Nonlinear Vibration
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Lecture - 36
Chaotic systems and control of chaos

So, welcome to today class of Non-linear Vibration. So, today we are going to complete this course and this is the last class of the last module. So, today class particularly we are going to summarize all the things besides studying few things what we have left that is the Chaotic system and control of chaos.

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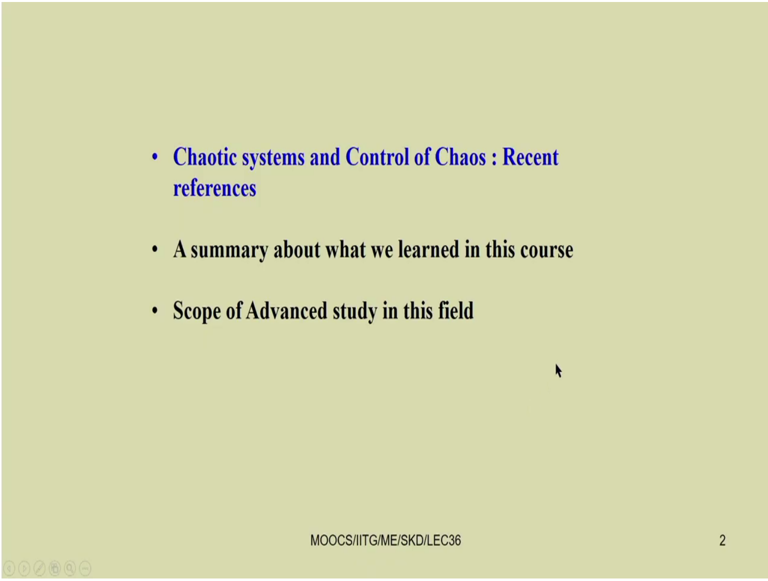

So, recent references I will give you. Then summary about what we have learned in this course we will discuss, then scope of advanced studies in this field also we are going to discuss in this class.

So, already you have studied different type of responses, fixed point response, periodic response, quasi periodic response and chaotic responses and how to characterize them also you have learned.

So, you know regarding the time response, phase portrait, Poincare section, FFT and Lyapunov exponents to characterize all these type of responses. So, particularly these study of chaos so, is an emerging field and you can have lot of literature related to these chaotic systems how you can control chaos.

So, already you know that chaos is a deterministic system. So, here you can determine the chaos. So, it has a lot of practical applications. So, this chaotic systems are controllable and different control objectives such as stabilization, synchronization and bifurcation control can be possible for these systems.

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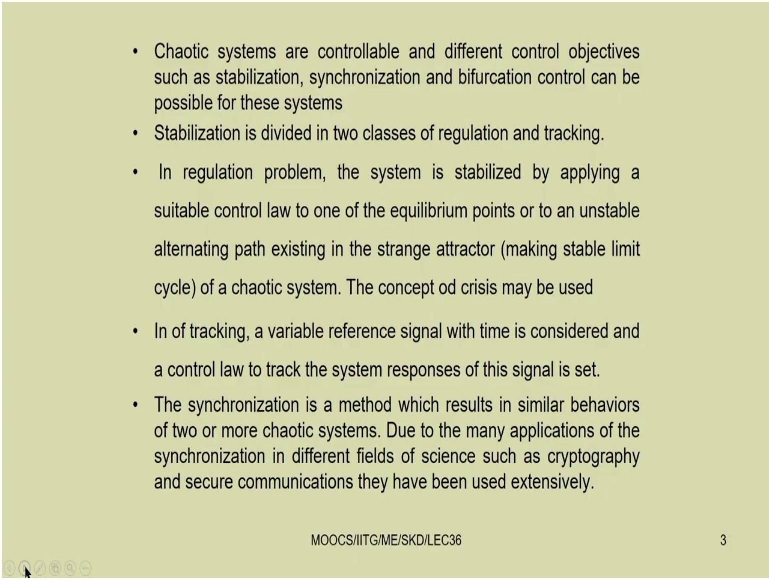

Stabilization is divided into two classes; so one is this regulation and other one is the tracking. So, in regulation problem the system is stabilized by applying a suitable control law to one of the equilibrium points or to an unstable alternating path existing in these strange attractor.

Sometimes you may use the concept of these the concept of these crisis to so concept of the crisis to control the bifurcation or control the chaotic system. So, already you know regarding these interior crisis, exterior crisis, and attractor merging crisis.

So, by so, when this chaotic attractor come in contact with some fixed point response unstable or stable fixed point response, sometimes it makes explode or sometimes it may disappear. So, by that way so, you can regulate the chaotic response. Also, by changing the system parameter, as you know by changing the system parameter. So you may have a periodic response, then it is two periodic, then four periodic.

So, you know the period doubling route to chaos. So, this if you change the system parameter a chaotic system can conveniently be reduced to its previous stage and it can be brought to a stable response, stable single periodic response also. So, in tracking a variable reference signal with time is considered and a control law to track the system response of the signal is set.

So, the synchronization is a method. So, which result in similar behaviour of two or more chaotic systems. So, duty due to the many applications of the synchronization in different fields of science particularly, in the cryptography and secure communications they have been used nowadays extensively. So, you can find 30, around 33 percent of the secure communications and the cryptography uses these chaotic control law.

So, this is an emerging field and you must learn this thing extensively to use this chaotic responses for different applications. So, already you know there are several systems. So, particularly when you are increasing the dimensions of the system or the number of state variables of the system. So you can see in a 2 degrees of freedom system or 3 degrees of freedom system, like this low range attractor or these four.

So, when you have more than three equations like the equations we have taken for 2 degrees of freedom system when they are converted to 4, first order differential equation so we got many chaotic responses.

So, those chaotic responses can conveniently be used to study all these type of synchronization and the stabilization properties of the system. So, these adaptive control, sliding mode control, backstepping control are used for synchronization.

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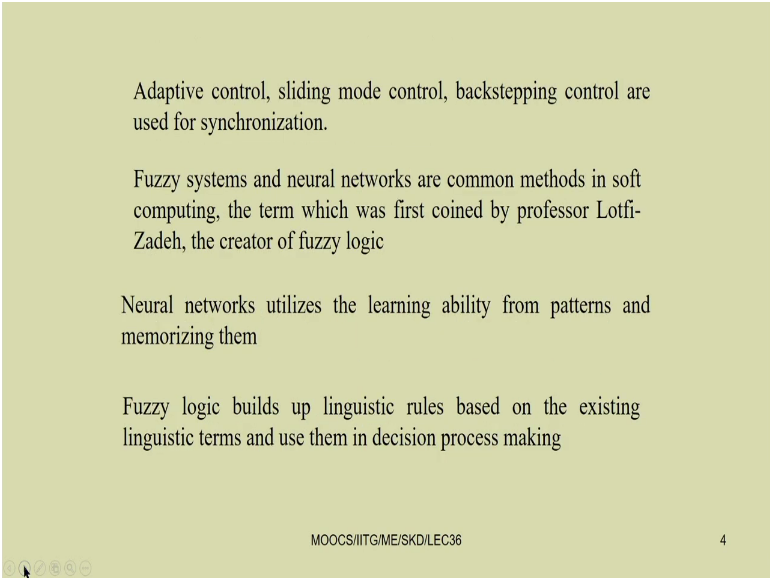

So, one may use this fuzzy system and neural networks are common methods in soft computing the terms which was first coined by Professor Lotfi-Zadeh, the creator of fuzzy logic. So, neural networks utilizes the learning ability from pattern and memorizing them. So, nowadays these deep learning process, is also used for many applications.

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So, these fuzzy logic built up linguistic rules based on the existing linguistic term and use them in decision making process. So, all these things what we have not covered in this course can be added up to what we have learned and can be utilized for different, many different applications.

So, many researchers have done or have used the Chen, Lu, Liu, Chua, Duffing, Genesio-Tesi, Lotka-Volterra, Newton-Leipnik, Rossler fractional order systems and based on these adaptive control, robust control, intelligent control, and combination of these methods. They have used these stabilization or the synchronization process of those systems. So, for example so, in image processing nowadays chaotic, these chaotic responses have been used for masking that images.

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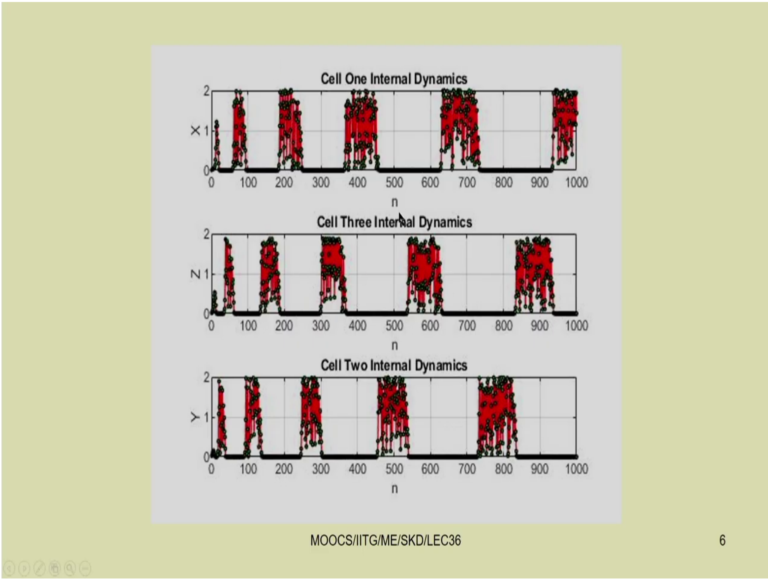

So, for example so, here I have given a simple code by using this logistic map, cubic order logistic map so which can be used for covering or masking the colour images or the images. So, here you just see these are the chaotic response. So, these chaotic response have been generated by a combination of these cubic order logistic map. So, you can see the codes, a code has been written in MATLAB.

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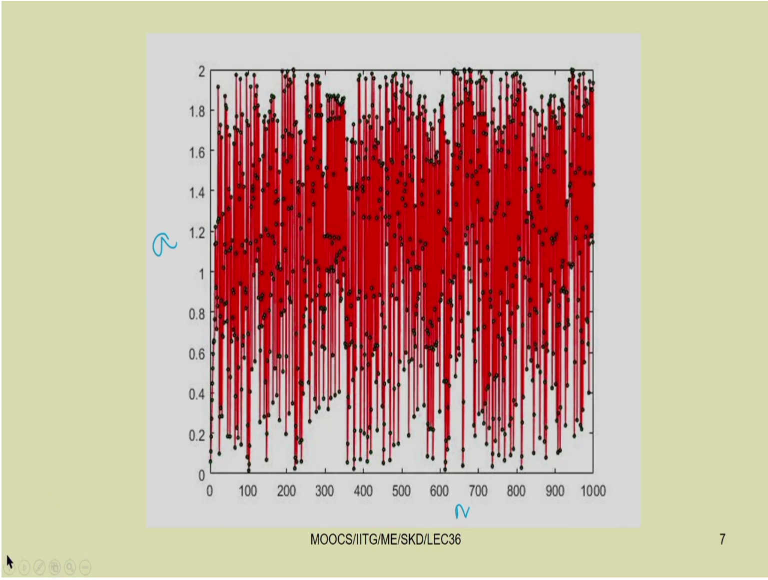

So, this is the logistic map also. So, where so, a combination of three logistic map have been shown here. So, this is the code, simple code, you can write a simple code for the logistic map.

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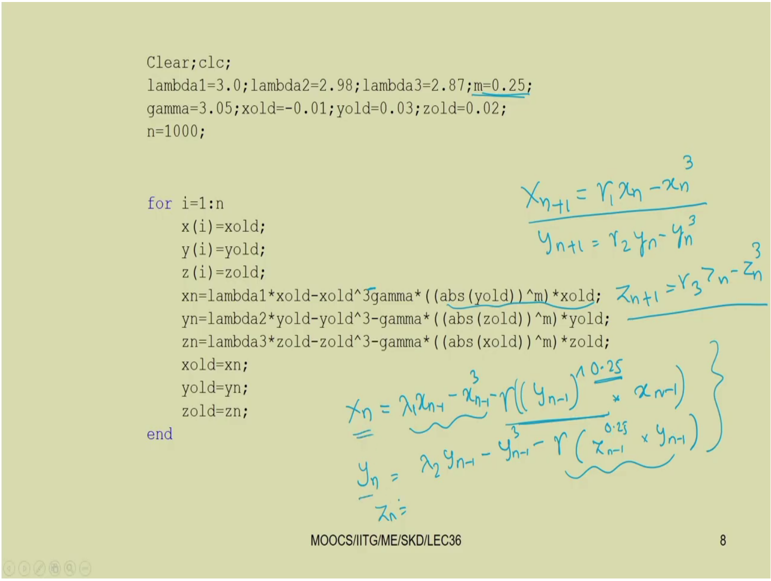

So, here this xn equal to lambda 1 into or you can write this ok. So, xold is one order less than this thing, if you are writing this is xn this xold is nothing, but xn minus 1. So, this is lambda 1 into xn minus 1 minus x m minus 1 cube.

So, minus so, we have a minus sign here. So, minus gamma into minus gamma into absolute. So, similarly so, I can write this y. So, x y z three state variables we have taken. So, here so, combination of two has been taken here combination of x and y. So, if I am writing a cubic order logistic map simply you can write this way.

So, X n plus 1 generally it is written in this way X n plus 1 can be written as lambda or let me write in a simpler form this is r into x n minus x n cube. So, this is cubic order. So, this is cubic order logistic map.

Similarly, for Y n plus 1 you can write. So, this will be equal to so, if you are writing this is r 1, so this is you can write r 2, so this is y n minus y n cube. Similarly, you can write Z n plus 1 will be equal to r 3 y r 3 into z n, z n minus z n cube.

So, these are the cubic order logistic map. Already, you have seen the other logistic maps also and train maps also and. So, you can using the you just see you can use either one equation or a combination of these equations.

So, here combination of these equations have been written. So, here you just see so, we have used these instead of a integer like this cube. So, here we have used a fraction. So, you just see this m is taken to be. So, you can see the value of m is taken to be 0.25. So, here it is taken a fraction order. So, this part is a fraction order is taken.

So, this is you just see, this is written gamma. So, you can put ok. So, gamma so this equation can be written as X, it is written X n equal to lambda x n minus 1 minus x n minus 1 cube minus gamma into gamma into so, absolute value of yold is taken. So, that is Y n minus 1 absolute value of Y n minus 1 to the power so 0.25. So, it is you just see this part is taken to be a fraction order.

