are allowed. You cannot take any random value of x between this range. So, the problem is the optimization algorithm has to find a way. So, that they can evaluate the objective function they can search for the new points by looking only for the desired or the discrete value of x.

Now, in the given example we have only five x values, but in real you may get a very large set of discrete x values and by finding which set of x value is going to give me an optimum solution it is going to be a challenging task.

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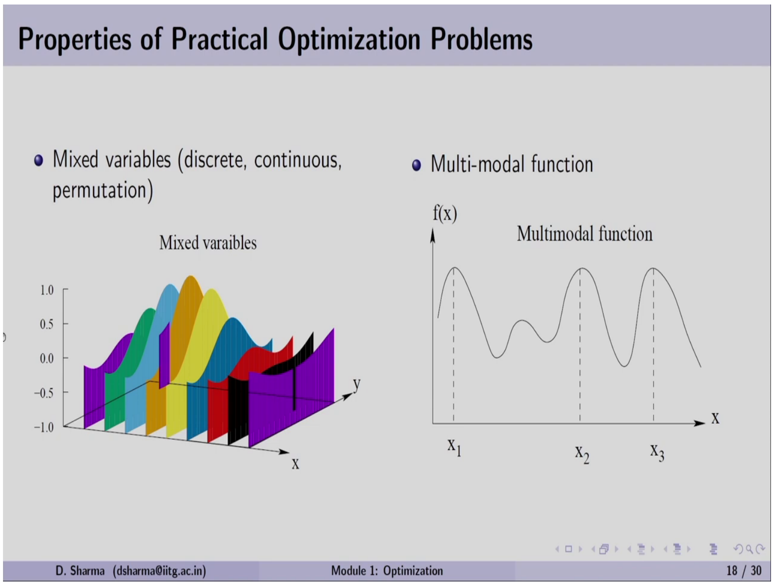


Now, mixed variable problem. There are so many optimization problem exist in the literature that needs different types of different types of variables. For example, it can has binary, it can has discrete or it can has a combination of these two with the real number.

So, one of the application say in electrical engineering is a unit commitment problem where the problem is mentioned as a binary as well as the real number. So, the unit commitment problem involves both of them. Suppose, if we want to design a gear. So, the designing a gear involves real number, but the number of a teeth includes the integer numbers.

So, therefore, these kind of a problem where we have multiple types of a variable then it poses, it poses lot of challenges to the optimization algorithm to find an optimum solutions. Coming to the multimodal functions these kind of functions are very popular why because at different values of x you will get the you will get the optimum value of $f(x)$.

Now, look at this function here. Here we have $x1$, $x2$ and $x3$. These are the three places. If we are going to maximize the function then these three points are the optimum solution. In general, when we solve such kind of a problem our optimization algorithm will give you one of the points.

But, since we since these points may be useful in some designing of some product or it may be used in engineering application or it can be involved in strategic decision making, so, we want to know what are these points; $x1$, $x2$ and $x3$ where the there is a same objective function which is maximum right now.

So, this poses a difficulty for optimization algorithm to find all these points for a given problem. So, as you maybe you can acknowledge here that when we have a objective function we do not know how many multimodal points will be there. So, the task is even more difficult because sometimes we may explore few points, but not all of them.

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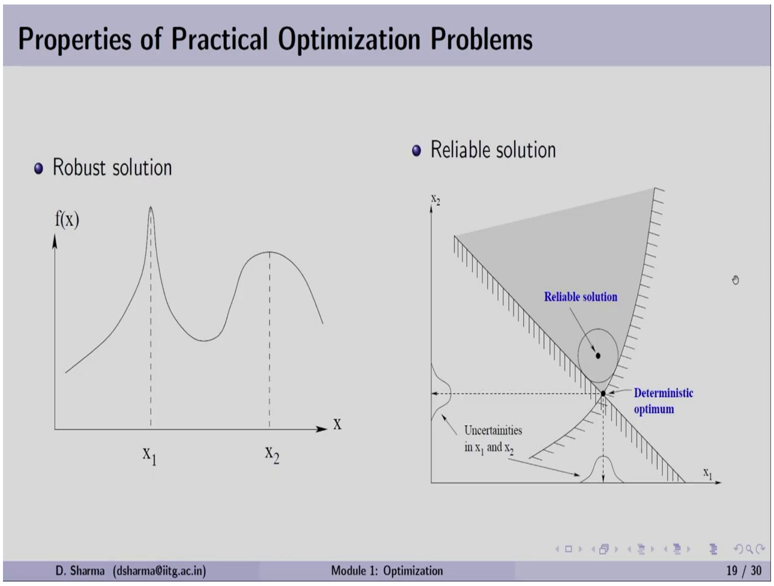


Coming to the another property which is called robust solution. Now, looking at the figure, suppose, we want to maximize the function now. So, if we maximize its very clear the optimization algorithm should give us $x1$. Now, we can take this $x1$, we can design something, we can take some strategic decision based on that.