So, 0.25 is taken. So, into then x n minus 1. So, this is the first equation written similarly the second equation what is written is so, this is lambda 1 taken. So, here lambda 2 Y n minus 1 minus Y n minus 1 cube Y n minus 1 cube then this is gamma into so, this whole thing is multiplied. So, gamma into absolute value of Z n minus 1 to the power Z n to the power 0.25 so, into so, Y n minus 1.

So, similarly for the Z n it is written. So, combination so, you can see. So, you can so, this is cubic order logistic map. So, along with that these fractional order is added here. Similarly, you can add these.

So, for Z n also similarly you can write and this absolute value of these X n, Y n and Z n if you add so, you can get the previous plot what shown. So, this is a versus. So, a equal to absolute of these X n square, Y n square Z n square root over. So, this is the number of iteration n you can put.

So, 1000 iterations we have taken. So, for that purpose we got this map. So, you just see so, here also so, you can see we have added these two pass two. So, this is X and Y we have added, then these Y and Z added and then Z and X added.

So, you just see these three combinations we have taken. So, we can combine these three to mask or the cover the any image or any secure security related data. So, this way you can use these chaotic responses for sending the secure data or for masking purpose in case of the colour image or any other data also.

So, you can write a simple code using either the logistic map or using some other. So, this is I have shown here the logistic map, but you can use any other map two-dimensional map or one-dimensional map. Actually one-dimensional map when we are using then only one parameter is there this r 1 is the only parameter. So, one can crack these parameters single parameter and one can get the secure code.

So, to avoid that thing generally you, one can go for these higher order equations, but while using this higher order equation as the number of unknowns are more. So, when he when on masking the thing so it will take more time and the cost of developing this thing is also huge. So, this is the code, complete code is written to write down these plot or find this plot.

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So, here you just see in this plot. So, we have used three subplots. In a single plot we are using three plots. So, we are using the subplots command. So, in this course so, you are going to learn this MATLAB and use this MATLAB for different assignment. So, you must learn how to plot all these things.

So, here the subplot command is used. In subplot 3, 1, 1; that means, so, we have 3 rows, 1 column and this is the first position. Similarly, we can have 3, 1, 2. So, in case of 3, 1, 2 this is the three row, one column and this is the second position and three row one column and this is the third position. So, you can write the plot command here.

So, here it is plotting these plot absolute value. So, absolute y so, absolute y versus. So, that is the number of iteration. So, as number of iteration is 1000. So, you can see we have put n equal to 1000. So, number of iteration is 1000. Sometimes you can put time also in place of this thing.

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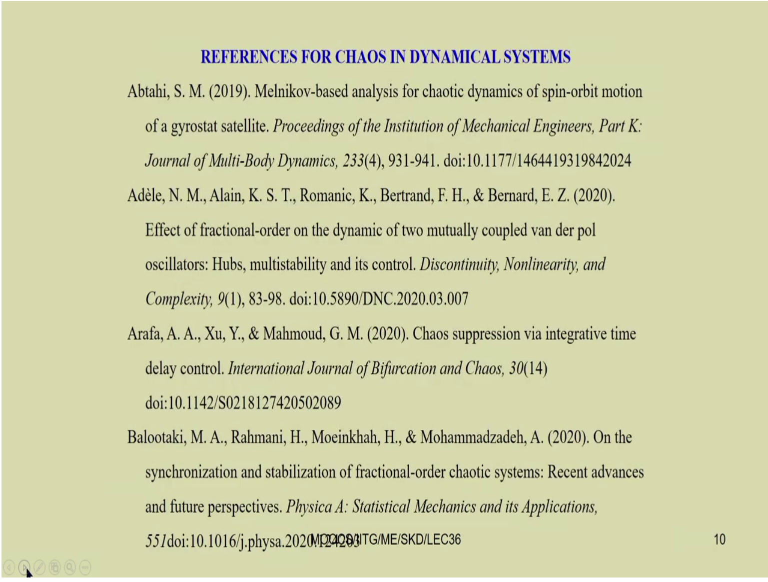

So, now let us see there are several references available in case of chaos in dynamical systems. So, few of them have been collected and it is very few recent papers have been collected and put it here. So, you can go through similar papers and you can have better understanding regarding these chaos process. So, for example, this is the this paper you can read this Melnikov-based analysis for chaotic dynamics of a spin orbit motion of a gyrostat satellite.

So, this paper is used for a satellite purpose. This paper effect of fractional order on the dynamic of two mutually coupled van der pol oscillators. So, you know regarding this van der pol oscillator who is gives rise to the periodic orbits or limit cycle, but by taking a fractional order a van der pol equation so easily you can get the chaotic response.

So, here it is studied for the hubs, multistability and control and another paper this you can see this chaos suppression via integrative time delay control. So, already you know regarding the delay system. So, in the applications particularly, in case of the turning operation on these vibration observers so we have studied or we have taken the example of the delay systems.

So, particularly this delay systems are used for reducing the vibration, sometimes it can be used as a negative damping also. So, you can use this delay for controlling these chaos. So, on the synchronization and stabilization of fractional order chaotic system recent advances and future perspective. So, this is a very good paper by Balootaki and Rahman, Balootaki, Rahmani, and Moeinkhah and Mohammadzadeh.. So, from this paper you can study regarding different stabilization process and synchronization process used in this fractional order chaotic systems.

So, this is a review paper and this paper you may see for better understanding regarding the stabilization. So, there are several references present in this paper which will be useful for integer as well as fractional order stabilization of the system, chaotic systems. So, there are some other papers like these bifurcation route to chaos.

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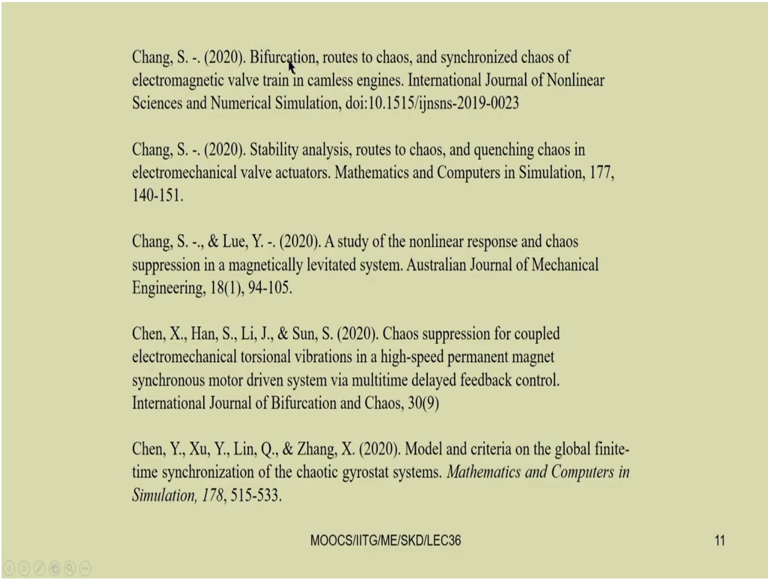

So, several bifurcation route to chaos also we have studied in this course. For example, we have discussed regarding period doubling route to chaos, then we have studied these torus doubling route to chaos, torus break down route to chaos. Then intermittency route to chaos. So, several crisis systems also we have studied for example, so, we have these interior crisis, exterior crisis and attractor merging crisis.

So, here you can study in this paper so bifurcation route to chaos and synchronized chaos of electromagnetic valve train in camless engine and the next one is the stability analysis, route to chaos and quenching chaos in electromechanical valve actuators. So, by using these external force these electromechanical. In electromechanical valves you can study the stability analysis and the different route to chaos and these quenching of chaos.

So, then by Chang and Lue you can study these paper, a study of a non-linear response and chaos operation in magnetically levitated system then Chen, Han and Sun so, chaos operation for coupled electromechanical torsional vibration in a high speed permanent magnet synchronization motor driven systems via multi time delay feedback control. So, then Chen-Lin and Zhang paper so model and criteria on the global finite time synchronization of the chaotic gyrostat system.

So, already you know the by synchronization so, we have two systems; one master system and one slave system. So, these master slave combinations can be utilized for synchronizing.

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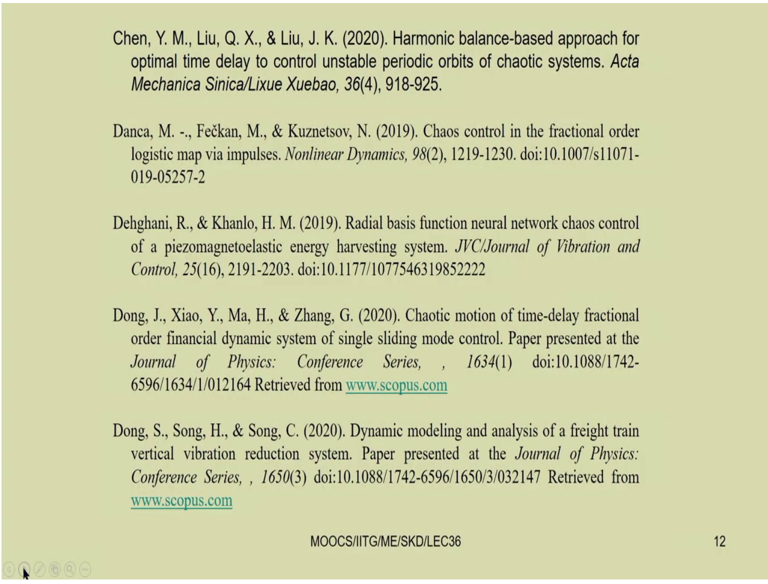

So, a particular type of motion so for example, a master so, a for example, you can take the low range attractor as the master one and some other attractor as the slave one and in particularly in case of the slave. So, you can take either a higher order that is greater than that of the master one or that less than that of the master one and you can find the synchronization procedure or different procedure to synchronize these chaotic motion.

So, the synchronization will be particularly useful for many applications already we have discussed. So, this another paper you can see here this harmonic balance based approach for optimal time delay to control the unstable periodic orbit of chaotic system.

So, this chaos control in the fractional order logistic map via impulses and this is by Danca and Danca, Feckan and Kuznetsov then this other paper you can see, this radial basic function is used here, radial basis function neural network chaos control of a piezo magneto elastic energy harvesting system. So, this work is by Dehghani and Khanlo.

So, then you can see the paper by Dong, Xiao, Ma and Zhang. So, the work on this chaotic motion of time delay fractional order financial dynamic systems of single sliding mode control. So, already we have seen sliding mode control can be used. So, this is these paper is on a financial dynamic systems.

So, though we have particularly discussed regarding these mechanical systems also, the same idea can be extended for financial or economic cases or study on the economics dynamics in economical systems also. In the dynamics of biological systems and also in the systems related to these electromechanical systems and systems related to data science also.

So, many applications can be found or many ways you can extend the ideas what you have learned in this course in many different directions. So, the next paper you can see this dynamic modelling and analysis of freight train vertical vibration reduction system.

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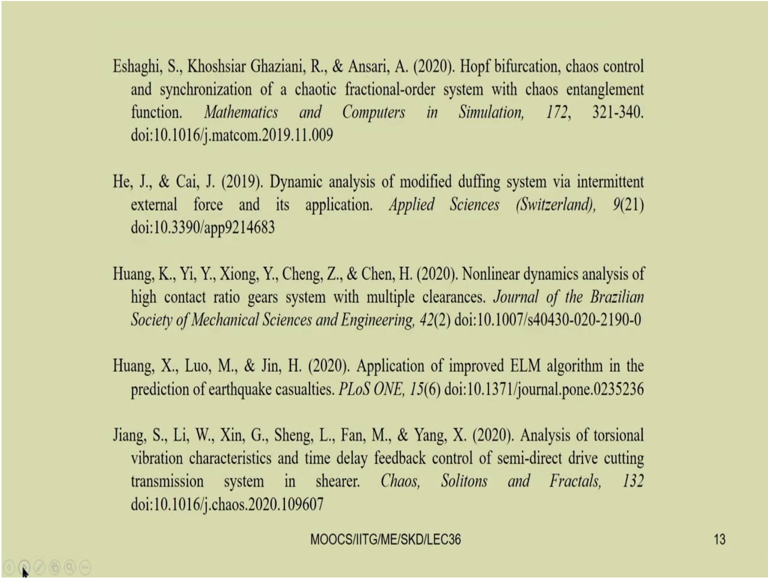

So, in train so you can have different type of motions so, those chaotic motions you can control by using these stabilization or synchronization method. So, sometimes you may required a models. So, these particularly these soft computing techniques are used when you are not sure about the dynamic model of the system.

So, when you know the dynamic model of the system. So, you can use other techniques, but the soft computing techniques particularly you can use when the model of the system is not clearly known to you. Particularly, in case of presence of friction in a system or some on modelled noise in the systems so it is difficult to model exactly, model the system.

So, you have to train the input and output of the system and taking those training data so you may use different soft computing techniques for synchronization of the systems so where chaotic phenomena occurs.

So, here a in this paper you can see it is on these Hopf bifurcation, chaos control and synchronization of a chaotic fractional order system with chaos entanglement function. So, sometimes these one orbit will entangle with the another orbit and it can give rise to a greater chaotic attractor.

So, already you are familiar with the Hopf bifurcation. So, in case of Hopf bifurcation from a periodic or from a fixed point response so you may get either a super harmonic super critical Hopf bifurcation of or the sub critical Hopf bifurcation. So, where either a stable periodic orbit or an unstable periodic orbit will emanates.

So, this Hopf bifurcation so, this is a dynamic bifurcation. So, particularly in case of these sub critical Hopf bifurcation so, you may get some chaotic system. So, near to that thing you may get some chaotic system and you can control that chaotic system by making these attractor or moving these attractor very close to these unstable fixed point or periodic response.

So, dynamic analysis and of modified duffing system via intermittent external force and applications and its applications. So, already you are familiar with duffing equation so, we have shown that when duffing equation the coefficient of x is negative so; that means, you are writing these equation in this way that is x double dot minus x plus alpha x q equal to f sin omega t or cos omega t.

So, in that case so, you can generally get a chaotic response. So, in this paper this dynamic analysis of modified duffing equation via intermittent external force has been discussed and some of the applications also have been discussed.

This paper by Huang, Xiong and Cheng and Chen non-linear dynamic analysis of high contact ratio gears systems with multiple clearances. So, here the gear system has been used.

So, in gear system non-linearity occur generally due to these clearance which give rise to backlash.

So, due to that thing so, the modelling itself will be non-linear. Non-linear governing equations you can get. So, in that non-linear equation so, when the system is of high contact ratio so that time this non-linear dynamics has been studied in this case and chaotic responses have been found.

So, this other paper by this one Huang, Luo and Jin, application of improved elm algorithm in the prediction of earthquake casualties. So, this is also a paper related to chaos. So, analysis of torsional vibration characteristic and time delay feedback control of semi direct drive cutting transmission system in shearer. So, this paper also discussed regarding chaos in torsional vibration systems, system with torsional vibration.