Now, you know that if there is a small change in a $x1$ value, the objective function will reduce drastically. So, since it is going to be reduced drastically, so, this point is very sensitive. So, if we are going to take a decision based on $x1$, this may be this may be this is going to be a problem for us in a future. So, what we can do is rather than finding the point x 1 we may look for a point say $x2$ which is kind of a stable.

Even if there is a small change in the x2 values, the objective function should not change much. So, therefore, we look for a robust solution like x2 as compared to x1, but optimization algorithm does not know. So, that is why finding such kind of a solution is a challenging task.

Coming to the reliable solutions. Now reliable solution, first of all we have to see that there are two lines which with the hatched pattern. Now, these two lines one is a straight line another is a curve; so, these two lines represents the constraints.

Now, suppose if we are going to solve this problem without considering anything a simple optimization problem you can find there is a deterministic optimum. So, this deterministic optimum is the solution if we solve this problem. However, in the real world application what you will realize that the set of variables which we take that can have certain uncertainty. So, because of that uncertainty this deterministic solution will become invisible.

So, the task is how we can find a reliable solution as shown in the figure by using the using or using the optimization algorithm. So, this poses a difficulty for our algorithm since the x1 and x2 variables are uncertain.

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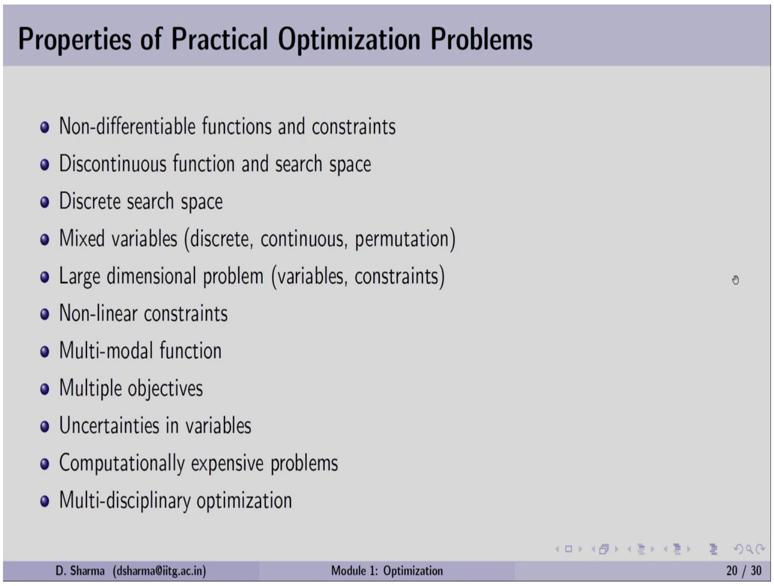


Now, putting all those properties in a one particular slide you will realize that that our function can be non-differentiable. So, the method which need gradient information may not be useful for solving such kind of a problem. It can be a discontinuous function or a search space there also the optimization algorithms which requires that the search space should be continuous that cannot be used.

Similarly, discreet search space; now handling discrete search space is a big challenge. So, we have to find a good algorithm that can solve such kind of a problem. Moving to the mixed variable; as I gave you an example of unit commitment problem or say gear design problem or it can be a pressure vessel design problem.

So, in these problems what you will realize that it requires the binaries binary variables as well as we need some time discrete variable along with the real variables. So, when you have a combination of such kind of a variable solving such those problems are really difficult.

Coming to the large dimensional problem; it means that the number of variable is too large. So, solving a practical problem having that can have lot many variables make a problem difficult to solve. Similarly, we can have lot many constraint for a given problem.

Non-linear constraints; so, this non-linear constrain it can be non-linear constraint as well as functions. Now, the non-linear constraints will pose a difficulty in a way that a some particular portion maybe will be a trap where our optimization algorithm can a stuck.

Those trap generally referred as local optimum solutions. So, in that case these nonlinearities will make a challenge that if optimization algorithm converges to the local optimum point how we can come out and find the global optimal point.

Multimodal function, as we have defined that there are different points where we can get the optimal objective function value. Since we do not know what are those point how an algorithm can find all those points so, that we can analyze those point in our decision making.