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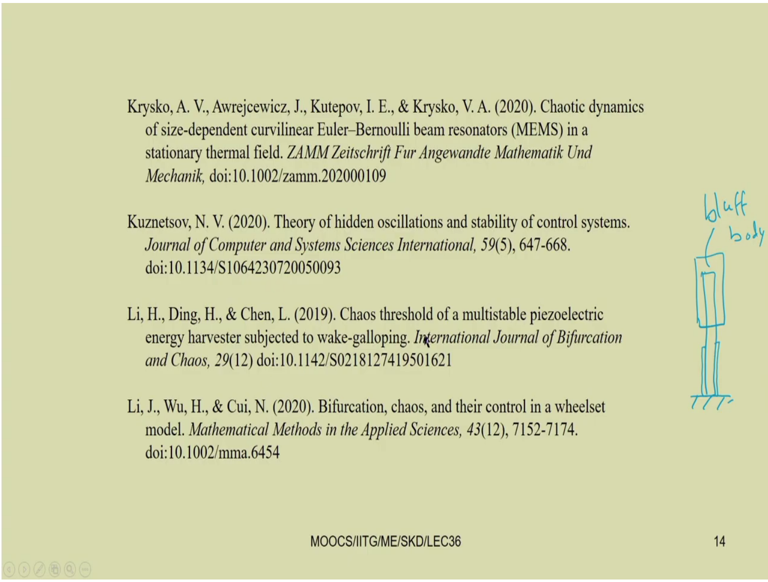

So, already you know so you may have a system where longitudinal vibration may occur so like in case of the spring mass system so you have longitudinal vibration.

So, in case of a willing of soft experiment so you have seen these transverse vibration and so, already you know regarding these torsional vibration. So, in case for example, you just take the example of a tri pillar system tri pillar. So, in tri pillar system three pillars or wires are suspended to hang a object and by rotating that object so, you can find the polar moment of inertia of that object.

So, generally this tri pillar suspension is used for finding the polar moment of inertia. So, here, instead of a small angular motion so if you give large angular motion so in that case nonlinearity will come into picture.

So, already you have seen the case of the simple pendulum so where that large theta give rise to non-linear phenomena. So, there are some many other papers are also there. So, few others include these chaotic dynamics of size dependent curvilinear Euler Bernoulli beam resonator. So, Euler Bernoulli beam equations so we have a continuous system.

So, in this work these chaotic dynamics has been studied for this Euler Bernoulli beam. So, then these Kuznetsov; so, he studied this theory of hidden oscillations and stability of control systems. So, then Li, Ding and Chen they studied this chaos threshold of a multi stable piezo electric energy harvester subjected to wake galloping. So, in this case they studied this galloping phenomena. So, particularly you have seen in case of this energy harvester that we may take this ambient vibration.

So, far this harvesting these energy here in case of galloping case so, particular for a above a particular wind speed the cantilever beam type of so, if one uses a cantilever beam type of observer so with some, let this is a obstacle mass or the ok so, bluff body. So, one can use a bluff body.

So, due to this wind force so wake will form behind this bluff body so which will give rise to motion in this harvester. So, in this harvester generally we put this piezoelectric patches. So, these are the piezoelectric patch. So, these piezoelectric patches will be used for harvesting energy and these this is the bluff body.

So, depending on the depending on the size and shape of the bluff body so we may have different lift and drag force. So, due to those force so different phenomena of the or different dynamics can be observed, different dynamics of the energy harvester can be observed.

So, one can get different type of resonance conditions particularly so, you have seen in the work of Anshul Garg where we have studied different resonance phenomena for example, so, we have taken this primary resonance condition, principle parametric resonance condition, combination parametric resonance condition for studying the response for a wide band of resonance frequency.

So, some other work you can see here. So, bifurcation chaos and their control in a wheelset model and this is by Liu, Wu and Cui.

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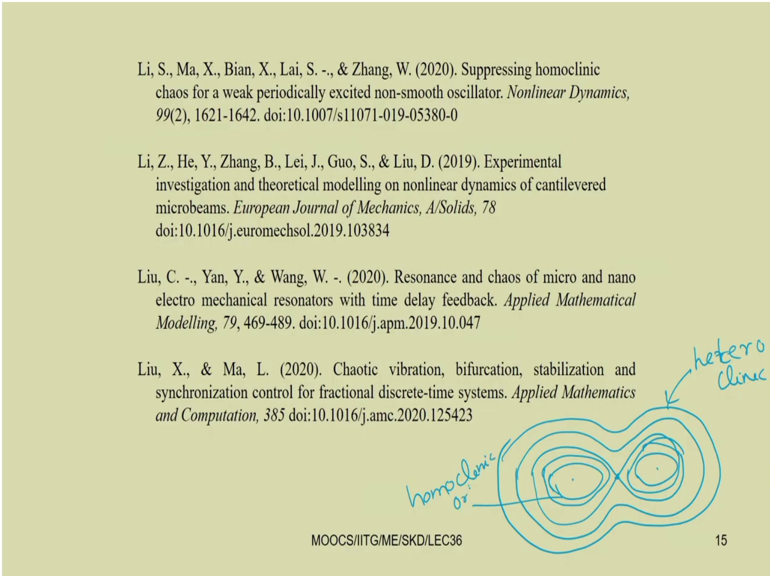

So, Li, Ma, Bian and Lai and Zhang, they studied these suppressing homoclinic chaos for a weak periodically excited non smooth oscillator. So, already you are familiar with these homoclinic and hetero clinic orbits. So, particularly in case of the simple pendulum case so, when you have plotted these x versus x dot that is phase portrait. So, you can see for so, you have seen. So, you have seen these homoclinic and hetero clinic orbits.

So, you have a saddle node point here and so, these are the homoclinic orbit and outside these things so you have the heteroclinic orbits. So, you have both homoclinic and heteroclinic orbits around the, these are the centre and you have a saddle node point here.

So, this is the heteroclinic. Hetero means dissimilar, hetero clinic orbit. So, the are going to two different orbits and inside these things so you have these homoclinic orbit. So, these are the homoclinic orbit. Homo means so, it is going to only single orbit. So, this is homoclinic orbit.

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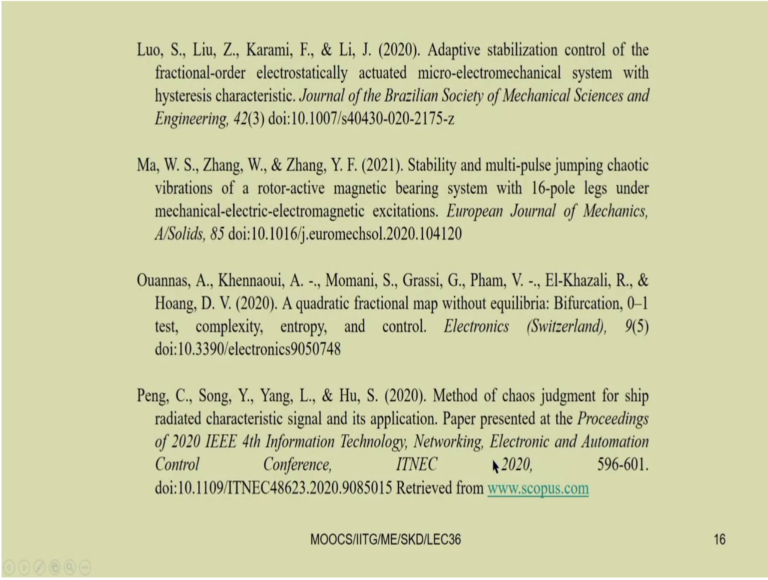

So, then there are many experimental works also available on the study of chaos. So, particularly nowadays people researchers are using these machine learning techniques for finding or characterizing these time responses of these chaotic systems. So, here one work by this Li, He, Zhang, Lei, Guo and Liu experimental investigation and theoretical modelling of non-linear dynamics of cantilever micro beams.

So, micro beams can be used for many applications particularly so, it can be used for detection of these bacteria and these microbes. So, it can be used in many application, it can be used in the cell manipulation, tissue engineering, and many biological applications and these work by Liu, Yan and Wang; so, resonance and chaos of micro and nano electromechanical resonators with time delay feedback.

So, then the work of Liu and Ma; chaotic vibration, bifurcation, stabilization and synchronization control of fractional discrete time system. So, you just see there a large number of publications and they have been published in many different journals. The journals you have seen are International Journal of non-linear mechanics, non-linear dynamics.

Then this European Journal of Mechanics, Journal of Brazilian Society of Mechanical Science and Engineering, IEEE Journals also are there. There are many ASME journals.

So, in almost you can find more than fifteen journals where these type of works have been published. So, some other works are here also given. So, adaptive stabilization control of the fractional order electro statically situated actuated micro electromechanical systems with hysteresis characteristics. So, already we know regarding the damping related to coulomb friction and also negative damping and we know regarding these viscous damping.

So, particularly in case of the different materials this hysteresis type of damping generally occurs. So, in this work they have studied these hysteresis characteristic of the system. So, the work by Ma, Zhang Zhang stability and multi pulse jumping chaotic vibrations of a rotor active magnetic bearing system with 16 pole legs under mechanical electric and electromagnetic excitation. So, then the some other papers are also you can find here a quadratic fractional map without equilibria.

So, here the bifurcation. So, generally 0-1 test has been carried out for chaotic system, 0-1 test, complexity, entropy, and control. So, then the work by Peng, Song, Yang and Hu this is method of chaos judgment for ship radiated characteristic signal and applications.

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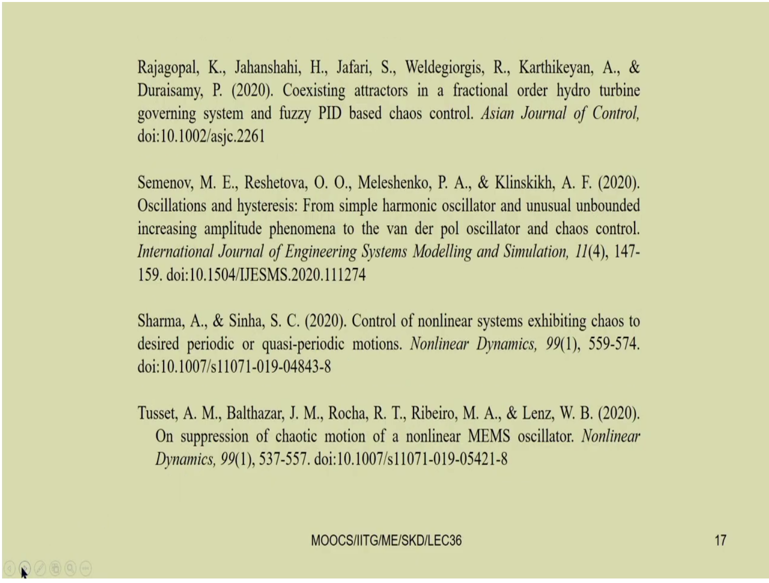

You can find some other paper like this; the coexisting attractors in a fractional order hydro turbine governing system and fuzzy PID based chaos control. So, already we have seen in case of our study the coexistence of fixed point response, periodic response, quasi periodic response and chaotic response.

Here also, this paper discusses this coexisting attractors in a fractional order system. So, the equations what we have studied so we have not taken these fractional order system, but you can extend your idea of these integer thing to this fractional order system. So, you here also you can use method of multiple scales or this Runge Kutta type of method, numerical all the numerical methods you have studied to study the fractional order systems.

So, oscillations and hysteresis from simple harmonic oscillation to unusual unbounded increasing amplitude phenomena to the van der pol oscillator and chaos control has been studied by Semenov, Reshetova, Meleshenko, Klinskikh.

So, in this work they have taken this van der pol oscillator and here, they have studied these chaos control. So, they have studied from the simple harmonic oscillator to these unbounded chaotic response and studied how to control these chaotic systems. So, Sharma and Sinha they have studied this control of non-linear system exhibiting chaos to desired periodic and quasi periodic motion. So, it is in non-linear dynamics.

So, this is another journal; International Journal of Engineering Systems Modelling and Simulation, Asian Journal of Control and these on suppression of chaotic motion of a non-linear MEMES oscillator MEMES, Micro Electromechanical Systems. So, microbeams previously you have seen. So, here Micro Electromechanical Systems so, these micro electromechanical systems are many applications.

So, some other papers are like these non-linear dynamic analysis of GTF gearbox, then non-linear dynamics analysis of GTF gearbox under frictional excitation with vibration characteristic recognition and control in frequency domain.

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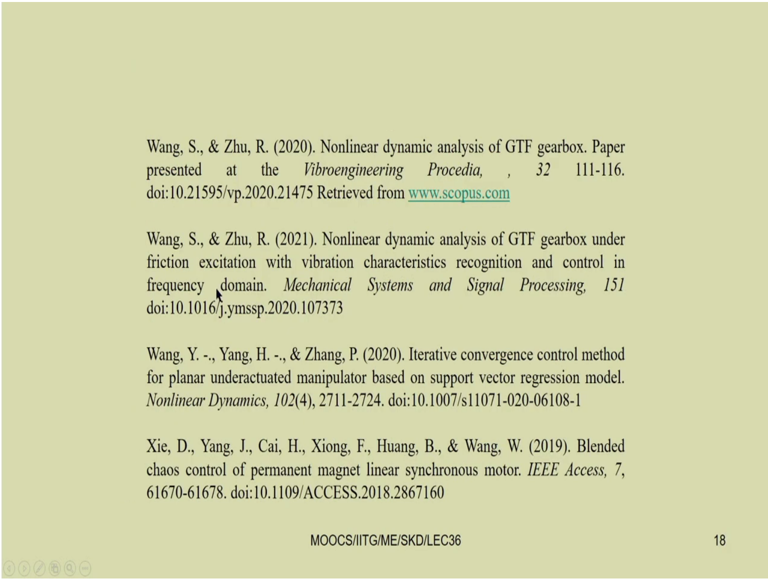

So, these are the work by one Wang and Zhu. So, then Wang and Zhang and Zhang they studied this iterative convergence control method for planar under actuated manipulator based on support vector regression model then Xie, Yang, Cai, Xiong, Huang and Wang.

They studied this blended chaos control of permanent magnet linear synchronous motor and we have few more paper, you can see this on period one motion to chaos in one dimensional time delay non-linear systems and then Xu, Chen and Luo so, they studied this period one motion to chaos in a non-linear flexible rotor system.

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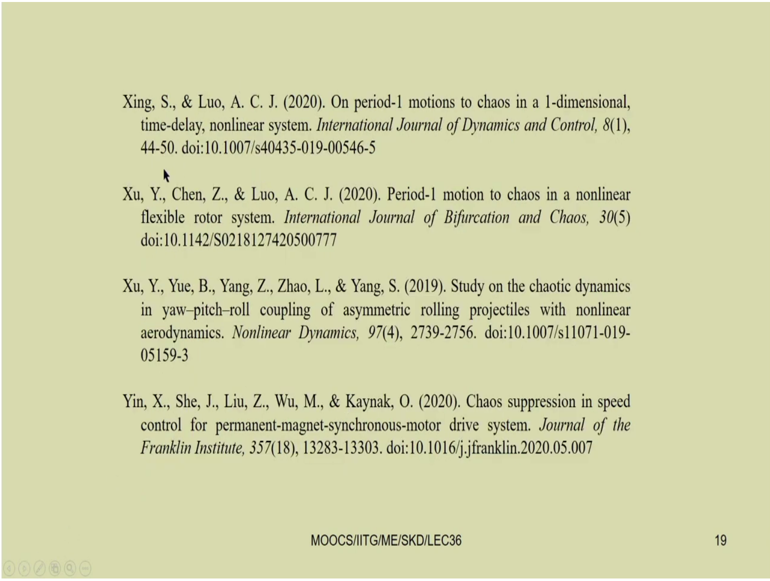

So, it is published in International Journal of Bifurcation and Chaos and this is the International Journal of Dynamics and Control. Xu, Yue, Yang, Zhao and Yang they have studied the chaotic dynamics in a yaw-pitch-roll coupling particularly this yaw-pitch-roll you can find in ship motion or in this wrist of a robotic handle. So, you can find these type of motion.

So, there you can study these chaotic response Yin, She, Liu, Wu and Kaynak. So, they studied these chaos operation in a speed, chaos operation in speed control for permanent magnet synchronization motor drive system. So, this is published in this Franklin Journal of the Franklin Institute.

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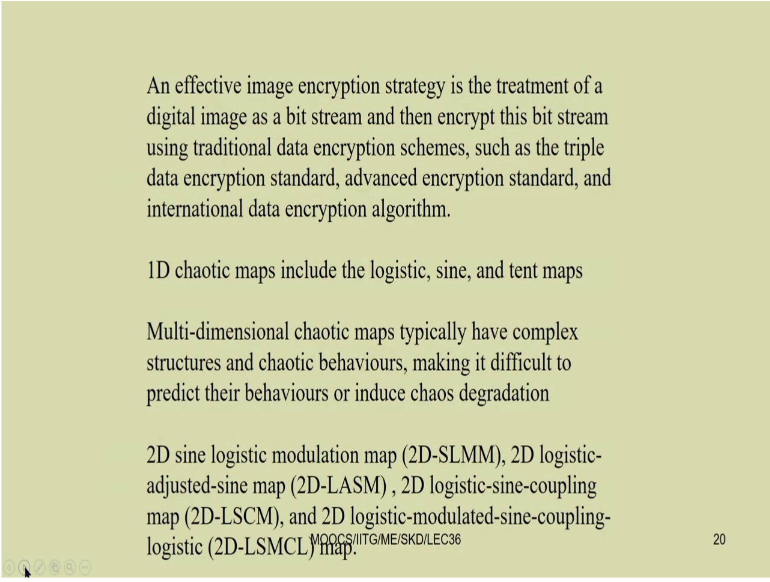

So, you have since there are several papers available. So, actually there are more than 10,000 papers you can find in scopus related to chaos. So, you have to take the particular keyword to study for example, I have given the keyword of chaos and vibration and I have collected only the, not all, but some of the paper in published in the year 2020 and 19 and you have seen so many papers are there.

So, if you see all the papers what are available so you can find many applications of these dynamic systems where these chao chaotic responses or other type of responses are there. So, sometimes these chao’s will be useful and sometimes these chaos is not useful particularly, for a chemical mixing. For example, you want to mix some paints so that time these chaotic so if you mix it in a chaotic way then the mixing will be proper.

So, in metallurgy particularly in case of many metallurgical systems or many other application systems so you may required chaos or chaos will be useful in construction also. So, when you are mixing so when you are mixing the materials. So if you mix in a chaotic way or use a chaotic function to mix them then the mixing will be proper.

So, sometimes it may not be useful. So, particularly in case of the turning operation or man manufacturing operation, turning, milling so, if there is some vibration or chaotic response then this chaotic mark will be on the surface of the work piece. So, then it is not required, but if you are making some texture on the work piece. So that time you can choose some pattern, chaotic pattern and you can have, you can use that thing there.

So, sometimes this chaotic response or the vibration will be required and sometimes you required to operate it. So, you have seen the application of these artificial muscle. So, where we have seen? So, we require these unstable range or where the system is unstable so that we can so with more air pressure so, the artificial muscle will bulge in this transfers reaction and there will be a reduction in the length in the longitudinal direction.

So, sometimes it is required and sometimes it may not be required. So, depending on the applications now you have to decide. So whether you have to use the vibration or you have to you have to suppress the vibration of the system. So, an effective so, also this chaotic system can be used in these in an effective way for image encryption.

So, already I told this thing. So, image encryption strategy is the treatment of a digital image as a bit stream and then encrypt this bit stream using traditional data encryption scheme such as the triple data encryption standard, advanced encryption standard, and the international data encryption algorithm.

So, 1D chaotic maps include this logistic map, sine map, and tent maps, they can be used, but already as I discussed so here, as the as you have only one or two parameters and it can be. So, one can unlock these parameters easily.

So, multi dimensional chaotic maps typically have complex structure and chaotic behaviours making it difficult to predict their behaviour or induce chaos degradation. So, 2D sine logistic modulation map, 2D-SLMM, 2D logistic adjusted sine map, 2D logistic sine coupling map, 2D logistic modulated sine coupling logistic map.

So, they are related to this 2D map. Similarly, you can have 3D map like this Lorenz Rossler. So, other type of equations are there. So or higher order maps are also there or higher order. So, you can either use the map or differential equations to find this chaotic response.

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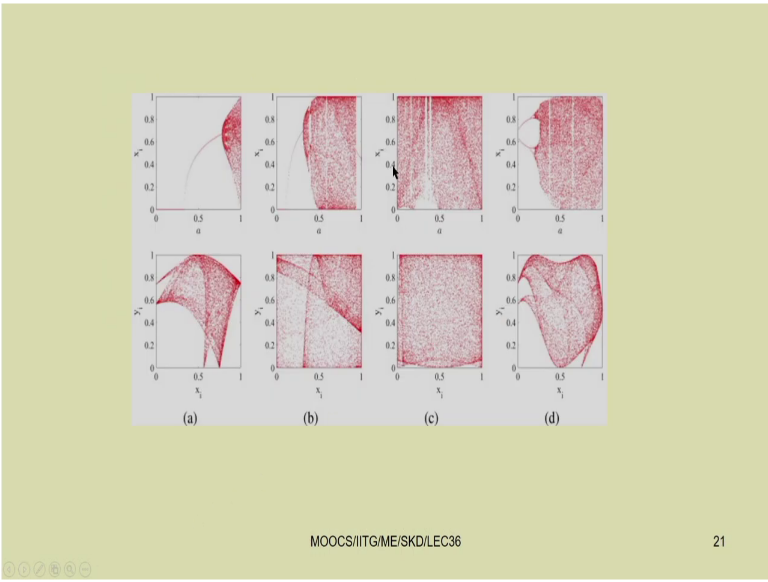

So, here some of the chaotic responses you can see. So, initially; so, these are the bifurcation diagram. So, initially so, this is the trivial state, then one has the non trivial state with single period, then it will reduce to 2 period, then 4 period, 8 period and period doubling route to chaos.

So, sometimes you can by changing the system parameter you may get this folding. So, you just see sometimes these chaotic response get folded and folding unfolding. So all those things you can see by varying different system parameter of the chaotic response.

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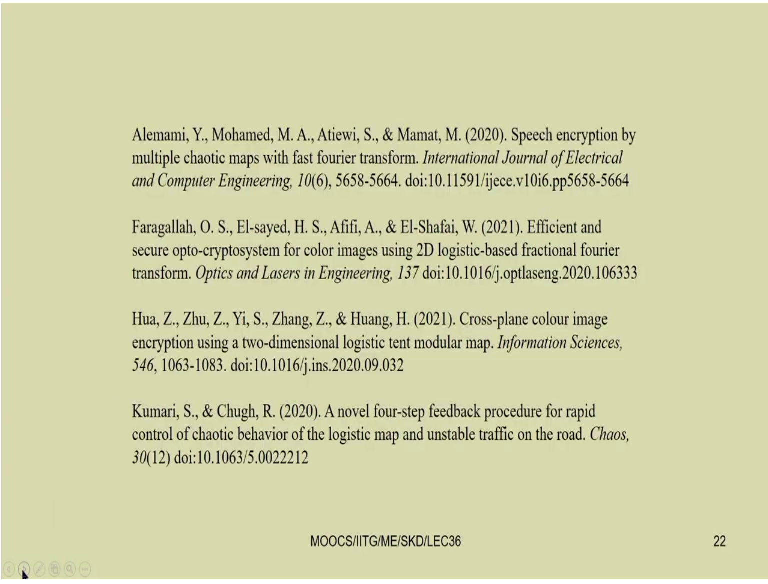

So, here some papers related to the encryption are there. So, by these Alemami, then Mohamed, Atiewi and Mamat so, a speech encryption by multiple chaotic maps with fast Fourier transform. So, by these Faragallah, EI Sayed and Afili and Shafai so, they have the paper related to these efficient and secure opto-cryptosystem system for colour image using 2D logistic based fractional Fourier transform.

Then the paper by these Hua, Zhu, Zhang, Huang, so, cross plane colour image encryption using a two dimensional logistic tent modular map. So, Kumari, Chugh so they have a paper on this novel four step feedback procedure for rapid control of a chaotic behaviour of logistic map and unstable traffic on the road.

So, here you can see some more papers. So, this paper is on Chaos, another paper on Information Science, another paper on these Optics and Laser in Engineering, these paper also you can see International Journal of Electrical and Computer Engineering. So, there are several other papers are also available in this field. So, you may read some of these papers to know more and more regarding these chaotic responses in the system.

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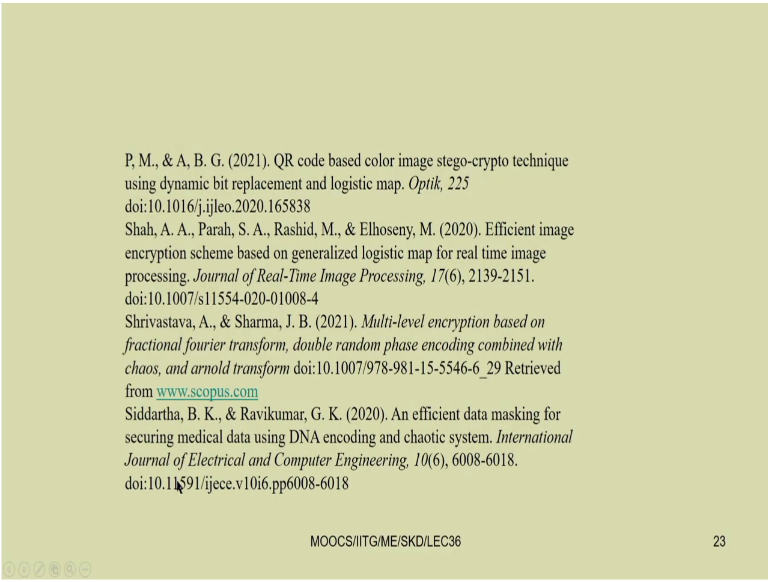

Some other papers this QR code based colour image stereo crypto technique using dynamic bit replacement and logistic map, efficient image encryption scheme based generalized logistic map for real time image processing.

Then you can have the paper wise Shrivastava and Sharma multi level encryption based on fractional Fourier transform, double random phase encoding combined with chaos, and Arnold transform then Siddartha and Ravi Kumar an efficient data masking for securing medical data using DNA encoding and chaotic system.

So, this is International Journal of Electrical and Computer Engineering ok. This is Journal of Real-Time Image Processing; Optik one journal is there also. So, you can see all these journals and this way you can study different chaotic system.

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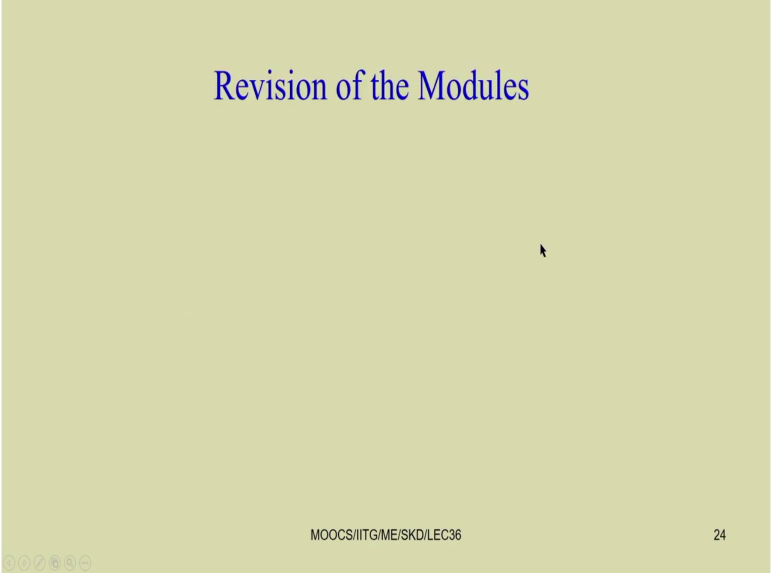

So, as we are coming to the end of these course and today, is the last class. So, let us revise what we have studied in all these models. So, we have started our study with a simple spring mass system.

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So, where these spring mass system, simple spring mass, linear spring mass system is having the periodic motion and you can see this harmonic motion in the simple spring mass system. Then we have taken this spring to be non-linear and we have increased the complexity of the system and we have studied this non-linear system.

So, before studying the non-linear system we did the revision regarding this linear system. So, in the linear system we have started with the single degree of freedom system; where we have plotted these or we know regarding this magnification vector.