Then comes multi objective; as our course is divided into two parts single objective optimization and multi objective optimization. So, multi objective optimization means when we are solving a problem which has more than one objective, it means that we have more number of a solutions which are optimal. Those solutions are generally referred as Pareto optimal solutions. So, can our algorithm gave those Pareto optimal solutions. So, that is a challenge for us.

Now, uncertainty in the variable as we discussed in the previous slide. So, this uncertainty can make our optimum solution infeasible. So, that is why we are looking for a reliable solution under the uncertainty in the variables. Some problems are computationally expensive.

Let us take an example. There is a problem called topology optimization. Now, if it is topology optimization of a structure it involves optimization plus finite element simulations. We know that finite element simulation requires time.

So, as we need for example, say 5 minutes to get 1 simulation of finite element and if we have to run our optimization algorithm safe for 100 iterations, so, basically we need 500 minutes to solve it which is a big number. So, this kind of a problem where we are including some simulations or the function evaluation itself is very computationally expensive.

So, those problems are going to be are going to be difficult to solve them in a limited time. Last, but not the least it is a multidisciplinary problem where we take the principles concepts from different disciplines like mechanical electrical civil engineering chemical engineering putting together and then we are solving some real word problems. So, when we are putting many concepts together the problems comes out to be very difficult to solve.

With this introduction on the scope of an optimization and the properties of optimization problems, we are now moving towards the problem formulation.

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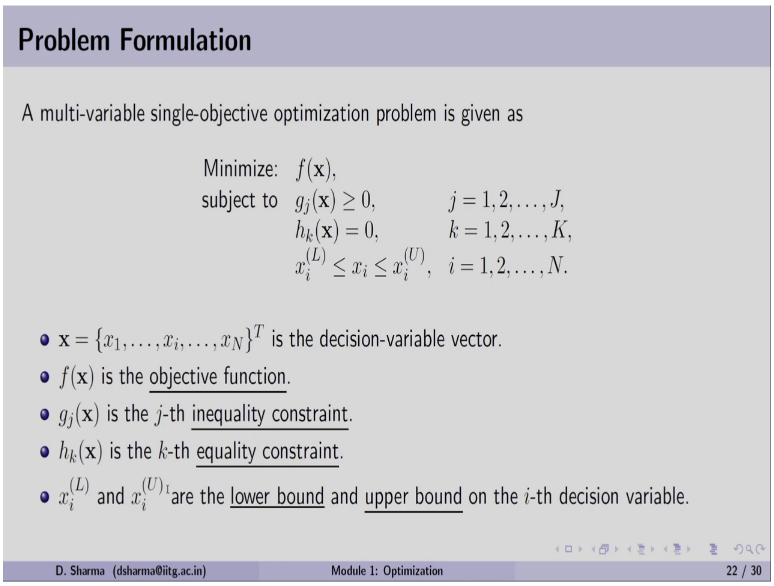


$$Minimize: \qquad f(x),$$

$$subject\ to: \qquad g_j(x) \geq 0, \ \ j = 1, 2, \ldots, J,$$

$$h_k(x) = 0, \; k = 1, 2, \ldots, K,$$

$$x_i^{(L)} \leq x_i \leq x_i^{U} \;, \; i = 1, 2, \ldots, N.$$

A problem formulation look like as we have one objective called f(x) and this problem is subjected to various kind of constraints. So, before we start let us see that the variable x is written as a capital X. So, that is why we have written it is a multi variable function.

And we also writing single-objective because we have just one objective which we want to minimize it and then we have various constraints. Now, looking at the constraint which is represented by $g_j(x)$ having the inequality sign and then this index j is varying from one to capital J. It signifies that we can have multiple inequality constraints. Similarly, if we look at the other constraint which is $h_k(x) = 0$, this is the equality constraint and the indices says that we can have multiple constraints in our problem.

And the last is the variable bounds, so, depending on the number of variables. So, the capital N suggest that a problem can have N number of a variable. So, each variable should have a variable bound. So, if we put together you can see that x is coming out to be a vector and generally we say it is a column vector. So, it consists of all the variables starting from x 1 to x n. Thereafter, since we are minimizing the function it will become the objective function for us.

Then we have inequality constraint. So, as we have defined the sign here with the $g_j(x)$ in the problem formulation tells us that the $g_j(x)$ is the inequality constraint. Similarly, for $h_k(x)$ looking at the sign where, left hand side equals to the right hand side suggests that the $h_k(x)$ is the equality constraint and $x_i^{(L)}$ or $x_i^{(U)}$ these are the upper and the lower bound on each variable.