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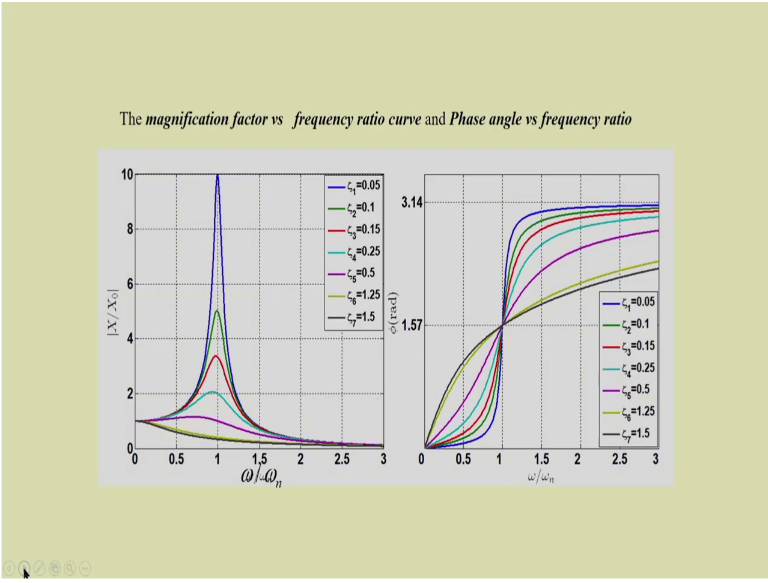

So, how the response is increasing and how the response can be controlled or how the response can be controlled by operating at a different frequency or by using different damping parameter.

So, by using different damping parameter we have seen one can conveniently reduce the response amplitude. So, one can study the response amplitude and phase particularly. So, we can see for higher value of damping so, we can have a phase angle of 180 degree and irrespective of damping. So, we can have for all irrespective of damping so, we can have these phase angle of 5 by 2. So, when omega equal to omega n so, here omega is the external frequency and omega n is the natural frequency of the system.

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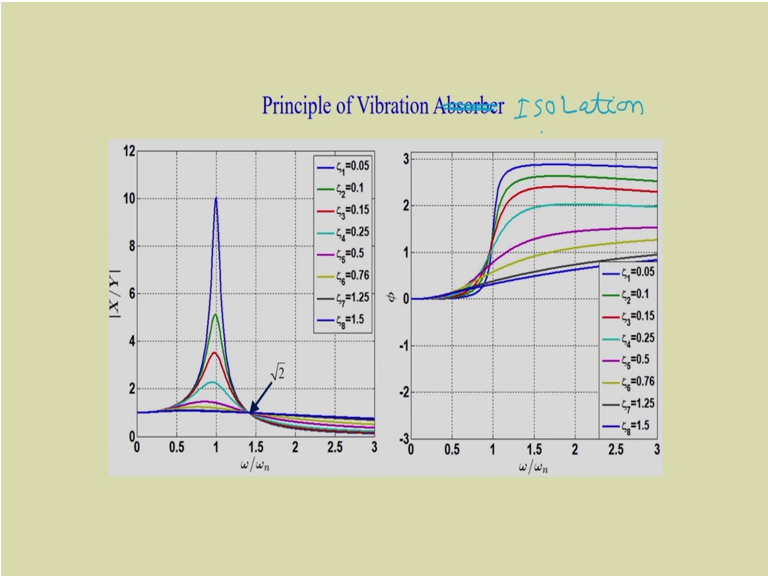

Further, we have studied this principle of vibration observer. So, we have to choose the spring and damper in or the stiffness and damper in such way that. So, the system has to be operated greater than root 2 to isolate the vibration. So, you can see this line is 1. So, the response amplitude is reduced or X by Y.

So, these so, this is principle of vibration isolation, this is not observer this is isolation, principle of vibration isolation is that the system has to be operated at a frequency greater than root 2 times the natural frequency of the system so that we will get less vibration in the system.

So, isolation is of two types either the machine will, the vibration of the machine is isolated so that it is not transmitted to the ground or the ground motion is not transmitted to the machine. So, both the cases are there. So, either then we can plot these X by Y or the force transmitted F t by a F. So, both will yield the same figure.

So, we have already seen these things. So, this is the principle of vibration isolation. So, the system has to be operated at a frequency greater than root 2 times the natural frequency of the system. So, you just see irrespective of the damping. So, all those are root 2. So, at root 2 all the lines are passing through these 1. So, that is X by Y equal to 1.

So, after this thing this X by Y is less than Y 1 Y is the amplitude ground motion, X is the motion of the machine. So, here so, force or motion is transmitted from the ground to the ground to the ground to the machine or the instrument which we want to isolate ok.

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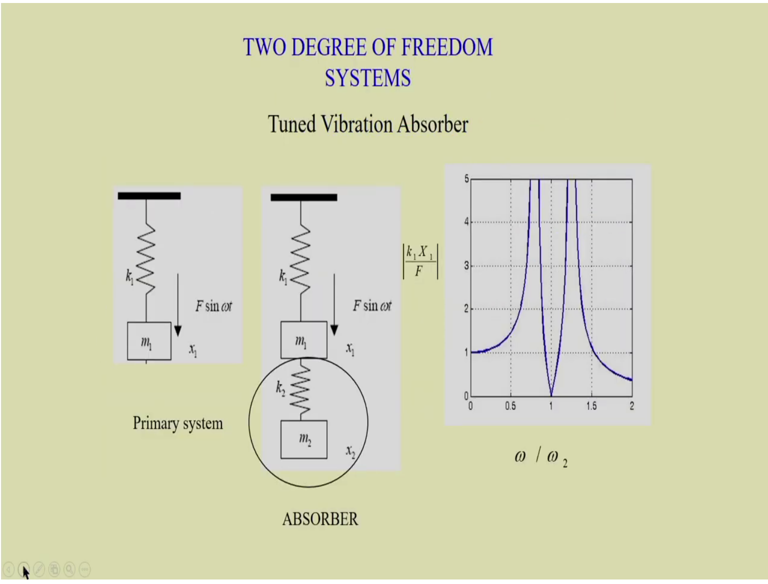

So, then we have studied regarding these 2 degrees of freedom systems. So, this is the primary system primary system may be subjected to a force of F sin omega t. So, we have seen we can control the vibration of these primary system or we can observe the vibration of the primary system by putting a secondary system that is k 2 m 2.

So, you can see. So, at omega equal to omega n so, when we have the primary system only so, it has in finite response now, by making it a 2 degree operating system at omega by omega n. So, here we have so, the response of the primary system equal to 0.

So, the primary system all the vibration of the primary system is observed, but in that way. So, we have two different resonant peak conditions. So, the natural frequency are shifted to left and right side of this 1. So, already we have studied in the application part we have studied these non-linear vibration observer. So, we have studied this passive and active type of vibration observer. Also, we have seen regarding these continuous system.

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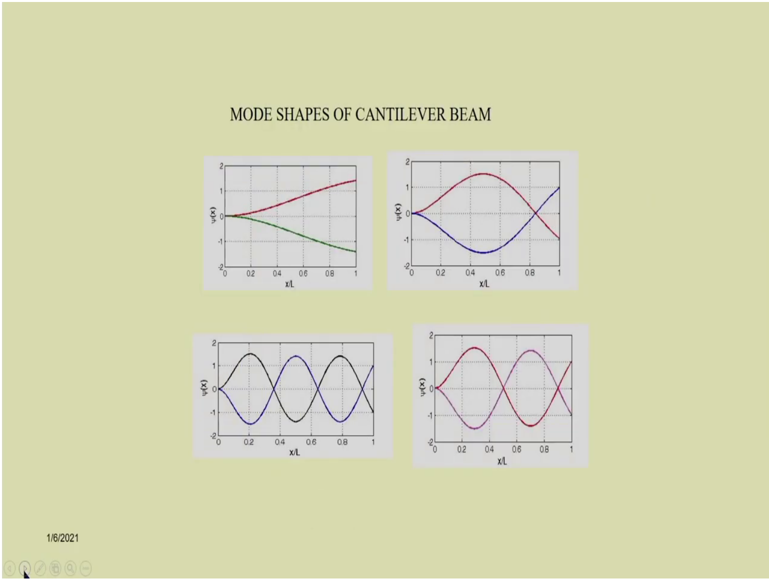

So, we know how to determine these mode shapes of the continuous system, then we studied the extended our idea of linear system to non-linear systems.

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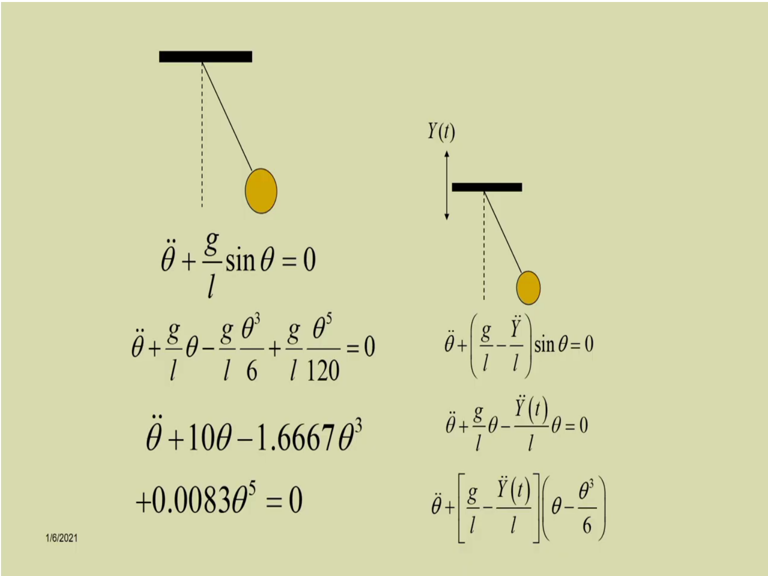

So, already we have differentiated between the linear and non-linear system by stating that the superposition rule cannot be applied to a non-linear system. So, though the in though in linear systems, but one can apply the superposition rule. So, in non-linear system it is not possible to apply the superposition rule.

So, every case of non-linear system has to be treated separately and their analysis must be done separately and to get the actual response of the system. So, once we will not extent the idea of one resonance condition to the other resonance condition so, which will yield the erroneous result. So, one should be very careful while studying these non-linear systems.

So, you just see the simple pendulum so, we have started the non-linear system with the simple pendulum. So, where we have derived this governing equation by using several methods, for example, Newton's second law, D'Alembert's principle, Lagrange principle can be used to derive these equation of motion.

So, the equation of motion becomes theta double dot plus g by l sin theta equal to 0. So, in the linear systems we have taken the sin theta equal to theta and these reduced to that of a have a system with harmonic motion. But if this theta is not small then we can expand the sin theta by theta cube by 6 plus theta 5th by 120. And you can see we have a non-linear system with cubic and quintic or fifth order polynomial. So, these non-linear systems can be conveniently solved either numerically or by using perturbation methods.

So, different perturbation methods we have studied for example, we have studied this straightforward expansion, lindstedt Poincare technique, then harmonic balance, harmonic balance is not the perturbation technique. So, we have studied this method of multiple scales, averaging method or a combination of these harmonic balance and these method of multiple scales. So, those are the recent methods which one can study.

So, this harmonic balance method also we have modified this harmonic balance method and we have studied we have applied to vibration observers. So, that thing you have seen and then so, here you just see so, we have so, the simple pendulum case so, if the platform of the simple pendulum case is moving up and down.

So, here you can see we have a different equation of motion where the coefficient where the coefficient of theta that is the response is a time varying term the first term is a constant term that is g by l that is a constant term, but the second term these Y double dot by l, Y double dot that is the acceleration. So, if the motion is periodic then Y double dot is also periodic.

So, the coefficient of theta and theta cube are time varying terms. So, that is why this is a parametrically excited system. So, this is a forced excitation or a direct excitation and this is parametrically excited system. So, we have studied both force and parametrically excited system in our study and we have studied the single degree of freedom system, multi degree of freedom system, and this continuous system.

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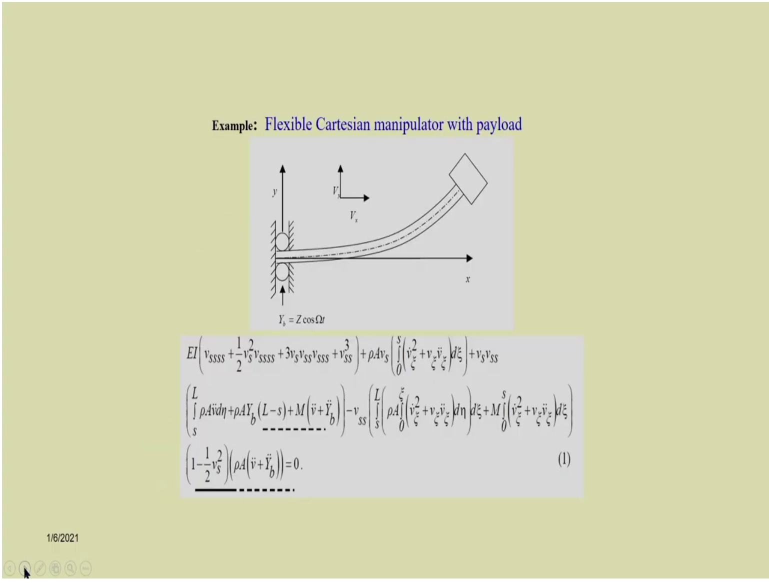

So, in case of continuous systems so, we got the so, we got the spatio temporal equation and these spatio temporal equations have been reduced to that of a temporal equation by applying these Galerkin's procedure.

So, generalized Galerkin’s technique has been used to convert this thing to that of a temporal equation and those temporal equations are solved by different method.

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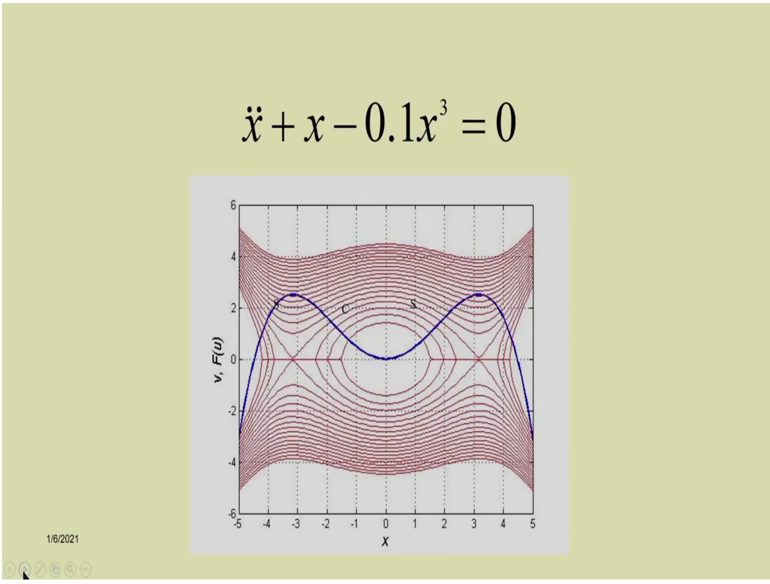

So, one can while you can make this qualitative analysis or one can start the analysis using a qualitative method. So, where one can plot the these potential function and from the potential function one can observe different characteristic.

For example, when the when this point is maximum so, when these potential is maximum so one can have a so this is the saddle point corresponding to so, corresponding to this maximum point this is the saddle point and this is the centre C. So, this is the centre C here and so, we can have another saddle point here.

So, one can have another saddle point here. So, corresponding to maximum corresponding to maximum potential energy so we have a saddle point and corresponding to minimum potential energy we have a centre. So, this shows the flow of the. So, x versus x dot that is displacement versus velocity or phase portrait or state space characteristic of the system.

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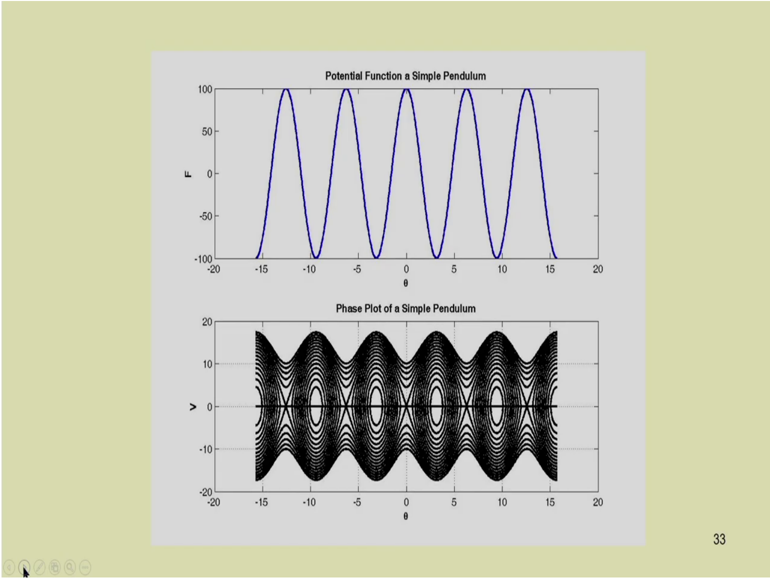

So, for a simple pendulum so, you can see so, different saddle and centre points. So, these are the homoclinic and hetero clinic orbits what I have told you just before.

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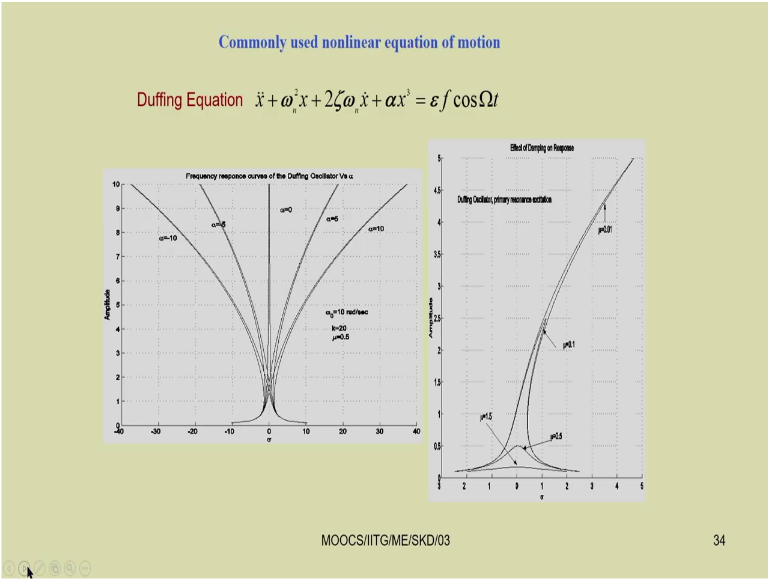

So, then we have studied this Duffing equation. Commonly, used non-linear equations we have studied that is Duffing equation. So, these are the backbone curves so for a different value of these non-linear parameter alpha coefficient of non-linear alpha so, you can plot the response. So, these are the skeleton response or the backbone curves. You just see alpha equal to 0 the system is linear. So, you can get these response infinite, the response will be infinite.

So, this side you can get these hardening effect and softening effect depending on this alpha to be positive or negative value. So, out of these things so, you just see we have this multiple response. So, out of these multiple response so we can have some response to be stable some may be unstable.

So, here we have the Duffing equation with different type of damping, damping increasing damping the response amplitude decreases. Here, you can see the non-linearity also decreases the response amplitude of the system.

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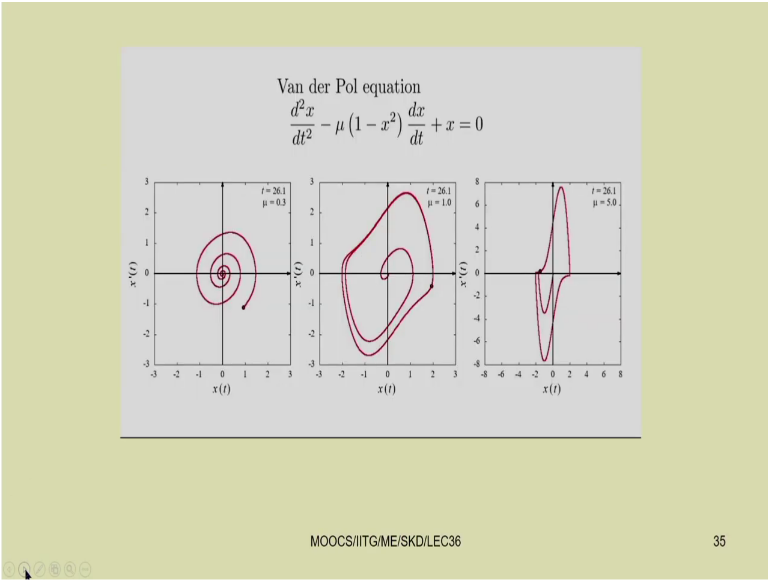

Further, we have studied this Van der Pol equation. So, were you can see we can getting the limit cycle. So, whether you are starting from the outside or from the inside. So, always it will go to the limit cycle. So, here you can see, it is going to the limit cycle.

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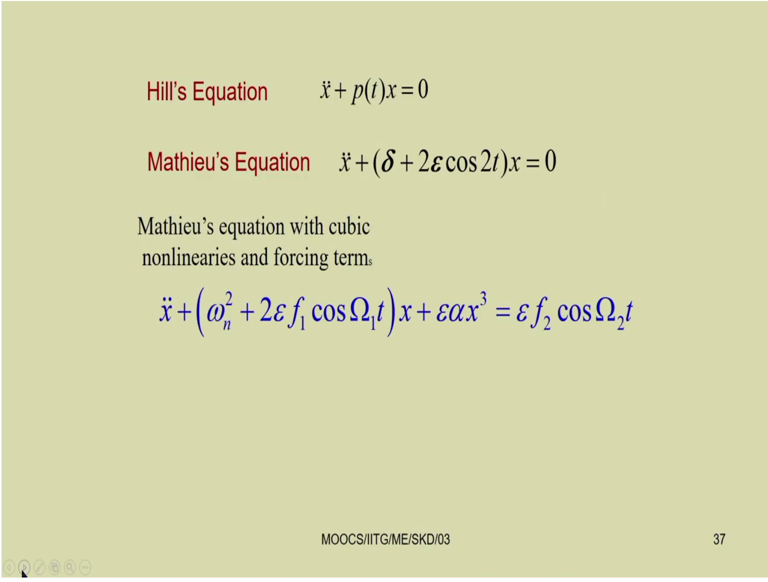

So, other equations we have studied are Hill's equation, Mathieu equation, combination of Hill's equation with Duffing equations also, Lorentz attractor also we have seen which yields different type of chaotic response.

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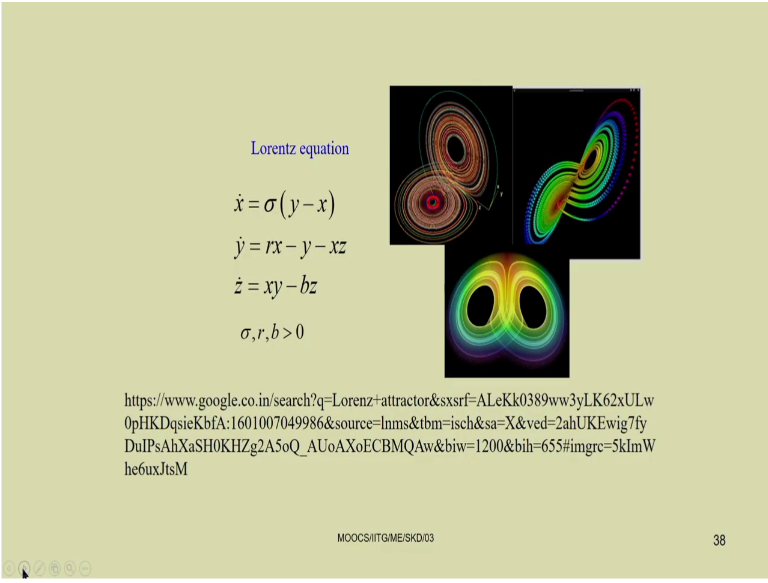

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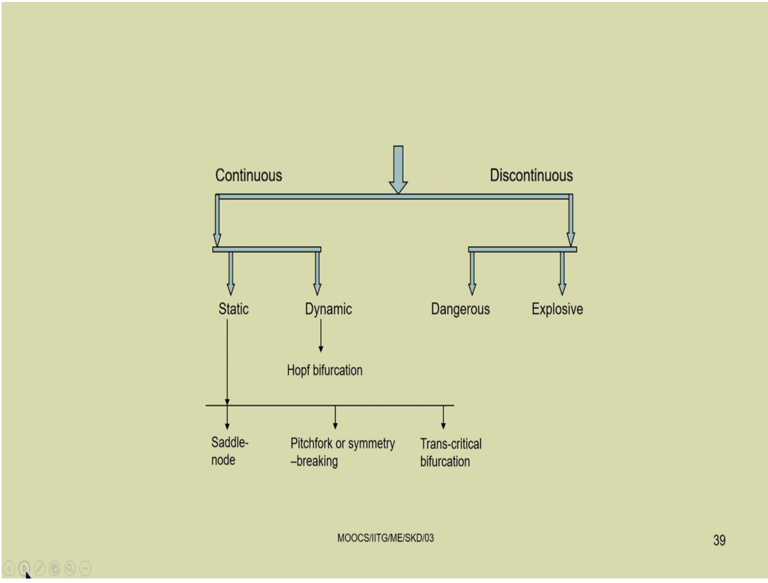

Also, we have studied regarding these bifurcation and different type of bifurcation.

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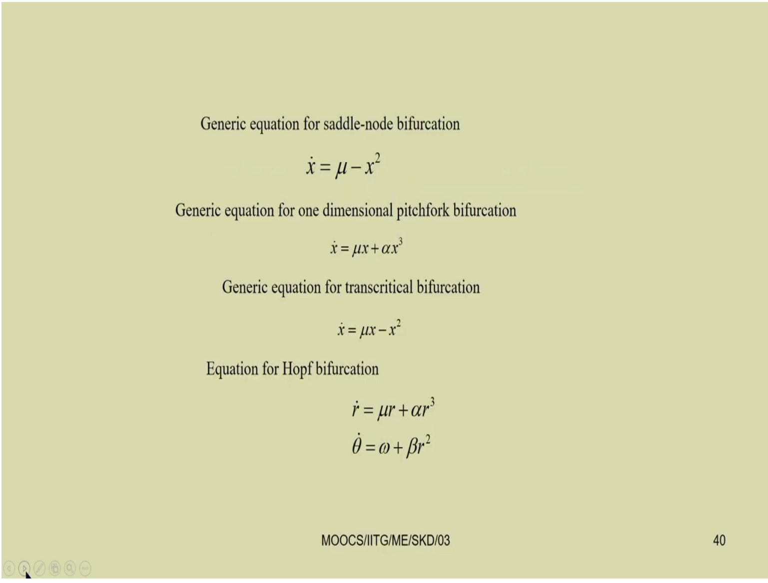

For example the saddle node bifurcation, pitchfork bifurcation, transcritical bifurcation, Hopf bifurcation and these are some of the generic form of these bifurcations. So, we have studied all those bifurcations in this course.

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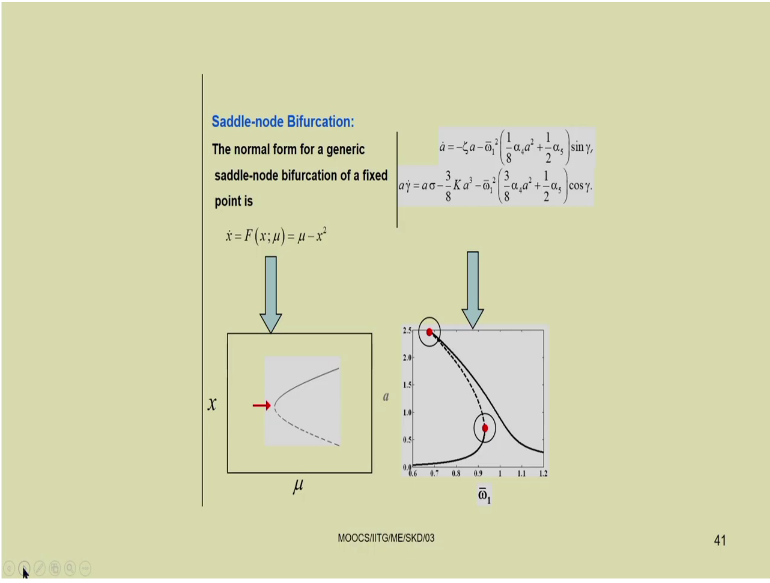

So, already you have studied all these things.