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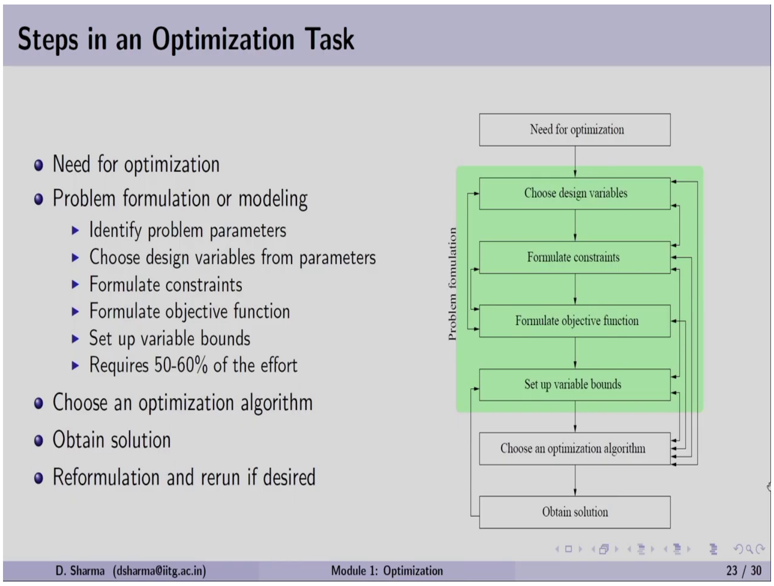


Now, let us talk about these steps in an optimization task. So, first we have to think that we need an optimization for a problem because there are certain problems that can be solved using simulations. If we are sure yes, we need an optimization there is a scope of an optimization then we go to the second step that is called problem formulation.

Now, problem formulation sometimes also referred as mathematical modeling. So, in this mathematical modeling first we have to identify the problem parameters. As you might have realized that when we are writing some mathematical equations those mathematical equations consist of various parameters some parameters are constant parameter some parameter can change their values.

So, our idea is we have to find all of these parameters, those are the sensitive parameters will become the design variable for our problem. Thereafter using the design variables as well as the constant parameters we can write our objective function, we can write our constraints.

Thereafter, we set up the variable bound. Now, if you look all these steps here that is exactly what we have written in our formulation the mathematical formulation. Say suppose you are working in a new problem right now. So, if we are going to make such kind of a mathematical modeling or problem formulation for an optimization it requires 50 to 60 percent of your effort which is a challenging task for us.

Thereafter, looking at the property of an optimization algorithm we choose the we choose the optimization algorithm now this optimization algorithm will be solving the problem and will be giving the solution. Now, if we look at the solution and if we believe that the solution is desired we can use it. So, we can stop this process.

However, if you believe that the solution is not as per our expectations then we have to reformulate. So, as you can see we have to reformulate and rerun. Now, looking at the figure on the right hand side what you will realize that say suppose the obtain solution is not desirable. So, what we are going to do is we will check the variable bounds because it can happen that the variable bounds which we have set the optimum solution is not lying there.

So, we look on the variable bounds. So, once we set the variable bounds and we are sure yes there we are going to search and find the optimum solution and still your solution is not desirable then we want to look again the objective function as well as the constraints. And a van while revisiting those constraints functions and objective functions we can also revisit whether the chosen design variables are appropriate or a not.

So, what you will realize that once we are not able to get the desired solution we have to look each and every step in the shaded region and once we get the desired solution then we can fix it we can use the optimal solution for our decision making.

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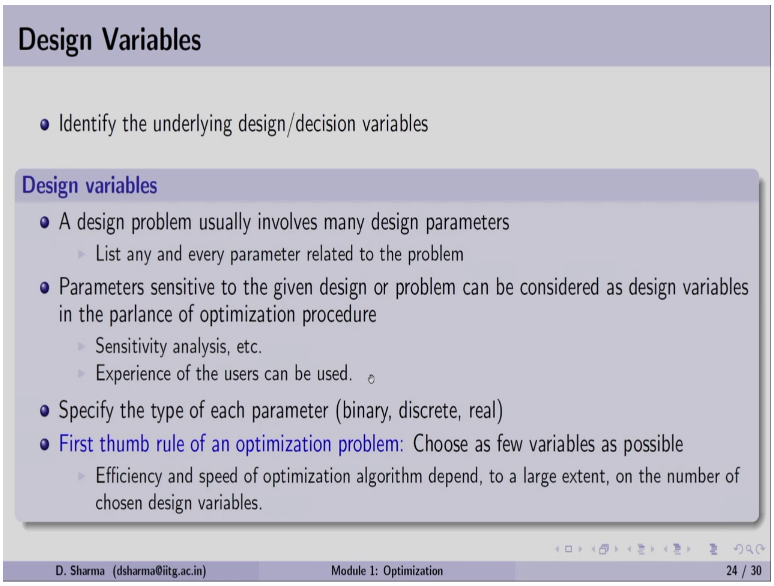

So, let us discuss how we can decide the design variables, constraints and objective function. Now, let us begin with the design variables right now. So, our first principle is we have to find all the design variables. Now, in this design variables a problem usually consists many design parameters.

As I explained you earlier that there are so many parameters involved. Some parameters are constant parameters some parameters can change in the problem. So, our task is we have to list each and every parameter relative to the problem. Thereafter, we look for those parameters which are sensitive to the problem.

Now, sensitive means that if I am going to change that parameter that is changing my objective function drastically. So, in this case we are going to take only those parameters which are sensitive as design variables and other parameters we can keep them as constant. How we can find whether this particular parameter is sensitive or a not? So, there is called sensitivity analysis where be we perturb that parameter value and try to find out what is the effect on objective function.

Similarly, we can use the experience of a person. So, someone who is working on a field or with the equipment and they are doing the optimization manually, so, we can take their experience for finding which parameter is the most sensitive for a given problem. Thereafter, we specify what type of parameter it is. So, it can be binary, discrete and real. So, each parameter means that includes the parameter as well as the decision variables.

Now, based on the based on the design variables, so, the first thumb rule of an optimization problem is we have to choose as few variables as possible, it is because the efficiency and speed of an optimization algorithm depend on the number of the chosen variable. So, it says that although there will be lot of parameters let us take only few design variables among them to construct our optimization problem, say for example, objective function and constraint.

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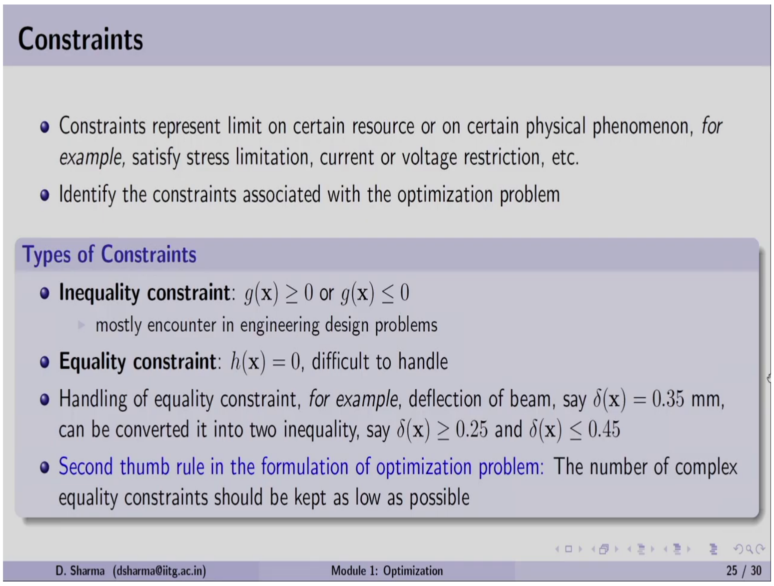


Let us move to the constraints now. Now, the constraints represent the limit on certain resources and physical phenomena. So, here when we look at when we are solving a problem we know that there is always a restriction on or there is some limit while we are where while we are doing their process. So, for example, we can have limitation on a stress, we can have restriction on current or a voltage. So, those restrictions we are going to write in terms of constraints.

So, here our task is we have to identify what kind of a constraints we needed. So, we have to note down all the constraints because those are going to be a part of our optimization problem. So, if you refer our problem formulation you remember that we have inequality constraints and equality constraints and we can have multiple types of inequality and equality constraints.

Now, let us talk about the type of inequalities. So, first is called inequality constraint. So, in the previously we discussed about greater than sign, but the inequality can be represented as greater then type or a smaller than types both type are referred as inequality constraint. Now, you will be realizing that the most of the engineering problems have the inequality constraint. The other type of constraint is the equality constraint, which are generally difficult to handle.

So, one way to handle the equality constraint is that we can convert our equality constraint into two inequality constraint as given in the example. Say for example, we have deflection of the beam say delta x is equals to 0.35. Now, this equality constraint where left hand side should be equal to the right hand side, we can convert into two inequality constraint say delta should be greater than and equals to 0.25 and delta should be smaller than 4.5.