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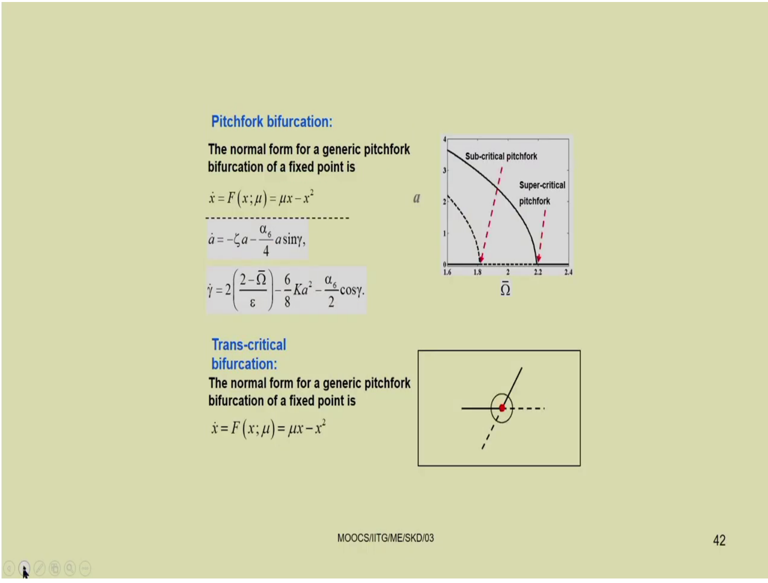

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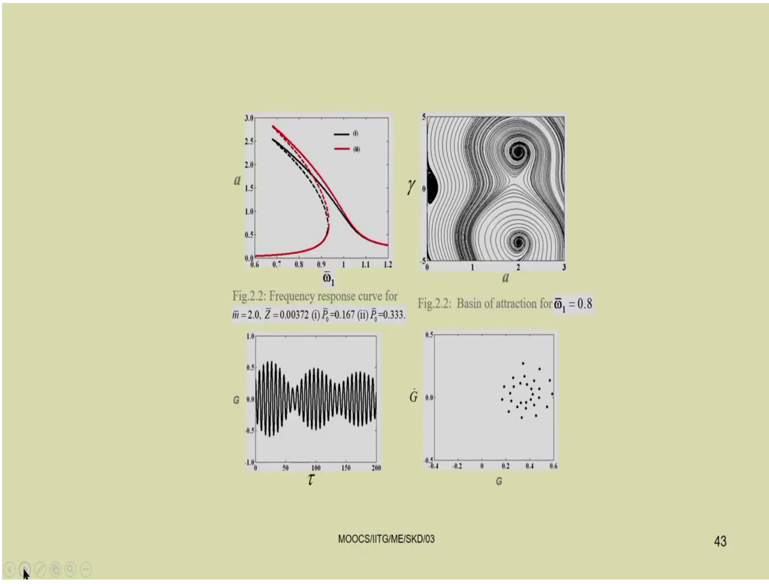

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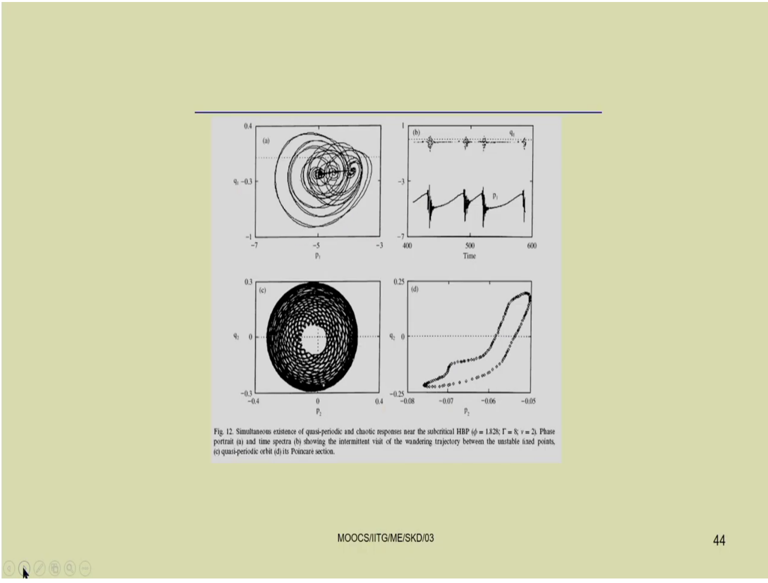

And also we have seen regarding this basin of attraction, phase portrait, time response, phase portrait, basin of attraction and also Poincare section. So, all these things we have studied in these course. So, different chaotic response so, here you can see this attractor merging crisis, also you have seen these intermittency route to chaos. So, this is the torus and torus breakdown route to chaos. So, here a torus and for the torus the Poincare section is a closed group you have seen.

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So, we perform certain experiments also ok.

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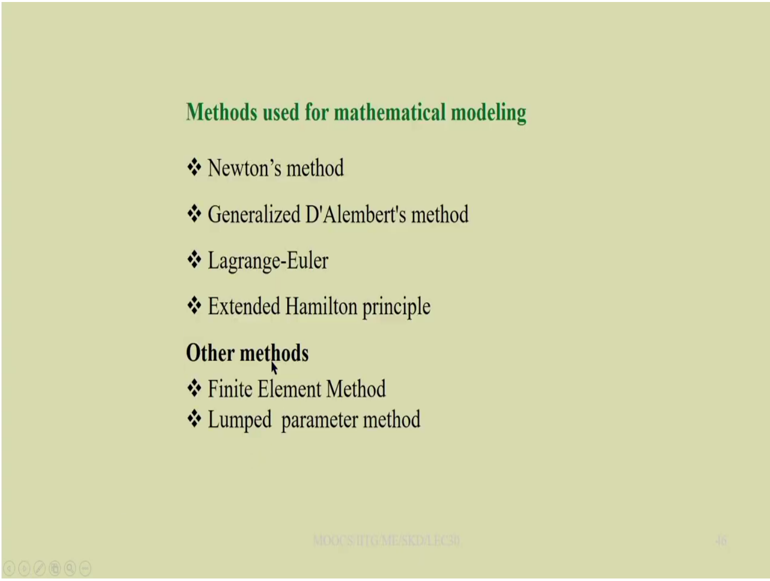

So, that we have used different methods already, we have discussed what are the main method studied and responses we have studied and this governing equation with different geometric, nonlinearities and inertia non-linearity terms are there, instability regions also we have studied.

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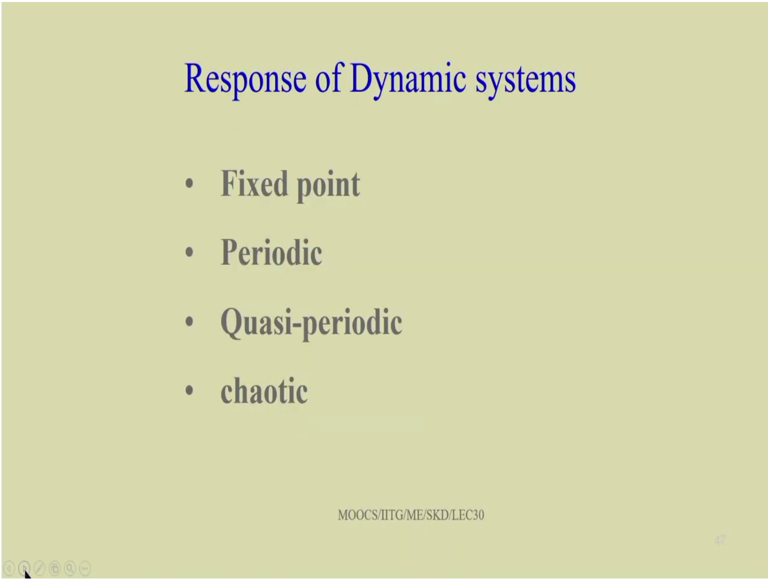

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Using generalized Galerkin's procedure
Governing Temporal equation becomes

$$\ddot{u}_n + 2\varepsilon\zeta_n\dot{u}_n + \omega_n^2 u_n - \varepsilon \sum_{m=1}^{\infty} f_{nm} u_m \cos\phi\tau$$

Parametric forcing term

$$+ \varepsilon \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \{\alpha_{klm}^n u_k u_l u_m + \beta_{klm}^n u_k \dot{u}_l \dot{u}_m$$

$$+ \gamma_{klm}^n u_k u_l \ddot{u}_m\} = 0, \quad n = 1, 2, \ldots, \infty \qquad (7)$$

Cubic geometric nonlinearities

Cubic inertial nonlinearities

Cubic inertial nonlinearities

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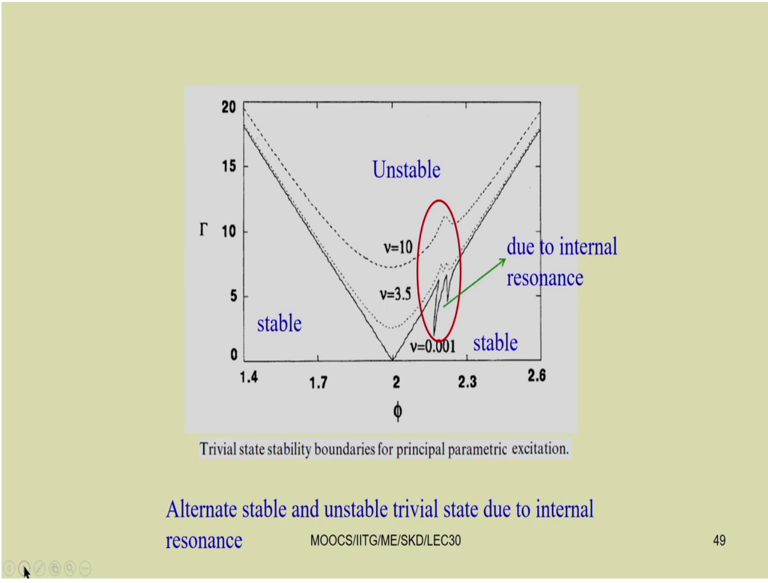

So, due to internal resonance conditions we have seen so how the instability regions is changing. So, in this different response clearly, we have shown different type of bifurcations.

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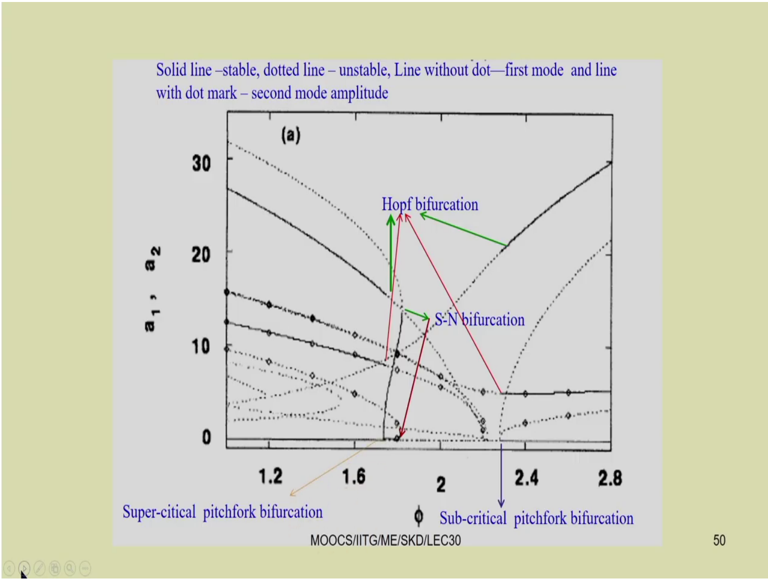

So, Hopf bifurcations, saddle node bifurcations, then subcritical supercritical pitch for bifurcations, then this is period doubling route to chaos.

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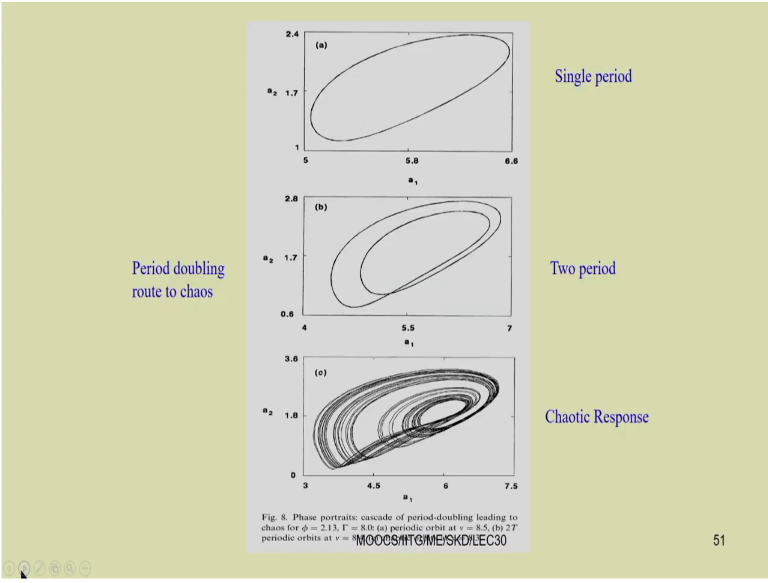

This is a Poincare section showing periodic, two periodic, four periodic, and chaotic response and in between you can see a window of periodic response.

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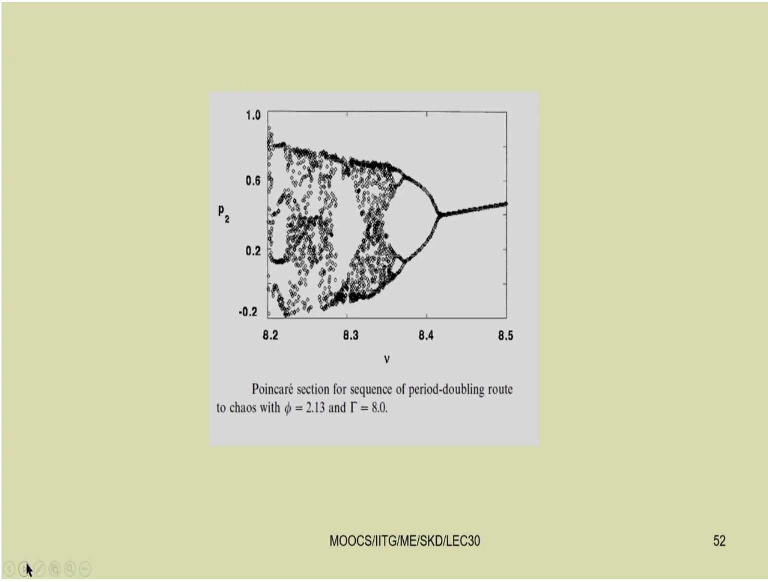

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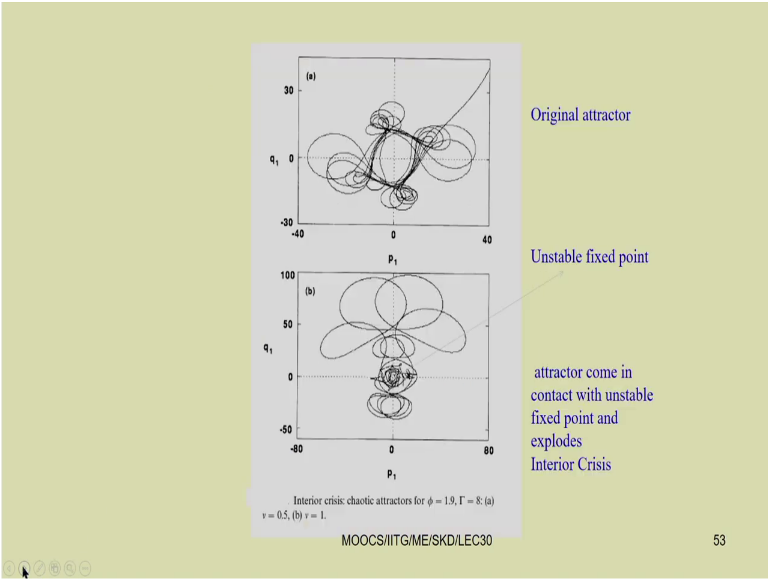

So, this is the attractor merging crisis and then this is attractor merging and the previous one is we have seen the interior crisis.