So, that will give some search space for an optimization algorithm to look for an optimal solution. So, based on the constraint the second thumb rule in the formulation of optimization problem is that the number of complex equality constraint should be kept as low as possible.

It is because you know that the equality constraints are difficult to handle and one equality constraint if we follow the procedure as defined here we have to change or convert the one equality constraint into two inequality constraints. That is making too many constraints for a problem. So, that is why we have to reduce these complex inequality constraints in our problem. Let us move to the objective function.

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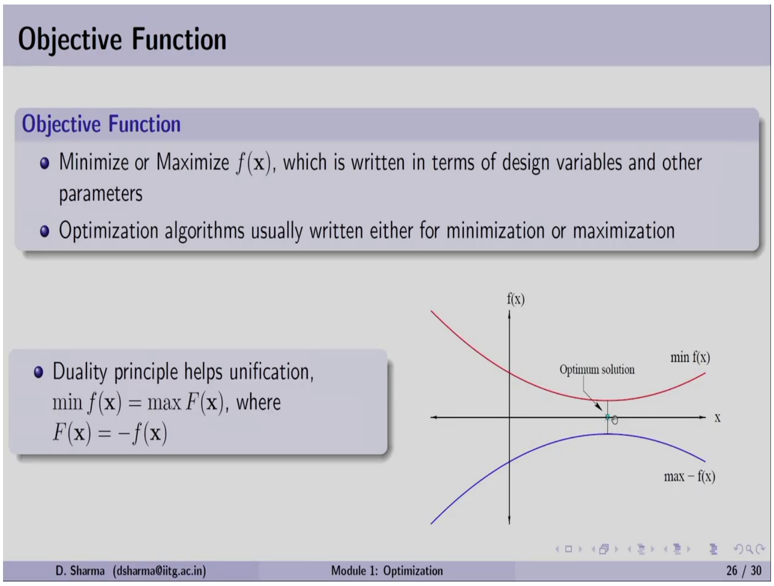


Now, as you know that the objective function can be of two type; either I will be minimizing the function or maximizing the function. How we can correlate? Say in an organization one particular organization is looking for minimizing the cost of a product. So, accordingly they will be designing everything.

But another organization may be looking for maximizing its profit. Now, both of the organization have their goal, but one is minimizing another one is maximizing. So, generally the objective functions are written in terms of design variables and other parameters. Now, what you will realize, that the algorithms generally are made using either minimization or for maximization. So, let us take an example of one particular algorithm that is designed for minimization.

So, if you supply a function which you want to minimize it this algorithm can able to give you an optimum solution and suppose we are working for an organization where we want to maximize the profit. Now, in this situation when we want to maximize the profit, but the algorithm is designed for minimization can we use it? So, the answer is yes. So, which method we can use it. So, here duality principle can help us to solve such kind of issues.

So, what duality principle says that as you can see here minimization of a function f(x) is equals to maximization of capital F(x), where capital F(x) is equals to minus times of minus times of f(x). What it says that? Let us look into the figure now. So, on the right hand side suppose I want to minimize the function f(x), which is drawn in the red color. If I am going to minimize it let us say we get the optimum solution as mentioned here.

Now, if you use; if you use the duality principle the same problem I can write as maximization of minus f(x). So, what we did here is we multiplied by minus and the minimization problem is converted into the maximization. Now, let us look at the nature of the blue line which is maximization of minus function. Now, here you can see that both of them have a different nature; basically they are mirror images.

However, when I am solving the problem of the blue color I will get the same optimum solution. As you know that we are looking for an optimum solution, so, in this case even if our algorithm is designed for minimization we can multiply by minus 1 to the profit of the company, convert that maximization problem into minimization, solve it, then we can get an optimum solution, so, the value of the design variables. We can take those design variable and we can find what will be the our profit.

And similar situation can happen if the optimization algorithm is designed for maximization by multiplying the minimization by multiplying minus 1 with a minimization problem, it will be converted into maximization and I can use my algorithm. Coming to the last part of the problem formulation that is variable bounds.