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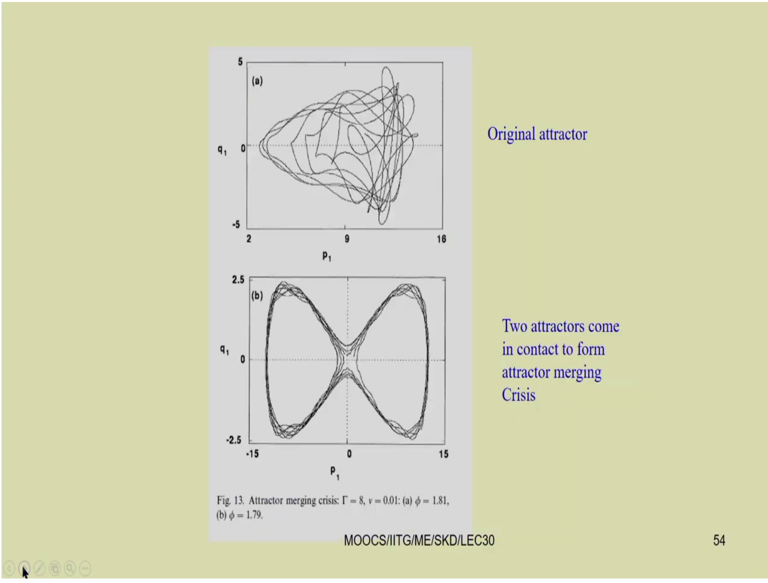

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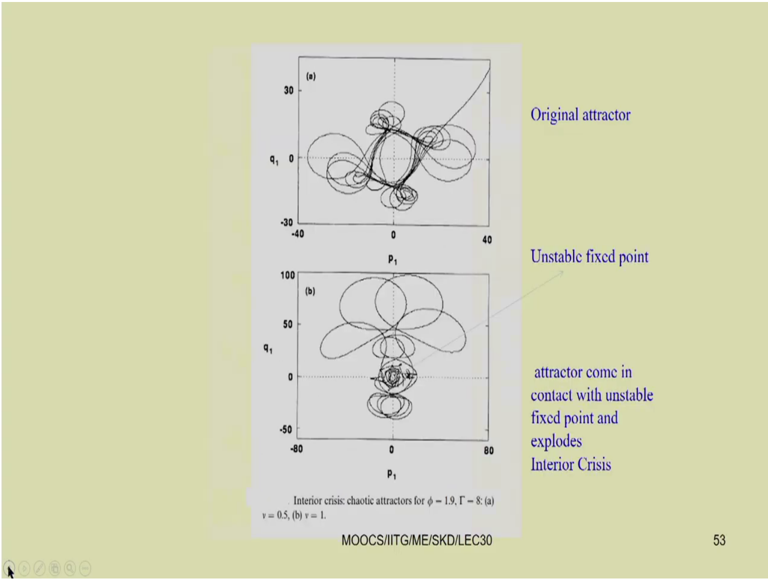

So, inside this thing you can see a unstable fixed point and these chaotic attractor come in contact with these unstable fixed point response to have explode to another greater chaotic attractor. So, this way we have seen different chaotic response.

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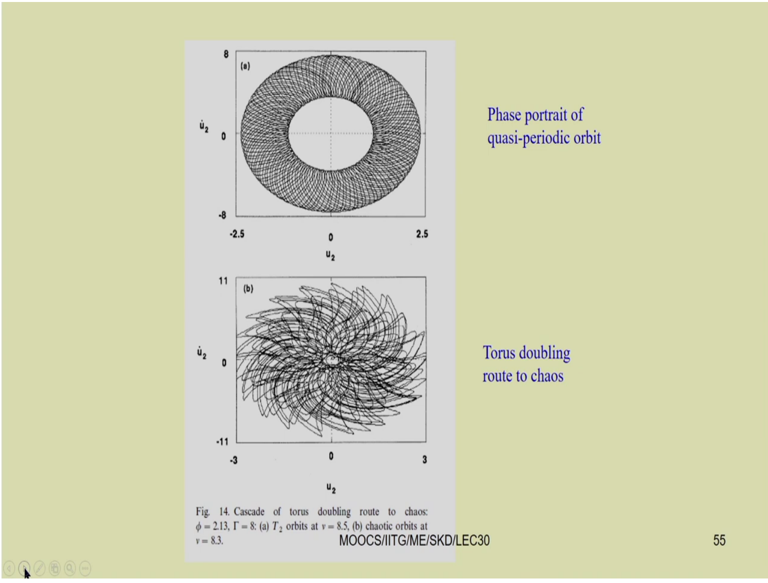

So, here the quasi-periodic route to chaos. We have seen, torus doubling route to chaos.

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Module	Module Name	Week	Lecture No	Title of the lecture	Assignments
1	Introduction to Nonlinear Mechanical Systems	1	1	Introduction to mechanical systems	Online: MCQ, Program, Problem solving
			2	Equilibrium points: potential function Time response, phase portraits	
			3	Simulation of Phase portraits from potential function	
2	Development of Nonlinear Equation of Motion *(Use of Symbolic Software to derive equation of Motion)*	2	1	Force and moment based Approach	
			2	Lagrange Principle	
			3	Extended Hamilton's principle	
3	Solution of Nonlinear Equation of Motion	3	1	Numerical solution method	
			2	Harmonic Balance method	
			3	Lindstd-Poincare' method	
		4	4	Method of Averaging	
			5	Method of multiple scales	
			6	Recent advanced method	
4	Vibration Analysis of Nonlinear SDOF	5	1	Application of Duffing Equation, frequency and	

So, in this course we have taken nine modules. So, the 1 st module we have introduced to the course, 2nd module we have derived this equation of motion, in 3rd model. So we have studied different solution procedure for these non-linear system, and in 4th module we have started analysis of these single degree of freedom system with weak excitation, 5th module we have covered these vibration analysis of non-linear single degree of freedom system with hard excitation.

In the 6th module we have covered these vibration analysis of parametrically excited system, 7 module we have studied analysis of system with periodic, quasi periodic and chaotic response. Then in module 8 we have studied numerical methods for non-linear system analysis and module 9 which we have covered in three weeks. So, we have studied different practical applications.

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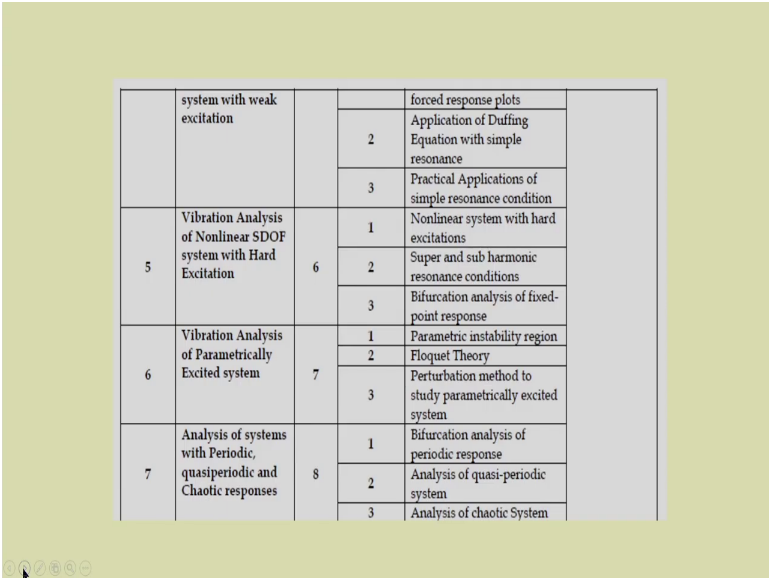

	system with weak excitation			forced response plots	
			2	Application of Duffing Equation with simple resonance	
			3	Practical Applications of simple resonance condition	
5	Vibration Analysis of Nonlinear SDOF system with Hard Excitation	6	1	Nonlinear system with hard excitations	
			2	Super and sub harmonic resonance conditions	
			3	Bifurcation analysis of fixed-point response	
6	Vibration Analysis of Parametrically Excited system	7	1	Parametric instability region	
			2	Floquet Theory	
			3	Perturbation method to study parametrically excited system	
7	Analysis of systems with Periodic, quasiperiodic and Chaotic responses	8	1	Bifurcation analysis of periodic response	
			2	Analysis of quasi-periodic system	
			3	Analysis of chaotic System	

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8	Numerical Methods for Nonlinear system Analysis	9	1	Solutions of a set of nonlinear equations	
			2	Numerical Solution of ODE and DDE equations	
			3	Poincare' section, FFT, Lyapunov exponent	
9	Practical Application 1: Nonlinear Vibration Absorber	10	1	Development of Equation of motion: symbolic software	
			2	Solution of EOM: Use of Harmonic Balance method	
			3	Program to obtain time and frequency response	
9	Application 2: Nonlinear Energy Harvester	11	1	Development of Equation of motion and its solution: symbolic software	
			2	Solution of EOM: Use of method of Multiple Scales	
			3	Program to obtain time and frequency response	
9	Practical Application 3: Analysis of electro-mechanical system	12	1	Development of Equation of motion and its solution	
			2	Use of Floquet theory	
			3	Parametric instability regions, Study of periodic, quasiperiodic and chaotic response	

So, in one week we have studied this Energy Harvester, next week we have studied vibration absorber and in this week we have particularly studied different systems particularly we have studied these turning operation and these also we have taken this farming operation that is rolling operation.

We have taken and today class we have briefly seen different applications of chaotic response and also we have revised these course and in this course particularly every week you are going to find these different problems will be you will find.

So, you have to solve all those problems using MATLAB. So, already we have so, already we have a tie off with these MATLAB company to give you, provide you this MATLAB so that you can use those MATLABs to solve your problems.

So, every week you will get these problems related to MATLAB so, which you can solve. You may use other software's also, but as we are providing you MATLAB. So, you can use conveniently this MATLAB to solve all these problems and we can have a project based, this course also will be a project based course.

So, were several projects will be given to you and based on the project response so you will be graded. In the final exam there will be 50 questions of 2 marks. So, these will be multiple choice type of questions and you have to solve those problems to get a better grade in this course.

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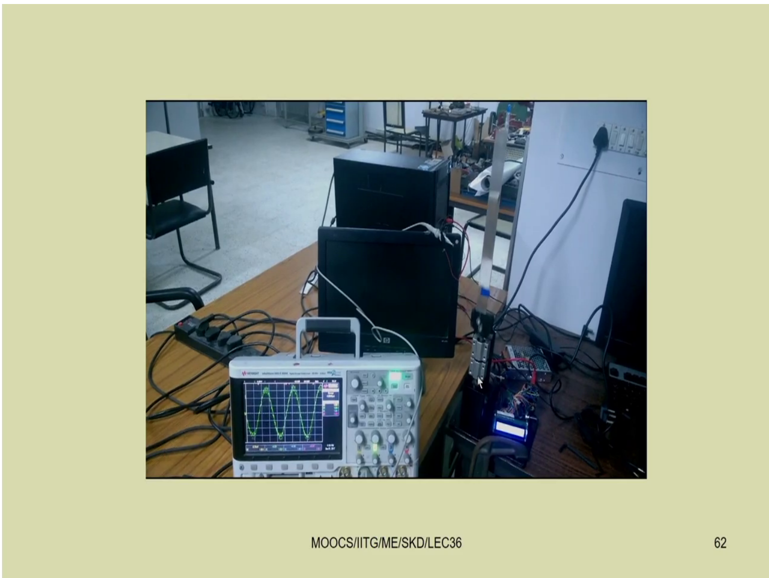

So, we did some experiments and previously, I was not able to show you. So, now, you can see these responses. So, some videos are given here. So, we have a base excited. So, you just see the base here the base is excited. So, we have this cantilever beam. So, in this cantilever beam you can see this is the attached mass. So, this is the when it is excited nearly equal to 2 times omega 1.

So, you can get this response that this is force vibration. So, when it is excited at a frequency twice the natural frequency, you can get the vibration, parametric vibration. So, this is the galloping type of things also what I have shown you or I discussed before.

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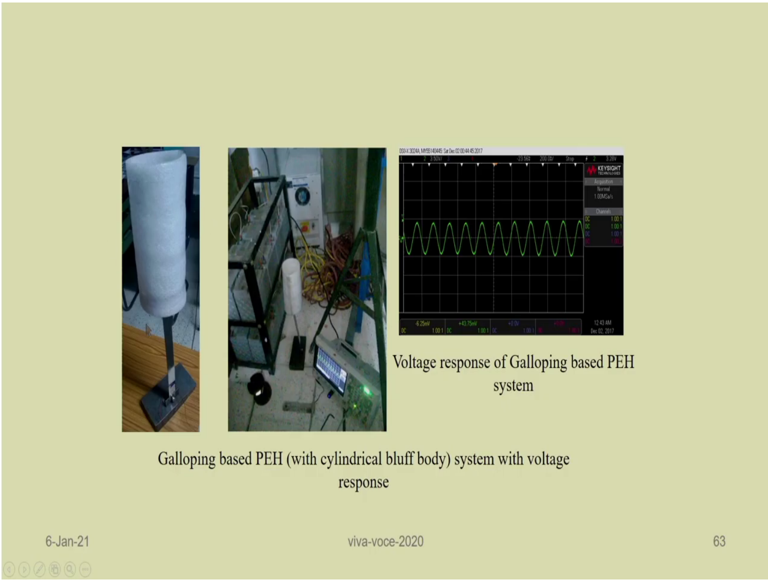

So, this is the bluff body and using this bluff body so when there will be wind is blowing across these things so wake formation will be there. So, due to that the beam will vibrate. So, here you have a piezoelectric patch. So, due to these piezoelectric patches, so you can harvest the energy. So, with this so, let us conclude this course and wishing you good luck for this course and I hope, you will do well in this course and we will have lot of interaction in this course in coming weeks.

So, if you have any doubts. So, we will be happy to answer your query. So, there are three tutors associated in this course. So, they will help you during these course. When you are not able to solve the assignment problems so, they will help you. So, always we are will be there to help you to clear your doubt. So, with this so thank you very much for your kind attention and listening the 36 lectures in this course.

So, thank you very much.