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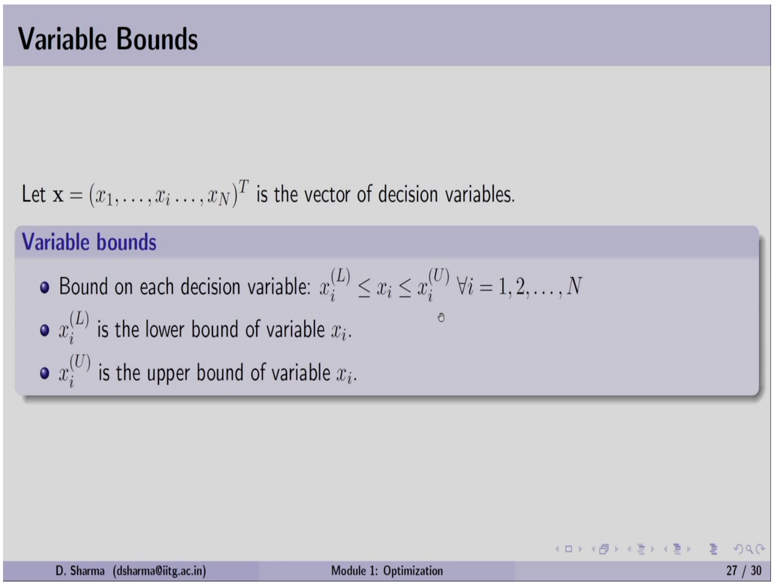


Now, as you can see that all the variables like x1, x2, .., xN they are putting into a one vector called capital X. Now, here you can realize that whether you are solving a problem from say electrical engineering, so you have voltage current and other things. If you are solving a mechanical problem you can have different parameters related to the machines.

So, in general we have various input or decision variables. So, we can put together and we can write in a vector form. However, for every variable whether it is x1 or xN? all variable should have the variable bounds which will be line from lower to the upper one. So, once we decided it we can have an optimization problem for optimization problem for our given problem.

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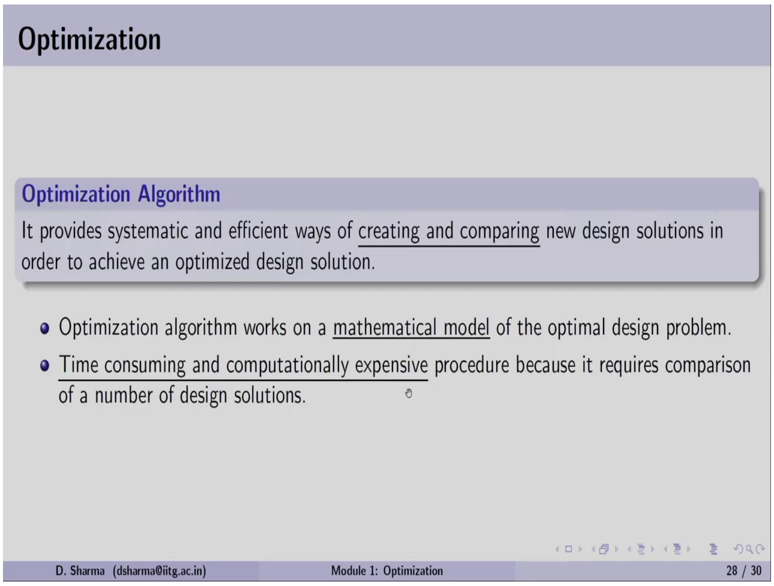


Now, as you know that up to this particular point we have we can have an optimization problem generally written in the mathematical form for solving it. If we solve it we can get an optimum solution. So, generally solving manually is not a desirable task because it is lot it is time consuming.

So, what we do here is to solving such problem we need algorithms. Those algorithms are generally referred as optimization algorithm. So, these algorithms, so basically an optimization algorithm provides systematic and efficient ways of creating and comparing new design solutions in order to achieve an optimized design solution. So, what you can see that in every iterations, we will be creating new solutions, comparing them so that we are sure that our solution is improving and finally, we will reach to the optimum solution.

Now, as I mentioned earlier that manually solving such kind of a problem is not easy so that is why we use optimization algorithm. Since, it involves creation and comparison, so, optimization algorithm sometimes can be computationally expensive and will lead more time to solve a problem.

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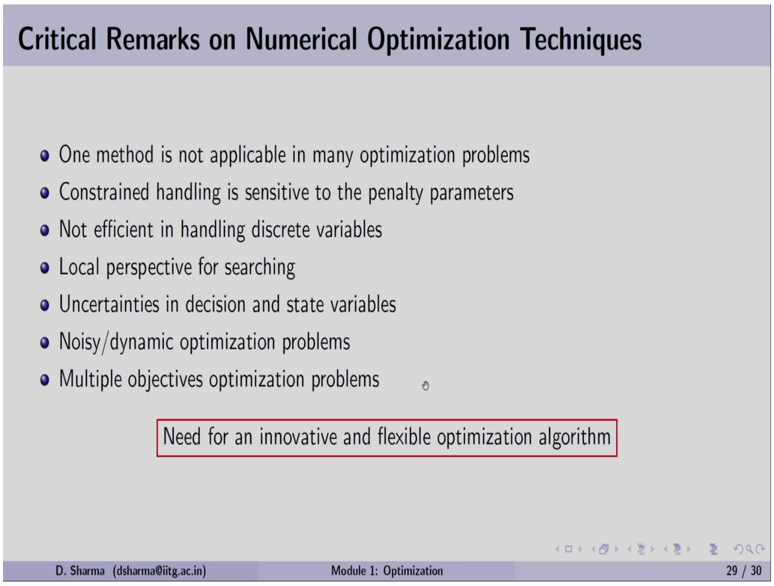


Now, with these all the background of practical optimization problem, the applications and the generalized problem formulations for a given problem as well as looking at the optimization algorithm we have critical remarks on numerical optimization techniques. Why numerical optimization technique? Because once the optimization formulation is ready we have to solve it.

Now, numerical optimization can be the choice or there are other algorithm which we can say as non-traditional optimization algorithm that we can use it. Since we are we will be working towards the nontraditional optimization algorithm, let us see how these numerical optimization techniques will behave.

So, as we know that the numerical optimization techniques one of the method may not be applicable for a large class of a problem. It is because for example, if our function is non differentiable, so, I cannot use those methods which needs gradient information. However, we are we can use some direct search methods.

Constraint handling is always difficult with the numerical optimization technique. It is because it involves lot of penalty parameters. So, these penalty parameters are very sensitive to our problem. So, in that case constraint handling is difficult with these kind of techniques. Now, when we have a discrete variable? Now, discrete variable is difficult whether it is direct search method or a gradient based methods. So, handling discrete variables are going to pose different challenges to us.

Generally, numerical optimization techniques use the some mathematical rule based on that they move from one point to the another point. So, they find the direction in which they move. So, in that case what you will find. So, generally that decision is taken with respect to the local perspective and that local perspective may not be may not be efficient and we may trap the solution we can get the solution in the local optimum.

Now, uncertainty; as you have seen that uncertainty in the design variable can post lot of challenges. So, that is why solving those problems with numerical optimization problem is a difficult task. Noisy and dynamic optimization problems are always difficult to solve with numerical optimization. It is because the optimization functions are keep on changing sometimes.

Then we can have a problem of multi objective optimization where we look for multiple solutions, as we know there could be multiple Pareto optimal solutions. So, solving such kind of a problem and generating all of them will be a difficult task with the numerical optimization techniques. By looking at all the remarks here what we can understand is we need for an innovative and flexible optimization algorithm that should cater our need; so, our need as written in terms of the remarks.

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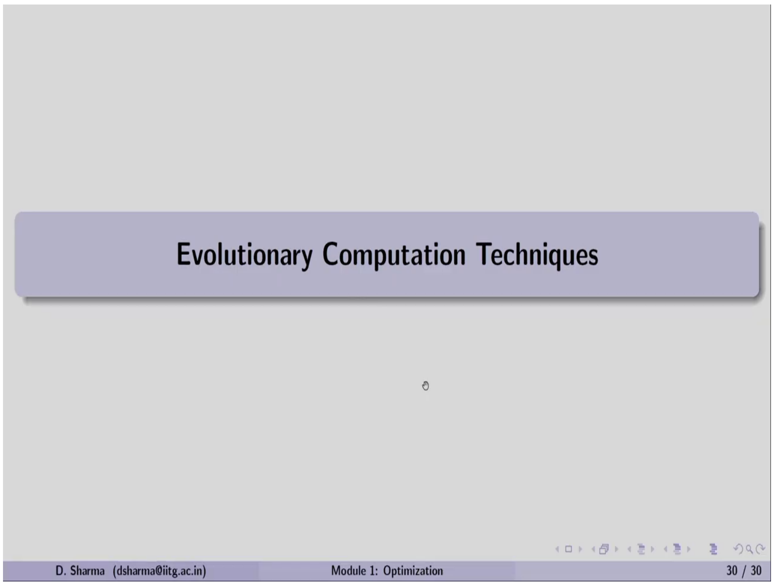


So, the answer to this question is we have evolutionary computation techniques that can be used for wide variety of a problem. They are direct search method, they do not need any gradient and that is why we can use it whether it is a discrete problem whether it is a multi objective problem or there is a uncertainty. The different kind of a complexity is in the problem, we can very well handle with evolutionary computation technique.

So, with this introduction in this session we can see in the later sessions and modules that evolutionary computation techniques are efficient to solve such kind of complex and large problem. With this node I conclude the session 1 of module 1.

Thank you.